

Blind Source Separation for OFDM with Filtering Colored Noise

TIEJ601 Postgraduate Seminar in Information Technology

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Introduction to my Doctoral Program and Research Background

- Moctoral Research: Enhancing Wireless Communication Systems with Different Signal Processing Methods (yet to be finalized).
- Main Contribution: Strengthening the signal detection capabilities of the air interface communication techniques.
- Supervisors: Profs Jyrki Joutsensalo, Timo Hamalainen, Tapani Ristaniemi
- First Phase: Based on the principles of Blind Source Separation of Gradient Algorithms. OFDM and DS-CDMA Systems were targeted.
 - Results are being published: One paper was presented. Some of the papers have been accepted. Rest of the publications are under review (Not yet available with IEEE Xplore).
- This presentation is based on the results of the first phase and the already presented paper "Blind Source Separation for OFDM with Filtering Colored Noise Out".



Content

- Introduction to BSS and applications of it.
- **BSS** models.
 - General BSS model.
 - BSS model for OFDM.
- Limitations and assumptions.
- Basic system model for simulations.
- Simulation results and evaluation of performance.
- Conclusions



Introduction to BSS and its Applications

- Theories of Blind Source or Signal Separation are used to recover unobserved signals or sources from several observed mixtures when no prior information is available about the transfer.
- The adjective "blind" stresses two scenarios.
 - Source signals are not observed.
 - Second is NO information is available about the mixture.
- Higher effective data rates are expected to be delivered by blind schemes when there is no training or pilot data sequence.
- Biomedical signal analysis and processing (ECG, EEG, MEG), acoustics (audio signal processing), geophysical data processing, data mining, speech recognition, image recognition and communications signal processing including wireless communications.



Independent or uncorrelated source symbols or signals,

$$\mathbf{s}(t) = [s_1(t), s_2(t), ..., s_b(t), ..., s_B(t)]^T$$

Coefficient values,

$$\mathbf{a}_{1} = \left[\mathbf{a}_{1,1}(t), \mathbf{a}_{2,1}(t), ..., \mathbf{a}_{f,1}(t), ..., \mathbf{a}_{F,1}(t)\right]^{T}$$

Additive independent and identically distributed (iid) white noise,

$$\mathbf{w}(t) = \left[w_1(t), w_2(t), ..., w_f(t), ..., w_F(t) \right]^T$$

• A is a FxB matrix and a mixture of coefficient values. $\mathbf{w}'(t)$ is colored noise. Then the receive signal matrix,

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t) = s_1(t)\mathbf{a}_1 + \mathbf{w}'(t)$$

• When appropriately shaped, mapped symbols are considered and the time delay is sufficiently shorter, let c1 to be defined as,

$$c_1 = \mathrm{E}\{s_1(t)s_1(t+\tau)\} \approx \mathrm{E}\{s_1(t)^2\}$$



- The differential correlation matrix taken with a small time difference τ , $\mathbf{C}(\tau) = \mathbf{E} \{\mathbf{x}(t)\mathbf{x}(t+\tau)^T\} = c_1\mathbf{a}_1\mathbf{a}_1^T + \mathbf{R}_{\tau}$
- The ordinary correlation matrix,

$$\mathbf{R} = \mathbf{E} \left\{ \mathbf{x}(t)\mathbf{x}(t)^T \right\}$$
$$= \mathbf{A}\mathbf{E} \left\{ \mathbf{s}(t)\mathbf{s}(t)^T \right\} \mathbf{A}^T + \sigma^2 \mathbf{I}$$
$$= c_1 \mathbf{a}_1 \mathbf{a}_1^T + \mathbf{R}_0$$

It is assumed that $\|\mathbf{R}_0\| > \|\mathbf{R}_\tau\|$ and noise variance is given by σ^2 , where $\|\cdot\|$ is Frobenius or 2-norm.

- The receiver output $\mathbf{y}(t)$ or \mathbf{y} with coefficient \mathbf{u} is given by, $\mathbf{y} = \mathbf{u}^T \mathbf{x}$
- The output power can be expressed as, $\mathbf{E} \{ \mathbf{y}(t)^2 \} = \mathbf{u}^T \mathbf{R} \mathbf{u}$



The scalar energy functions $J(\mathbf{u}_1)$ and $J(\mathbf{u}_2)$ are dependent of the measured signal values $\mathbf{x}(t)$ or \mathbf{x} . \mathbf{u}_1 and \mathbf{u}_2 are respective coefficient values similar to \mathbf{u} . The differential cross correlation matrix and the Lagrangian multipliers are represented by \mathbf{C} , λ_1 and λ_2 respectively.

$$J(\mathbf{u}_1) = \mathbf{u}_1^T \mathbf{C} \mathbf{u}_1 + \lambda_1 \left(\mathbf{I} - \mathbf{u}_1^T \mathbf{R}^{-1} \mathbf{u}_1 \right)$$
$$J(\mathbf{u}_2) = \mathbf{u}_2^T \mathbf{C} \mathbf{u}_2 + \lambda_2 \left(\mathbf{I} - \mathbf{u}_2^T \mathbf{R} \mathbf{u}_2 \right)$$

