Blind Source Separation for OFDM with Filtering Colored Noise Out

M. G. S. Sriyananda, J. Joutsensalo, and T. Hämäläinen
Department of Mathematical Information Technology,
Faculty of Information Technology,
P.O. Box 35, FI-40014 University of Jyväskylä,
FINLAND
Email: {gamage.m.s.sriyananda, jyrki.j.joutsensalo, timo.t.hamalainen}@jyu.fi

***Abstract*—Two blind algorithms that are developed with the intention of improving the symbol detection of Orthogonal Frequency Division Multiplexing (OFDM) techniques are proposed in this paper. OFDM systems are easy to equalize in implementations. The schemes are based on the theories of blind source separation (BSS). They are among the premier mechanisms used for extracting unobserved signals from observed mixtures in signal processing. In this study noise component of the received signal mixture is tried to be filtered out. A scalar energy function with the iterative fixed point rule for receive signal is used in determining the filter coefficients while taking the time correlation properties of the channel to advantage for BSS. The methods are tested in a single input single output slow fading channel with a receiver containing Equal Gain Combining (EGC) for treating the channel state information (CSI) values. These solutions can be introduced as low computational complexity approaches.**

Keywords — Blind Source Separation, OFDM, Slow Fading

I. Introduction

Distortion and mixing of signals during a process of communication is a common scenario. Having a prior knowledge about the captured data to recover information symbols is advantageous but not a compulsory requirement. Theories of Blind Source or Signal Separation are used to recover unobserved signals or sources from several observed mixtures when no prior information is available about the transfer. The adjective blind stresses two scenarios. First is that the source signals are not observed. Second is no information is available about the mixture. Higher effective data rates are expected to be delivered by blind schemes when there is no training or pilot data sequence. Typically the observation values are obtained at the output of a set of sensors, where each sensor receives a different combination of the source signals. The lack of prior knowledge about the mixture is compensated by a statistically strong but often physically plausible assumption of independence between the source signals. Non availability of the prior information is the strength of the BSS model, making it a versatile tool for exploiting the spatial diversity provided by an array of sensors.

Orthogonal Frequency Division Multiplexing (OFDM) techniques and derivatives of them are the most widely used category of technologies in the sphere of wireless communication at this moment. They are capable of using the spectrum very efficiently while increasing the data rates without consuming additional bandwidth or power as usual. These multicarrier techniques are very robust to channel multipath propagation and simple equalization can be used.

The paper is organized as five main sections. Theoretical background for this study followed by a customized solution for OFDM systems is given in the section II. While system model is explained in section III, section IV is reserved for system parameters and simulation results. Conclusion is presented in section V.

II. BSS Models

Progression of BSS models and algorithms are in many directions. Scenarios with more sensors than sources are considered as the category which is of much practical importance [1], [2]. Generally captured data is a combination of noisy observations, complex signals and mixtures. Blind source separation (BSS) theories are applicable to the standard narrow band array processing or beamforming models and convolutive mixtures. Multichannel blind deconvolution problems can be solved using this approach. Only stationary non-Gaussian signals can be separated by some of these schemes. Due to these limitations, poor performance is obtained when dealing with some real sources, like audio signals. But there are different approaches which take advantage of the non stationarity of sources in order to achieve better performance than the conventional methods [3].

Biomedical signal analysis and processing (ECG, EEG, MEG) [4], [5], acoustics (audio signal processing), geophysical data processing, data mining, speech recognition, image recognition and communications signal processing including wireless communications [6] are some of the much beneficial areas that can be effectively developed by the BSS concepts. As an example, usefulness of BSS in the context of event-related MEG measurements is demonstrated by [7]. In this case BSS is applied to artifact identification of raw data. In one lesser impressive application, BSS is used to answer the problems in power line communications [8].

978-1-4577-1878-6/11/$26.00 ©2011 IEEE

A. General BSS Model

For the independent linear instantaneous output signal mixtures, separation of the sources is done by combining the observations by means of a matrix adapted [3]. It is expected or assumed that the sources also must be independent.

