Math 494 - Mathematical Statistics

Midsemester exam 2

March 29, 2016

Time permitted: 50 minutes

This exam has 4 questions worth a total of 50 marks.

This paper has 2 pages including this one.

Full working will be required to achieve full credit.

No calculators are allowed in this examination.

Students are permitted to bring one (double-sided) sheet of notes.

1. Suppose Jeff collected the following data on how many koalas his car ran over per day during a 10 day road trip.

   30 15 23 22 17 20 20 17 22 20

   (a) Use a sign test to test the hypothesis that the median number of koalas Jeff runs over on a given day is 21.

   (b) Use a rank test to test the hypothesis that the median number of koalas Jeff runs over on a given day is 21.

   (c) What extra assumption do we need to use the rank test (compared to the sign test)? What is the major advantage of using the rank test over the sign test?

   [15 marks]

2. (a) If U is a uniform random variable on $[0, 1]$ and X is a continuous random variable with distribution function $F_X(x)$, show that $F_X^{-1}(U)$ follows the same distribution as X.

   (b) Suppose X has the following density function

   $$f_X(x) = \frac{x^5}{6}, \quad x \in [0, 1]; \quad 0, \quad \text{otherwise.}$$

   Using the inverse transform method, explain how you would simulate a random variable which follows density $f_X(x)$.

   [10 marks]

3. The *X-tra X-ponential* distribution has the following density:

   $$f(x) = \frac{2}{\Gamma(\frac{3}{4})\theta^{3/4}} x^2 e^{-x^4/\theta}, \quad x \in \mathbb{R}.$$

   (a) Suppose I had a random sample $X_1, \ldots, X_n$ which follow the X-tra X-ponential distribution. Find a sufficient statistic Y for θ.

   (b) Define what it means for a sufficient statistic to be complete.

   (c) Is your sufficient statistic Y complete? Explain why or why not.

   (d) Find the unbiased estimator that attains the minimum variance bound for θ. Furthermore, calculate the minimum variance.

   [20 marks]

4. What is the multiple comparisons problem? Name two potential methods for calculating simultaneous/family confidence intervals.

   [5 marks]

End of Examination