Homework 3: Due Wednesday 16th of Feb

1. Consider a uniform but non-cubical die. Assume that $\mathbb{P}(1) = \mathbb{P}(6) = p$, and $\mathbb{P}(2) = \mathbb{P}(3) = \mathbb{P}(4) = \mathbb{P}(5) = p'$. Suppose that the die is rolled n times and the uppermost face observed, resulting in frequencies f_1, f_2, f_3, f_4, f_5 and f_6,where $f_1 + f_2 + f_3 + f_4 + f_5 + f_6 = n$.

   (a) Express p' in terms of p.

   (b) Find the MM estimate of p.

   (c) Find the ML estimate of p.

   (d) Suppose that the observed frequencies are $f_1 = 22, f_2 = 11, f_3 = 13, f_4 = 15, f_5 = 15$ and $f_6 = 24$. Estimate p and compute the expected frequencies under the assumed model.

2. A random sample of n observations is obtained on a discrete random variable X which has pmf

x	0	1	2
$p(x)$	θ^2	$\theta(1-\theta)$	$1-\theta$

   (a) Find the maximum likelihood estimate for θ.

   (b) Find the variance of the maximum likelihood estimator when n is large.

   (c) If a sample of 200 observations gives $\hat{\theta} = 0.5$, find an approximate 95% confidence interval for θ.

3. In a population in which the frequency of the gene for colour blindness is θ, genetic theory indicates that the probability that a male is colour blind is θ, and the probability that a female is colour blind is θ^2. A random sample of m males is found to include x who are colour blind, and a random sample of n females is found to include y who are colour blind.

   (a) Specify the likelihood function based on both samples, and show that θ can be obtained as a root of a quadratic equation.

   (b) Derive a large sample approximation for the variance of the maximum likelihood estimator.

4. If a sample of 20 observations from $X \stackrel{d}{=} \text{Po}(\lambda)$ yields $\sum_{i=1}^{20} x_i = 15$, test the null hypothesis that $H_0 : \lambda = 0.6$ against a two sided alternative. State the P-value.

5. Problems from the textbook: 6.2.1, 6.2.8, 4.5.3, 4.6.5, 4.6.7.