A communication system containing source signal $\mathbf{s}(t)$, $\mathbf{s}(t) = [s_1(t), s_2(t), ..., s_b(t), ..., s_B(t)]^T$ is considered. $s_b(t)$ is the signal element b at time t. $s_2(t), s_3(t), ..., s_b(t), ..., s_B(t)$ are independent or uncorrelated symbols or signals. Signal $s_1(t)$ is time correlated. $\mathbf{A}$ is a $F \times B$ matrix and a mixture of coefficient values. The element f, b of it is given by $\mathrm{a}_{f,b}$. $\mathbf{a}_1$, $\mathbf{a}_1 = [\mathrm{a}_{1,1}, \mathrm{a}_{2,1}, ..., \mathrm{a}_{f,1}, ..., \mathrm{a}_{F,1}]^T$ is the coefficient value matrix of $s_1(t)$. Additive independent and identically distributed (iid) white noise is denoted by $\mathbf{w}(t) = [w_1(t), w_2(t), ..., w_f(t), ..., w_F(t)]^T$, and colored noise is given by $\mathbf{w}'(t)$. $w_f(t)$ is the noise element f at time t. Then the receive signal matrix $\mathbf{x}(t)$, $\mathbf{x}(t) = [x_1(t), x_2(t), ..., x_f(t), ..., x_F(t)]^T$ of the basic BSS model at time t with B unknown input or sources and F output or sensor observations can be given as [3].

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t) = s_1(t)\mathbf{a}_1 + \mathbf{w}'(t) \tag{1}$$

When appropriately shaped, mapped symbols are considered and the time delay is sufficiently shorter, let c_1 to be defined as,

$$c_1 = \mathrm{E}\left\{s_1(t)s_1(t+\tau)\right\} \approx \mathrm{E}\left\{s_1(t)^2\right\} \tag{2}$$

The differential correlation matrix [9] taken with a small time difference τ is given by $\mathbf{C}(\tau)$,

$$\mathbf{C}(\tau) = \mathrm{E}\left\{\mathbf{x}(t)\mathbf{x}(t+\tau)^T\right\} = c_1\mathbf{a}_1\mathbf{a}_1^T + \mathbf{R}_\tau \tag{3}$$

where matrix transpose is denoted by ^{T}. The ordinary correlation matrix $\mathbf{R}$ is,

$$\begin{aligned}\mathbf{R} &= \mathrm{E}\left\{\mathbf{x}(t)\mathbf{x}(t)^T\right\} \\ &= \mathbf{A}\mathrm{E}\left\{\mathbf{s}(t)\mathbf{s}(t)^T\right\}\mathbf{A}^T + \sigma^2\mathbf{I} \\ &= c_1\mathbf{a}_1\mathbf{a}_1^T + \mathbf{R}_0\end{aligned} \tag{4}$$

It is assumed that $\|\mathbf{R}_0\| > \|\mathbf{R}_\tau\|$ and noise variance is given by σ^2, where $\|\cdot\|$ is Frobenius or 2-norm.

The receiver output $\mathbf{y}(t)$ or $\mathbf{y}$ with coefficient $\mathbf{u}$ is given by,

$$\mathbf{y} = \mathbf{u}^T\mathbf{x} \tag{5}$$

The output power $\mathrm{E}\left\{\mathbf{y}(t)^2\right\}$ can be derived by (4) and (5),

$$\mathrm{E}\left\{\mathbf{y}(t)^2\right\} = \mathbf{u}^T\mathbf{R}\mathbf{u} \tag{6}$$

The scalar energy functions $J(\mathbf{u}_1)$ and $J(\mathbf{u}_2)$ are dependent of the measured signal values $\mathbf{x}(t)$ or $\mathbf{x}$. $\mathbf{u}_1$ and $\mathbf{u}_2$ are respective coefficient values similar to $\mathbf{u}$. The differential cross correlation matrix estimated from the equation (3) and the Lagrangian multipliers are represented by $\mathbf{C}$, λ_1 and λ_2 respectively.

