人力资源分配问题

问题描述

某项目包含 n 个子任务，记完成第 i 个子任务所需投入的工时为 D_i（人*天），任意时刻投入第i个子任务的人数都不得超过 M_i人，共有劳动力X人，如何分配可使完成所有任务的总时间T（天）最短，T是多少？$(X, T \in R)$

思路一

尽量使每人每天都有活儿干即可。记t时刻分配到第i个任务的劳动力为$S_i(t)$，约束条件：$\sum_{i=1}^{n} S_i(t) \le X$ 且 $0 \le S_i(t) \le M_i$。令$\int_0^{T_i} S_i(t)dt = D_i$，则 $T = max(T_i)$ (i=1,...,n)

目标： min T

若$X \in R$，在无约束条件下，以所有子任务同时完工为目标，可使$T = \sum_{i=1}^{n} D_i / X$,显然$S_i \equiv \frac{D_i}{\sum_{i=1}^{n} Di} X$；在约束条件下，若$\frac{D_i}{\sum_{i=1}^{n} Di} X > M_i$，$S_i = M_i$，最终完工时间为$T = max(\frac{D_i}{S_i})$.

```
X=randi(30); D=ceil(abs(randn(1,25)*5+5)); M=randi([1,6],1,25);
S=D/sum(D)*X;
S(S>M)=M(S>M);
T=max(D./S)
```

```
T = 13
```

```
%用规划求解器验证结果
d=size(D);
[~,~,T2]=fminimax(@(x) D'./x,ones(d)',ones(d),X,[],[],zeros(d),M')
```

```
Local minimum possible. Constraints satisfied.

fminimax stopped because the size of the current search direction is less than
twice the default value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
T2 = 13.0000
```

思路二 A greedy method

Greedy is not correct! A counter example: D=[6,1], M=[3,1], X=3

记t时刻分配到第i个任务的劳动力为$S_i(t)$，$\sum_{i=1}^{n} S_i(t) \le X$且$0 \le S_i(t) \le M_i$

对子任务i=1,...,n， 按降序排列 $\frac{D_i}{M_i}$ 及其相应的子任务i，记排列后的任务为i=1,...,n。排列后，从i=1到i=n依次分配劳力，每次都尽可能多的分配劳力给每个子任务。某子任务完成后，空闲出的劳力继续分配给后续任务。

At time t=0, find the first k s.t $X \le \sum_{i=1}^{k} M_i$, the first k sequences would then run first. for i=1,...,k-1 $S_i = M_i$, $S_k = X - \sum_{i=1}^{k-1} S_i$, at time $t = t + min(\frac{D_1}{S_1}, ..., \frac{D_k}{S_k})$，update as follow:

1. find task j which has $min(\frac{D_1}{S_1}, ..., \frac{D_k}{S_k})$,

2. transfer out S_j workers into S_k first, S_{k+1} second and so on, till the first p s.t. $S_{k+p} = X - \sum_{i=1}^{k+p-1} S_i$,

3. set k=k+p then

Repeat until all tasks are done