Bayesian Final Project

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2.1 Proposal

Project Proposal: Investigating the Effects of Mother's Gestation Characteristics on Newborn's Birth Weight using Bayesian Inference

The objective of this project is to explore the effects of mother's gestation characteristics on newborn's birth weight using Bayesian analysis. We will analyze publicly available birth data from the National Bureau of Economic Research to gain insights into the relationship between various gestation characteristics of newborn's mother and birth weight. By employing Bayesian linear regression models, we aim to provide a comprehensive understanding of these gestation characteristics and their impact on birth weight.

To analyze the data, we will utilize prior knowledge to set informative prior parameters for the Bayesian linear regression models. We plan to compare different models, starting with a baseline model without interaction terms and incorporating interaction terms to explore more complex relationships. After fitting the model using Markov Chain Monte Carlo (MCMC) sampling, we evaluate model performance using various metrics, including visualization, leave-one-out (LOO) comparison, and cross-validation.

We decide to only focus on the birth weight of female new born in our actual analysis.

2.2 Attribution

Tianyi Li: Project leader, team management, prior predictive simulation, build statistical model

Jade Gu: Raw data summary, model fitting and selection, summarize key converge diagnostics

Jiashu Liu: Raw data summary, assisting in model fitting and selection

Jeremy Lu: Presentation preparation, assisting in model selection, project paper write-up

2.3 Raw Data Summary

2.3.1 Data Origin

Below is a link to a compressed CSV file containing US birth data for the year 2018. The data is provided by the National Center for Health Statistics and can be accessed from the National Bureau of Economic Research website at https://data.nber.org/natality/2018/natl2018.csv.zip.

2.3.2 Data Clean (Include Variable Selection)

This 2018 birth data contains 3801534 observations of 240 variables. First, based on our research topic, we will keep girls' data only. Second, we will pick the six variables commonly associated with infant birth weight and mortality rates: dbwt (infant's birthweight in grams), magerc(mother's age in years), dmar (mother's martial status), cig_rec (mother's smoking status), bmi (mother's body mass index), combgest (mother's gestation length). We also removed missing values and treated all categorical variables as factors.

2.3.3 Data Sampling

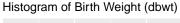
Since our post-filter birth data contains 1591008 observations which is still extremely large, we obtain a random sample of this data that includes only 0.1% of all observations, which results in 1591 observations.

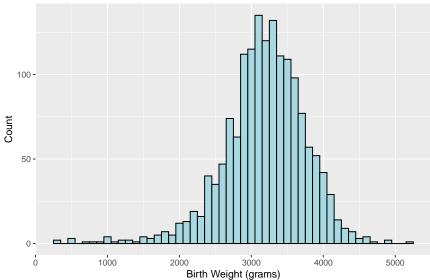
2.3.4 Exploratory data analysis

In our birth data, we have four numerical variables and two factored categorical variables.

Numerical variables:

dbwt (Birth weight)





Above is a histogram of birth weight of female babies. The data is shaped roughly normally with a center around 3200. Values ranged from 320g (babies born prematurely) to 5220g. We computed some descriptive statistics: Median = 3220; Mean = 3183; SD = 574.74.

mager (Mother's age)

Mother's age in years, as an integer. In general as women age, the risk for pregnancy related complications as well as chromosomal conditions tend to be higher. Several studies have also shown that there are associations between age and child birth weight. Values ranged from 14 to 47 yrs old. We computed some descriptive statistics: Median = 29; Mean = 28.59; SD = 5.86.

bmi (Body Mass Index)

Mother's body mass index. BMI, which is calculated as a formula involving weight and height, is also shown to be associated with birth weight. A 2019 meta-analysis in China found that high maternal BMI was associated with fetal overgrowth, and underweight mothers had increased risk of low birth weight. Values

ranged from 13.60 to 69.10. We computed some descriptive statistics: Median = 25.70; Mean = 27.29; SD = 7.00.