$$J(\mathbf{u}_1) = \mathbf{u}_1{}^T\mathbf{C}\mathbf{u}_1 + \lambda_1\left(\mathbf{I} - \mathbf{u}_1{}^T\mathbf{R}^{-1}\mathbf{u}_1\right) \tag{7}$$

$$J(\mathbf{u}_2) = \mathbf{u}_2{}^T\mathbf{C}\mathbf{u}_2 + \lambda_2\left(\mathbf{I} - \mathbf{u}_2{}^T\mathbf{R}\mathbf{u}_2\right) \tag{8}$$

The signal $s_1(t)\mathbf{a}_1$ is targeted to be extracted while noise component of the interference $\mathbf{w}'(t)$ is removed. The projections of $\mathbf{u}_1$ and $\mathbf{u}_2$ to the effective space spanned by the matrix $\mathbf{C}^{-1}$ is tried to be minimized by the first terms of (7) and (8), where matrix inverse is referred by $^{-1}$. The output power to that kind of inverse process is minimized by $\mathbf{u}_1$ and $\mathbf{u}_2$ where the correlation matrix $\mathbf{C}^{-1}$ is produced. Time correlated signal portion $s_1(t)\mathbf{a}_1$ is included with the differential correlation matrix $\mathbf{C}$. Because the inverse matrix is used, resulting $\mathbf{u}_1{}^T\mathbf{C}^{-1}\mathbf{u}_1$ and $\mathbf{u}_2{}^T\mathbf{C}^{-1}\mathbf{u}_2$ are in minimum. Therefore it is expected that the signal $s_1(t)\mathbf{a}_1$ is passed quite well by $\mathbf{u}_1$ and $\mathbf{u}_2$. The projection of $\mathbf{u}_1$ and $\mathbf{u}_2$ is tried to be minimized by the second term. So $\mathrm{E}\left\{\mathbf{y}(t)^2\right\}$ is tried to be minimized. But without differential correlation, much of the interference part $\mathbf{R}_0$ is included with $\mathbf{R}$. The desired signal $s_1(t)\mathbf{a}_1$ is filtered well again by $\mathbf{u}_1$ and $\mathbf{u}_2$.

Series of solutions and approaches attempting to remove or minimize the effect of noise or interference components under different conditions can be seen. Similar kind of two solutions where noise component is tried to be removed are presented bellow.

- *Algorithm 1*

Considering the partial derivative $\frac{\partial J(\mathbf{u}_1)}{\partial \mathbf{u}_1} = 0$ of (7) for the optimization,

$$\lambda\mathbf{u}_1 = \mathbf{R}\mathbf{C}\mathbf{u}_1 \tag{9}$$

Colored noise is filtered out.

Using the iterative fixed point rule as follows,

$$\mathbf{g}_1(t) = \mathbf{R}\mathbf{C}\mathbf{u}_1(t) \tag{10}$$

$$\mathbf{u}_1(t+1) = \frac{\mathbf{g}_1(t)}{\|\mathbf{g}_1(t)\|} \tag{11}$$

- *Algorithm 2*

Considering the partial derivative $\frac{\partial J(\mathbf{u}_2)}{\partial \mathbf{u}_2} = 0$ of (8) for the optimization,

$$\lambda\mathbf{u}_2 = \mathbf{C}\mathbf{R}^{-1}\mathbf{u}_2 \tag{12}$$

Colored noise is filtered out.

