CS325_HW1

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1.

After simplification, we can just compute the function $n = 8\log_2(n)$, and we got the two results,
$n \approx 1.1$
$n \approx 43.5593$

And we found that while n is bigger than 1.1 but smaller than 43.5593, the insertion running time is smaller than merge, which is faster. So, the answer is [1.1 – 43.5593].

2.

a. O(g(n)), because the limit as n goes to infinity of f(n) / g(n) is 0.

b. Ω(g(n)), because the limit as n goes to infinity of f(n) / g(n) is infinity.

c. Θ(g(n)), because the limit as n goes to infinity of f(n) / g(n) is a constant.

d. Θ(g(n)), because the limit as n goes to infinity of f(n) / g(n) is a constant.

e. O(g(n)), because the limit as n goes to infinity of f(n) / g(n) is 0.

f. O(g(n)), because the limit as n goes to infinity of f(n) / g(n) is 0.

g. Θ(g(n)), because the limit as n goes to infinity of f(n) / g(n) is a constant.

h. O(g(n)), because the limit as n goes to infinity of f(n) / g(n) is 0.

i. Ω(g(n)), because the limit as n goes to infinity of f(n) / g(n) is infinity.

j. O(g(n)), because the limit as n goes to infinity of f(n) / g(n) is 0.

3.

a. Proves, because f1(n) =Θ (g(n)) so, f1(n)=c1g(n) and f2(n)= Θ (g(n))=c2g(n) we can get f1(n)/f2(n) is equally a Constant c so f1(n)= Θf2(n) is prove

b. disproves, if f1(n)=f2(n)=2^n g1(n)=g2(n)=20^n so, lim n->infinity, (f1(n)+f2(n))/(g1(n)+g2(n)) is 0. It’s O(g(n)) instead of Θ(g(n)), there is a contradiction.

4.

See teach files

5.

a.

insertTime.py

```python
from random import randint
from time import time

MIN_INT = 0
MAX_INT = 10000

def insertsort(array):
    #go through each element in the array
    for j in range (1,len(array)):
        key = array[j];
        i = j - 1
        while i >= 0 and array[i] > key:
            array[i+1] = array[i]
            i = i -1
        array[i+1] = key
    return array

if __name__ == "__main__":
    begin = 10000
    increment = 5000
    values = [begin + i * increment for i in range(7)]
    list   = [[randint(MIN_INT,MAX_INT) for _ in range(value)] for value in values]

    print ("value          running time")

    for array in list:
        array_size = len(array)
```

```
        start_time = time()

        insertsort(array)

        print("{}           {}".format(array_size,time()-start_time))
```

mergeTime.py

```
from random import randint
from time import time

MIN_INT = 0
MAX_INT = 10000

def merge(left,right):
    #create empty array to hold sorted values
    array = []
    #keep merge until one side has no element left
    while len(left) != 0 and len(right) != 0:
        #if the first element of left array is smaller merge it into empty array
        #and remove the merged element
        if left[0] < right[0]:
            array.append(left[0])
            left.remove(left[0])
        # the situation that is equal or bigger than right side
        else:
            array.append(right[0])
            right.remove(right[0])
```

```
    #if the left side array is empty, then append the right side
    if len(left) == 0:
        array += right;
    else:
        array += left;
    return array
#define mergesort algorithm
def mergesort(array):
    array_size = len(array)

    if array_size <= 1:
        return array
    else:
        array_split = array_size//2
        left_array = mergesort(array[:array_split])
        right_array = mergesort(array[array_split:])
        return merge(left_array,right_array)

if __name__ == "__main__":
    begin = 10000
    increment = 5000
    values = [begin + i * increment for i in range(7)]
    list   = [[randint(MIN_INT,MAX_INT) for _ in range(value)] for value in values]

    print ("value           running time")

    for array in list:
        array_size = len(array)
```

```
start_time = time()

mergesort(array)

print("{}          {}".format(array_size,time()-start_time))
```

b.

Here is the table result from my algorithms, I took three tables and got the average running time.

insertTime table

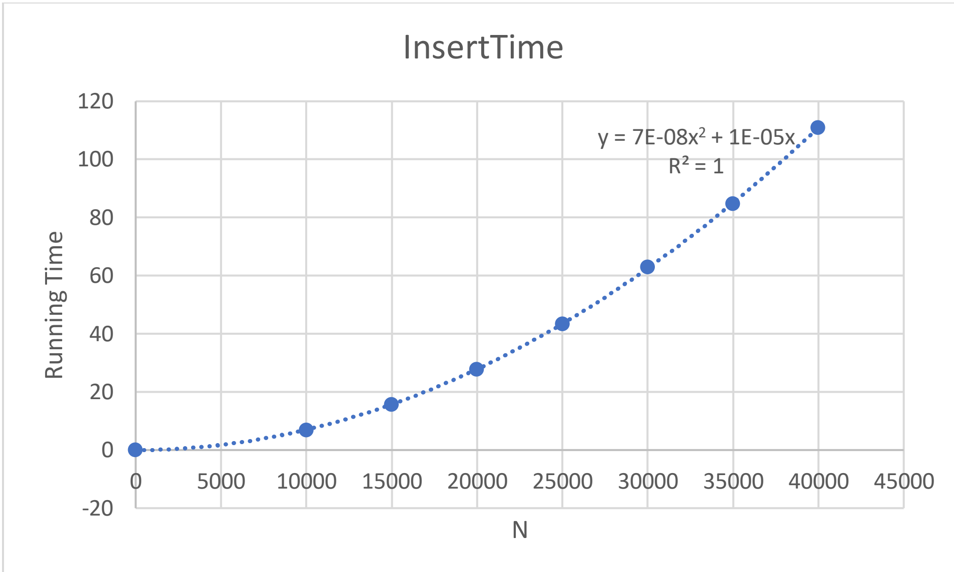
N	averageT
0	0
10000	6.9
15000	15.69
20000	27.78
25000	43.33
30000	62.93
35000	84.89
40000	110.91