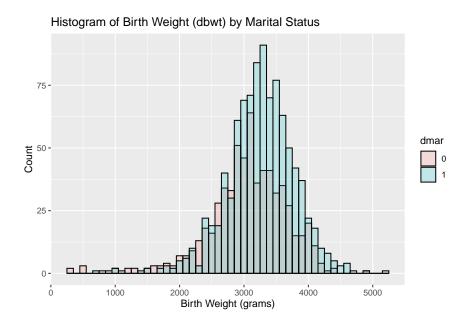
combgest (Gestation length)

Mother's gestation length in weeks. In general, the average human gestation is around 40 weeks, and any babies born before 37 weeks are considered premature. Babies that are born after 42 weeks will be considered overdue. Values ranged from 20.00 to 47.00. We computed some descriptive statistics: Median = 39.00; Mean = 38.55; SD = 2.49.

Categorical variables:

dmar (Marital status)

Mother's martial status. "1" stands for married, and "0" stands for unmarried. In our data, unmarried mothers tended to be younger, as well as having a slightly lower birthweight. According to the birth data, 956 out of 1591 observations (60.09%) are married, and the remaining 635 observations (39.91%) are unmarried. The mean birth weight of married moms is 3260g and median is 3280g, while the mean and median birth weight of unmarried is 3068g and 3105g.



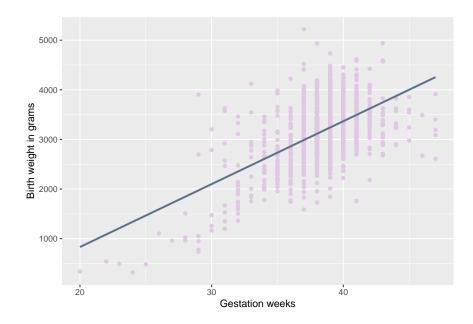
cig_rec

Whether the mother smokes or not. "1" stands for smoke, and "0" stands for non-smoke. According to the birth data, 106 out of 1591 observations (6.66%) are smokers, and the remaining 1481 observations (93.34) are non-smokers. The mean birth weight of moms who smoke is 2999g and median is 3023g, while the mean and median birth weight of mom who don't smoke is 3197g and 3232g.

2.3.5 More visualization

```
ggplot(birth, aes(x = combgest, y = dbwt)) +
geom_point(color = "#e0c7e3") +
geom_smooth(method = "lm", se = FALSE, color = "#5b7288") +
labs(x = "Gestation weeks", y = "Birth weight in grams")
```

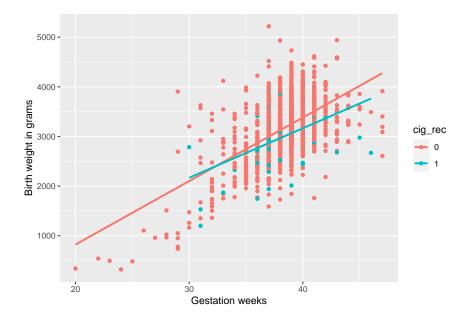
'geom_smooth()' using formula = 'y ~ x'



We observe a positive linear relationship between the mother's combined gestation length and the female newborn's birth weight. This is expected, as mothers with longer combined gestation are expected to get heavier baby girls. However, there might be some variations and potential outliers. Let's add whether or not the mom smokes into account:

```
ggplot(birth, aes(x = combgest, y = dbwt, color = cig_rec)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  xlab("Gestation weeks") +
  ylab("Birth weight in grams")
```

'geom_smooth()' using formula = 'y ~ x'



Based on the fitted lines for mothers who smoke and do not smoke, we can see that the slope for smoking mothers is slightly smaller than the slope for non smoking mothers, indicating that there is an interaction effect between the predictor variables and the relationship between gestation weeks and birth weight is different between moms who smoke and those who don't smoke.

2.4 Statistical Model

We aim to model the relationship between the birth weight of female newborns and maternal characteristics of age, marital status, smoking status, body mass index, and gestation weeks.