Using the iterative fixed point rule as follows,

$$\mathbf{g}_2(t) = \mathbf{C}\mathbf{R}^{-1}\mathbf{u}_2(t) \tag{13}$$

$$\mathbf{u}_2(t+1) = \frac{\mathbf{g}_2(t)}{\|\mathbf{g}_2(t)\|} \tag{14}$$

For the iteration m of any of the solutions,

$$\mathbf{u}(t+m) = \frac{\mathbf{g}(t+m-1)}{\|\mathbf{g}(t+m-1)\|} \tag{15}$$

B. BSS for OFDM

Contributions of BSS algorithms in symbol recovery at the receiver are tested for OFDM. Attempts taken to remove noise component and interference components can be identified as two of the main variants of basic and conventional approaches developed with the intention of improving the symbol recovery for OFDM transmissions.

A basic OFDM [10], [11] system equipped with N subcarrier (N-point inverse discrete Fourier transform (IDFT)/discrete Fourier transform (DFT)) transmitter is considered. Channel is modeled with L independent supportive paths with exponentially decaying delay profiles for each tap. Channel impulse response for the path l of the channel tap k of the multipath frequency-selective Rayleigh fading channel within the same duration is denoted by $h_{k,l}(t)$. Frequency response of the channel state information (CSI) of subcarrier n within an OFDM symbol duration t is given by $H_n(t)$.

$$H_n(t) = \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} h_{k,l}(t) e^{-j\frac{2\pi kn}{N}} \tag{16}$$

Transmit symbol and normalized additive white Gaussian noise of subcarrier n for the period t are given by $d_n(t)$ $v_n(t)$ correspondingly. The standard deviation to the additive white Gaussian noise is denoted by σ. Then the receive signal $r_n(t)$ on subcarrier n during t can be expressed as in (17).

$$r_n(t) = H_n(t) d_n(t) + \left(\frac{\sigma}{\sqrt{2}} v_n(t) \right) \tag{17}$$

Due to slow fading it can be assumed that for a given symbol frame path gain $H_n(t) = H_n$ and $h_{k,l}(t) = h_{k,l}$.

$$r_n(t) = H_n d_n(t) + \left(\frac{\sigma}{\sqrt{2}} v_n(t) \right) \tag{18}$$

Receive signal samples of subcarrier n at time period containing symbol $d_n(t)$ can be expressed as $r_n(t), r_n(t+\tau), ..., r_n(t+(p-1)\tau), ..., r_n(t+(P-1)\tau)$ where P samples are taken with small time shifts of τ within each information symbol duration. Row matrix containing these samples as the elements is represented by the notation $\mathbf{r}_n(t)$. These samples are equivalent to $x_1(t), x_2(t), ..., x_f(t), ..., x_F(t)$ in (1). Symbol sample of $d_n(t)$ and additive white Gaussian noise of sample p of $r_n(t)$ are indicated by $d_n(t+(p-1)\tau)$ and $v_n(t+(p-1)\tau)$ correspondingly. Using (18), sample p of receive signal $r_n(t)$ can be expressed as,

$$\begin{aligned} r_n(t+(p-1)\tau) &= H_n d_n(t+(p-1)\tau) \\ &+ \left(\frac{\sigma}{\sqrt{2}} v_n(t+(p-1)\tau) \right) \end{aligned} \tag{19}$$

For any symbol sample on subcarrier n within a time duration of a transmit information symbol $d_n(t)$, $d_n(t+(p-1)\tau) = d_n(t)$.

$$\begin{aligned} r_n(t+(p-1)\tau) &= H_n d_n(t) \\ &+ \left(\frac{\sigma}{\sqrt{2}} v_n(t+(p-1)\tau) \right) \end{aligned} \tag{20}$$

The differential correlation matrix $\mathbf{C}_n(\tau)$ at a small time difference τ and ordinary correlation matrix $\mathbf{R}_n$ for subcarrier n according to (3) and (4) are,

$$\mathbf{C}_n(\tau) = \mathrm{E}\left\{ \mathbf{r}_n(t) \mathbf{r}_n(t+\tau)^\dagger \right\} \tag{21}$$

$$\mathbf{R}_n = \mathrm{E}\left\{ \mathbf{r}_n(t) \mathbf{r}_n(t)^\dagger \right\} \tag{22}$$

Matrix complex conjugate transpose is denoted by †.

