

第五章 运算方法与运算器(一)

秦磊华 计算机学院

本章主要内容



基于补码数据表示研究运算方法和设计运算器(简)

- 5.1 定点加法运算
- 5.2 加法运算器设计
- 5.3 定点减法运算
- 5.4 定点减法运算器设计





1.补码加法运算方法

$$[X + Y]_{\lambda h} = [X]_{\lambda h} + [Y]_{\lambda h}$$

- 1) 理解意义
- 2) 公式证明思路及可用的方法

$$[X]_{\frac{1}{n}} = \begin{cases} X & 0 \le X < 2^{n} \\ 2^{n+1} + X & -2^{n} \le X < 0 \end{cases}$$

$$[X]_{\frac{1}{n}} = \begin{cases} X & 0 \le X < 1 \end{cases}$$

$$[X]_{\frac{1}{n}} = \begin{cases} X & 0 \le X < 1 \end{cases}$$

$$[X]_{\frac{1}{n}} = \begin{cases} 2 + X & -1 \le X < 0 \end{cases}$$



$$[X + Y]_{\stackrel{?}{\uparrow}h} = [X]_{\stackrel{?}{\uparrow}h} + [Y]_{\stackrel{?}{\uparrow}h}$$

- 1) X >0, Y >0 (可直接理解)
- 2) X > 0, Y < 0
- 3) X < 0, Y > 0 (2/3证明相同)
- 4) X < 0, Y < 0



1) X > 0, Y > 0

因为: x+y>0

所以: $x + y = [x + y]_{i}$ (mod 2^{n+1})



2) X > 0, Y < 0

所以:
$$[x]_{N} = x$$
 $[y]_{N} = 2 + y$ $[x]_{N} + [y]_{N} = x + 2 + y$ $= 2 + (x + y)$ 当 $x + y < 0$ 时 $2 + (x + y) = [x + y]_{N}$ (mod 2) 当 $x + y > 0$ 时 $2 + (x + y) > 2$ 模含去 $[x]_{N} + [y]_{N} = 2 + (x + y) = x + y$ (mod 2) $= [x + y]_{N}$ (mod 2)



4)
$$X < 0$$
, $Y < 0$

$$[x]_{}^{}$$
 = 2 + x $[y]_{}^{}$ = 2 + y
 $[x]_{}^{}$ + $[y]_{}^{}$ = (2 + x) + (2 + y)
 $= 4 + x + y$
 $= 2 + (2 + x + y) \mod 2$
 $-2 \le x + y < 0$
故 $0 \le 2 + x + y < 2$
故 $2 + (2 + x + y) \mod 2 = (2 + x + y)$
 $= [x + y]_{}^{}$ $\mod 2$

7



例1 已知 X = 0.10101, Y = 0.01000, 求 X+Y

解: $[X]_{\stackrel{?}{N}} = 0.10101$, $[Y]_{\stackrel{?}{N}} = 0.01000$

X+Y = 0.11101



例2 已知 X = -10111, Y =-1000, 求X+Y

解: $[X]_{\stackrel{?}{=}} = 101001$, $[Y]_{\stackrel{?}{=}} = 11000$

$$X+Y = -111111$$



2.补码减法运算方法

$$[X-Y]_{\dot{\uparrow}\dot{\uparrow}} = [X]_{\dot{\uparrow}\dot{\uparrow}} - [Y]_{\dot{\uparrow}\dot{\uparrow}} = [X]_{\dot{\uparrow}\dot{\uparrow}} + [-Y]_{\dot{\uparrow}\dot{\uparrow}}$$

- 1) 理解意义
- 2) 公式证明思路及可用的方法

$$[X-Y]_{N}^{*} = [X+(-Y)]_{N}^{*}$$

$$= [X]_{N}^{*} + [-Y]_{N}^{*} !$$

$$[Y-Y]_{N}^{*} = [Y]_{N}^{*} + [-Y]_{N}^{*}$$

$$[Y]_{N}^{*} + [-Y]_{N}^{*} = 0$$



2.补码减法运算方法

$$[X-Y]_{\grave{\imath} \backprime} = [X]_{\grave{\imath} \backprime} - [Y]_{\grave{\imath} \backprime} = [X]_{\grave{\imath} \backprime} + [-Y]_{\grave{\imath} \backprime}$$

例3 已知
$$[X]_{\stackrel{}{N}} = 101001$$
,求 $[-X]_{\stackrel{}{N}}$
由 $[X]_{\stackrel{}{N}} = 101001$
 $\longrightarrow X = -10111$
 $\longrightarrow -X = 10111$
 $\longrightarrow [-X]_{\stackrel{}{N}} = 010111$
 $[X]_{\stackrel{}{N}} = 101001$

11



例4 已知 X = -10111, Y =-1000, 求X - Y

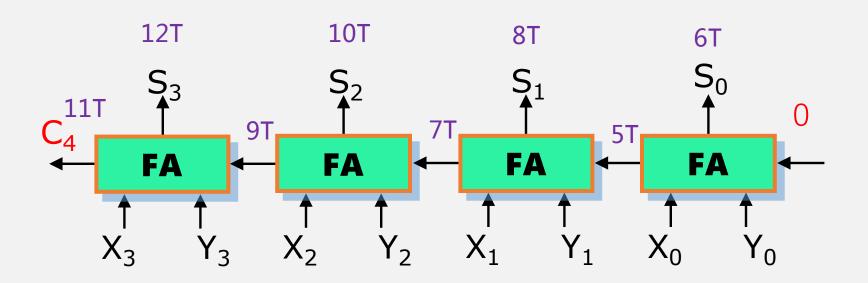
解: $[X]_{\stackrel{}{N}} = 101001$, $[Y]_{\stackrel{}{N}} = 11000$ $[-Y]_{\stackrel{}{N}} = 01000$

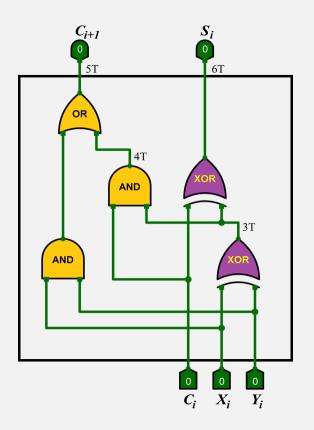
1 0 1 0 0 1 + 0 0 1 0 0 0 0 0 1 0 0 0 1 1 0 0 0 1

X-Y = -1111



3. 基本补码加法运算器设计

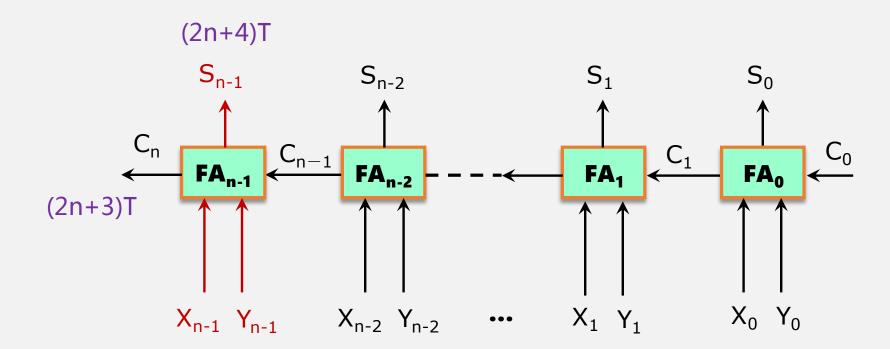




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3. 基本补码加法运算器设计



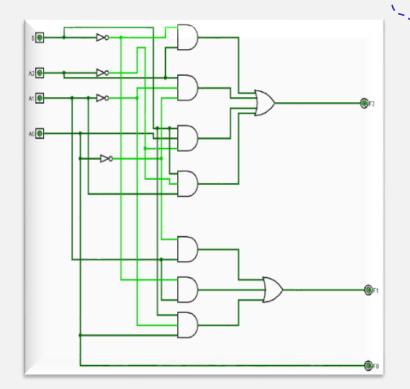
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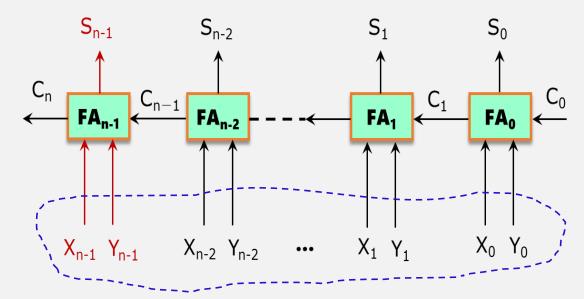


