Math 459 HW1

Due Tuesday, Feb. 2

Guidelines:

- You must show your work to get credit.
- Include your **R** code and the output (just copy+paste into a text file).

1. Suppose you have an i.i.d. sample $X_1, \ldots, X_n$ from a Poisson distribution with parameter λ, i.e.

$$\Pr(X = x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad \lambda > 0, \quad x = 0, 1, \ldots$$

(a) Find the MLE for λ.

(b) Find the posterior mean, assuming a gamma prior, Gamma(α, β), with hyperparameters α and β, i.e.

$$p(\lambda|\alpha, \beta) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\lambda\beta}}{\Gamma(\alpha)}, \quad \lambda, \alpha, \beta > 0.$$

To be clear, you should view α and β as fixed; treat them as constants in your calculations.

(c) Using **R**, generate a sample of size 40 from a Poi$(\lambda = 6)$ distribution. Find the MLE using the `optim` function in the **R** package `stats`; submit the **R** code.

(d) Find the posterior mode for hyperparameter values $\alpha = 2$, $\beta = 3$, again using a sample of size 40 from a Poi$(\lambda = 6)$ distribution. Do not use a package which does this for you; either write your own function or use the `optim` function. Submit the **R** code.

2. Consider the Gaussian model from Lecture 3. Assume $X_1, \ldots, X_n$ are i.i.d. $\mathcal{N}(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Find the marginal posterior for μ, assuming the prior

$$p(\mu, \sigma^2) = p(\mu|\sigma^2)p(\sigma^2)$$

where $p(\sigma^{-2}) = e^{-\sigma^{-2}}, \quad \sigma^{-2} > 0$. You will need to use a change of variables to find $p(\sigma^2)$. Also, assume that $p(\mu|\sigma^2) = (2\pi\sigma^2)^{-1/2} \exp\{-\mu^2/(2\sigma^2)\}$. You may ignore constants of proportionality. Can you identify which type of distribution this marginal posterior density represents?

3. Suppose you agree with a Bayesian that we should quantify our prior beliefs by specifying a prior density on the unknown parameter, and a sampling model to describe the data generating process. However, you are not convinced that Bayes's Rule is the right way to update your beliefs after observing new information. Suggest an alternative rule, with a brief justification, and give a mathematical (probabilistic) expression for this rule.