💐 Algorithm 1

Considering the partial derivative
$$\frac{\partial J(\mathbf{u}_1)}{\partial \mathbf{u}_1} = 0$$

$$\lambda_1 \mathbf{u}_1 = \mathbf{RC} \mathbf{u}_1$$

$$\mathbf{g}_1(t) = \mathbf{RC} \mathbf{u}_1(t)$$

$$\mathbf{u}_1(t+1) = \frac{\mathbf{g}_1(t)}{\|\mathbf{g}_1(t)\|}$$



Algorithm 2

Considering the partial derivative $\frac{\partial J(\mathbf{u}_2)}{\partial \mathbf{u}_2} = 0$ $\lambda_2 \mathbf{u}_2 = \mathbf{C} \mathbf{R}^{-1} \mathbf{u}_2$ $\mathbf{g}_2(t) = \mathbf{C} \mathbf{R}^{-1} \mathbf{u}_2(t)$ $\mathbf{u}_2(t+1) = \frac{\mathbf{g}_2(t)}{\|\mathbf{g}_2(t)\|}$

 \blacksquare For the iteration m of any of the solutions,

$$\mathbf{u}(t+m) = \frac{\mathbf{g}(t+m-1)}{\|\mathbf{g}(t+m-1)\|}$$



BSS Models: BSS Model for OFDM

Channel impulse response for the path l of the channel tap k of the multipath frequency-selective Rayleigh fading channel within the same duration is denoted by $h_{k,l}(t)$. Frequency response of the channel of subcarrier n within an OFDM symbol duration t is given by,

$$H_n(t) = \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} h_{k,l}(t) e^{-j\frac{2\pi kn}{N}}$$

Symbol sample on subcarrier n within a time duration of a transmit information symbol $d_n(t)$, $d_n(t + (p - 1)\tau) = d_n(t)$. Transmit symbol and normalized additive white Gaussian noise of subcarrier n for the period t are given by $d_n(t)$ and $v_n(t)$ correspondingly. Due to slow fading it can be assumed that for a given symbol frame path gain $H_n(t) = H_n$ and $h_{k,l}(t) = h_{k,l}$.

$$r_n(t+(p-1)\tau) = H_n d_n(t) + \left(\frac{\sigma}{\sqrt{2}} v_n(t+(p-1)\tau)\right)$$



BSS Models: BSS Model for OFDM

Algorithm 1

$$g_{1,n}(t) = \mathbf{R}_n \mathbf{C}_n u_{1,n}(t)$$
$$g_{1,n}(t+m-1) = \mathbf{R}_n \mathbf{C}_n u_{1,n}(t+m-1)$$

Algorithm 2

$$g_{2,n}(t) = \mathbf{C}_n \mathbf{R}_n^{-1} u_{2,n}(t)$$

$$g_{2,n}(t+m-1) = \mathbf{C}_n \mathbf{R}_n^{-1} u_{2,n}(t+m-1)$$

Coefficient $u_n(t+m)$ for iteration m for subcarrier n of any of the solutions,

$$u_n(t+m) = \frac{g_n(t+m-1)}{\|g_n(t+m-1)\|}$$

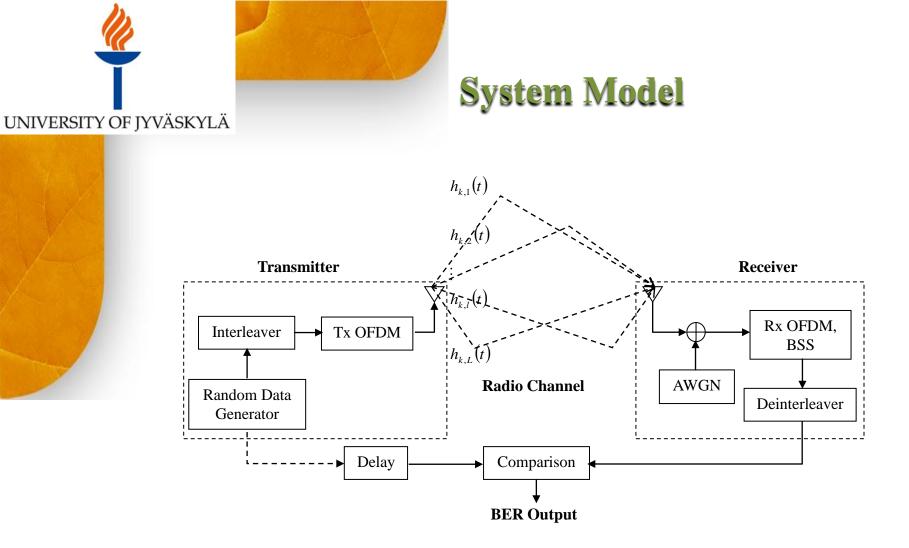
Signal after Equal Gain Combining (EGC) can be expressed as,

$$y_n(t) = \frac{u_n(t+m)^{\dagger}}{P} \frac{H_n^{\dagger}}{|H_n|} \sum_{p=0}^{P-1} r_n(t+(p-1)\tau)$$