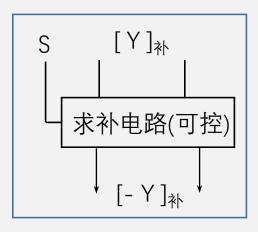
3. 基本补码减法运算器设计

$$[X + Y]_{\dot{\gamma}h} = [X]_{\dot{\gamma}h} + [Y]_{\dot{\gamma}h}$$

$$[X - Y]_{\dot{\gamma}h} = [X]_{\dot{\gamma}h} - [Y]_{\dot{\gamma}h} = [X]_{\dot{\gamma}h} + [-Y]_{\dot{\gamma}h}$$





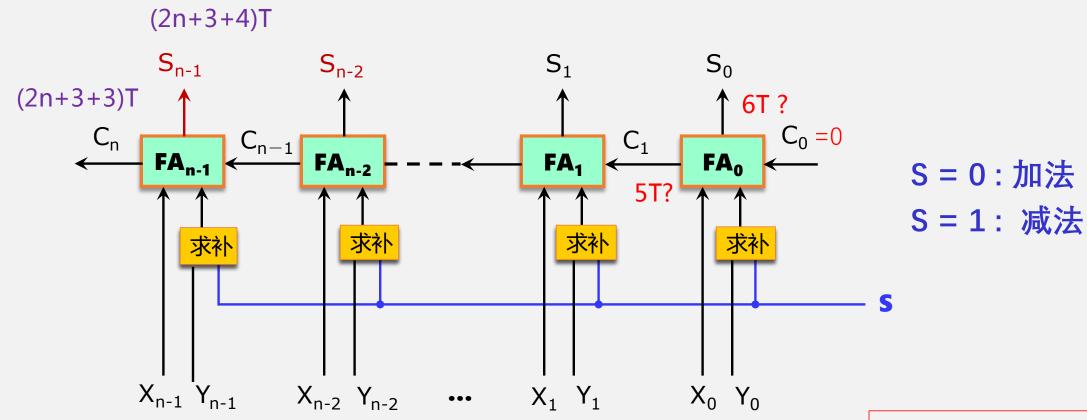


计算机组成原理

3T



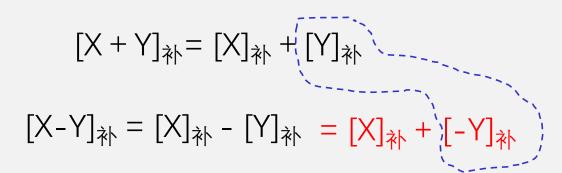
3. 基本补码减法运算器设计

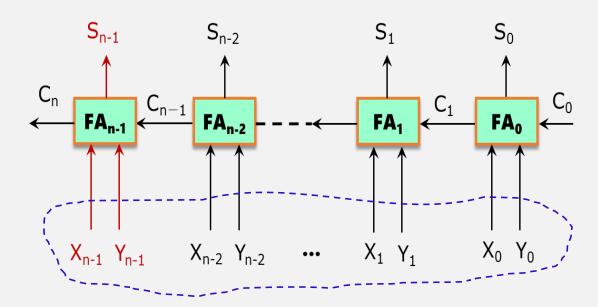


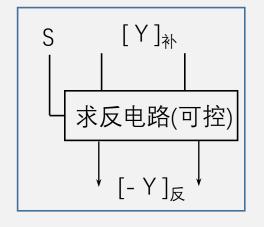
减法不满足交换律



3. 基本补码减法运算器设计









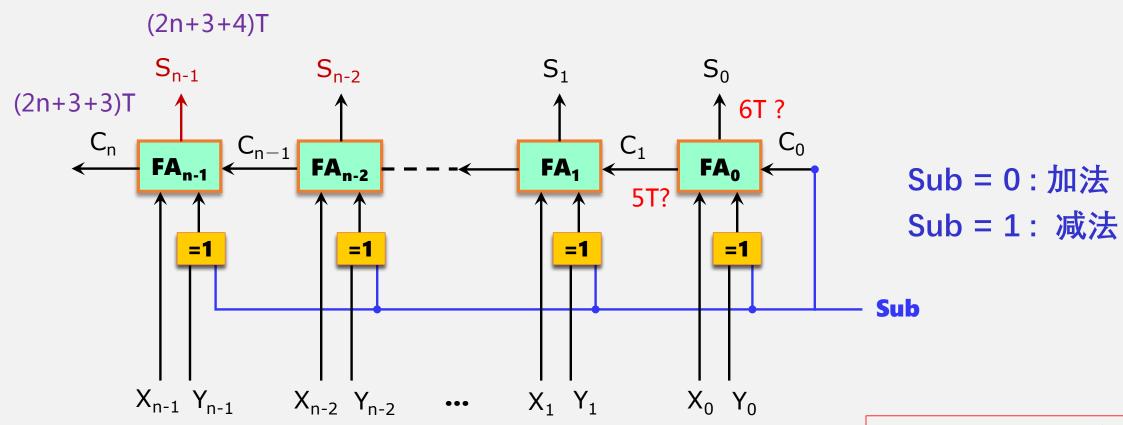
1101101 ① 1111111 0010010

1101101 ① 0000000 1101101

计算机组成原理



3. 基本补码减法运算器设计



减法不满足交换律

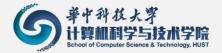


4. 补码加减法运算溢出

例5 已知 X = -11011, Y = -1000 求 X + Y

解: $[X]_{\stackrel{?}{N}} = 100101$, $[Y]_{\stackrel{?}{N}} = 11000$

X+Y = +11101



4.补码加减活运算溢出

解:
$$[X]_{\stackrel{}{N}} = 010101$$
, $[Y]_{\stackrel{}{N}} = 01000$

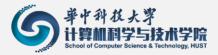
$$X+Y = -10011$$



4.补码加减法运算溢出

溢出: 运算结果超出对应数据类型和机器字长的数据表示范围

2



5.补码加减法运算溢出检测

- ◆根据概念,溢出只发生在同号数相加时,包括[X]_补与[Y]_补,[X]_补与[-Y]同号
 - 1) 方法1:对操作数和运算结果的符号位进行检测 当结果的符号与操作数的符号不同时表明发生了溢出