$g_n(t)$ and $g_n(t+m-1)$ pairs of subcarrier n for the two algorithms can be introduced by using the iterative fixed point rule similar to (10) and (13),

- *Algorithm 1*

$$g_{1,n}(t) = \mathbf{R}_n \mathbf{C}_n u_{1,n}(t) \tag{23}$$

$$g_{1,n}(t+m-1) = \mathbf{R}_n \mathbf{C}_n u_{1,n}(t+m-1) \tag{24}$$

- *Algorithm 2*

$$g_{2,n}(t) = \mathbf{C}_n \mathbf{R}_n^{-1} u_{2,n}(t) \tag{25}$$

$$g_{2,n}(t+m-1) = \mathbf{C}_n \mathbf{R}_n^{-1} u_{2,n}(t+m-1) \tag{26}$$

Coefficient $u_n(t+m)$ for iteration m for subcarrier n of any of the solutions given for OFDM is stated as,

$$u_n(t+m) = \frac{g_n(t+m-1)}{\|g_n(t+m-1)\|} \tag{27}$$

Refined receive signal similar to (5) is,

$$y_n(t+(p-1)\tau) = u_n(t+m)^\dagger r_n(t+(p-1)\tau) \tag{28}$$

Signal after Equal Gain Combining (EGC) can be expressed as,

$$y_n(t) = \frac{u_n(t+m)^\dagger}{P} \frac{H_n{}^\dagger}{|H_n|} \sum_{p=0}^{P-1} r_n(t+(p-1)\tau) \tag{29}$$

Let $\mathbf{u}_{OFDM}$ be a $N \times N$ diagonal matrix containing elements $u_1(t+m), u_2(t+m), ..., u_n(t+m), ..., u_N(t+m)$ on the diagonal and $\mathbf{x}_{OFDM}$ be a column matrix where element n can be given by,

$$x_n = \frac{1}{P} \frac{H_n{}^\dagger}{|H_n|} \sum_{p=0}^{P-1} r_n(t+(p-1)\tau) \tag{30}$$

The simplified matrix format output $\mathbf{y}(t)$ or $\mathbf{y}$ of the receiver with coefficients $\mathbf{u}$ for these OFDM based techniques is given as,

$$\mathbf{y}_{OFDM} = \mathbf{u}_{OFDM}^T \mathbf{x}_{OFDM} \tag{31}$$

III. System Model

An OFDM transmitter receiver setup consisting N subcarriers [10], [11] is used for the simulations where the complete discrete time base band simulation system model is given in Figure 1. It is assumed that transmit receiver setup is properly synchronized under all the conditions and operated in a slow fading frequency-selective Rayleigh channel. Multiple samples for each receive symbol on every subcarrier are taken for testing the iterative BSS algorithms. Same system model is used in order to test and verify the results of standard OFDM system [10], [11] with no multiple sampling for the

