

## Homework 5: Due Wednesday 9th of March

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1. Let  $X$  follow distribution with density\*

$$f_X(x) = \frac{\sin(x)}{1.41\sqrt{x(1-x)}}, \quad x \in [0, 1].$$

- (a) The arcsine distribution has probability density function

$$f_Y(y) = \frac{1}{\pi\sqrt{y(1-y)}}, \quad y \in [0, 1],$$

and distribution function

$$F_Y(y) = \frac{2}{\pi} \arcsin(\sqrt{y}), \quad y \in [0, 1]$$

Explain how to simulate an observation from the arcsine distribution and then explain how to use this for an accept-reject algorithm to simulate an observation from  $X$ .

- (b) Calculate the probability that a randomly generated point from the arcsine distribution will be accepted in your algorithm defined in part (a).
2. Show that if  $Y$  is a sufficient statistic for a parameter  $\theta$ , then if  $\phi$  is a one-to-one function,  $\phi(Y)$  is also a sufficient statistic.

Hence conclude that there exist infinitely many sufficient statistics.

3. Problems from the textbook: 4.8.6\*\*, 4.8.9\*\*, 4.8.19, 7.2.1, 7.2.3, 7.2.4, 7.2.6.

\* 1.41 isn't exactly what is required for  $f_X$  to be a density but just pretend it is.

\*\* Do not write the R functions.