# DSAA 5002: Knowledge Discovery and Data Minig in Data Science

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# **Chapter 2: Getting to Know Your Data**

Data Objects and Attribute Types



- Basic Statistical Descriptions of Data
- Measuring Data Similarity and Dissimilarity
- Summary

# Types of Data Sets

- Record
  - Relational records
  - Data matrix, e.g., numerical matrix, crosstabs

 Document data: text documents: termfrequency vector

- Transaction data
- Graph and network
  - World Wide Web
  - Social or information networks
  - Molecular Structures
- Ordered
  - Video data: sequence of images
  - Temporal data: time-series
  - Sequential Data: transaction sequences
  - Genetic sequence data
- Spatial, image and multimedia:
  - Spatial data: maps
  - Image data:
  - Video data:

n-	team	coach	pla y	ball	score	game	n n	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

# Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

#### **Data Objects**

- Data sets are made up of data objects.
- A data object represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called samples, examples, instances, data points, objects, tuples.
- Data objects are described by attributes.
- Database rows -> data objects; columns ->attributes.

#### **Attributes**

- Attribute (or dimensions, features, variables): a data field, representing a characteristic or feature of a data object.
  - E.g., customer \_ID, name, address
- Types:
  - Nominal
  - Binary
  - Numeric: quantitative
    - Interval-scaled
    - Ratio-scaled

# **Attribute Types**

- Nominal: categories, states, or "names of things"
  - Hair\_color = {auburn, black, blond, brown, grey, red, white}
  - marital status, occupation, ID numbers, zip codes

#### Binary

- Nominal attribute with only 2 states (0 and 1)
- Symmetric binary: both outcomes equally important
  - e.g., gender
- Asymmetric binary: outcomes not equally important.
  - e.g., medical test (positive vs. negative)
  - Convention: assign 1 to most important outcome (e.g., HIV positive)

#### Ordinal

- Values have a meaningful order (ranking) but magnitude between successive values is not known.
- Size = {small, medium, large}, grades, army rankings

# **Numeric Attribute Types**

- Quantity (integer or real-valued)
- Interval
  - Measured on a scale of equal-sized units
  - Values have order
    - E.g., temperature in C°or F°, calendar dates
  - No true zero-point
- Ratio
  - Inherent zero-point
  - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K° is twice as high as 5 K°).
    - e.g., temperature in Kelvin, length, counts, monetary quantities



### Discrete vs. Continuous Attributes

#### Discrete Attribute

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Sometimes, represented as integer variables
- Note: Binary attributes are a special case of discrete attributes

#### Continuous Attribute

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables

# **Chapter 2: Getting to Know Your Data**

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data



- Measuring Data Similarity and Dissimilarity
- Summary

# **Basic Statistical Descriptions of Data**

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube

# **Measuring the Central Tendency**

- Mean (algebraic measure) (sample vs. population):  $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$   $\mu = \frac{\sum x}{N}$ Note: n is sample size and N is population size.
  - Weighted arithmetic mean:
  - Trimmed mean: chopping extreme values  $\bar{\chi}$  =

#### Median:

- Middle value if odd number of values, or average of the middle two values otherwise
- Estimated by interpolation (for grouped data):

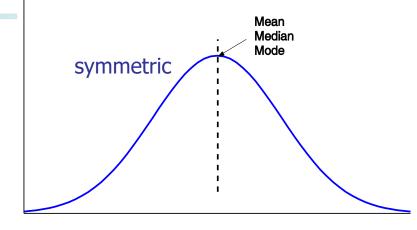
$median = L_1 + (\frac{n}{-})$	$/2 - (\sum freq)l$	)width
$\mathbf{L}_{1}$	$freq_{\it median}$	jvvioliti

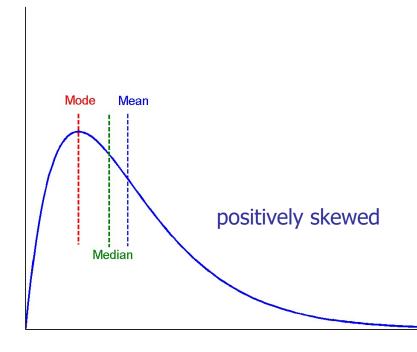
- Mode
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal
  - Empirical formula:  $mean mode = 3 \times (mean median)$

