Math 494 - Mathematical Statistics

Midsemester exam 2

March 25, 2015

Time permitted: 50 minutes

This exam has 4 questions worth a total of 50 marks.

This paper has 3 pages including this one.

Full working will be required to achieve full credit.

No calculators are allowed in this examination.

Students are permitted to bring one (double-sided) sheet of notes.

1. (a) A survey of Americans was undertaken and they were asked two questions:

    - 1. Do you think kangaroos are cute?
    - 2. Do you think wombats are fat?

    Let A and B denote if the answers to questions 1 and 2 (respectively) are yes, and A' and B' if the answer is no. The data is then summarised in the following table.

obs freq	B	B'	
A	50	20	70
A'	25	5	30
	75	25	100

    Using a χ^2 test, examine if there is a relationship between A and B.

   (b) Jeff the professional drinker of 'water' claims that the median number of drinks of 'water' he has in a day has a median of 3. He takes note of how much he drinks for 100 days. The results are summarised below.

x	0	1	2	3	4	5
obs. freq. f_x	0	18	14	23	25	20

    Using a sign test, test the hypothesis that the median is 3 against the alternate hypothesis that the median is greater than 3.

[10 marks]

2. The density function of a standard exponential random variable is as follows

$$f(x) = e^{-x}, \text{ if } \ x \geq 0, \quad 0, \text{otherwise.}$$

   (a) Given the ability to generate R$[0,1]$ random variables, explain how you would simulate an observation from the exponential distribution using the inverse-transform method.

   (b) The *folded normal* distribution has the following density

$$g(x) = \frac{2}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}, \text{ if } \ x \geq 0, \quad 0, \text{otherwise.}$$

    Using your answer to part (a), explain how you would simulate an observation from this distribution using an accept-reject algorithm.

   (c) Using your answer to part (b) explain how one can simulate from the standard normal distribution.

[15 marks]

3. (a) Given a random sample $X_1, X_2, \ldots, X_n$ from common distribution $\mathrm{Po}(\lambda)$, using the Fisher-Neyman factorisation theorem, find a sufficient statistic Y for λ. Recall that the Poisson distribution has mass function

$$\mathbb{P}(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x \in \{0, 1, \ldots\}.$$

(b) Define what it means for a sufficient statistic to be complete.

(c) Is your sufficient statistic Y complete?

(d) Using part (c) or otherwise, give the unbiased estimator for λ that attains the minimum variance bound.

[15 marks]

4. The following is an excerpt of an ANOVA from the (slightly modified) data taken in class. The question of interest is whether one's beliefs about Han's ability to gamble are impacted by one's gender.

```
Analysis of Variance Table

Response: money earned
          Df  Sum Sq
gender     1      49
Residuals 35 1551635
```

(a) What are the assumptions used in this ANOVA?

(b) Give an estimate for σ^2.

(c) Explain how one would test the hypothesis that one's gender has no impact on one's beliefs about Han's ability to gamble. What underlying principle was used to derive the form of this test?

[10 marks]

End of Examination