0 1 0 1 0 1 + 0 1 1 0 0 0 1 0 1 1 0 1 1 0 0 1 0 1 + 1 1 1 0 0 0 1 0 1 1 1 0 1

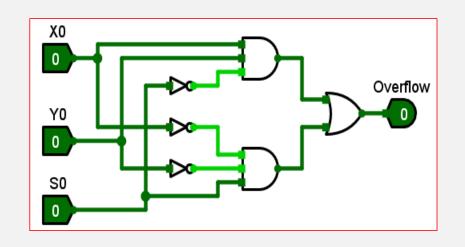


5.补码加减法运算溢出检测

设Xn, Yn 为参加运算数的符号位, Sn 为结果符号位

Overflow =
$$X_n Y_n S_n + \overline{X}_n Y_n S_n$$

当Overflow=1时,结果溢出,根据逻辑表达式,得到相应电路



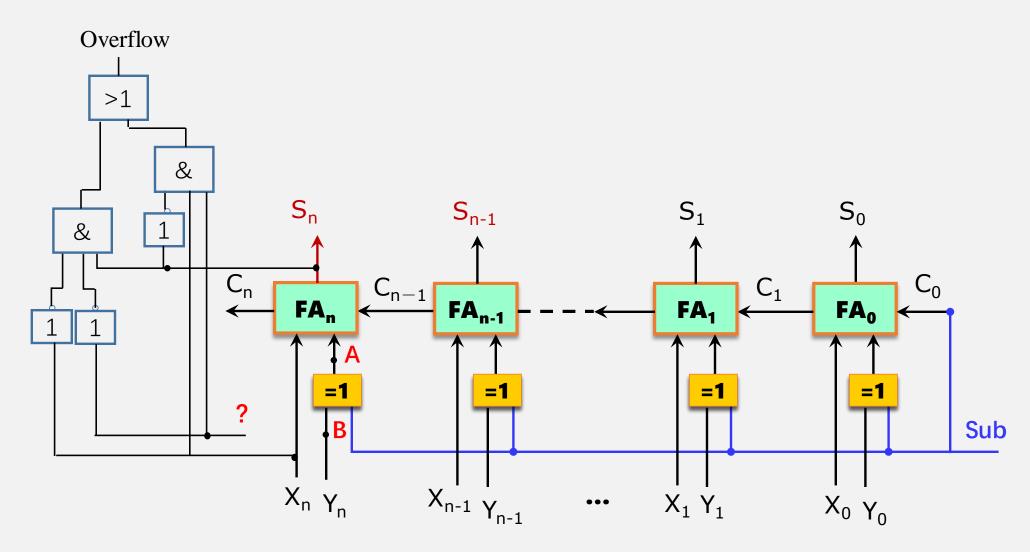
代价分析:

6个逻辑门, 3级时延



5.补码加减法运算溢出检测

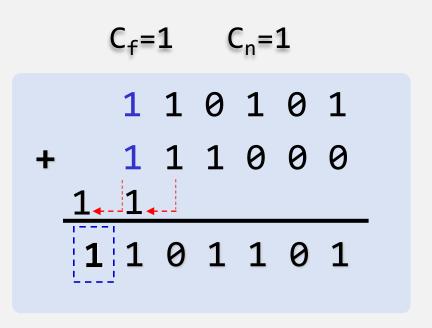
Overflow =
$$X_n Y_n \overline{S}_n + \overline{X}_n \overline{Y}_n S_n$$





5.补码加减法运算溢出检测

2) 方法2:通过运算中最高位数据的进位与符号位的进位是否同步来检测 假**符号位进位**为Cf,**最高数据位进位**为Cn





5.补码加减法运算溢出检测

2) 方法2:通过运算中最高位数据的进位与符号位的进位是否同步来检测设符号位进位为Cf,最高数据位进位为Cn

$$C_f = 0$$
 $C_n = 1$

$$C_f = 1$$
 $C_n = 0$

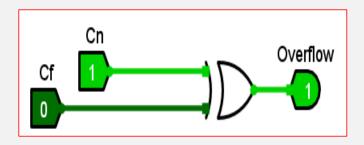
$$Overflow = C_f \oplus C_n$$



5.补码加减法运算溢出检测

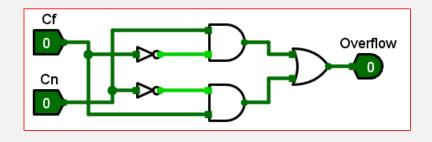
2) 方法2:通过运算中最高位数据的进位与符号位的进位是否同步来检测设符号位进位为Cf. 最高数据位进位为Cn

 $Overflow = C_f \oplus C_n$



代价分析:

1个异或逻辑门, 3T时延



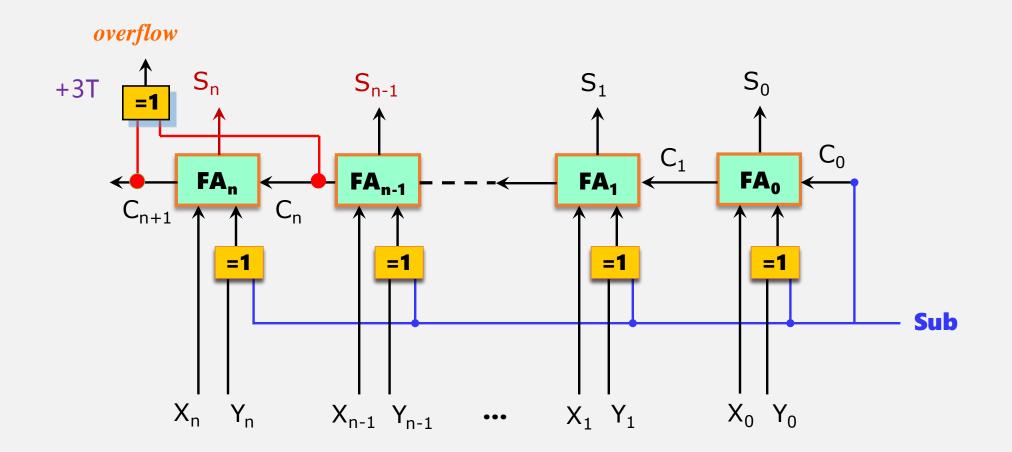
代价分析:

5个逻辑门, 3T时延



5.补码加减法运算溢出检测

方法2: $Overflow = C_f \oplus C_n$





5.补码加减活运算溢出检测

3) 方法3: 用变型补码的双符号位,设双符号位为 f_1 f_2 , 其中 f_1 为最高符号位

$$f_1 = 0$$
 $f_2 = 0$



5.补码加减活运算溢出检测

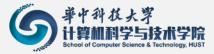
3) 方法3: 用变型补码的双符号位,设双符号位为 f_1 f_2 , 其中 f_1 为最高符号位

$$f_1=0$$
 $f_2=1$ 正溢出(上溢)

$$Overflow = f_1 \oplus f_2$$

$$f_1=1$$
 $f_2=0$ 负溢出(下溢)

第一符号位是结果的正确符号位



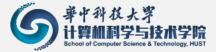
5.补码加减活运算溢出检测

3) 方法3: 用变型补码的双符号位,设双符号位为 f₁ f₂, 其中f₁为最高符号位

 $Overflow = f_1 \oplus f_2$

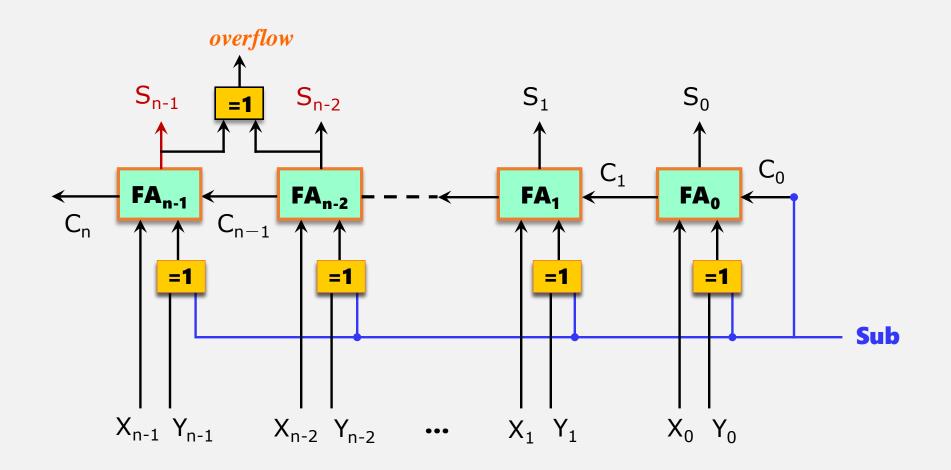
溢出检测电路和成本分析同方法2。

与方法2不同的是,需要增加了一位符号位,会影响数据表示范围吗?