From the histogram of birth weight before, we can assume that the birth weight of each female newborn follows a normal distribution. Also, from the scatterplot above, there is a positive linear relationship between the gestation length and birth weight. Let Y_i denotes the birth weight for observation i. Since Y_i is numerical and we assume there is a linear relationship between Y and the predictor variables, a Normal Bayesian regression model is the most appropriate model to use. With 5 predictor variables and 1 outcome variable, our model looks like:

data:
$$Y_i|\beta_0, \beta_1, ..., \beta_5, \sigma \stackrel{ind}{\sim} N\left(\mu_i, \sigma^2\right)$$
 with $\mu_i = \beta_0 + \beta_1 X_{i1} + ... + \beta_5 X_{i5}$ priors:
$$\beta_0 \sim N\left(m_0, s_0^2\right)$$

$$\beta_1 \sim N\left(m_1, s_1^2\right)$$
 ...
$$\beta_5 \sim N\left(m_5, s_5^2\right)$$

$$\sigma \sim \operatorname{Exp}(l).$$
 (1)

For observation of female new-born i, Y_i is her birth weight, X_{i1} is her mother's gestation length in week, X_{i2} is a binary outcome indicating whether her mom smokes, X_{i3} is her mom's bmi index, X_{i4} is her mom's age, X_{i5} is a binary variable of whether her mom is married.

We have to specify the prior mean and sd of each variable. For simplicity, we assume that the prior distributions are independent. First, we center all numerical predictor variables by subtracting the value of each observation by its mean. Now the intercept β_0 represents the birth weight of an average female new-born whose mother is unmarried and does not smoke. According to Medical News Today, the average birth weight of a full-term female is 3.2 kg, which normally ranges from 2.5 kg to 4.0 kg; however, it can also

be below 1.5 kg or above 4.5 kg. As a result, we take 3200 and 750 as prior mean and standard deviation for β_0 .

- For β_1 of X_1 , the gestation week, from prior scientific understanding, on average, one week longer in gestation length is associated with an increase in birth weight by 10 to 225 grams, so we take 108 and 54 as mean and sd for β_1 .
- For β_2 of X_2 , whether mom smokes or not, from prior scientific research, on average, the birth weight of female baby whose mother smokes could be 200 to 300 grams lower than baby whose mother doesn't smoke. We take -250 and 25 for prior mean and sd of β_2 .
- For β_3 of X_3 , the bmi index of baby's mother, from prior scientific research findings, on average, one unit increase in the bmi index is associated with 20 to 30 grams increase in birth weight. We take 25 and 2.5 as prior mean and sd for β_3 .
- For β_4 of X_4 , the mom's age. We employ weakly informative criteria and set 0 and 2.5 as prior mean and sd.
- For β_5 of X_5 , the marriage status of baby's mom, prior scientific study suggests that the average birth weight of female infants born to married mothers may be around 50 grams to 100 grams higher. We take 75 and 12.5 as prior mean and sd for β_5
- Finally, for the error term, a plausible exponential rate parameter is 0.003. We have $\sigma \sim Exp(0.003)$.

2.5 Prior Predictive Simulation

Now let's start simulating some data based on our prior parameters. First we define our prior model function in R.