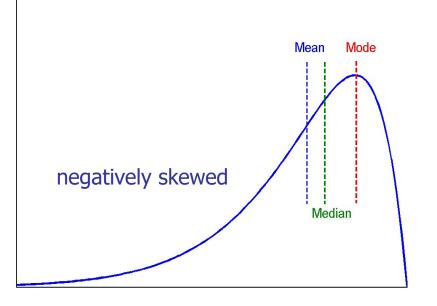
51–80 700 81–110 44

# Symmetric vs. Skewed Da

 Median, mean and mode of symmetric, positively and negatively skewed data







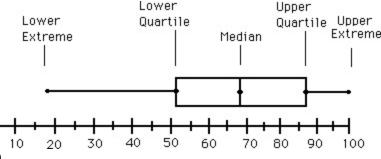
# Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - **Quartiles**: Q<sub>1</sub> (25<sup>th</sup> percentile), Q<sub>3</sub> (75<sup>th</sup> percentile)
  - Inter-quartile range:  $IQR = Q_3 Q_1$
  - Five number summary: min,  $Q_1$ , median,  $Q_3$ , max
  - Boxplot: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - Outlier: usually, a value higher/lower than 1.5 x IQR
- **Outlier**: usually, a value nigner/lower unan 1.3 x 1Q/x Variance and standard deviation (sample: s, population: σ)
  - **Variance**: (algebraic, scalable computation)

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2} = \frac{1}{n-1} \left[ \sum_{i=1}^{n} x_{i}^{2} - \frac{1}{n} \left( \sum_{i=1}^{n} x_{i} \right)^{2} \right] \qquad \sigma^{2} = \frac{1}{N} \sum_{i=1}^{n} (x_{i} - \mu)^{2} = \frac{1}{N} \sum_{i=1}^{n} x_{i}^{2} - \mu^{2}$$

**Standard deviation** s (or  $\sigma$ ) is the square root of variance  $s^2$  (or  $\sigma^2$ )

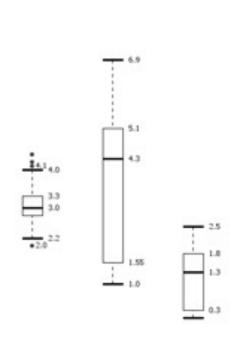
# **Boxplot Analysis**



- Five-number summary of a distribution
  - Minimum, Q1, Median, Q3, Maximum

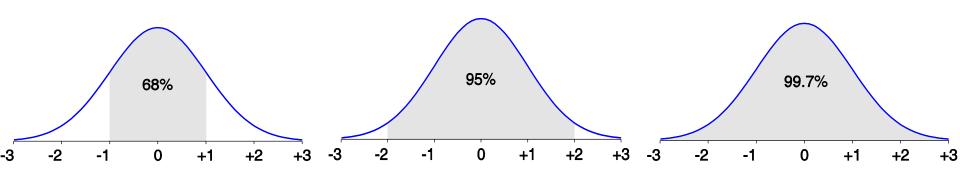
#### Boxplot

- Data is represented with a box
- The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- Whiskers: two lines outside the box extended to Minimum and Maximum
- Outliers: points beyond a specified outlier threshold, plotted individually



# **Properties of Normal Distribution Curve**

- The normal (distribution) curve
  - From  $\mu$ – $\sigma$  to  $\mu$ + $\sigma$ : contains about 68% of the measurements ( $\mu$ : mean,  $\sigma$ : standard deviation)
  - From  $\mu$ –2 $\sigma$  to  $\mu$ +2 $\sigma$ : contains about 95% of it
  - From  $\mu$ –3 $\sigma$  to  $\mu$ +3 $\sigma$ : contains about 99.7% of it



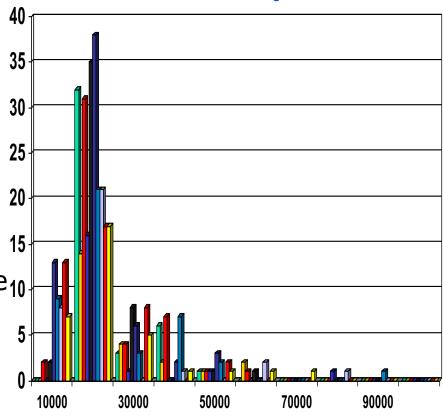
# Graphic Displays of Basic Statistical Descriptions

- Boxplot: graphic display of five-number summary
- Histogram: x-axis are values, y-axis repres. frequencies
- **Quantile plot**: each value  $x_i$  is paired with  $f_i$  indicating that approximately 100  $f_i$ % of data are  $\leq x_i$
- Quantile-quantile (q-q) plot: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- Scatter plot: each pair of values is a pair of coordinates and plotted as points in the plane