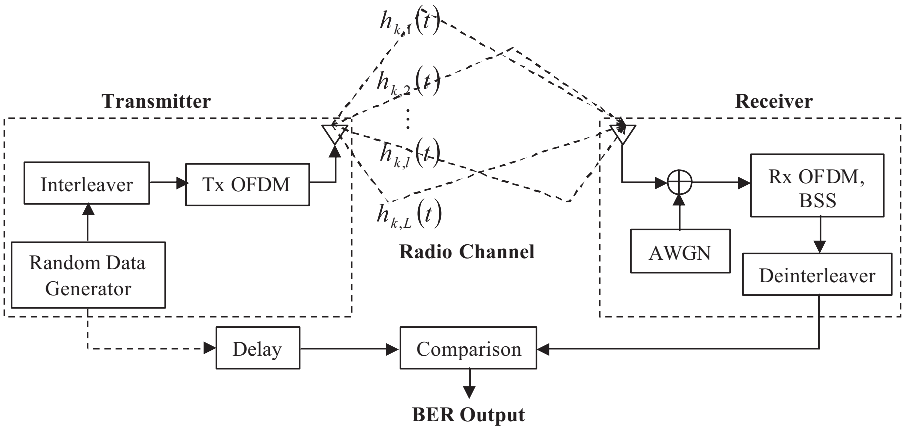


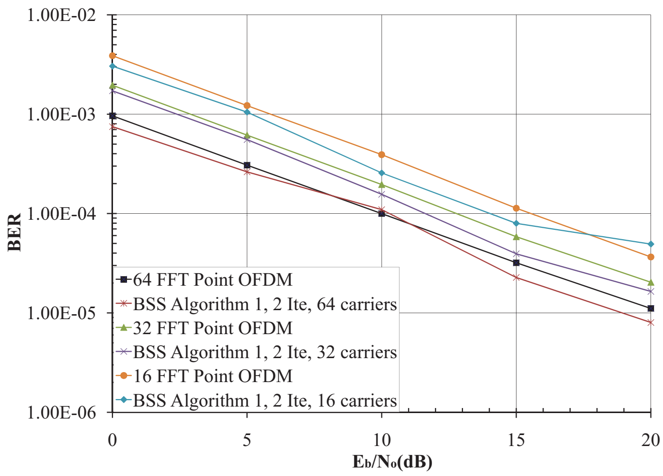
Fig. 1. Block diagram of the lowpass equivalent system model

receive signals. Among various combining techniques, stable conventional combining technique namely EGC is used with perfect path gain values at the receiver.

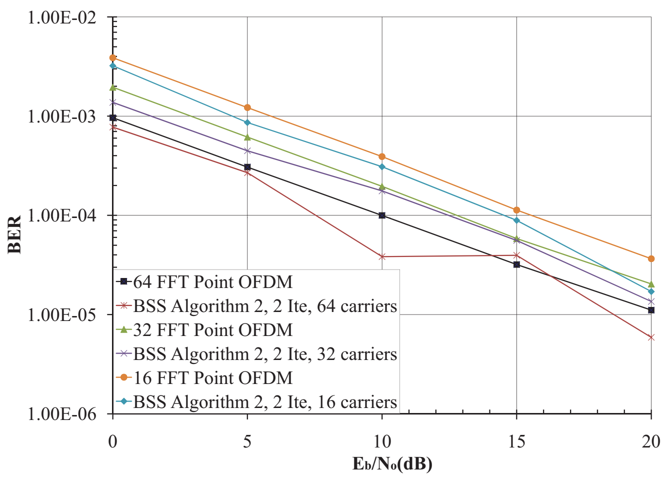
IV. System Parameters and Simulation Results

Performance curves obtained for OFDM receiver with two BSS schemes and standard OFDM systems. Binary information bits are converted to BPSK symbols at the 64 (32 and 16 in the cases of testing performance of the algorithms) subcarrier basic OFDM transmitters. Then interleaving is applied before they are been serial to parallel converted among subcarriers. A slow fading channel is used for the transmissions. Multiple signal samples are taken from each of the receive symbol on every subcarrier that are to be used for the BSS solutions. Considering the properties of the channel [12] and time correlation properties of the binary wave forms [9], samples are taken only within the first 20% of time duration of an information symbol. 4 independent paths are used for each channel tap. No any other variation of the signal is considered during a symbol duration. Systems with the BSS schemes are operated for a number of iterations using the fixed point rule (Initially $g_n(t) = 1$). Single transmit and receive antenna setup is used.

The OFDM systems namely the system with two noise removal BSS algorithms and the standard OFDM system with different number of subcarriers are tested and where the results are presented in Figure 2. All the other parameters and conditions are maintained as the same. The results of the BSS schemes worked out for 16, 32 and 64 subcarriers employing algorithms 1 and 2 are given by Figure 2 - (a) and (b) accordingly. Receive signal sampling is limited to 15 samples for each of the receive symbol on every subcarrier. In this scenario the OFDM systems with BSS algorithms are capable of outperforming the corresponding standard OFDM system. That the iterative fixed point rule contributes negatively for the process of recovery of corrupted signals under these parameters.