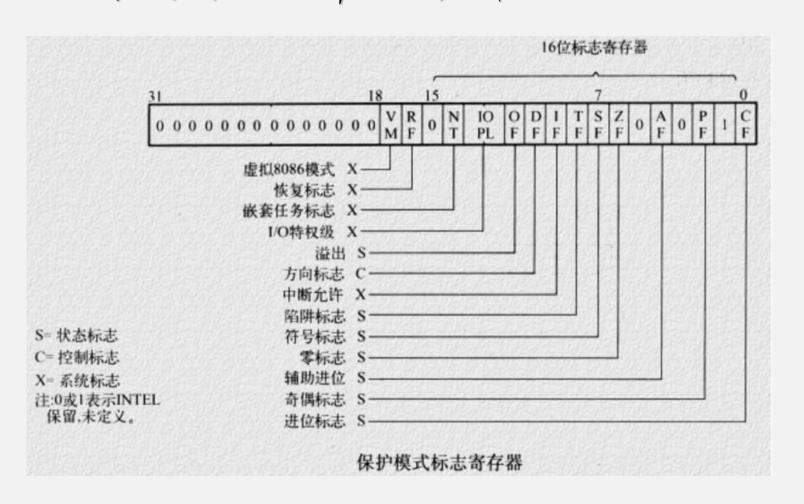
5.补码加减法运算溢出检测

方法3: $Overflow = f_1 \oplus f_2$





5.补码加减法运算溢出检测



MOVE AX, 0XFFFFF

ADD AX, BX

JO LOOP

.....

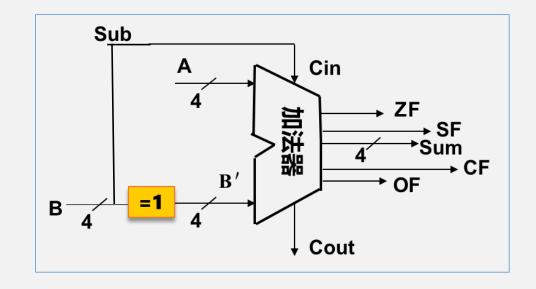
LOOP:

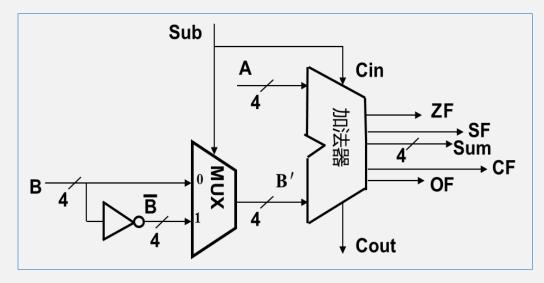
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计算机组成原理

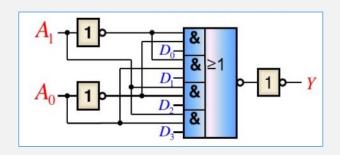


5.补码加减法运算溢出检测





比较两种方案哪种更好?为什么?



34



5.补码加减法运算溢出检测

问题: 若CPU中未设计上述溢出检测电路或未设置标志寄存器,如何检测溢出?

```
int tadd_ok(int x,int y)
{
  int sum= x+y;
  int neg_over= x<0&&y<0&&sum >=0;
  int pos_over= x>=0&&y>=0&&sum<0;
  return !neg_over&&!pos_over;
}</pre>
```

- ◆体会软硬溢出检测的逻辑等效性
- ◆ 体会软硬溢出检测的差异性
- ◆ 对你未来的程序设计有何启示?
- ◆ 这就是软硬协同的系统观



6. 无符号教加减法运算溢出检测

 $\begin{array}{c} 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\\ +\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\\ \hline 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\\ \end{array}$

加法未溢出 Cout=0

1 1 1 1 1 1 1 1 255 + 1 1 1 1 1 1 1 0 254 1 1 1 1 1 1 1 0 1 253

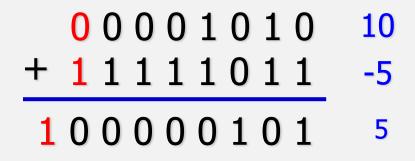
加法溢出 Cout=1 和数小于加数



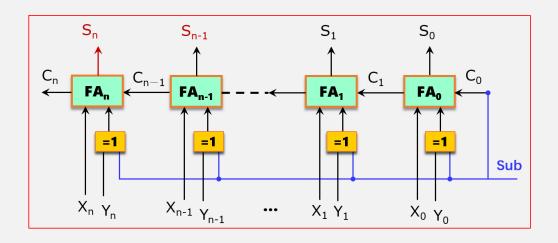
6. 无符号数加减法运算溢出检测

$$\begin{array}{c} 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\\ +\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\\ \hline 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1\\ \end{array}$$

减法溢出 Cout=0 差大于被减数



减法未溢出 Cout=1



无符号溢出检测 = SUB ⊕ Cout



6. 无符号数加减法运算溢出检测

$$\begin{array}{c} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ + & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

- ◆ 无符号数加, 65+66 = 131, 正常
- ◆ 有符号数加, 65+66=-125, 溢出
- ◆ 无符号数减, 65-190 = 131, 溢出
- ◆ 有符号数减, 65-(-66) =-125, 溢出

无符号溢出检测 = SUB ⊕ C_{out}

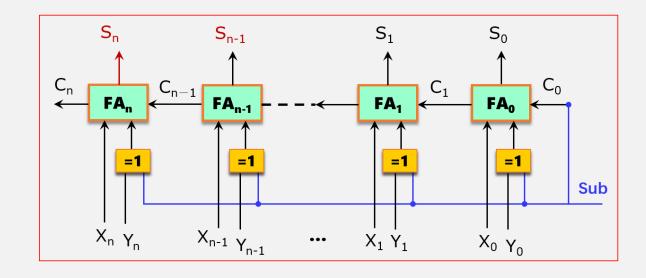


7. 快速加洁器设计

$$S_i = X_i \oplus Y_i \oplus C_i$$

$$C_{i+1} = X_i Y_i + (X_i \oplus Y_i) C_i$$

S_n时延: (2n+4+3)T

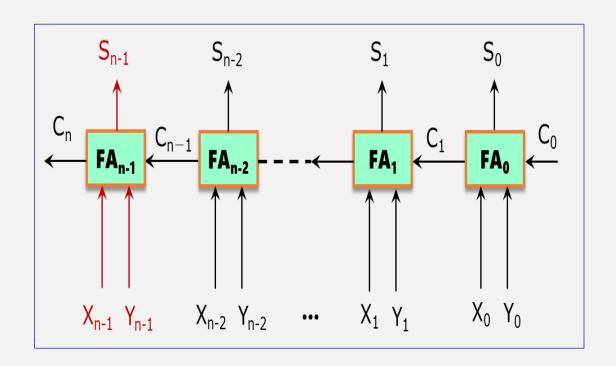


$$G_i = X_i Y_i$$
 进位生成函数 Generate

$$C_{i+1} = \mathbf{G_i} + \mathbf{P_i} C_i$$

等中科技大字 计算机科学与技术学院 School of Computer Science & Technology, HUST

7. 快速加洁器设计



高位依赖于低位进位,形成了串行进位链

$$G_{i} = X_{i}Y_{i}$$
 $P_{i} = X_{i} \oplus Y_{i}$
 $C_{1} = G_{0} + P_{0}C_{0}$
 $C_{2} = G_{1} + P_{1}C_{1}$
 $C_{3} = G_{2} + P_{2}C_{2}$

.....

