Math 459 Lecture 2

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Estimating the Exponential Parameter

We generate random samples of different sizes from an $\mathrm{Exp}(\lambda = 3.5)$ distribution.

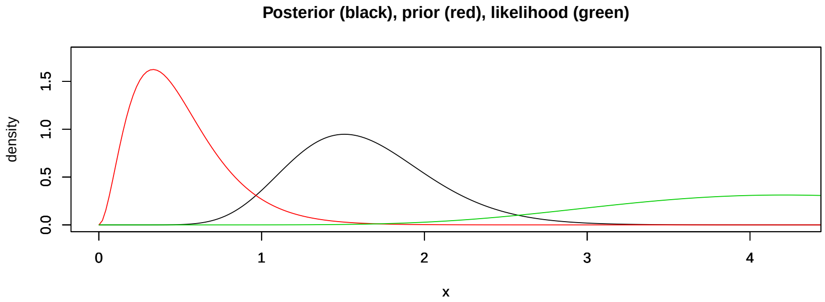
- What is the MLE here?
- We will try Gamma priors with different values of α and β (shape and rate).
- We will compute the MLE, prior mean and posterior mean in each case.
- We will also plot the prior, posterior and likelihood in each case.

Small Sample (n=11): Estimates for Gamma(3,6) Prior

- prior mean is $\alpha/\beta = 1/2$
- MLE is 4.2069428
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 1.6251244
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 1.509044

Small Sample (n=11): Plots for Gamma(3,6) Prior

```
eg.trip(x1,3,6)
```

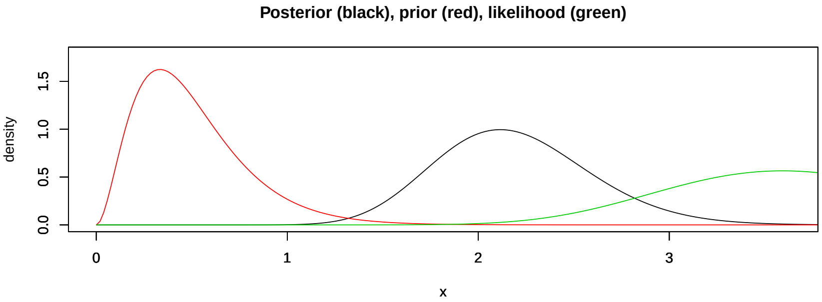


Moderate Sample (n=26): Estimates for Gamma(3,6) Prior

- prior mean is $\alpha/\beta = 1/2$
- MLE is 3.5917468
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 2.1905279
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 2.1149925

Moderate Sample (n=26): Plots for Gamma(3,6) Prior

```r
eg.trip(x2,3,6)
```

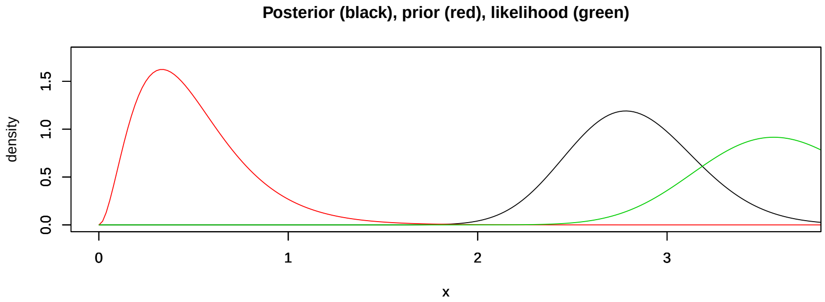


Large Sample (n=67): Estimates for Gamma(3,6) Prior

- prior mean is $\alpha/\beta = 1/2$
- MLE is 3.5627092
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 2.8219078
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 2.7815948