(a)



(b)

Fig. 2. OFDM communication system with BSS and EGC : **(a)** algorithm 1 and **(b)** algorithm 2

Results of the simulations done for OFDM receiver with two BSS algorithms with different number of receive samples and standard OFDM system is presented by Figure 3. Other parameters and conditions of the models are held as the same. Performance of the BSS schemes carried out taking 5, 8 and 10 receive signal samples for the two algorithms are shown by Figure 3 - (a) and (b) respectively. Results of the both algorithms with 10 samples are presented in 3 - (c). The simulation curves of the two schemes together with the standard or the normal OFDM system fall closer to each other. It can be observed that all these algorithms are capable of outperforming the standard OFDM system when higher numbers of samples are taken. Increasing the number of samples contributes positively for the process of refinery of corrupted symbols under these parameters.

V. Conclusion

Information symbol recovery for the symbols transmitted in OFDM transmissions with aid of two BSS solutions was considered. Reduction of destructive influence caused within free space communication was targeted specifically focusing on to removal of noise component from the receive signal mixture. These algorithms were tested with variable number of samples taken from each receive symbol on every sub-carrier and variable number of subcarriers for each scheme. These schemes can be further developed to cater to the real environment after more analysis with properties like Doppler effect.

References

[1] J. -F. Cardoso, "Blind Signal Separation: Statistical Principles," *Proceedings of the IEEE,* vol. 86(10), pp. 2009-2025, October 1998.

[2] M. E. Sahin, I. Guvenc, and H. Arslan, "Uplink User Signal Separation for OFDMA-Based Cognitive Radios," *EURASIP Journal on Advances in Signal Processing,* Hindawi Publishing Corporation, vol. 2010, Article ID 502369.

[3] F. Abrard, and Y. Deville, "A Timefrequency Blind Signal Separation Method Applicable to Underdetermined Mixtures of Dependent Sources," *Elsevier Journal on Signal Processing,* Elsevier B. V., vol. 85, pp. 1389-1403, 2005.

[4] A. K. Barros, R. Vigário, V. Jousmäki, and N. Ohnishi, "Extraction of Event-related Signals from Multichannel Bioelectrical Measurements," *IEEE Transaction on Biomedical Engineering,* vol. 47, no. 5, pp. 583-588, 2000.

[5] R. L. Adler, J. -P. Dedieu, J. Y. Margulies, M. Martens, and M. Shub, "Newtons method on Riemannian manifolds and a geometric model for the human spine," *IMA Journal of Numerical Analysis,* vol. 22(3), pp. 359-390, July 2002.

[6] M. E. Waheed, "Blind Signal Separation using an Adaptive Weibull Distribution," *International Journal of Physical Sciences,* vol. 4(5), pp. 265-270, May 2009.

[7] K. -R. Müller, R. Vigário, F. Meinecke, and A. Ziehe, "Blind Source Separation Techniques for Decomposing Event-related Brain Signals," *International Journal of Bifurcation and Chaos,* World Scientific Publishing Company, vol. 14, no. 2, pp. 773-791, 2004.

[8] H. Szu, P. Chanyagorn, and I. Kopriva, "Sparse Coding Blind Source Separation through Powerline," *Elsevier Neurocomputing Letters,* Elsevier Science B. V., vol. 48, pp. 1015-1020, October 2002.

[9] S. Haykin, *Communication Systems*. John Wiley & Sons, Inc. 4th Edition, 2001.

[10] C. K. Sung, and I. Lee, "Multiuser Bit-Interleaved Coded OFDM with Limited Feedback Information," *IEEE Vehicular Technology Conference,* Dallas, Texas, USA, September 2005.