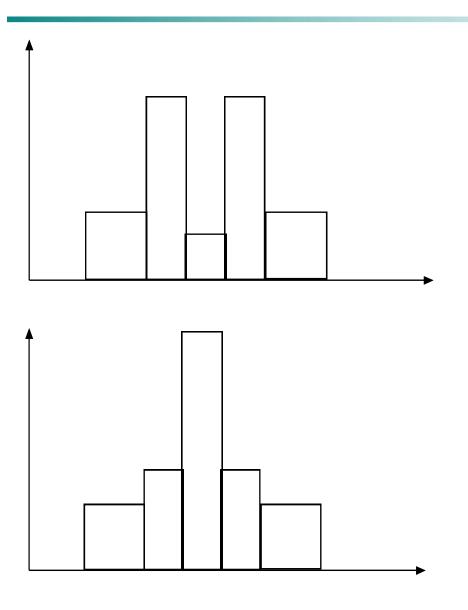
# Histogram Analysis

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- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that it is the area of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



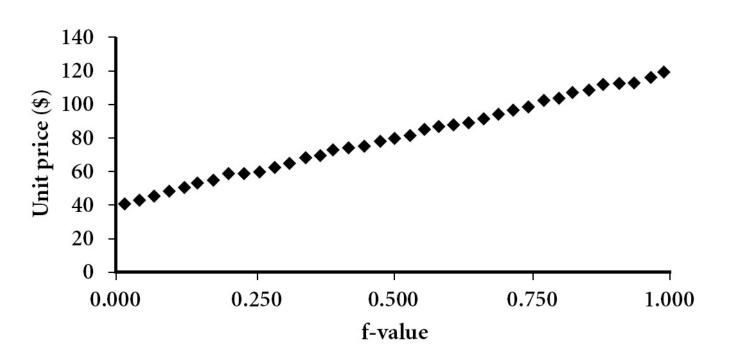
# **Histograms Often Tell More than Boxplots**



- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

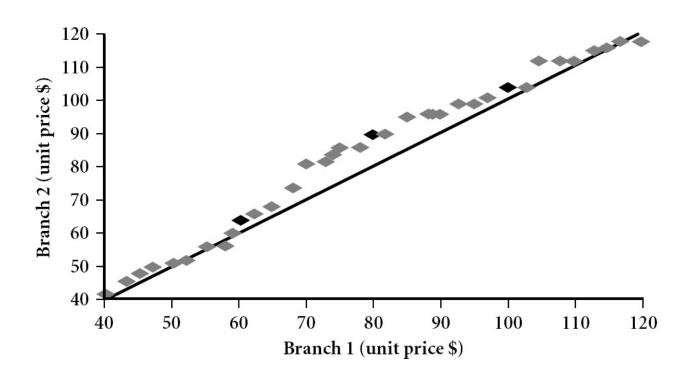
# Quantile Plot 分后数义

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots quantile information
  - For a data  $x_i$  data sorted in increasing order,  $f_i$  indicates that approximately 100  $f_i$ % of the data are below or equal to the value  $x_i$



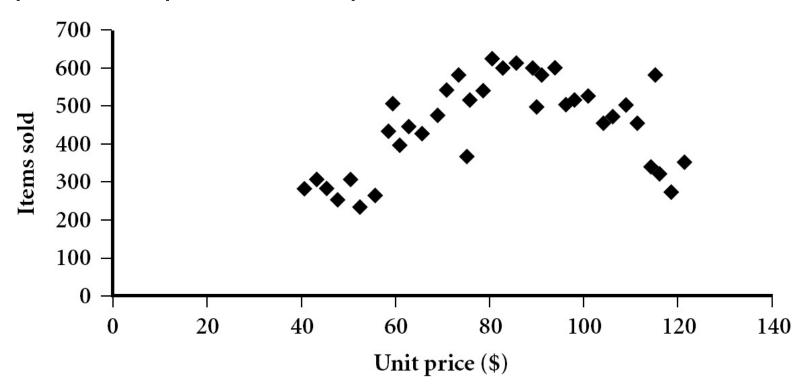
# Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

