



# Training Deep Feedforwards Networks

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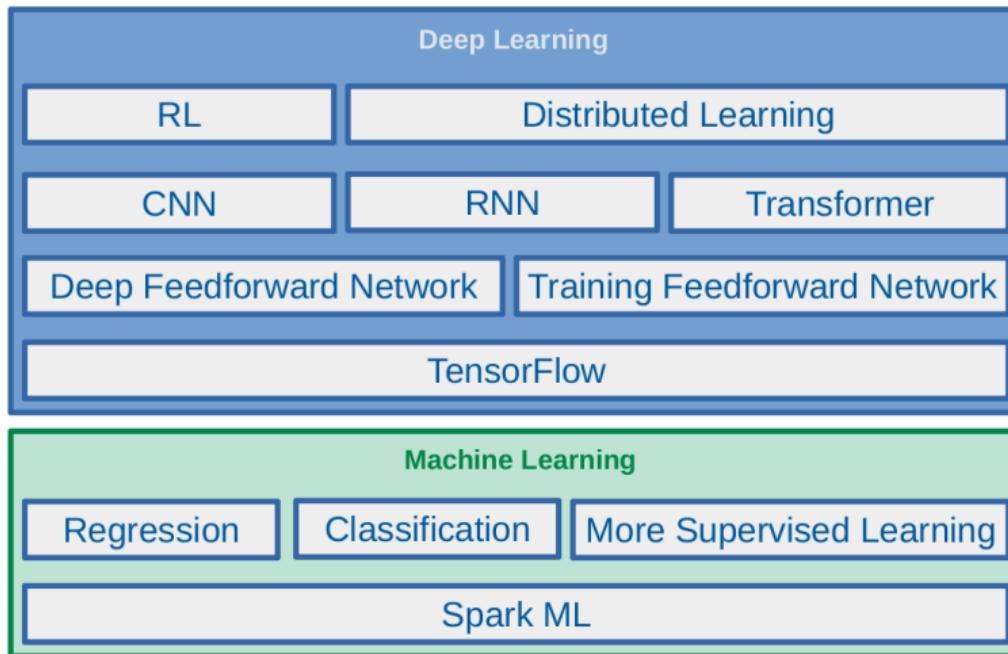