[11] L. Hanzo, L -L. Yang, E -L. Kuan, and Yen, *Single- and Multi-Carrier DSCDMA: Multi-User Detection, Space-Time Spreading, Synchronisation, Networking and Standards*. John Wiley & Sons, Inc. 2003.

[12] J. G. Proakis, *Digital Communications*. McGraw-Hill, Inc. 4th Edition, 2001.

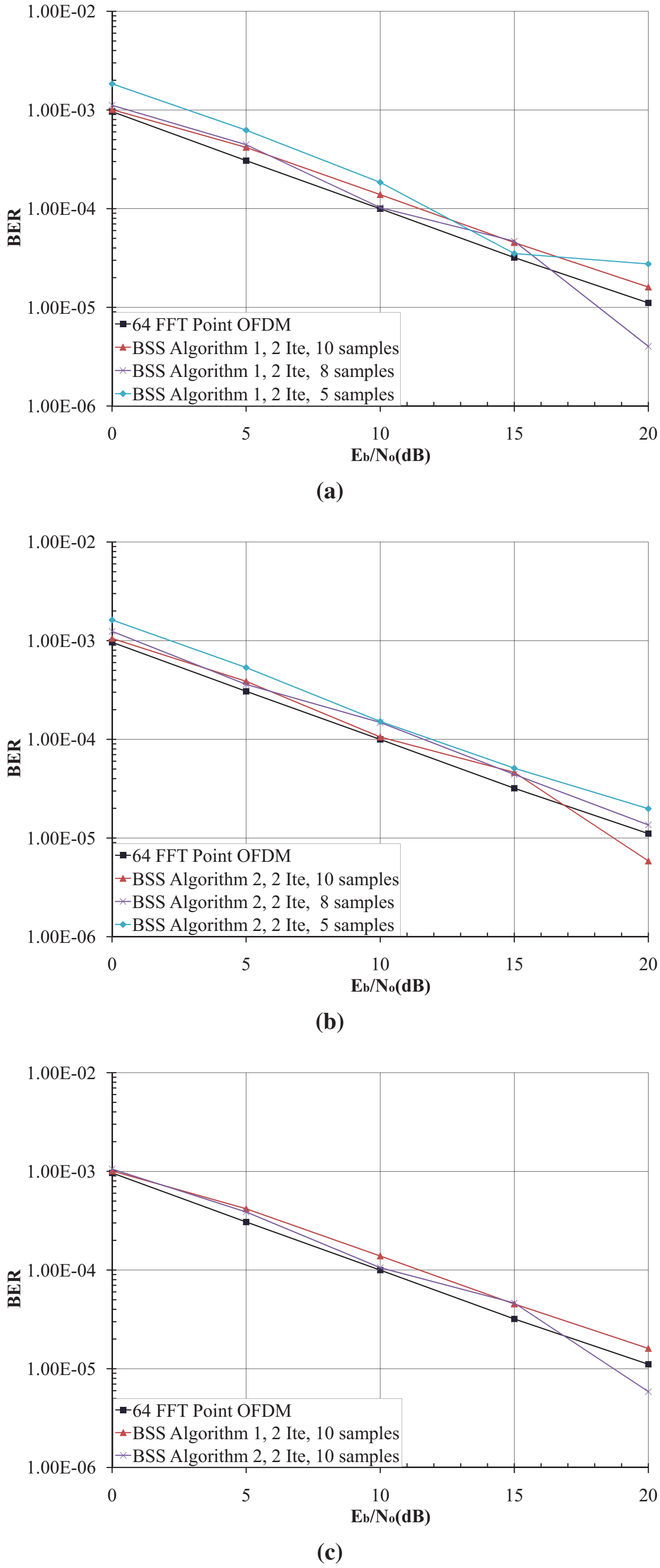


Fig. 3. OFDM system with BSS and EGC : **(a)** algorithm 1, **(b)** algorithm 2 and **(c)** two algorithms with 10 samples