

Math 459 Lecture 9

Todd Kuffner

Professor Kuffner's Interpretations of Historical Events

George Box (1978)

All models are wrong but some are useful

John von Neumann (1947)

truth ... is much too complicated to allow anything but approximations

Why Priors Matter: The Marriage of Carla Bruni

Model, singer, former First Lady of France (though she is Italian)



The Bayesian Suitors



The Model of the Model

Poor Carla has amnesia; every few minutes her memory resets.

- ▶ Mick, Donald and Nicolas propose to her once each day
- ▶ Carla will say yes with some probability θ (could be different for each suitor)
- ▶ Each suitor has some prior belief about how much Carla loves him, and this is expressed as a prior distribution on the probability θ that she will say ‘yes’
- ▶ They agree in advance on a maximum n
- ▶ Each suitor will stop asking her if his posterior probability that $\theta > 0.5$ falls below 0.5

Sampling Model

For each suitor, each proposal X_{ij} , $j = \{M, D, S\}$, $i = 1, \dots, n$, is an independent Bernoulli trial with parameters $\theta_M, \theta_D, \theta_S$ (could also assume they are the same...)

$$X_{ij} \stackrel{i.i.d.}{\sim} \text{Ber}(\theta)$$

$$\begin{aligned} Y_j &= \sum_{i=1}^n X_{ij} \\ &\sim \text{Bin}(n, \theta_j) \end{aligned}$$

Mick Jagger's Prior

Mick believes that

*you can't always get what you want, but if you try sometime, you just might find **you get what you need***

$$p_M(\theta_M) = \theta_M + 0.5, \quad \theta_M \in (0, 1)$$

Thus

$$\Pr_M(\theta_M > 0.5) = \int_{0.5}^1 p_M(\theta_M) d\theta_M = 0.625$$

Nicolas Sarkozy's Prior

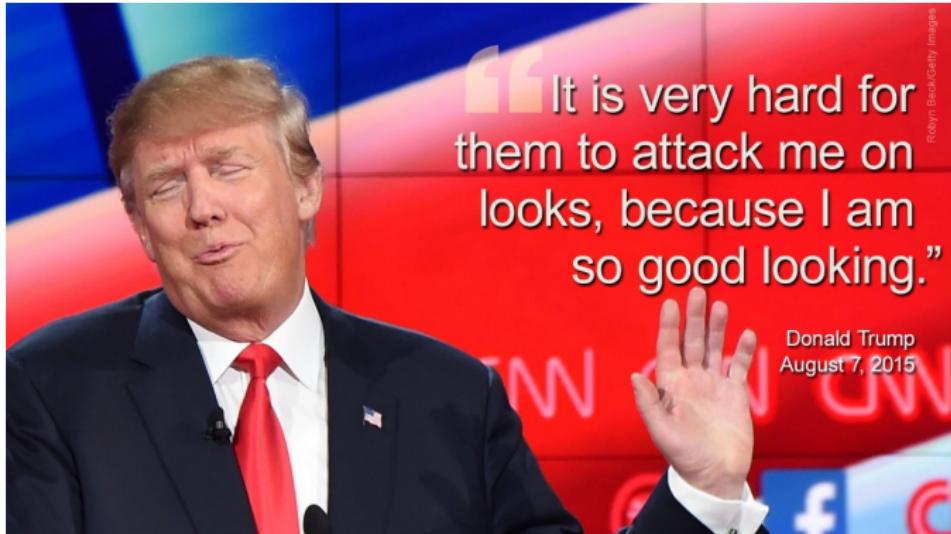
Nicolas Sarkozy is supremely confident.

$$p_N(\theta_S) = 10, \quad \theta \in (0.9, 1)$$

Thus

$$\Pr_S(\theta_S > 0.5) = 1.$$

Donald Trump's Prior



Donald doesn't think **he** should have to ask.

He is *still* waiting for **her** to ask.

Mick Jagger's Result

Carla says no to Mick on day one.

$$\begin{aligned} p(\theta_M | y_M = 0) &\propto p(\theta_M) f(y_M | \theta_M) \\ &\propto \frac{\theta_M}{2} - \theta_M^2 + \frac{1}{2} \end{aligned}$$

Can solve for the normalizing constant (12/5), so

$$\Pr_{\theta_M | Y_M} (\theta_M > 0.5) = \int_{0.5}^1 p_M(\theta_M | y_M) d\theta_M = 0.35$$

Mick gives up after one try.

Nicolas ‘Failure is NOT an option’ Sarkozy

$$\Pr_{\theta_S|y_S=0}(\theta_S > 0.5) = 1$$

The normalizing constant in the posterior is 20 (note $\theta \in (0.9, 1)$), so

$$\Pr_{\theta_S|Y_S}(\theta_S > 0.5) = 20 \int_{0.5}^1 10(1 - \theta_S)d\theta_S = 1.$$

He will never give up.

Cromwell (1599-1658), Charles I's buddy

Cromwell's Rule (Lindley, 1991): if prior excludes some value of θ , then it is also excluded in posterior.

*I beseech you, in the bowels of Christ, think it possible
that you may be mistaken.*

Oliver Cromwell, letter to Church of Scotland on August 5, 1650



A Second Example: two experiments

Fisher's 'lady tasting tea' experiment (1935).

We may speak of this hypothesis as the 'null hypothesis', and it should be noted that the null hypothesis is never proved or established, but is possibly disproved, in the course of experimentation.

- ▶ Muriel Bristol claimed she could tell whether tea or milk was added to a cup first.
- ▶ H_0 : she is wrong

Consider 10 random Bernoulli trials (randomize whether milk or tea are put first), and she gets 9 correct—correctly identifying which was poured first.

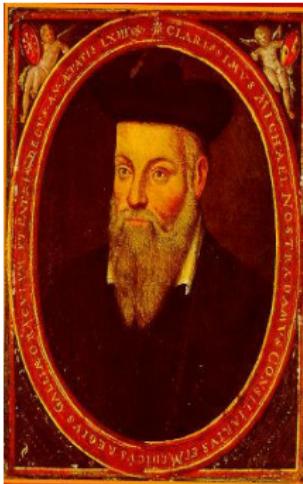
Analysis

What is the probability that, just by sheer chance, she gets at least 9/10 correct?

$$\sum_{k=9}^{10} \binom{10}{k} 0.5^k (1 - 0.5)^{10-k} = 10 \times 0.5^{10} + 1 \times 0.5^{10} \approx 0.0107$$

Maybe she is telling the truth...

Extrasensory Perception (ESP)



Michel de Nostredame ‘Nostradamus’ (1503-1566)

- ▶ Test his psychic powers by seeing if he can predict 10 coin flips.

What Bayes Adds

If Nostradamus gets 9 correct, a frequentist would think it is **just as plausible** that Nostradamus has ESP as she would that Muriel can taste whether milk or tea was poured first.

- ▶ **Priors** allow us to incorporate **common sense**.

Sidenote

From NSF's April 2002 report on the state of science understanding among adult Americans:

- ▶ 60% believe in ESP (even when considering only college graduates)
- ▶ 30% believe UFOs are space vehicles from other civilizations
- ▶ 32% believe in lucky numbers
- ▶ 40% think that astrology is scientific

Goldstein (2006), Shermer (2002) ‘Smart People Believe Weird Things’