 $C_{n} = G_{n-1} + P_{n-1}C_{n-1}$
 $C_{n+1} = G_{n} + P_{n}C_{n}$



 $G_i = X_i Y_i$

 $P_i = X_i \oplus Y_i$

$$C_1 = G_0 + P_0C_0$$

$$C_2 = G_1 + P_1C_1$$

$$= G_1 + P_1(G_0 + P_0C_0) = G_1 + P_1G_0 + P_1P_0C_0$$

$$C_3 = G_2 + P_2C_2$$

$$= G_2 + P_2(G_1 + P_1G_0 + P_1P_0C_0)$$

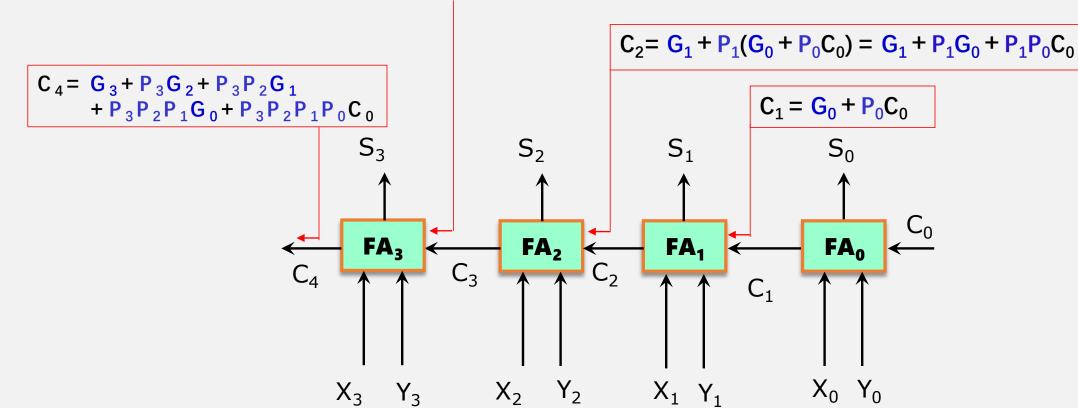
$$= G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0$$

$$C_4 = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0$$









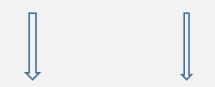


7. 快速加波器设计

$$C_4 = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0$$

$$C_{16} = G_{15} + P_{14}G_{13} + \dots + P_{15}P_{14} \dots P_1P_0C_0$$

17个输入端的与门

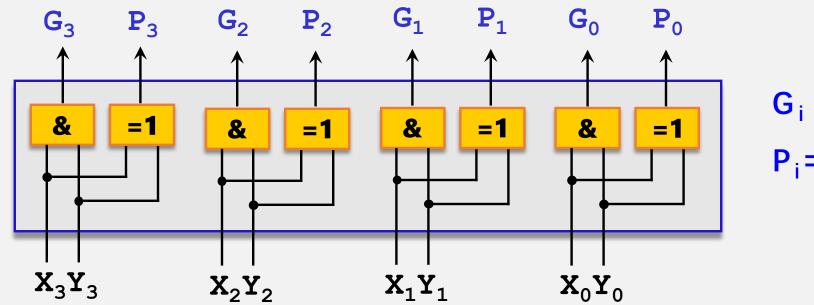


扇入系数:逻辑门的输入端数量

扇出系数:逻辑门直接连接的输出端数量

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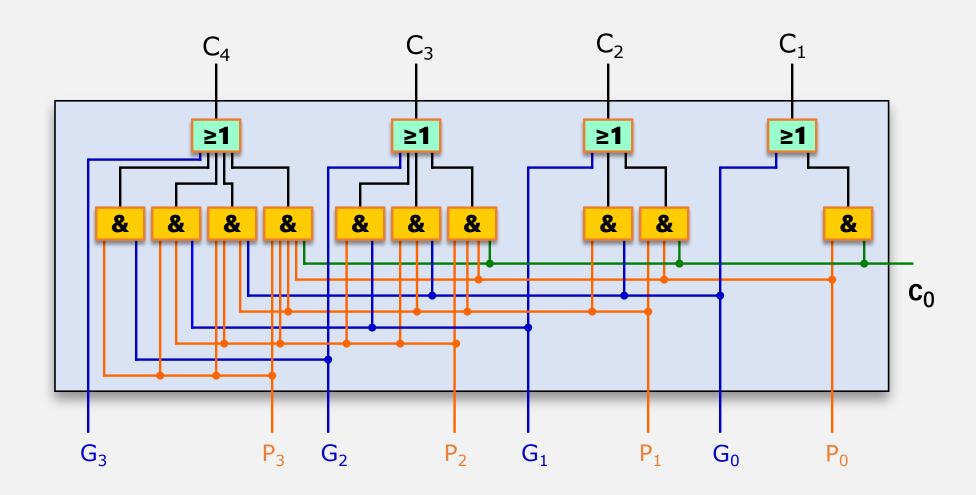
7. 快速加波器设计



$$G_i = X_i Y_i$$

 $P_i = X_i \oplus Y_i$

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$$G_{i} = X_{i}Y_{i}$$

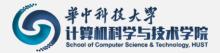
$$P_{i} = X_{i} \oplus Y_{i}$$

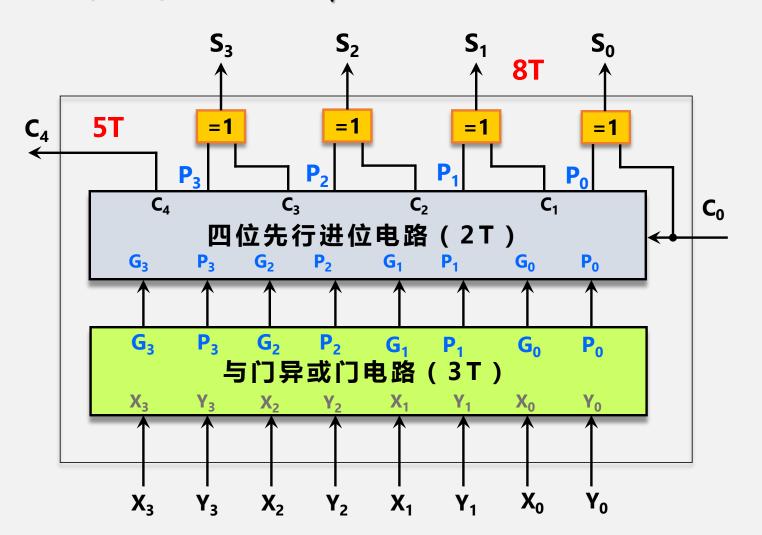
$$S_{3} = X_{3} \oplus Y_{3} \oplus C_{3} = P_{3} \oplus C_{3}$$

$$S_{2} = X_{2} \oplus Y_{2} \oplus C_{2} = P_{2} \oplus C_{2}$$

$$S_{1} = X_{1} \oplus Y_{1} \oplus C_{1} = P_{1} \oplus C_{1}$$

$$S_{0} = X_{0} \oplus Y_{0} \oplus C_{0} = P_{0} \oplus C_{0}$$

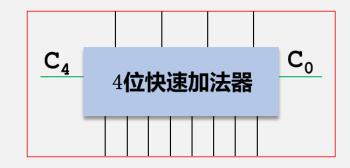




$$S_3 = X_3 \oplus Y_3 \oplus C_3 = P_3 \oplus C_3$$

$$C_3 = G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0$$

$$P_i = X_i \oplus Y_i$$
 $G_i = X_i Y_i$

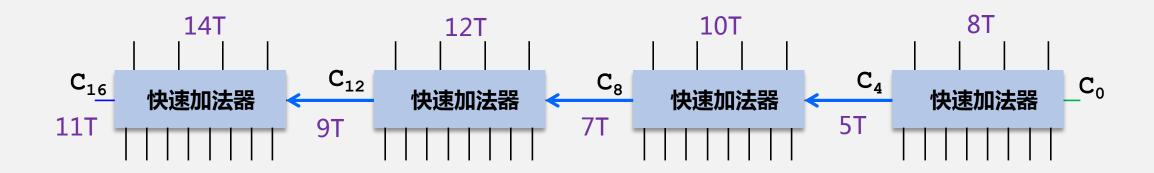




7. 快速加滤器设计



- ◆ 组内先行进位
- ◆组间? 串行进位
- ◆可否组间并行?