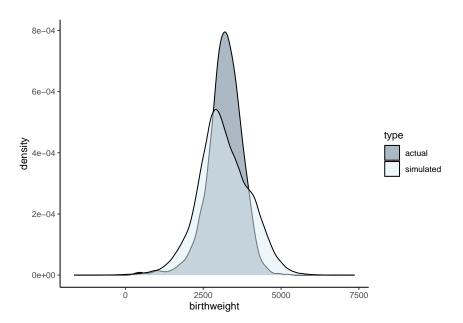
```
set.seed(84735)
# Define function
prior_pred <- function(data) {</pre>
  beta_0 <- rnorm(1, 3200, 750)
  beta 1 <- rnorm(1, 108, 54)
  beta_2 \leftarrow rnorm(1, -250, 25)
  beta_3 \leftarrow rnorm(1, 25, 2.5)
  beta_4 \leftarrow rnorm(1, 0, 2.5)
  beta_5 <- rnorm(1, 75, 12.5)
  sigma \leftarrow rexp(1, 0.003)
  1 <- nrow(data)</pre>
  y <- numeric(1)
  for (i in 1:1) {
    # Convert factors to numeric
    cig_rec_num <- as.numeric(data$cig_rec[i])</pre>
    dmar_num <- as.numeric(data$dmar[i])</pre>
    mu <- beta 0 +
      beta_1 * data$centered_combgest[i] +
      beta_2 * cig_rec_num +
      beta_3 * data$centered_bmi[i] +
      beta 4 * data$centered mager[i] +
      beta_5 * dmar_num
    y[i] <- rnorm(1, mu, sigma)
```

```
return(y)
}

# Simulate 100 times
n <- 100
pr_p <- replicate(n = n, prior_pred(birth))
dim(pr_p)</pre>
```

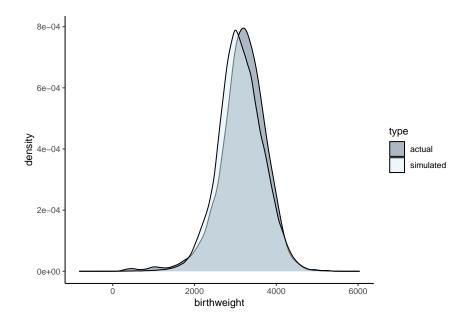
[1] 1591 100

To get a sense of how our prior parameters perform, we draw a density plot of the original data and our simulated data.



The overall shape of our simulated data is normal and centered only slightly off from the actual data . However, the simulated data indicates a higher variability compared to the actual data. So we decide to lower the standard deviation of our prior parameters.

Here is the density plot of the original data and simulated data after we adjust the standard deviation and error term of the prior parameters:



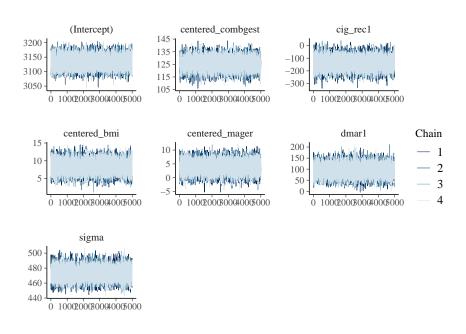
With the change in prior parameters, the density plot of the simulated data are now almost identical to the actual data we observe.

2.6 Modeling Fitting, PPCs, and Model Selection

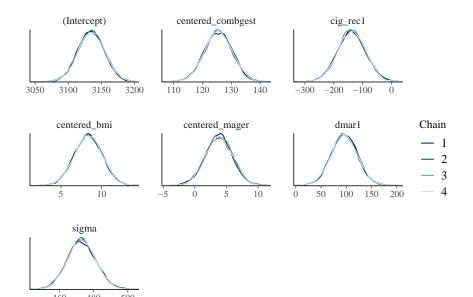
Now, let's fit the posterior model using MCMC sampling.

Then, we create and interpret both visual and numerical diagnostics of our MCMC simulation.

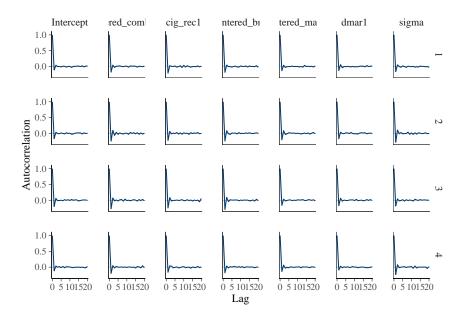
```
# Visual diagnostics of MCMC simulation
mcmc_trace(birth_model, size = 0.1)
```



mcmc_dens_overlay(birth_model)

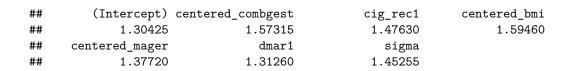


mcmc_acf(birth_model)