# The Course Web Page

<https://id2223kth.github.io>  
<https://tinyurl.com/6s5jy46a>

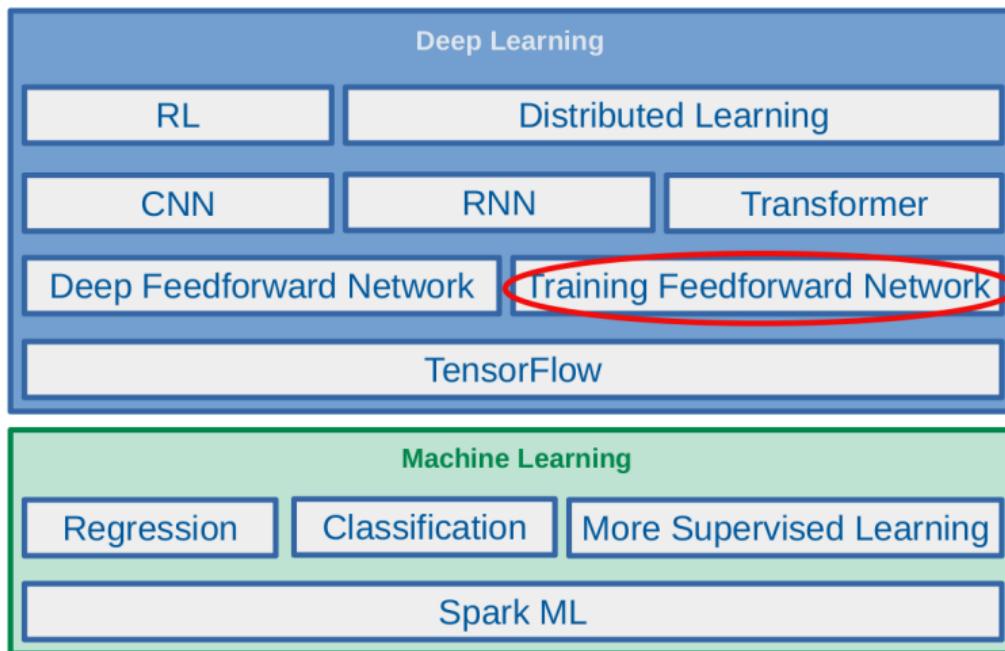


# Where Are We?





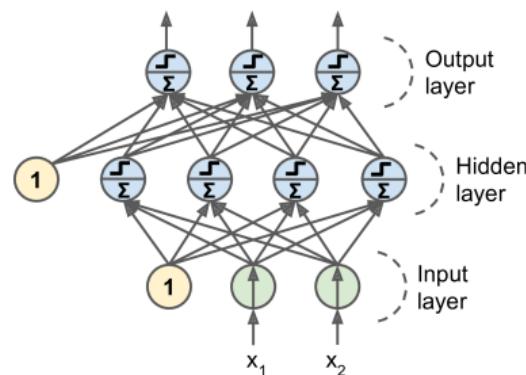
# Where Are We?



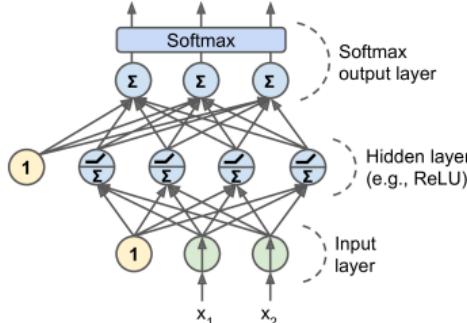
# Feedforward Neural Network Architecture

- ▶ A **feedforward neural network** is composed of:

- One **input layer**
- One or more **hidden layers**
- One final **output layer**



# Feedforward Network in TensorFlow



```
n_output = 3
n_hidden = 4
n_features = 2

model = keras.models.Sequential()
model.add(keras.layers.Dense(n_hidden, input_shape=(n_features,), activation="relu"))
model.add(keras.layers.Dense(n_output, activation="softmax"))

model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

# Challenges of Training Feedforward Neural Networks

- ▶ Challenges ...
- ▶ Overfitting: risk of overfitting a model with large number of parameters.
- ▶ Vanishing/exploding gradients: hard to train lower layers.
- ▶ Training speed: slow training with large networks.



# Overfitting



# High Degree of Freedom and Overfitting Problem

- ▶ With **large number of parameters**, a network has a **high degree of freedom**.
- ▶ It can **fit** a huge variety of **complex datasets**.
- ▶ This **flexibility** also means that it is **prone to overfitting on training set**.
- ▶ Let's **reduce** the degree of freedom a model.



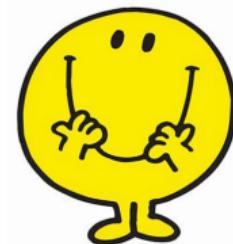
# Avoiding Overfitting

- ▶ Early stopping
- ▶  $\ell_1$  and  $\ell_2$  regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



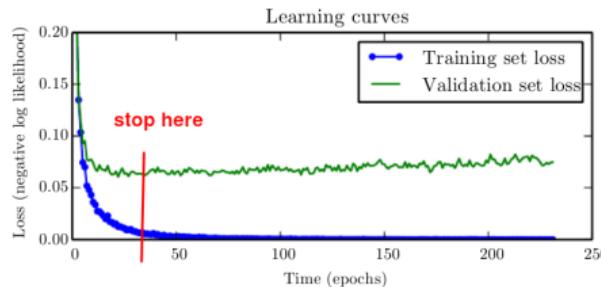
# Avoiding Overfitting

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## Early Stopping (1/2)

- ▶ As the **training steps go by**, its prediction error on the **training/validation set** naturally **goes down**.
- ▶ After a while the **validation error stops decreasing** and **starts to go back up**.
  - The model has started to **overfit the training data**.
- ▶ In the **early stopping**, we **stop training** when the **validation error reaches a minimum**.





## Early Stopping (2/2)

```
from tensorflow.keras.callbacks import EarlyStopping

model = tf.keras.models.Sequential(...)