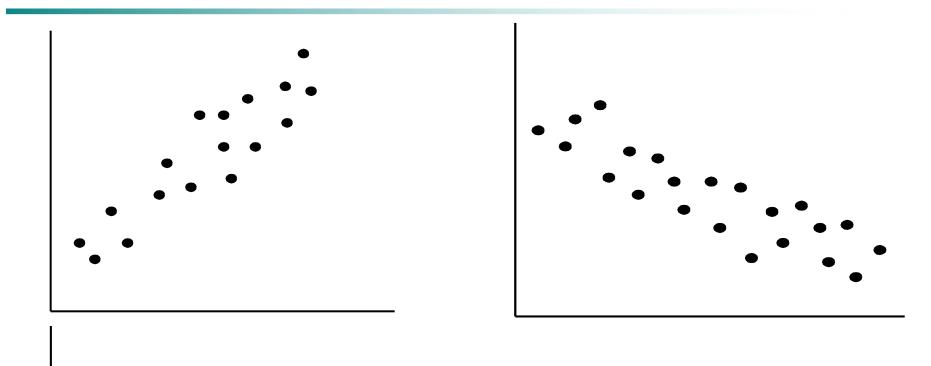


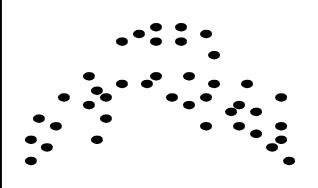
# **Scatter plot**

- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



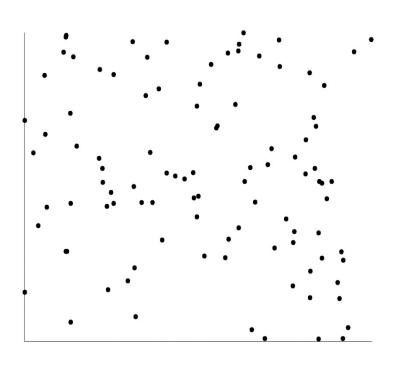
# Positively and Negatively Correlated Data



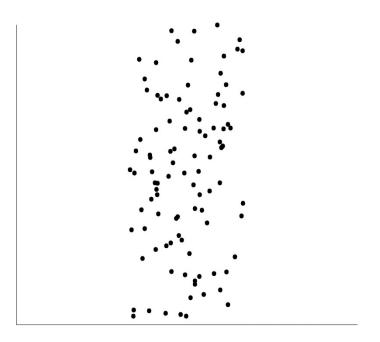


- The left half fragment is positively correlated
- The right half is negative correlated

# **Uncorrelated Data**







# **Chapter 2: Getting to Know Your Data**

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- Basic Statistical Descriptions of Data
- Measuring Data Similarity and Dissimilarity



Summary

# Similarity and Dissimilarity

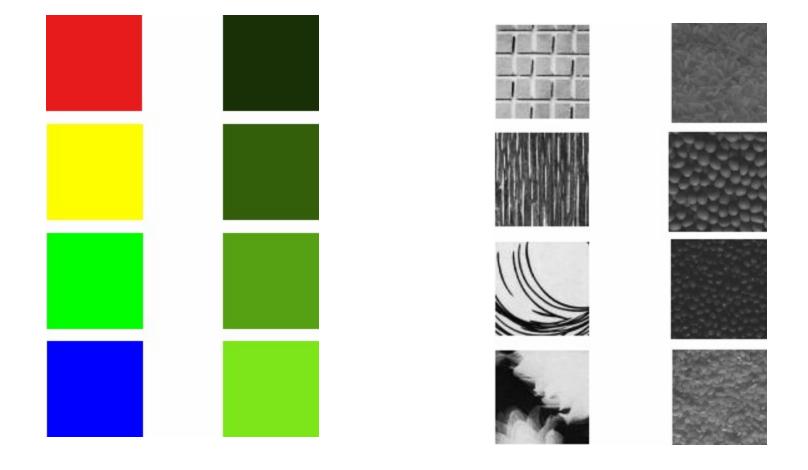
#### Similarity

- Numerical measure of how alike two data objects are
- Value is higher when objects are more alike
- Often falls in the range [0,1]
- Dissimilarity (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

# **Visual Similarity**

Color

Texture



# **Uses for Visual Similarity Measures**

- Classification
  - Is it a horse?
- - Show me pictures of horses.



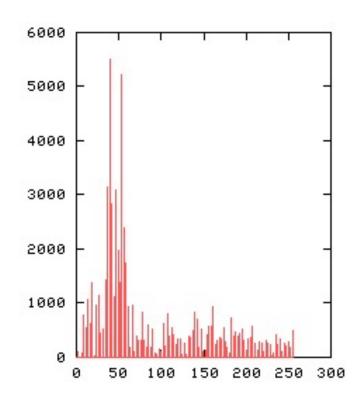


- Unsupervised segmentation
  - Which parts of the image are grass?