${\it \# Numerical \ diagnostics \ of \ MCMC \ simulation}$

neff_ratio(birth_model)



rhat(birth_model)

##	(Intercept)	centered_combgest	cig_rec1	centered_bmi
##	0.9999253	0.9998844	0.9998714	0.9999122
##	centered_mager	dmar1	sigma	
##	0.9999747	0.9998369	0.9998523	

Regarding the assessment of our MCMC simulation, the visual diagnostics provide valuable insight into the behavior of the Markov chains. In particular, we have examined the trace plots, density plots, and autocorrelation plots for each parameter. Based on the trace plots, we can observe that the lines for each parameter fluctuate randomly without exhibiting any clear patterns. Additionally, all four chains appear to mix well, which is a positive indication that the simulation is properly exploring the parameter space. Furthermore, the density plots show that each parameter has a smooth, bell-shaped distribution, which is a desirable property of a well-fitting model. Moreover, the autocorrelation plots exhibit a quick decay in correlation as the lag increases for each parameter, which is indicative of good autocorrelation properties.

Moving onto the numerical diagnostics of our MCMC simulation, we have calculated the neff_ratio and R-hat statistics for each parameter. In terms of numerical diagnostics, the neff_ratio statistic shows that all parameters have a ratio larger than 0.5, indicating a good effective sample size. Additionally, the R-hat statistic shows that all parameters have a value close to 1, indicating good convergence of the chains. These numerical diagnostics further confirm that our posterior simulation has stabilized.

Taken together, the results of both the visual and numerical diagnostics provide strong evidence that our posterior simulation has sufficiently stabilized, and that we can trust the results of our Bayesian analysis.

2.6.2 Tidy summary

Next, we will produce a tidy() summary of this model. In addition, we will interpret the non-intercept coefficients' posterior median values in context.

```
## # A tibble: 8 x 5
                         estimate std.error conf.low conf.high
     term
     <chr>>
##
                            <dbl>
                                        <dbl>
                                                  <dbl>
                                                             <dbl>
## 1 (Intercept)
                          3135.
                                        20.3
                                               3109.
                                                          3161.
## 2 centered_combgest
                           126.
                                         4.82
                                                119.
                                                            132.
                                                -200.
                                                            -75.0
## 3 cig_rec1
                          -138.
                                        48.2
## 4 centered_bmi
                             8.37
                                         1.72
                                                   6.18
                                                             10.6
                             3.82
                                                   1.05
## 5 centered_mager
                                         2.15
                                                              6.62
## 6 dmar1
                            95.8
                                        25.9
                                                  62.5
                                                            129.
## 7 sigma
                           473.
                                         8.34
                                                462.
                                                            484.
## 8 mean PPD
                          3184.
                                        16.7
                                               3162.
                                                           3205.
```

Interpretations of coefficients of some key variables:

- The posterior median value of the intercept in the model is 3135, indicating that on average, the expected weight of a female newborn whose mother has average age, bmi, gestation length, is not married and doesn't smoke is 3135 grams.
- The posterior median value of the centered_combgest coefficient is 125, suggesting that for each additional gestational week, the expected weight of the female newborn increases by 125 grams, holding all other predictors in the model constant.
- The posterior median value of the cig_rec1 coefficient is -138, indicating that mothers who smoke during pregnancy are expected have female newborns that weigh around 138 grams less on average than non-smoking mothers, holding all other predictors in the model constant.