model.compile(optimizer='sgd', loss='sparse_categorical_crossentropy', metrics=['accuracy'])

earlystop_callback = EarlyStopping(monitor='accuracy', min_delta=0.05, patience=1)

model.fit(x_train, y_train, epochs=500, callbacks=[earlystop_callback])
```

# Avoiding Overfitting

- ▶ Early stopping
- ▶  $\ell_1$  and  $\ell_2$  regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation





## $\ell_1$ and $\ell_2$ Regularization (1/3)

- ▶ Penalize **large values** of weights  $w_j$ .

$$\tilde{J}(w) = J(w) + \lambda R(w)$$

- ▶ Two questions:
  1. How should we define  $R(w)$ ?
  2. How do we determine  $\lambda$ ?



## /1 and /2 Regularization (2/3)

- **/1 regression:**  $R(w) = \lambda \sum_{i=1}^n |w_i|$  is added to the cost function.

$$\tilde{J}(w) = J(w) + \lambda \sum_{i=1}^n |w_i|$$

```
keras.layers.Dense(100, activation="relu", kernel_regularizer=keras.regularizers.l1(0.1))
```



## /1 and /2 Regularization (3/3)

- **/2 regression:**  $R(w) = \lambda \sum_{i=1}^n w_i^2$  is added to the **cost function**.

$$\tilde{J}(w) = J(w) + \lambda \sum_{i=1}^n w_i^2$$

```
keras.layers.Dense(100, activation="relu", kernel_regularizer=keras.regularizers.l2(0.01))
```

# Avoiding Overfitting

- ▶ Early stopping
- ▶  $\ell_1$  and  $\ell_2$  regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation





# Max-Norm Regularization

- ▶ Max-norm regularization: constrains the weights  $w_j$  of the incoming connections for each neuron  $j$ .
  - Prevents them from getting too large.
- ▶ After each training step, clip  $w_j$  as below, if  $\|w_j\|_2 > r$ :

$$w_j \leftarrow w_j \frac{r}{\|w_j\|_2}$$

- $r$  is the max-norm hyperparameter
- $\|w_j\|_2 = (\sum_i w_{i,j}^2)^{\frac{1}{2}} = \sqrt{w_{1,j}^2 + w_{2,j}^2 + \dots + w_{n,j}^2}$

```
keras.layers.Dense(100, activation="relu", kernel_constraint=keras.constraints.max_norm(1.))
```

# Avoiding Overfitting

- ▶ Early stopping
- ▶  $\ell_1$  and  $\ell_2$  regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



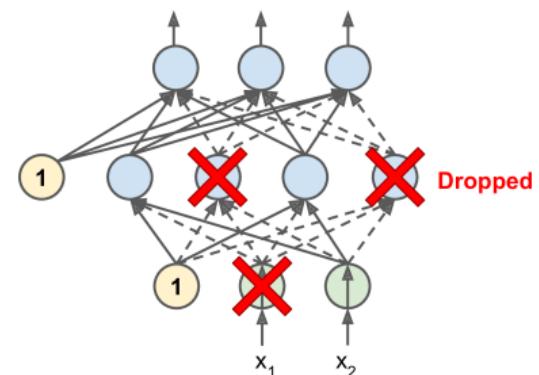
## Dropout (1/4)

- ▶ Would a **company** perform better if its employees were told **to toss a coin** every morning to decide **whether or not to go to work**?



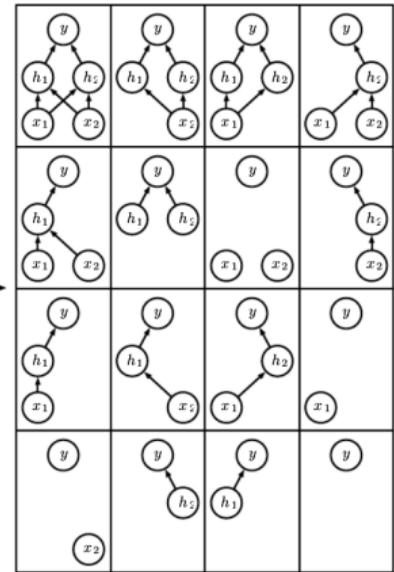
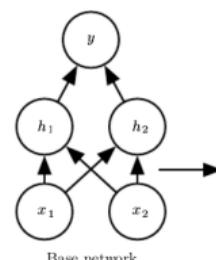
## Dropout (2/4)

- ▶ At each **training step**, each neuron drops out temporarily with a probability  $p$ .
  - The **hyperparameter  $p$**  is called the **dropout rate**.
  - A neuron will be **entirely ignored** during **this training step**.
  - It may be **active** during the **next step**.
  - Exclude the **output neurons**.
- ▶ After training, neurons don't get dropped anymore.



# Dropout (3/4)

- ▶ Each neuron can be either **present or absent**.
- ▶  **$2^N$  possible networks**, where **N** is the total number of **droppable neurons**.
  - **N = 4** in this figure.



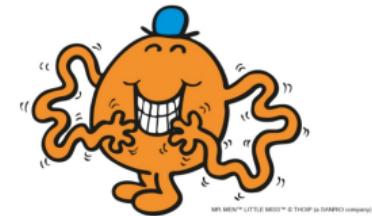


## Dropout (4/4)