7. 快速加洁器设计

$$C_4 = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0$$

$$G^* = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0$$
 成组进位生成函数

$$P^* = P_3 P_2 P_1 P_0$$

成组进位传递函数

$$C_4 = G^* + P^*C_0$$

两式逻辑表达式完全一样!

$$C_1 = G_0 + P_0C_0$$

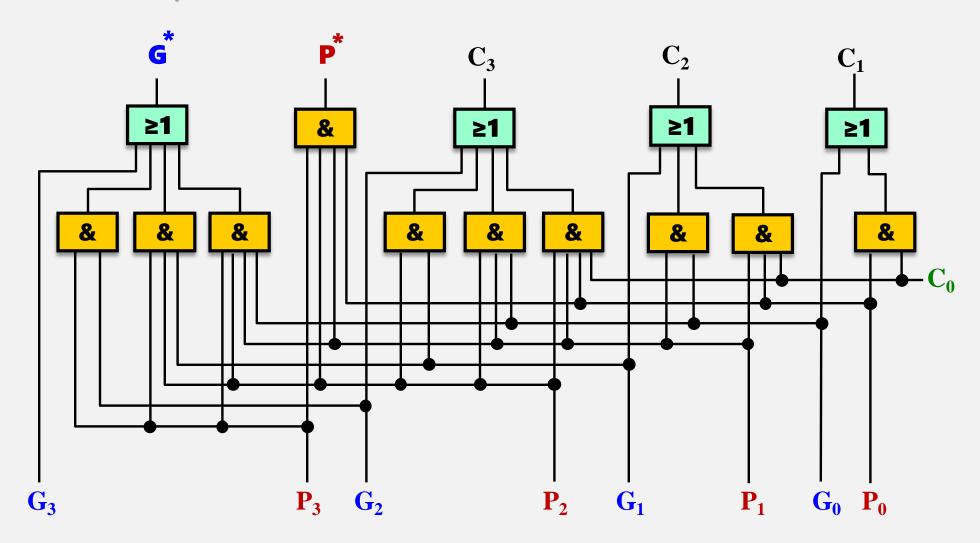
? 也应该完全一样!