2.6.3 Posterior Summary

Here, we will use posterior_interval() to produce 95% credible intervals for the model parameters. After that, we will check any association between independent variables and dependent variables.

```
# posterior summarizes
posterior_interval(birth_model, prob = 0.95)
```

```
##
                             2.5%
                                         97.5%
## (Intercept)
                     3094.7804055 3174.672507
## centered_combgest 116.0942049
                                   134.783062
## cig rec1
                     -232.4722567
                                   -44.244782
## centered_bmi
                                    11.726993
                        5.0466606
## centered mager
                       -0.3839561
                                      8.055304
## dmar1
                       44.9740960
                                   148.003314
## sigma
                      457.0615312 489.612834
```

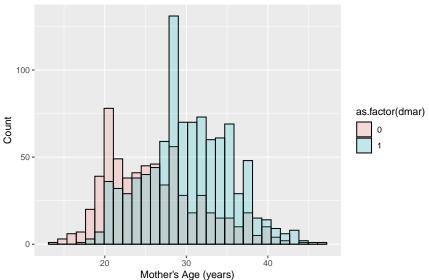
When controlling for the other predictors in the model, the 95% posterior credible intervals for the coefficient of centered_combgest, centered_bmi, and dmar1 lie entirely above 0, suggesting that centered_combgest, comtered_bmi, and dmar1 have significant positive associations with dbwt. The 95% posterior credible intervals for the coefficient of centered_mager contains 0, suggesting that centered_mager has no association with dbwt. The 95% posterior credible intervals for the coefficient of cig_rec1 lie entirely below 0, suggesting that cig_rec1 has significant negative association with dbwt. In other words, the model suggests that increasing the gestational period (centered_combgest) and maternal BMI (centered_bmi) are associated with higher birthweights of female newborns, as is being married (dmar1), and smoking (cig_rec1) is associated with a significant decrease in the birthweight.

2.6.4 More models

Next, let's try fit additional models to include new information that we know and take interaction terms into account. For our new model, we are going to add two interaction terms: cig_rec&bmi and mager&dmar since the age of new-born's mothers and their marital status might be correlated, and whether mom smokes or not might also be correlated with her BMI.

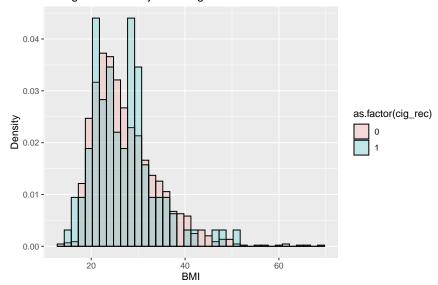
'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.

Histogram of Mother's Age by Marital Status



```
## Warning: The dot-dot notation ('..density..') was deprecated in ggplot2 3.4.0.
## i Please use 'after_stat(density)' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.
```

Histogram of BMI by Smoking Status



To fit this new model, we will use weakly informative priors throughout.

```
# Use weakly informative criteria throughout
# Add interaction: cig_rec*centered_bmi, centered_mager*dmar
birth_model2 <- stan_glm(dbwt ~
                           centered_combgest +
                           cig_rec +
                           centered_bmi +
                           centered_mager +
                           dmar +
                           cig_rec:centered_bmi +
                           centered_mager:dmar,
                        data = birth,
                        family = gaussian,
                        prior_intercept = normal(3200, 500, autoscale = TRUE),
                        prior = normal(0, 2.5, autoscale = TRUE),
                        prior_aux = exponential(0.1, autoscale = TRUE),
                        chains = 4, iter = 5000*2, seed = 84735,
                        prior_PD = FALSE)
```

Next, to determine the necessity of interaction terms, we will check the 80% posterior credible interval for interaction coefficients.

```
# Posterior summary statistics
tidy(birth_model2, effects = c("fixed", "aux"))
## # A tibble: 10 x 3
##
                            estimate std.error
      term
##
      <chr>
                               <dbl>
                                         <dbl>
## 1 (Intercept)
                             3145.
                                         21.3
## 2 centered_combgest
                                          4.88
                              125.
## 3 cig_rec1
                             -140.
                                         48.3
## 4 centered_bmi
                                7.71
                                          1.73
## 5 centered_mager
                                7.68
                                          3.22
## 6 dmar1
                                         26.5
                               91.2
## 7 cig_rec1:centered_bmi
                                6.31
                                          7.14
## 8 centered_mager:dmar1
                               -7.05
                                          4.39
## 9 sigma
                              473.
                                          8.44
                                         16.7
## 10 mean_PPD
                             3183.
# Posterior credible interval for the interaction terms
posterior_interval(birth_model2, prob = 0.80,
                   pars = "cig_rec1:centered_bmi")
## cig_rec1:centered_bmi -2.75577 15.47533
posterior_interval(birth_model2, prob = 0.80,
                   pars = "centered_mager:dmar1")
##
                                        90%
                              10%
## centered_mager:dmar1 -12.53636 -1.365554
```