```
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dropout(rate=0.2),
    keras.layers.Dense(128, activation="relu"),
    keras.layers.Dropout(rate=0.2),
    keras.layers.Dense(10, activation="softmax")
])
```

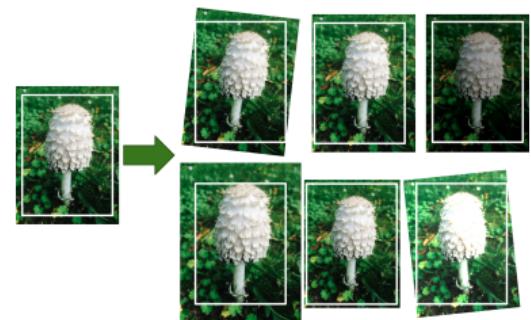
# Avoiding Overfitting

- ▶ Early stopping
- ▶  $\ell_1$  and  $\ell_2$  regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



# Data Augmentation

- ▶ One way to make a model **generalize better** is to **train it on more data**.
- ▶ This will **reduce overfitting**.
- ▶ Create **fake data** and add it to the **training set**.
  - E.g., in an **image classification** we can slightly shift, rotate and resize an image.
  - Add the resulting pictures to the **training set**.



# Vanishing/Exploding Gradients





## Vanishing/Exploding Gradients Problem (1/4)

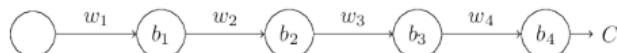
- ▶ The backpropagation goes from output to input layer, and propagates the error gradient on the way.

$$w^{(\text{next})} = w - \eta \frac{\partial J(w)}{\partial w}$$

- ▶ Gradients often get smaller and smaller as the algorithm progresses down to the lower layers.
- ▶ As a result, the gradient descent update leaves the lower layer connection weights virtually unchanged.
- ▶ This is called the vanishing gradients problem.

## Vanishing/Exploding Gradients Problem (2/4)

- ▶ Assume a network with just a single neuron in each layer.



- $w_1, w_2, \dots$  are the **weights**
- $b_1, b_2, \dots$  are the **biases**
- $C$  is the **cost function**

- ▶ The output  $a_j$  from the  $j$ th neuron is  $\sigma(z_j)$ .

- $\sigma$  is the **sigmoid** activation function
- $z_j = w_j a_{j-1} + b_j$
- E.g.,  $a_4 = \sigma(z_4) = \text{sigmoid}(w_4 a_3 + b_4)$

## Vanishing/Exploding Gradients Problem (3/4)

- Lets compute the **gradient** associated to the **first hidden neuron** ( $\frac{\partial C}{\partial b_1}$ ).



$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial z_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial z_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial z_1}{\partial b_1}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial w_4 a_3 + b_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial w_3 a_2 + b_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial w_2 a_1 + b_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial w_1 a_0 + b_1}{\partial b_1}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3} \times w_3 \times \frac{\partial a_2}{\partial z_2} \times w_2 \times \frac{\partial a_1}{\partial z_1} \times 1$$

## Vanishing/Exploding Gradients Problem (4/4)

- ▶ Now, consider  $\frac{\partial C}{\partial b_3}$ .



$$\frac{\partial C}{\partial b_3} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3} \times w_3 \times \frac{\partial a_2}{\partial z_2} \times w_2 \times \frac{\partial a_1}{\partial z_1} \times 1$$

- ▶ Assume  $w_3 \times \frac{\partial a_2}{\partial z_2} < \frac{1}{4}$  and  $w_2 \times \frac{\partial a_1}{\partial z_1} < \frac{1}{4}$ 
  - The gradient  $\frac{\partial C}{\partial b_1}$  be a factor of 16 (or more) smaller than  $\frac{\partial C}{\partial b_3}$ .
  - This is the essential origin of the vanishing gradient problem.

# Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping





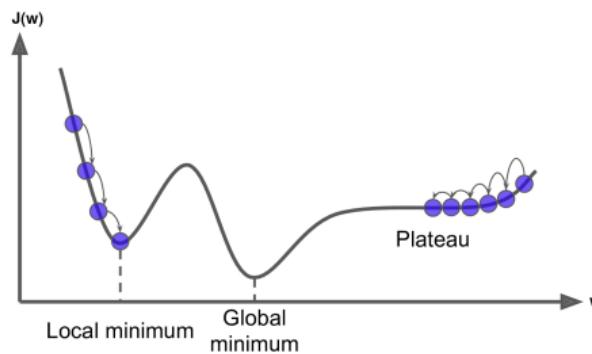
# Overcoming the Vanishing Gradient

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## Parameter Initialization Strategies (1/4)

- ▶ The **non-linearity** of a neural network causes the **cost functions** to become **non-convex**.
- ▶ The stochastic gradient descent on **non-convex cost functions** performs is **sensitive** to the values of the **initial parameters**.
- ▶ Designing initialization strategies is a **difficult task**.





## Parameter Initialization Strategies (2/4)

- ▶ The **initial parameters** need to **break symmetry** between **different units**.
- ▶ **Two hidden units** with the **same activation function** connected to the **same inputs**, must have **different** initial parameters.
  - The goal of having each unit **compute a different function**.
- ▶ It motivates **random initialization** of the parameters.
  - Typically, we set the **biases** to **constants**, and initialize only the **weights randomly**.



## Parameter Initialization Strategies (3/4)

- ▶ We need the signals to flow properly in **both** directions.
- ▶ The **Glorot and Bengio initialization** proposed that:
  - The **variance** of the outputs of each layer to be **equal** to the **variance** of its inputs.
  - The **gradients** to have **equal variance before and after** flowing through a layer in the reverse direction.
- ▶ It is not possible to guarantee both unless each layer has an **equal number of inputs and neurons**.
- ▶ Based on the **Xavier initialization**, the weights are **initialized** using **normal distribution** with **mean 0** and the following **standard deviation**.



## Parameter Initialization Strategies (4/4)

- ▶  $\text{fan}_{\text{in}}$  and  $\text{fan}_{\text{out}}$  are the number of inputs and neurons for the layer whose weights are being initialized.
- ▶  $\text{fan}_{\text{avg}} = \frac{2}{\text{fan}_{\text{in}} + \text{fan}_{\text{out}}}$
- ▶ Glorot initialization, for none, logistic, sigmoid, and tanh:  $\sigma^2 = \frac{1}{\text{fan}_{\text{avg}}}$
- ▶ He initialization, for ReLU:  $\sigma^2 = \frac{2}{\text{fan}_{\text{in}}}$