# **Histogram Example**





#### Slides from Dave

# **Cumulative Histogram**

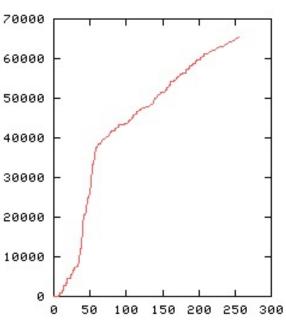


Normal Histogram 6000 5000 4000 3000 2000 1000

100

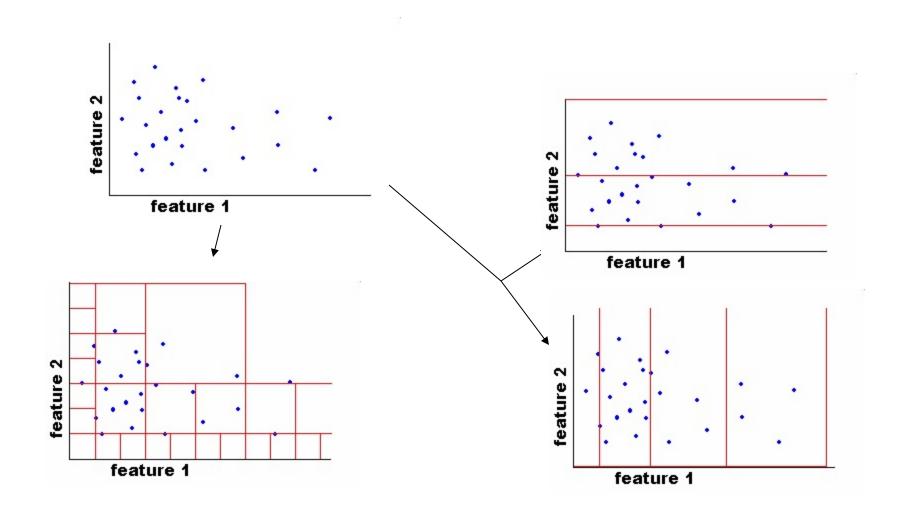
150 200 250





Slides from Dave

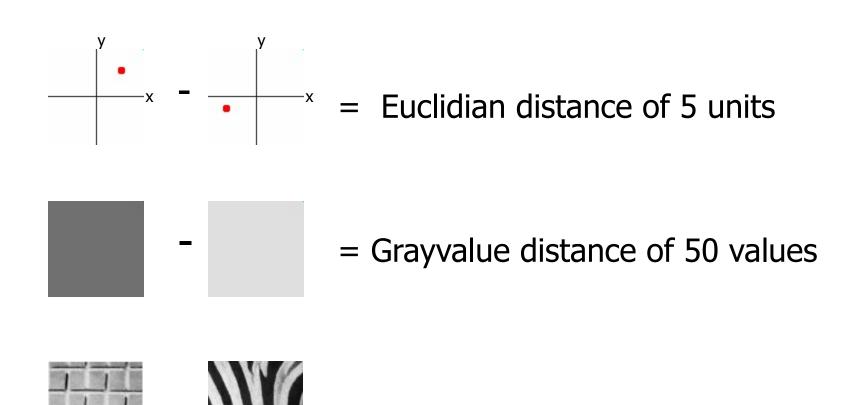
# **Adaptive Binning**



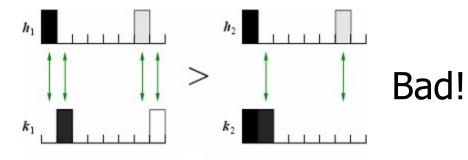
# **Higher Dimensional Histograms**

- Histograms generalize to any number of features
  - Colors
  - Textures
  - Gradient
  - Depth

#### **Distance Metrics**

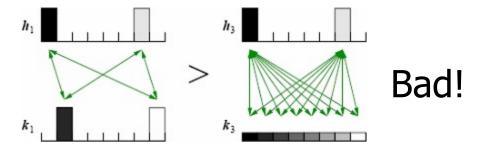


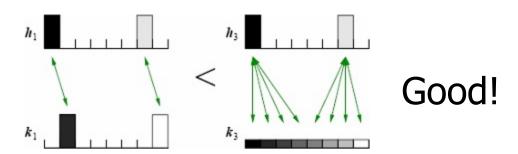
# Bin-by-bin



Good!