The 80% posterior credible interval for interaction coefficient β_6 contains 0, suggesting that the association between smoking and BMI is not significant. Thus, this interaction term is not necessary. The 80% posterior credible interval for interaction coefficient β_7 is entirely and well below 0, suggesting a negative association between mother's age and marriage status. And we believe that we should include this interaction term in our new model birth_model3.

```
# Use weakly informative criteria throughout
# Add interaction: centered_mager*dmar
birth_model3 <- stan_glm(dbwt ~</pre>
                            centered_combgest +
                            cig_rec +
                            centered_bmi +
                            centered_mager +
                            dmar +
                            centered_mager:dmar,
                         data = birth,
                         family = gaussian,
                        prior_intercept = normal(3200, 500, autoscale = TRUE),
                        prior = normal(0, 2.5, autoscale = TRUE),
                        prior_aux = exponential(0.1, autoscale = TRUE),
                         chains = 4, iter = 5000*2, seed = 84735,
                         prior_PD = FALSE)
```

2.6.5 Model Comparison

pp_check(birth_model)
pp_check(birth_model2)

To compare our 3 models and decide which one is the best, we will use three different approaches, including posterior predictive checks, cross-validation, and ELPD.

The first method is to evaluate predictive accuracy using pp_check.

According to pp_check() output, all three models preform well Although the simulations are not perfect, but they do reasonably capture the features of the observed data.

The second method is to evaluate predictive accuracy using cross-validation.

```
# Run a 10-fold cross-validation
cv_procedure1 <- prediction_summary_cv(model = birth_model, data = birth, k = 10)
cv_procedure2 <- prediction_summary_cv(model = birth_model2, data = birth, k = 10)</pre>
```

Based on the cross-validated metrics provided, birth_model3 has the lowest mae, lowest mae_scaled, highest within_50, and highest within_95 than the other two models. Its lowest mae and mae_scaled value indicate that the this model on average makes most accurate predictions, and its highest within_50 and within_95 value indicate that the this model is more likely to make accurate predictions within the selected interval. Therefore, the best model is birth_model3, which contains an interaction between age and marital status.

The third method is to evaluate predictive accuracy using ELPD.

```
# Calculate ELPD for the 3 models
set.seed(84735)
loo_1 <- loo(birth_model)</pre>
loo_2 <- loo(birth_model2)</pre>
loo_3 <- loo(birth_model3)</pre>
# Results
c(loo_1$estimates[1], loo_2$estimates[1], loo_3$estimates[1])
## [1] -12060.26 -12060.69 -12060.04
# Compare the ELPD for the 3 models
loo_compare(loo_1, loo_2, loo_3)
##
                 elpd_diff se_diff
## birth_model3 0.0
                            0.0
## birth_model -0.2
                            1.6
## birth_model2 -0.6
                            0.9
```

According to loo_compare() output, birth_model3 again has the highest ELPD, indicating it is the best model out of three.

2.7 Discussion

Overall, the best model was the linear regression with all the terms and an interaction for marital status and the mother's age. We learned that there are positive associations between child weight and bmi, gestation period, and marital status, and also negative associations with smoking status. An interaction term with marital status and age also proved to improve the model. Additionally, despite smoking and bmi both having associations independently with female child birth weight, we learned that as an interaction term, they were actually not relevant.

One limitation is the computing power of rstanarm and our devices. The whole dataset for baby girls is over 190,000 observations, and it would take a long time to fit models on that much information. While one of the advantages of Bayesian Inference is needing less data if we have good priors, using all the data might have given us for more accurate results.