```
keras.layers.Dense(10, activation="relu", kernel_initializer="he_normal")
```

# Overcoming the Vanishing Gradient

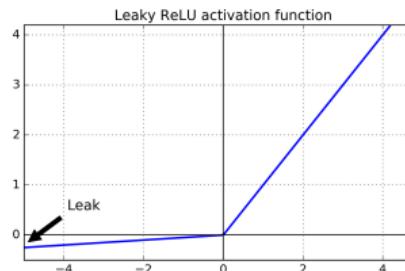
- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping

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# Nonsaturating Activation Functions (1/4)

- ▶  $\text{ReLU}(z) = \max(0, z)$
- ▶ The **dying ReLUs** problem.
  - During **training**, some neurons **stop outputting anything other than 0**.
  - E.g., when the **weighted sum of the neuron's inputs is negative**, it starts outputting 0.
- ▶ Use **leaky ReLU** instead:  $\text{LeakyReLU}_\alpha(z) = \max(\alpha z, z)$ .
  - $\alpha$  is the **slope** of the function for  $z < 0$ .



## Nonsaturating Activation Functions (2/4)

### ► Randomized Leaky ReLU (RReLU)

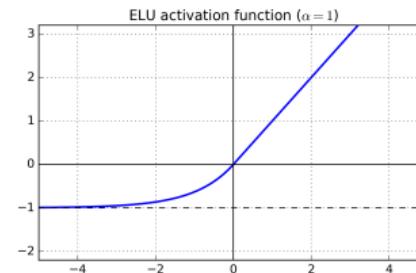
- $\alpha$  is picked **randomly** during training, and it is **fixed** during testing.

### ► Parametric Leaky ReLU (PReLU)

- Learn  $\alpha$  **during training** (instead of being a hyperparameter).

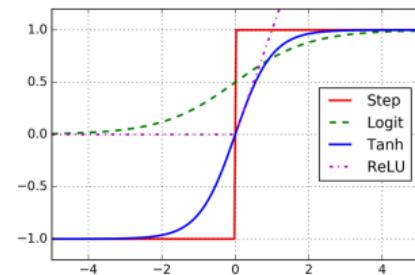
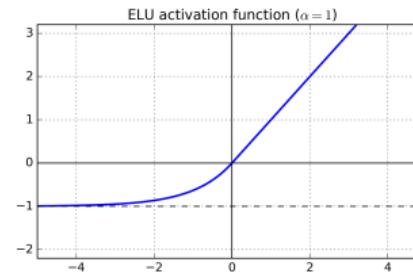
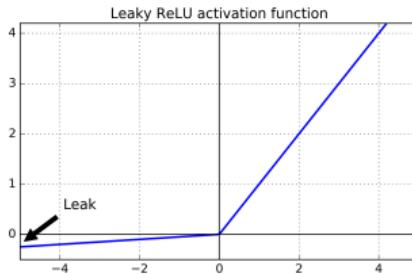
### ► Exponential Linear Unit (ELU)

$$\text{ELU}_\alpha(z) = \begin{cases} \alpha(\exp(z) - 1) & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$



## Nonsaturating Activation Functions (3/4)

- ▶ Which activation function should we use?
- ▶ In general logistic < tanh < ReLU < leaky ReLU (and its variants) < ELU
- ▶ If you care about runtime performance, then leaky ReLUs works better than ELUs.





## Nonsaturating Activation Functions (4/4)

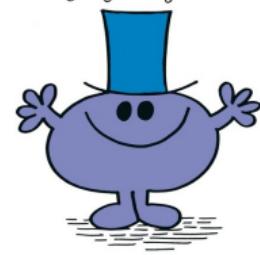
```
# elu
keras.layers.Dense(10, activation="elu")
```