mergeTime table

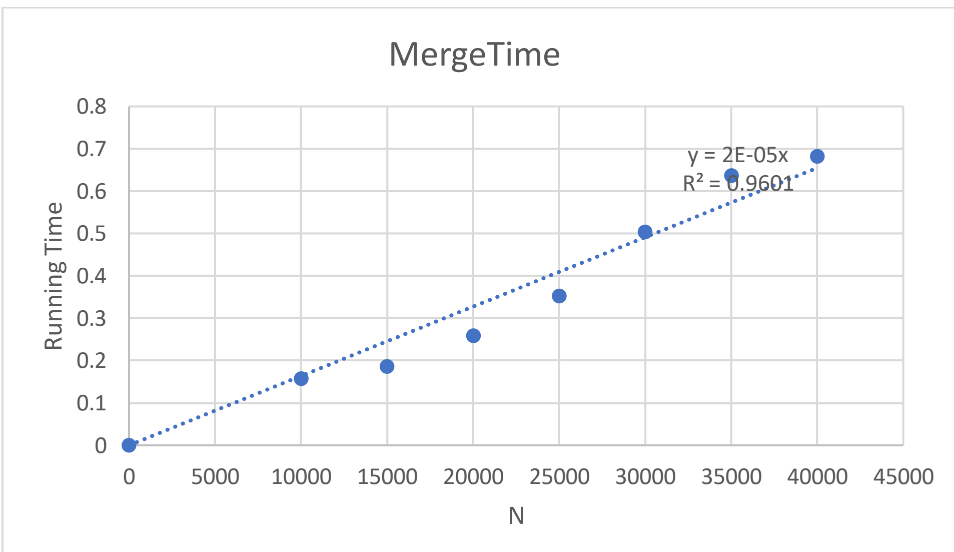
N	averageT
0	0
10000	0.1578
15000	0.1857
20000	0.2593
25000	0.3531
30000	0.5042
35000	0.6367
40000	0.682

c.

I used the polynomial curve to fit the insertTime graph, r^2 is 1, which means it fits perfectly.

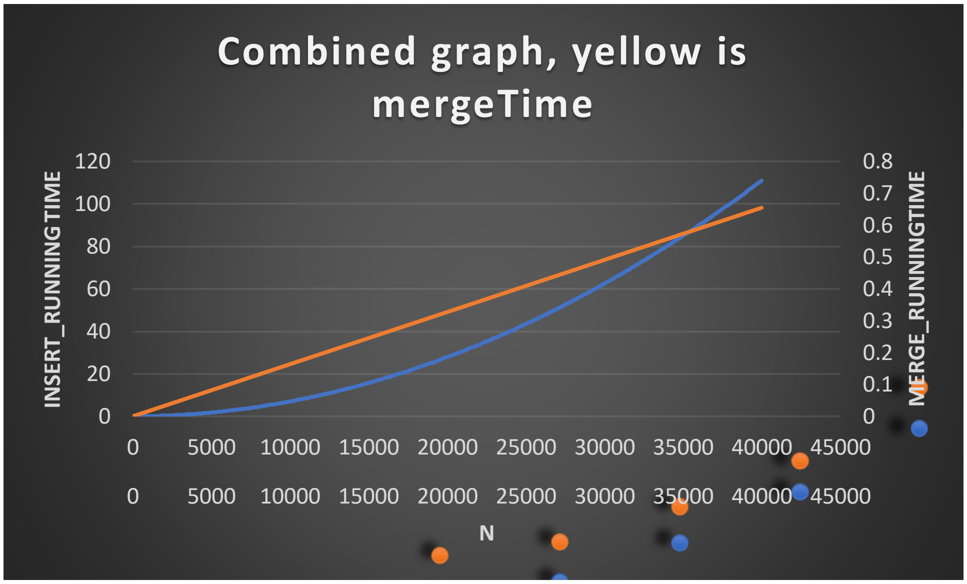


And I used linear curve to fit the mergeTime graph, r^2 is 0.9601, which is also considered as extreme good.



d.

The combined graph, the yellow line is mergeTime, the blue one is insertTime, and there are two Y-aixs.



e.

For the Insert Sort algorithm, when using the worst-case input, the experimental running times appeared to be very close to the theoretical worst case, O(n^2), as expected. The curve fit for the above graph points to an equation of the f(n) = n^2 nature. When the best-case input was use, the experimental running times appeared to be very close to the theoretical best case, O(n), as expected.

For the Merge Sort algorithm, when using the worst-case input, the experimental running times appeared to be very close to O(n lgn), as expected. There technically isn't a worst case for the Merge Sort algorithm, but it was discovered an array setup that would be the hardest to sort. See link at the bottom of mergesort_worst.py The curve fit for the above graph points to an equation of the f(n) = n nature. When the best-case input was use, the experimental running times appeared to be very close to O(n lgn), as expected. Again, there technically isn't a best case for the Merge Sort algorithm but the results of this "best case" were faster than the "worst case".