Limitations and Assumptions

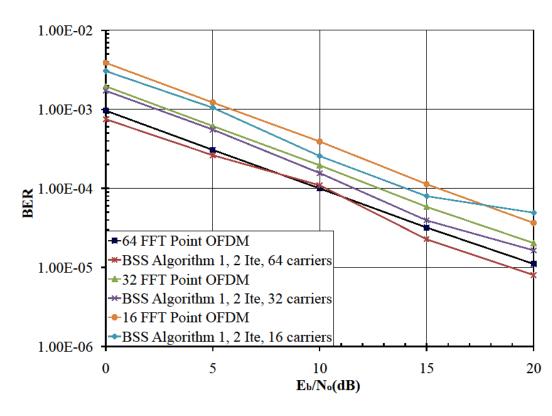
- The channel is assumed to be known at the receiver.
- Free space propagation models (e.g., Hata model), which are used to address the issues such as distance and Doppler shifts are not taken in to account.
- Ideal channel characteristics are assumed.
- Perfect synchronization in time and frequency is assumed.
- Issues related to the peak to average power ratio are not considered.
- Data flow rates are considered to be constant through out the operation of the system (ie no RQ and ARQ schemes are considered).



Block diagram of the lowpass equivalent system model

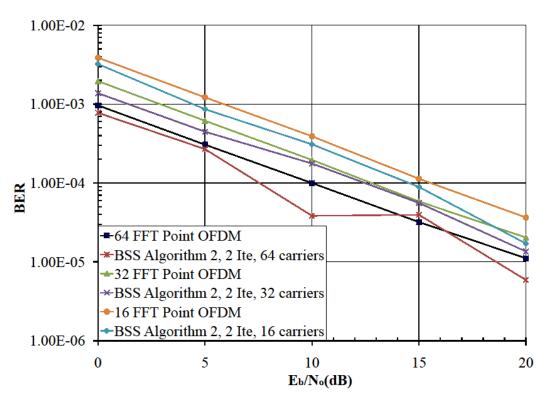


Simulation Results Different Numbers of Subcarriers



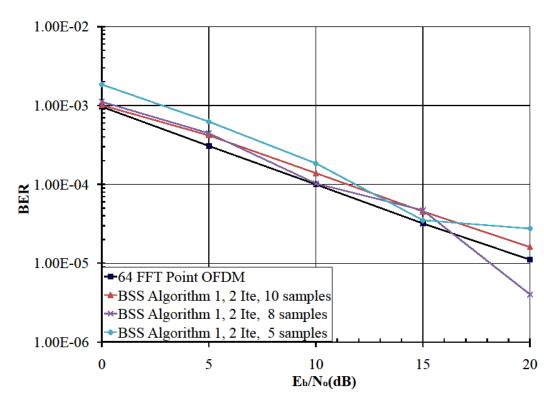


Simulation Results Different Numbers of Subcarriers



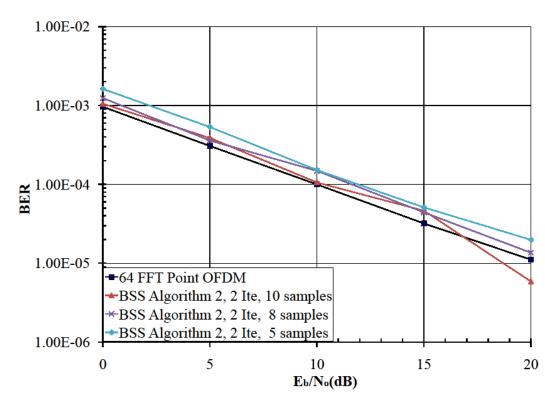


Simulation Results - Different Numbers of Samples



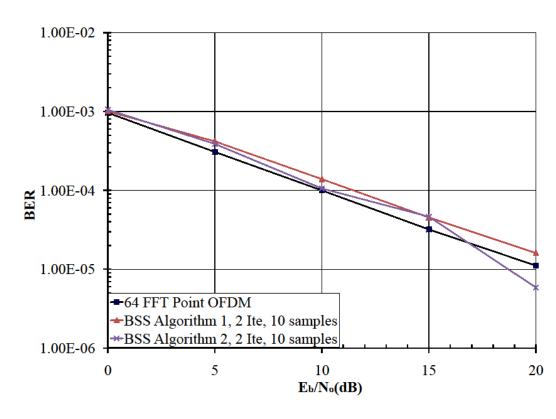


Simulation Results - Different Numbers of Samples





Simulation Results -Same Numbers of Samples



OFDM system with BSS and EGC: Performance of two algorithms with 10 samples



Conclusions

- These schemes can be further developed to cater to the real environment after more analysis with properties like Doppler effect.
- Even though it is not quantified, these algorithms can be considered as relatively low computationally complex algorithms.



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