```
# leaky relu
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dense(128, kernel_initializer="he_normal"),
    keras.layers.LeakyReLU(),
    keras.layers.Dense(10, activation="softmax")
])
```

# Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ **Batch normalization**
- ▶ Gradient clipping

by Roger Hargreaves





## Batch Normalization (1/4)

- ▶ The gradient tells how to **update each parameter**, under the assumption that **the other layers do not change**.
  - In practice, we update all of the layers **simultaneously**.
  - However, **unexpected results can happen**.
- ▶ **Batch normalization** makes the **learning of layers** in the network more **independent of each other**.
  - It is a technique to address the problem that the **distribution of each layer's inputs** changes **during training**, as the parameters of the previous layers change.
- ▶ The technique consists of **adding an operation** in the model just **before the activation function** of each layer.



## Batch Normalization (2/4)

- ▶ It's zero-centering and normalizing the inputs, then scaling and shifting the result.
  - Estimates the inputs' mean and standard deviation of the current mini-batch.

$$\mu_B = \frac{1}{m_B} \sum_{i=1}^{m_B} x^{(i)}$$

$$\sigma_B^2 = \frac{1}{m_B} \sum_{i=1}^{m_B} (x^{(i)} - \mu_B)^2$$

- ▶  $\mu_B$ : the empirical mean, evaluated over the whole mini-batch  $B$ .
- ▶  $\sigma_B$ : the empirical standard deviation, also evaluated over the whole mini-batch.
- ▶  $m_B$ : the number of instances in the mini-batch.



## Batch Normalization (3/4)

$$\hat{x}^{(i)} = \frac{x^{(i)} - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$
$$z^{(i)} = \gamma \hat{x}^{(i)} + \beta$$

- ▶  $\hat{x}^{(i)}$ : the zero-centered and normalized input.
- ▶  $z^{(i)}$ : the output of the BN operation, which is a scaled and shifted version of the inputs.
- ▶  $\gamma$ : the scaling parameter vector for the layer.
- ▶  $\beta$ : the shifting parameter (offset) vector for the layer.
- ▶  $\epsilon$ : a tiny number to avoid division by zero.
- ▶  $\otimes$ : represents the element-wise multiplication.

## Batch Normalization (4/4)

```
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.BatchNormalization(),
    keras.layers.Dense(128, activation="relu"),
    keras.layers.BatchNormalization(),
    keras.layers.Dense(10, activation="softmax")
])
```

# Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ **Gradient clipping**

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# Gradient Clipping

- ▶ **Gradient clipping:** clip the gradients during **backpropagation** so that they **never exceed some threshold**.