# Cross-bin





#### **Distance Measures**

- Heuristic
  - Minkowski-form
  - Weighted-Mean-Variance (WMV)
- Nonparametric test statistics
  - $\chi^2$  (Chi Square)
  - Kolmogorov-Smirnov (KS)
  - Cramer/von Mises (CvM)
- Information-theory divergences
  - Kullback-Liebler (KL)
  - Jeffrey-divergence (JD)
- Ground distance measures
  - Histogram intersection
  - Quadratic form (QF)
  - Earth Movers Distance (EMD)



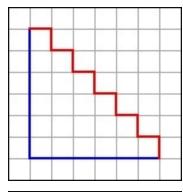


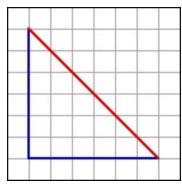
# **Heuristic Histogram Distances**

Minkowski-form distance L<sub>p</sub>

$$D(I,J) = \left(\sum_{i} \left| f(i,I) - f(i,J) \right|^{p} \right)^{1/p}$$

- Special cases:
  - L<sub>1</sub>: absolute, cityblock, or
     Manhattan distance
  - L<sub>2</sub>: Euclidian distance
  - L<sub>∞</sub>: Maximum value distance





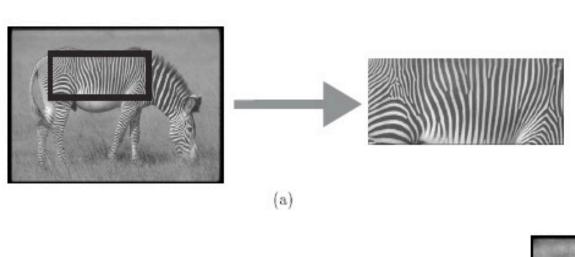
## **More Heuristic Distances**

- Weighted-Mean-Variance
  - Only includes minimal information about the distribution

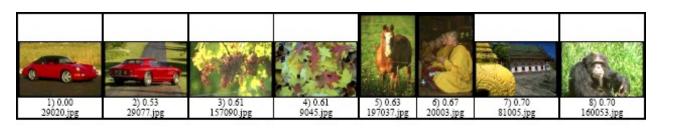
$$D^{r}(I,J) = \frac{\left|\mu_{r}(I) - \mu_{r}(J)\right|}{\left|\sigma(\mu_{r})\right|} + \frac{\left|\sigma_{r}(I) - \sigma_{r}(J)\right|}{\left|\sigma(\sigma_{r})\right|}$$

# Examples

- Using
  - Color (CIE Lab)
  - Color + XY
  - Texture (Gabor filter bank)







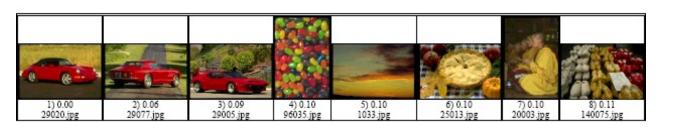
#### L1 distance

					<b>P</b>	THE REAL PROPERTY.	8
1) 0.00	2) 0.26	3) 0.43	4) 0.61	5) 0.72	6) 0.73	7) 0.75	8) 0.77
29020.jpg	29077.jpg	29017.jpg	29005.jpg	197037.jpg	77047.jpg	197097.jpg	20003.jpg