同理 可得: C_8 VS C_2 C_{12} VS C_3 , C_{16} VS C_4 的表达式也一样



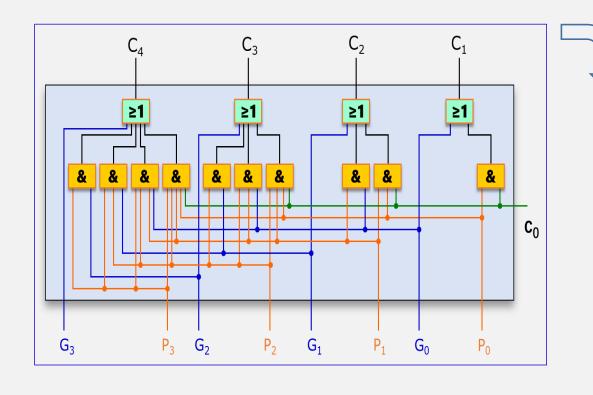
7. 快速加洁器设计

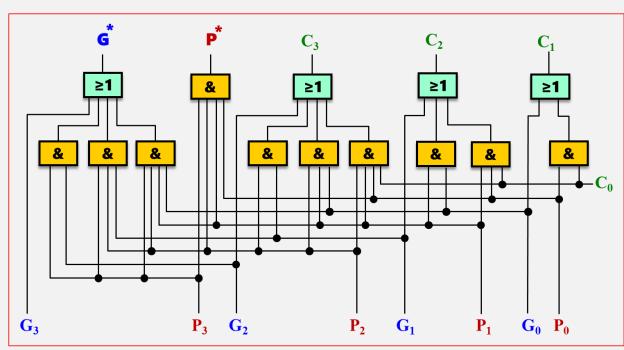
两级先行进位电路



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7. 快速加洁器设计

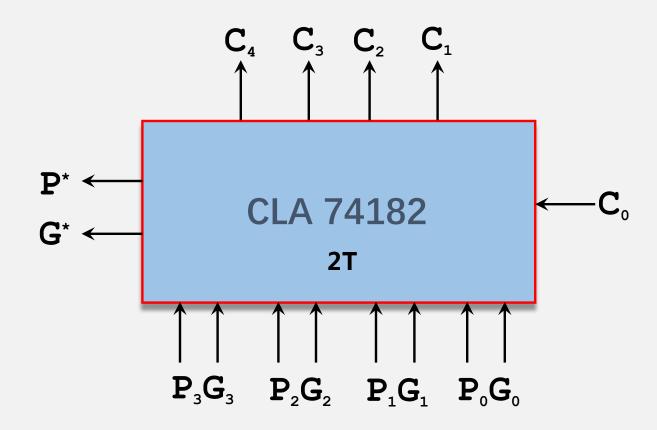




[']51

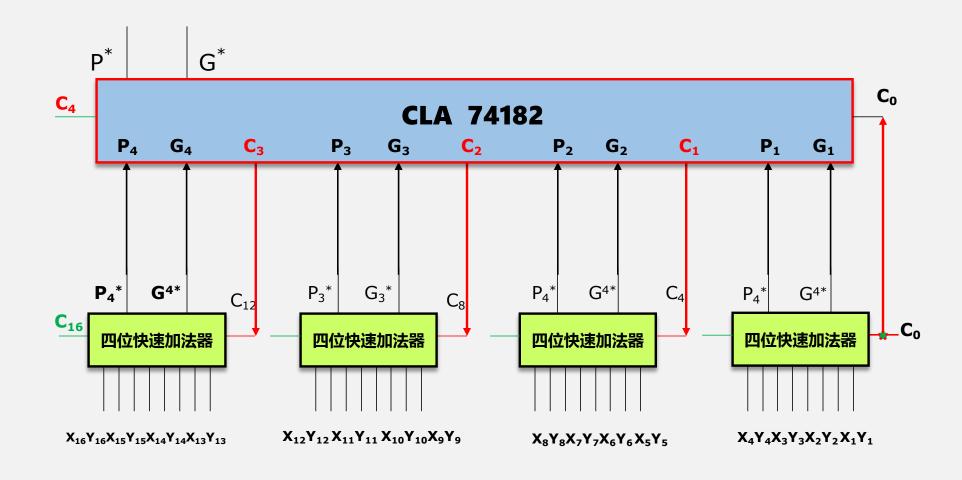


7. 快速加洁器设计



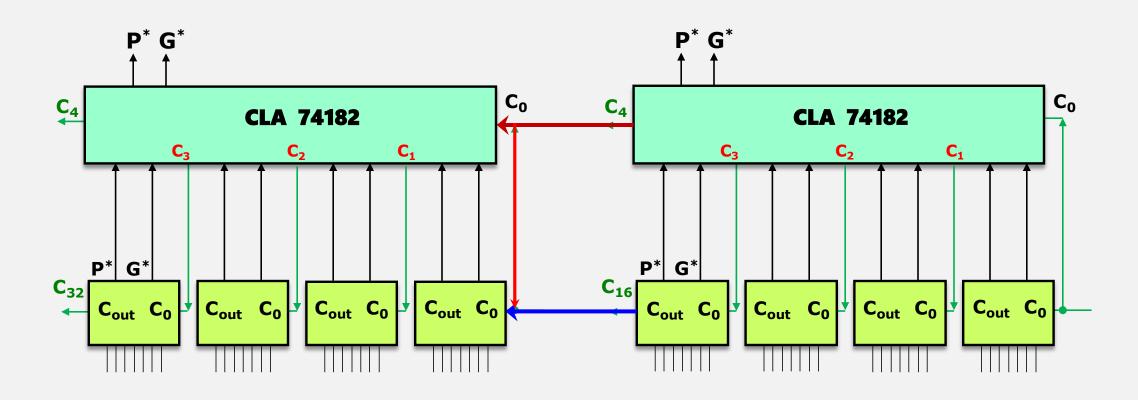


7. 快速加洁器设计





7. 快速加洁器设计

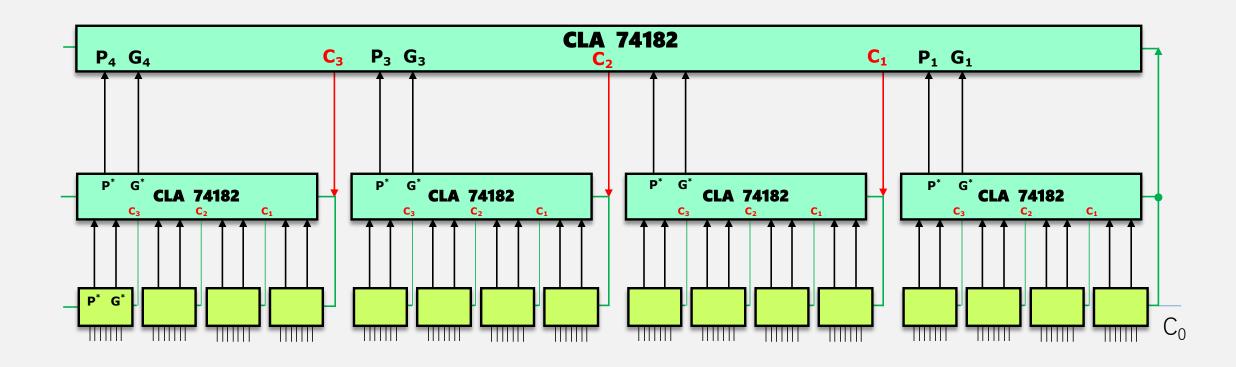


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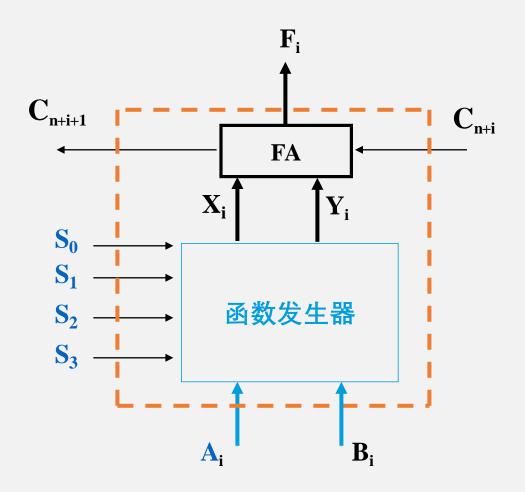
55

7. 快速加波器设计





8. 四位运算器芯片 5N74181



FA的逻辑表达式:

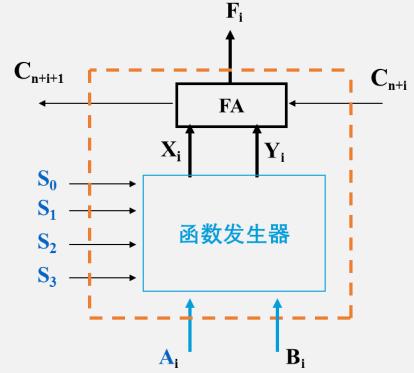
$$F_i = X_i \oplus Y_i \oplus C_{n+i}$$

$$C_{n+i+1} = X_i Y_i + Y_i C_{n+i} + X_i C_{n+1}$$



8. 四位运算器芯片 SN74181

S0 S1	Y_{i}	S2 S3	X _i
0 0	$\overline{\overline{A}}_{i}$	0 0	1
0 1	$\overline{\overline{A}}_{i}B_{i}$	0 1	$A_i + B_i$
1 0	$A_i B_i$	1 0	$\overline{\mathbf{A_i}}$ + $\mathbf{B_i}$
1 1	0	1 1	$\overline{\mathbf{A_i}}$



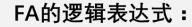
$$X_{i} = \overline{S}_{2}\overline{S}_{3} + \overline{S}_{2}S_{3}(\overline{A}_{i} + \overline{B}_{i}) + S_{2}\overline{S}_{3}(\overline{A}_{i} + B_{i}) + S_{2}S_{3}\overline{A}_{i}$$

$$Y_{i} = \overline{S}_{0}\overline{S}_{1}\overline{A}_{i} + \overline{S}_{0}S_{1}\overline{A}_{i}B_{i} + S_{0}\overline{S}_{1}\overline{A}_{i}\overline{B}_{i}$$