```
optimizer = keras.optimizers.SGD(clipvalue=1.0)
model.compile(loss="mse", optimizer=optimizer)
```

- ▶ Setting the **clipvalue** or **clipnorm** argument when creating an optimizer.
- ▶ **clipvalue=1.0** and **clipnorm=1.0**: values between -1.0 and 1.0.
- ▶ **clipvalue=1.0**:  $[0.9, 100.0] \Rightarrow [0.9, 1.0]$
- ▶ **clipnorm=1.0**:  $[0.9, 100.0] \Rightarrow [0.00899964, 0.9999595]$

# Training Speed

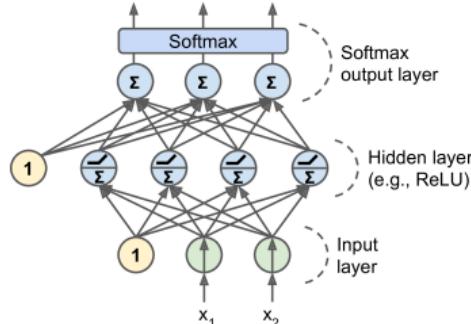




## Regular Gradient Descent Optimization (1/2)

- ▶ Gradient descent optimization algorithm
- ▶ It updates the weights  $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ Better optimization algorithms to improve the training speed

## Regular Gradient Descent Optimization (2/2)



```
n_output = 3
n_hidden = 4
n_features = 2

model = keras.models.Sequential()
model.add(keras.layers.Dense(n_hidden, input_shape=(n_features,), activation="relu"))
model.add(keras.layers.Dense(n_output, activation="softmax"))

model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

# Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam Optimization





# Optimization Algorithms

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- ▶ Nesterov momentum
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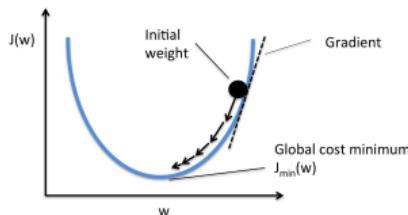
## Momentum (1/3)

- ▶ Momentum is a concept from physics: an object in motion will have a tendency to keep moving.
- ▶ It measures the resistance to change in motion.
  - The higher momentum an object has, the harder it is to stop it.



## Momentum (2/3)

- ▶ This is the very simple idea behind **momentum optimization**.
- ▶ We can see the **change in the parameters  $w$**  as **motion**:  $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ We can thus use the concept of momentum to give the update process a **tendency to keep moving** in the same direction.
- ▶ It can help to **escape from bad local minima pits**.





## Momentum (3/3)

- ▶ Regular gradient descent optimization:  $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ **Momentum optimization** cares about what previous gradients were.
- ▶ At each iteration, it adds the **local gradient** to the **momentum vector m**.

$$\begin{aligned}m_i &= \beta m_i + \eta \frac{\partial J(w)}{\partial w_i} \\w_i^{(\text{next})} &= w_i - m_i\end{aligned}$$

- ▶  $\beta$  is called **momentum**, and it is between 0 and 1.

```
optimizer = keras.optimizers.SGD(lr=0.001, momentum=0.9)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

# Optimization Algorithms

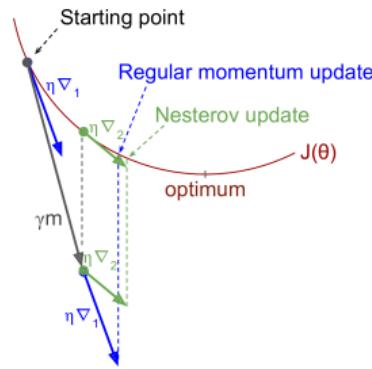
- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization

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## Nesterov Momentum (1/2)

- ▶ Nesterov Momentum is a small variant to Momentum optimization.
- ▶ Faster than vanilla Momentum optimization.
- ▶  $\nabla_1$  represents the gradient of the cost function measured at the starting point  $w$ , and  $\nabla_2$  represents the gradient at the point located at  $w + \beta m$ .





## Nesterov Momentum (2/2)

- ▶ Measure the gradient of the cost function slightly ahead in the direction of the momentum (not at the local position).

$$\begin{aligned} m_i &= \beta m_i + \eta \frac{\partial J(w + \beta m)}{\partial w_i} \\ w_i^{(\text{next})} &= w_i - m_i \end{aligned}$$

