Homework 9: Due Wednesday April 20

You may use R to assist you in your solutions for questions marked with *.

1. * The following exam marks were obtained by students in a course: x represents the first semester mark and y the second semester mark.

x	54	47	69	87	65	73	83	81	72	74
y	61	22	55	78	45	75	56	66	59	70

   Assuming these observations are from a bivariate normal population:

   (a) Estimate the regression lines $\mathbb{E}(Y|x)$ and $\mathbb{E}(X|y)$.

   (b) Plot the data and the fitted regression lines. (It is probably easiest to draw the lines by hand.)

   (c) Test the hypothesis $\rho = 0$.

2. Suppose that the independent normally distributed random variables Y_1, Y_2, Y_3, Y_4 have means given by $\mathbb{E}(Y_1) = \alpha+2\beta, \mathbb{E}(Y_2) = 3\beta, \mathbb{E}(Y_3) = \alpha-\beta, \mathbb{E}(Y_4) = 2\alpha+\beta$, and equal variances, denoted by σ^2. The following observations are made:

$$y_1 = 4, y_2 = 4, y_3 = 5, y_4 = 3.$$

   (a) Express this in the form $\underset{\sim}{y} = A\underset{\sim}{\theta}+\underset{\sim}{e}$, and hence estimate α, β and σ^2 using the method of least squares.

   (b) Find a 95% confidence interval for $\mathbb{E}(Y_1)$.

3. For a one-way anova with k groups and n observations in each group, and the parameterisation $y_{ij} = \mu + \alpha_i + e_{ij}$,

   (a) In the general linear model setup, what is the design matrix A for all the parameters $\mu, \alpha_1, \ldots, \alpha_k$?

   (b) Is the matrix $A^T A$ full rank? If not, suggest a solution such that $A^T A$ becomes full rank.

   (c) Derive the form of the estimator for $\hat{\underset{\sim}{\theta}}$.

   (d) What is the variance-covariance matrix for the parameter vector $\hat{\underset{\sim}{\Theta}}$.

4. For a quadratic regression with n observations $y_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + e_i$,

   (a) In the general linear model setup, what is the design matrix A?

   (b) Derive the form of the estimator for $\hat{\underset{\sim}{\theta}}$.

   (c) What is the variance-covariance matrix for the parameter vector $\hat{\underset{\sim}{\Theta}}$.

5. Problems from the book: 9.6.11, 9.6.14, 9.7.2.