Large Sample (n=67): Plots for Gamma(3,6) Prior

```
eg.trip(x3,3,6)
```

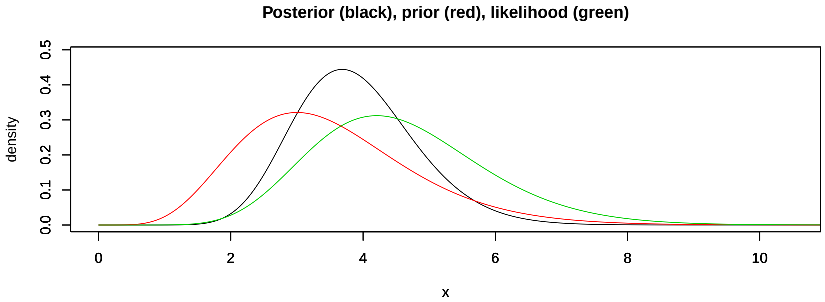


Small Sample (n=11): Estimates for Gamma(7,2) Prior

- prior mean is $\alpha/\beta = 3.5$
- MLE is 4.2069428
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 3.9005571
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 3.6838595

Small Sample (n=11): Plots for Gamma(7,2) Prior

```r
eg.trip(x1,7,2)
```

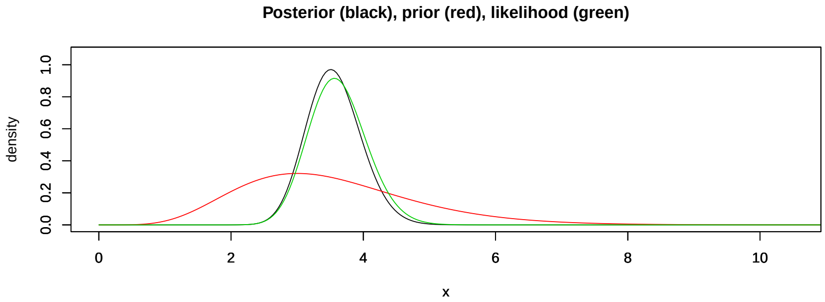


Large Sample (n=67): Estimates for Gamma(7,2) Prior

- prior mean is $\alpha/\beta = 3.5$
- MLE is 3.5627092
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 3.5566812
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 3.5086179

Large Sample (n=67): Plots for Gamma(7,2) Prior

```
eg.trip(x3,7,2)
```

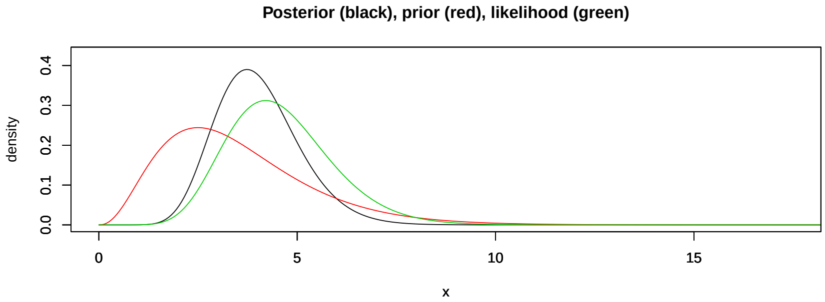


Small Sample (n=11): Estimates for Gamma(3.5,1) Prior

- prior mean is $\alpha/\beta = 3.5$
- MLE is 4.2069428
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 4.0113697
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 3.7347236

Small Sample (n=11): Plots for Gamma(3.5,1) Prior

```
eg.trip(x1,3.5,1)
```



Large Sample (n=67): Estimates for Gamma(3.5,1) Prior

- prior mean is $\alpha/\beta = 3.5$
- MLE is 3.5627092
- posterior mean is $(\alpha + n)/(\beta + \sum x_i)$ or 3.559543
- posterior mode is $(\alpha + n - 1)/(\beta + \sum x_i)$ or 3.5090531

Large Sample (n=67): Plots for Gamma(7,2) Prior

```
eg.trip(x3,3.5,1)
```