```
optimizer = keras.optimizers.SGD(lr=0.001, momentum=0.9, nesterov=True)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```



# Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization





## AdaGrad (1/2)

- ▶ AdaGrad keeps track of a learning rate for each parameter.
- ▶ Adapts the learning rate over time (adaptive learning rate).
- ▶ Decays the learning rate faster for steep dimensions than for dimensions with gentler slopes.



## AdaGrad (2/2)

- ▶ For each feature  $w_i$ , we do the following steps:

$$s_i = s_i + \left( \frac{\partial J(w)}{\partial w_i} \right)^2$$

$$w_i^{(\text{next})} = w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(w)}{\partial w_i}$$

```
optimizer = keras.optimizers.Adagrad(lr=0.001)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

# Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization





## RMSProp (1/2)

- ▶ AdaGrad often stops too early when training neural networks.
- ▶ The learning rate gets scaled down so much that the algorithm ends up stopping entirely before reaching the global optimum.
- ▶ The RMSProp fixed the AdaGrad problem.
- ▶ It is like the AdaGrad problem, but accumulates only the gradients from the most recent iterations (not from the beginning of training).



## RMSProp (2/2)

- ▶ For each feature  $w_i$ , we do the following steps:

$$s_i = \beta s_i + (1 - \beta) \left( \frac{\partial J(w)}{\partial w_i} \right)^2$$
$$w_i^{(\text{next})} = w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(w)}{\partial w_i}$$

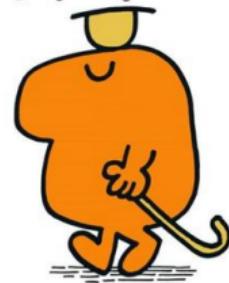
```
optimizer = keras.optimizers.RMSprop(lr=0.001, rho=0.9)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```



# Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization

by Roger Hargreaves





## Adam Optimization (1/3)

- ▶ Adam (Adaptive moment estimation) combines the ideas of Momentum optimization and RMSProp.
- ▶ Like Momentum optimization, it keeps track of an exponentially decaying average of past gradients.
- ▶ Like RMSProp, it keeps track of an exponentially decaying average of past squared gradients.



## Adam Optimization (2/3)

$$1. \quad m^{(\text{next})} = \beta_1 m + (1 - \beta_1) \nabla_w J(w)$$

$$2. \quad s^{(\text{next})} = \beta_2 s + (1 - \beta_2) \nabla_w J(w) \otimes \nabla_w J(w)$$

$$3. \quad m^{(\text{next})} = \frac{m}{1 - \beta_1^T}$$

$$4. \quad s^{(\text{next})} = \frac{s}{1 - \beta_2^T}$$

$$5. \quad w^{(\text{next})} = w - \eta m \oslash \sqrt{s + \epsilon}$$

- ▶  $\otimes$  and  $\oslash$  represent the element-wise multiplication and division.
- ▶ Steps 1, 2, and 5: similar to both Momentum optimization and RMSProp.
- ▶ Steps 3 and 4: since  $m$  and  $s$  are initialized at 0, they will be biased toward 0 at the beginning of training, so these two steps will help boost  $m$  and  $s$  at the beginning of training.



## Adam Optimization (3/3)

```
optimizer = keras.optimizers.Adam(lr=0.001, beta_1=0.9, beta_2=0.999)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```



# Summary

# Summary

- ▶ Overfitting
  - Early stopping,  $\ell_1$  and  $\ell_2$  regularization, max-norm regularization
  - Dropout, data augmentation
- ▶ Vanishing gradient
  - Parameter initialization, nonsaturating activation functions
  - Batch normalization, gradient clipping
- ▶ Training speed
  - Momentum, nesterov momentum, AdaGrad
  - RMSProp, Adam optimization





## Reference

- ▶ Ian Goodfellow et al., Deep Learning (Ch. 7, 8)
- ▶ Aurélien Géron, Hands-On Machine Learning (Ch. 11)



# Questions?