$$\Rightarrow X_i$$

$$X_{i} = \overline{S_{3}A_{i}B_{i} + S_{2}A_{i}\overline{B}_{i}}$$

$$Y_{i} = \overline{A_{i} + S_{0}B_{i} + S_{1}\overline{B}_{i}}$$

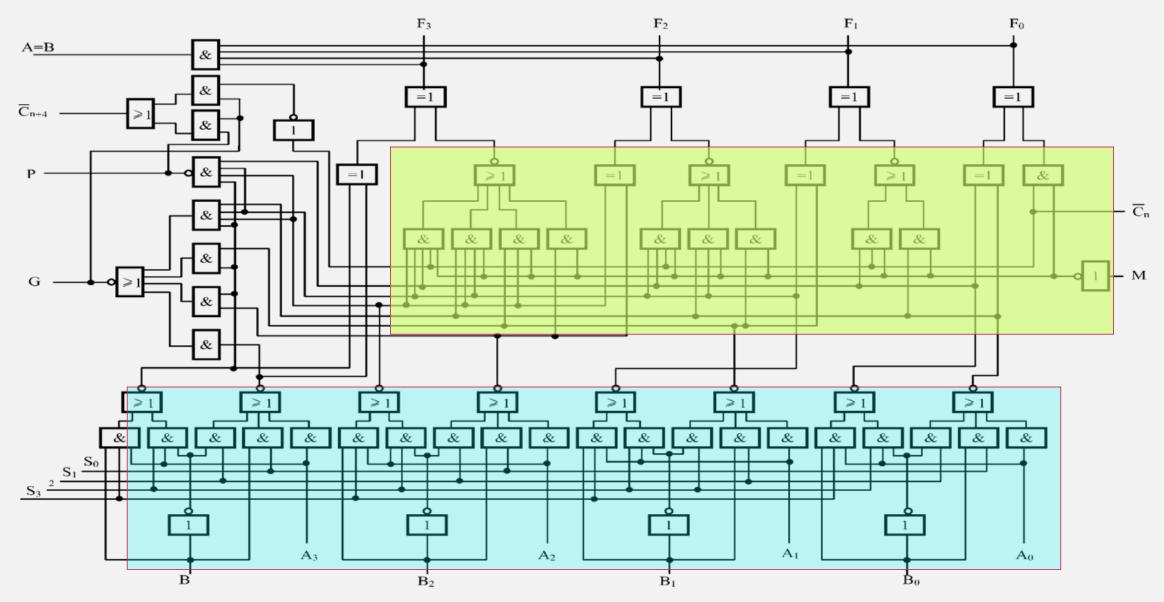


$$F_i = X_i \oplus Y_i \oplus C_{n+i}$$

$$C_{n+i+1} = X_i Y_i + Y_i C_{n+i} + X_i C_{n+1}$$

8·四位运算器芯片 SN74181





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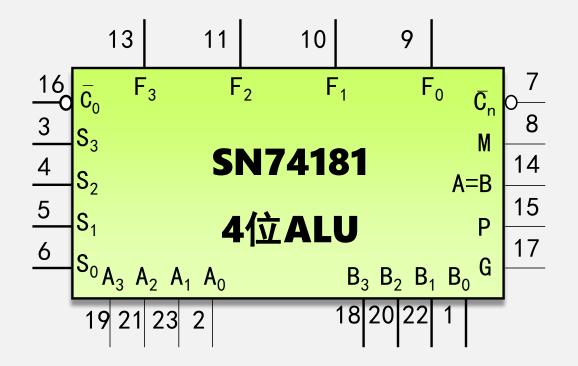
8. 四位运算器芯片 SN74181

- ◆加、减: 算术运算, + :逻辑运算;
- ◆M=H时逻辑运算, M=L 时算术运算;
- ◆算术运算结果和过程均用补码表示;
- ◆ 当S₃S₂S₁S₀ =LHHL, M=L, C=0时, F=?

$\overline{S_3S_2S_1S_0}$	逻辑运算 M=H	算术运算 M=L C _n =H
LLLL	F=Ā	F=A
LLLH	$F=\overline{A+B}$	F=A+B
LLHL	F=AB	F=A+B
LLHH	F=逻辑 0	F= -1
LHLL	F=AB	F=A加 AB
LHLH	F=B	F=(A+B)加 AB
LHHL	F=A⊕B	F=A 减 B 减 1
LHHH	$F=A\overline{B}$	F=AB减1
HLLL	$F=\overline{A}+B$	F=A加 AB
HLLH	F=Ā⊕B	F=A加B
HLHL	F=B	F=(A+B)加 AB
HLHH	F=AB	F=AB减1
HHLL	F=1	F=A加 A
HHLH	$F=A+\overline{B}$	F=(A+B)加A
HHHL	F=A+B	F=(A+B)加 A
нннн	F=A	F=A减1



8·四位运算器芯片 SN74181



前面讲过的三种溢出检测方法可 直接用于74181吗?

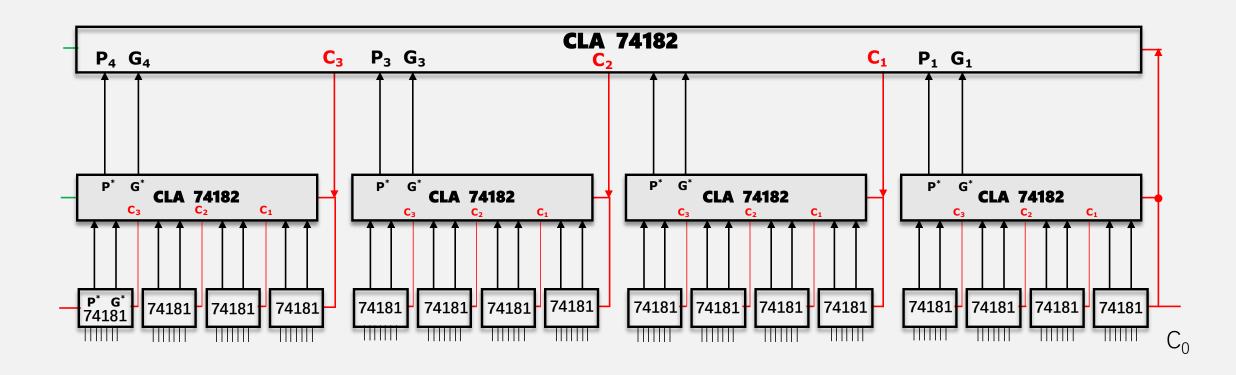
Overflow₁ =
$$X_n Y_n \overline{S}_n + \overline{X}_n \overline{Y}_n S_n$$

Overflow₂ =
$$C_f \oplus C_n$$

Overflow₃ =
$$f_1 \oplus f_2$$



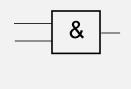
8. 四位运算器芯片 SN74181

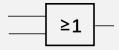


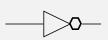
61

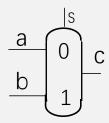


9. 运算器设计其他方法(结构)

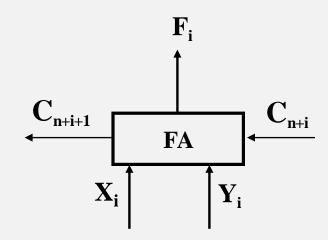








if (s = 0, c = a; else c = b)

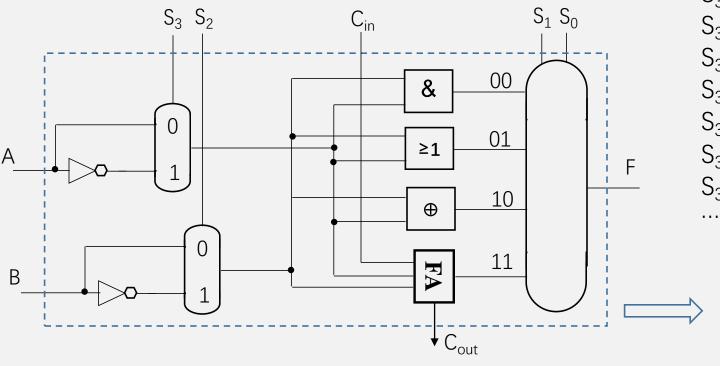


II 3

3.1 定点数加/减运算



9. 运算器设计其他方法(结构)



 $S_3S_2S_1S_0 = 0000$: F=AB

 $S_3S_2S_1S_0 = 1100$: $F = \overline{A}\overline{B}$

 $S_3S_2S_1S_0 = 1000$: $F = \overline{A}B$

 $S_3S_2S_1S_0 = 0100$: $F = A\overline{B}$

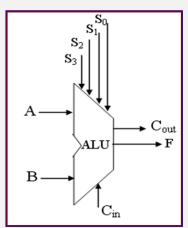
 $S_3S_2S_1S_0 = 0001$: F=A+B

 $S_3S_2S_1S_0 = 0010$: $F = A \oplus B$

 $S_3S_2S_1S_0 = 0011$ **C**_{in}=0: F=A加B

S₃S₂S₁S₀= 0111 **C**_{in}=1: F=A減B

.





9. 运算器设计其他方法(硬件描述语言: Verilog)

```
moudle add_4 (x,y, sum ,cin, cout )
input[3:0] x,y;
outpu[3:0] sum ;
output cout ;
input cin ;
assign {cout, sum } = x + y + cin;
endmoudle
```

```
moudle add_4 (x,y, sum ,cin, cout )
input[3:0] x,y;
outpu[3:0] sum;
output cout;
input cin;
sum < = x xor y xor cin;
cout < = (x and y) or (x and cin) or (y and cin)
endmoudle
```

配合FPGA使用 (Field Programmable Gate Array)