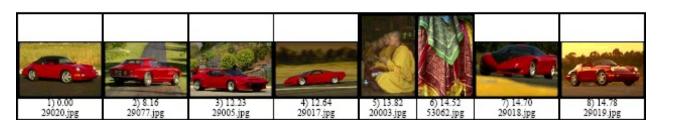
Jeffrey divergence



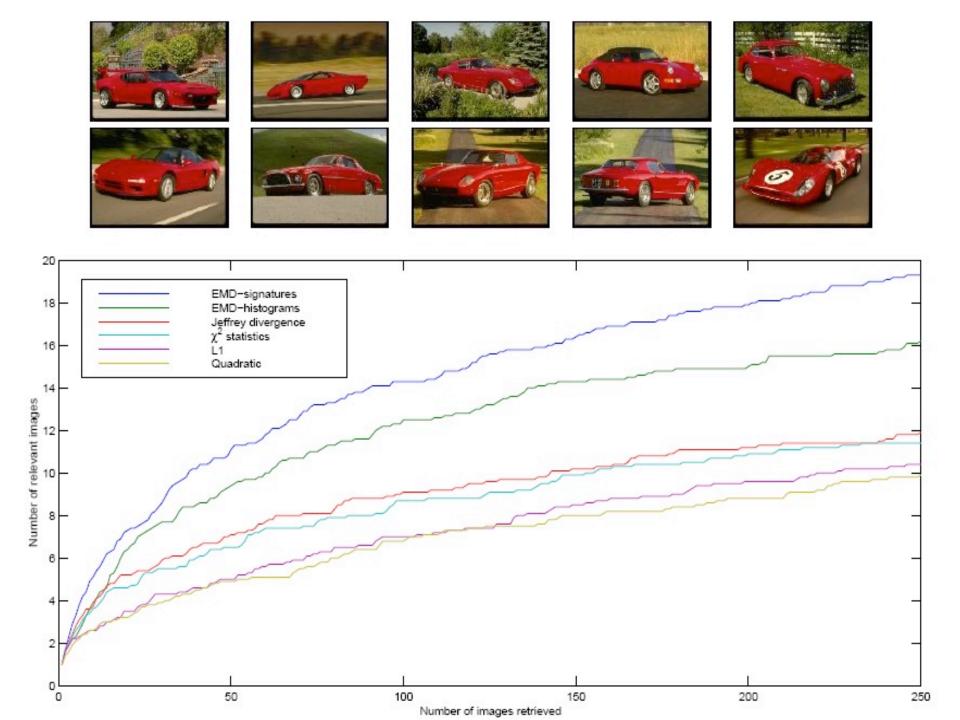
 $\chi^2$  statistics



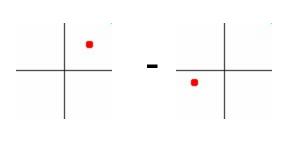
Quadratic form distance

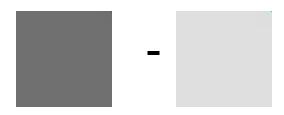


Earth Mover Distance

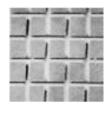


# **Concluding thought**





= it depends on the application





# **Data Matrix and Dissimilarity Matrix**

#### Data matrix

- n data points with p dimensions
- Two modes

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

#### Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix
- Single mode

$$\begin{bmatrix} 0 & & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

# **Proximity Measure for Nominal Attributes**

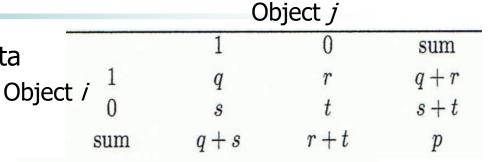
- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - m: # of matches, p: total # of variables

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the M nominal states

# **Proximity Measure for Binary Attributes**

- A contingency table for binary data
- Distance measure for symmetric binary variables:
- Distance measure for asymmetric binary variables:
- Jaccard coefficient (similarity)
  measure for asymmetric binary
  variables):



$$d(i,j) = \frac{r+s}{q+r+s+t}$$

$$d(i,j) = \frac{r+s}{q+r+s}$$

$$sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$$

Note: Jaccard coefficient is the same as "coherence":

$$coherence(i,j) = \frac{sup(i,j)}{sup(i) + sup(j) - sup(i,j)} = \frac{q}{(q+r) + (q+s) - q}$$

# Dissimilarity between Binary Variables

#### Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N 🝮	N	N 🕡
Mary	F	Y	N	P	N	P -	N
Jim	M	Y	P -	N -	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

# **Standardizing Numeric Data**

• Z-score: 
$$z = \frac{x - \mu}{\sigma}$$

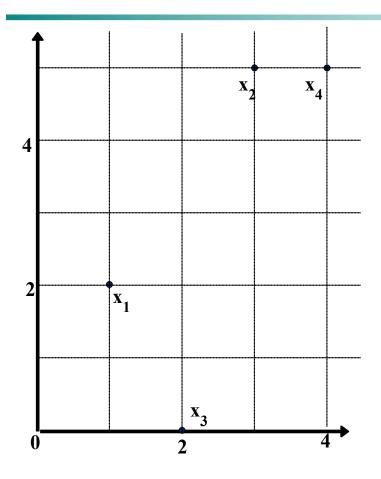
- X: raw score to be standardized, μ: mean of the population, σ: standard deviation
- the distance between the raw score and the population mean in units of the standard deviation
- negative when the raw score is below the mean, "+" when above
- An alternative way: Calculate the mean absolute deviation

$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$$
 where 
$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + ... + x_{nf})$$
 standardized measure (*z-score*): 
$$z_{if} = \frac{x_i - m_f}{s_f}$$

Using mean absolute deviation is more robust than using standard deviation

# **Example:**

# **Data Matrix and Dissimilarity Matrix**



#### **Data Matrix**

point	attribute1	attribute2
x1	1	2
<i>x2</i>	3	5
<i>x3</i>	2	0
<i>x4</i>	4	5

#### **Dissimilarity Matrix**

(with Euclidean Distance)

