494 Mid Sem Exam 2 Solutions.

1a.

exp	B	B'
A	52.5	17.5
A'	22.5	7.5

$$\Rightarrow \chi^2 = \frac{2.5^2}{52.5} + \frac{2.5^2}{17.5} + \frac{2.5^2}{22.5} + \frac{2.5^2}{7.5}$$

Compare this to $C_{0.95}(\chi^2_1)$

b. $H_0 : m = 3$
$H_1 : m > 3$

Let $Z = \text{freq}(X > 3)$

Under H_0, $Z \overset{d}{=} \text{Bin}(77, 0.5)$

$z = 45$ (observed)

$\Rightarrow P = P(Z \geq 45)$

If $P < 0.05 \Rightarrow$ reject H_0.

2. a. $y = 1 - e^{-x}$

$1 - y = e^{-x}$

$x = -\log(1-y)$

$\Rightarrow$ to simulate use $-\log(1-U)$

b. - Generate an observation X from the exponential distribution in part a.

- Accept this if $U \leq \frac{g(x)}{M f(x)}$

where $M = \max_{x \geq 0} \frac{g(x)}{f(x)}$

$$h(x) = \frac{g(x)}{f(x)} = \frac{2}{\sqrt{2\pi}} e^{-\frac{x^2}{2} + x}$$

$$h'(x) = \frac{2}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2} + x\right)} \cdot (-x + 1) = 0$$

$\Rightarrow x = 1.$

$$\Rightarrow M = \frac{g(1)}{f(1)} = \frac{2}{\sqrt{2\pi}} e^{\frac{1}{2}}.$$

- If rejected start again.

c. For each simulated Y, simply allow a 50/50 chance of setting it to be $-Y$ or Y.

3a. $f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x \in \{0, 1, \dots\}$

$$f_{x_1}(x_1|\lambda) \cdots f(x_n|\lambda) = \frac{e^{-\lambda}\lambda^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n (x_i)!}$$

$$= \underbrace{\frac{1}{\prod (x_i)!}}_{\text{not } x_i} \cdot \underbrace{e^{-\lambda} \cdot \lambda^{\sum x_i}}_{f^n \text{ of } \lambda \text{ \& } \sum x_i}$$

$\Rightarrow Y = \sum_{i=1}^n X_i$ is a suff stat.

b. Y is ~~sufficient~~ complete if it comes from a family of distributions s.t. if $E(u(Y)) = 0$ for all values of θ implies $u = 0$.

c. $f(x|\theta) = \exp\{\underbrace{-\lambda}_{q(\lambda)} + \underbrace{x \log\lambda}_{k(x)\cdot p(\lambda)} - \underbrace{\log(x!)}_{H(x)}\}$.

$\Rightarrow Y = \sum k(x) = \sum x_i$ is complete

d. $E(Y) = n \cdot \lambda$

$\Rightarrow E(Y/n) = \lambda$.

$\Rightarrow Y/n$ is MVB.

4. a. - Observations are ~~[illegible]~~ inclept

- Observations are normally distributed with same variance.
- mean differs by group.

b. $s^2 = 44332$.

c. $F = \frac{49}{44332}$.

Compare this to $F_{1,35}$ distribution.