	<i>x1</i>	<i>x</i> 2	<i>x3</i>	<i>x4</i>
<i>x1</i>	0			
<i>x2</i>	3.61	0		
<i>x3</i>	2.24	5.1	0	
<i>x4</i>	4.24	1	5.39	0

#### Distance on Numeric Data: Minkowski Distance

Minkowski distance: A popular distance measure

$$d(i,j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \dots + |x_{ip} - x_{jp}|^h}$$

where  $i = (x_{i1}, x_{i2}, ..., x_{ip})$  and  $j = (x_{j1}, x_{j2}, ..., x_{jp})$  are two p-dimensional data objects, and h is the order (the distance so defined is also called L-h norm)

- Properties
  - d(i, j) > 0 if  $i \neq j$ , and d(i, i) = 0 (Positive definiteness)
  - d(i, j) = d(j, i) (Symmetry)
  - $d(i, j) \le d(i, k) + d(k, j)$  (Triangle Inequality)
- A distance that satisfies these properties is a metric

# Special Cases of Minkowski Distance

- h = 1: Manhattan (city block, L<sub>1</sub> norm) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + ... + |x_{i_p} - x_{j_p}|$$

• h = 2: (L<sub>2</sub> norm) Euclidean distance

$$d(i,j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + ... + |x_{ip} - x_{jp}|^2)}$$

- $h \to \infty$ . "supremum" ( $L_{max}$  norm,  $L_{\infty}$  norm) distance.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{h \to \infty} \left( \sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_{f} |x_{if} - x_{jf}|$$

# **Example: Minkowski Distance**

#### **Dissimilarity Matrices**

point	attribute 1	attribute 2
<b>x1</b>	1	2
<b>x2</b>	3	5
х3	2	0
x4	4	5

	AU	2			U		
	<b>x4</b>		4		5		
			-				
1							
					X,	X <sub>4</sub>	
					4	4	
4							
	***************************************						
2					••••••		
		<b>X</b> <sub>1</sub>					

# Manhattan (L<sub>1</sub>)

L	<b>x1</b>	<b>x2</b>	х3	x4
<b>x</b> 1	0			
<b>x2</b>	5	0		
х3	3	6	0	
<b>x4</b>	6	1	7	0

#### **Euclidean (L<sub>2</sub>)**

L2	<b>x1</b>	<b>x2</b>	х3	<b>x4</b>
<b>x1</b>	0			
<b>x2</b>	3.61	0		
х3	2.24	5.1	0	
<b>x4</b>	4.24	1	5.39	0

#### **Supremum**

$L_{\infty}$	<b>x</b> 1	<b>x2</b>	х3	<b>x4</b>
<b>x1</b>	0			
<b>x2</b>	3	0		
х3	2	5	0	
<b>x</b> 4	3	1	5	0

#### Ordinal Variables

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- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace  $x_{if}$  by their rank  $r_{if} \in \{1,...,M_f\}$
  - map the range of each variable onto [0, 1] by replacing
     i-th object in the f-th variable by

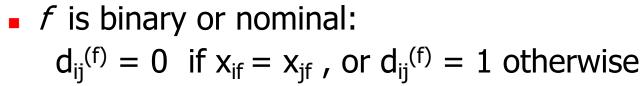
$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

 compute the dissimilarity using methods for intervalscaled variables

# **Attributes of Mixed Type**

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$



- f is numeric: use the normalized distance
- f is ordinal
  - Compute ranks r<sub>if</sub> and
  - Treat z<sub>if</sub> as interval-scaled

$$Z_{if} = \frac{r_{if} - 1}{M_f - 1}$$



# no Osition Cosine Similarity

 A document can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	team	coach	hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If  $d_1$  and  $d_2$  are two vectors (e.g., term-frequency vectors), then

$$cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1|| ||d_2||,$$

where  $\bullet$  indicates vector dot product, ||d||: the length of vector d

# **Example: Cosine Similarity**

- $cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1|| ||d_2||$ , where • indicates vector dot product, ||d|: the length of vector d
- Ex: Find the similarity between documents 1 and 2.

$$d_{1} = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$$

$$d_{2} = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$$

$$d_{1} \bullet d_{2} = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25$$

$$||d_{1}|| = (5*5+0*0+3*3+0*0+2*2+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5}$$

$$= 6.481$$

$$||d_{2}|| = (3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5} = (17)^{0.5}$$

$$= 4.12$$

$$\cos(d_{1}, d_{2}) = 0.94$$

# **Chapter 2: Getting to Know Your Data**

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary



# Summary

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratioscaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing.
- Many methods have been developed but still an active area of research.

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