Supplementary Online Material

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1 RNG test with Chi-Square

Using a statistical method, it's possible to prove that a random number generator distributes in a fair manner. A distribution can be qualified as 'good' when each possible sequence is equally likely to appear. This means that a good random number generator might also produce sequences that look non-random and still fail any statistical test, but actually are random.

One of the possible methods to statistically determine if the random numbers are really random, is the hypothesis Chi-Square (χ^2) method [15] [16]. This method has the following steps:

1. State null (H_0) and alternative (H_1) hypothesis.
2. Choose level of significance (α).
3. Find critical values.
4. Find test statistic.
5. Draw your conclusion.

Starting with an example, we'll use the Chi-Square method to determine if a random die can be considered as fair. Since we expect that the die is fair, we can state our $\mathbf{H_0}$ hypothesis: the used die is a fair die. On the other hand, the alternative hypothesis, $\mathbf{H_1}$, will be: the used die is not a fair die.

The next step is to choose the level of significance. The level of significance, the Greek letter α, is the probability of rejecting the null hypothesis, H_0. A significance level of 0.05 means a 5% risk of concluding that a difference exists when there is not an actual difference. The level of 0.05 is quite often chosen for many applications and we will use this number for our example.

After choosing the level of significance, a critical value (abbreviated as c.v.) should be determined. This can be done by using a Chi-Square table. This requires the look up of a value corresponding a degrees of freedom. The number of degrees of freedom is the number of values in the final calculation of a statistic that are free to vary. Most of the time it is abbreviated as d.f.. To calculate the value, we use equation 1.

$$d.f. = n - 1 \tag{1}$$

For a die, this would result in $n = 6$ and $d.f. = 6 - 1 = 5$. Since all the necessary values are now known, the table 1 can be used to find the critical value. Which will be **11.07**.

d.f.	$\chi^2{}_{\mathbf{0.99}}$	$\chi^2{}_{\mathbf{0.1}}$	$\chi^2{}_{\mathbf{0.05}}$	$\chi^2{}_{\mathbf{0.001}}$
1	0.000	2.706	3.841	6.635
2	0.020	4.605	5.991	9.210
3	0.115	6.251	7.815	11.34
4	0.297	7.779	9.488	13.28
5	0.554	9.236	11.07	15.09
6	0.872	10.64	12.59	16.81
7	1.239	12.02	14.07	18.48
8	1.646	13.36	15.51	20.09
9	2.088	14.68	16.92	21.67
10	2.558	15.99	18.31	23.21

Table 1: Chi-square critical values. [2]

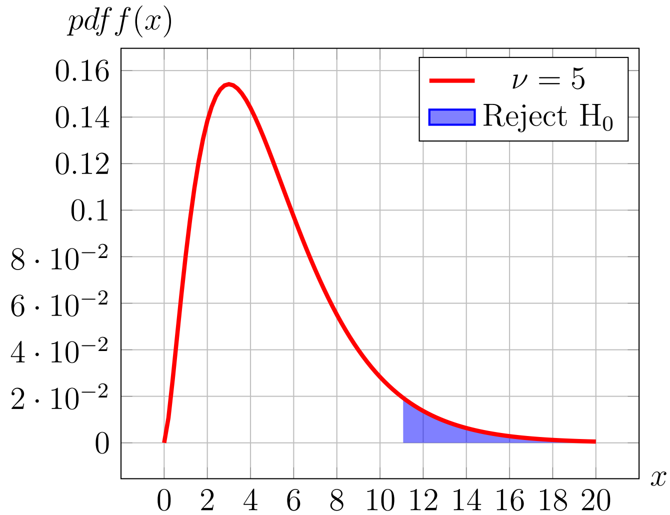

Figure 1: The χ^2 (Chi-Square) distribution.

In figure 1 we have drawn the line of degrees of freedom and have marked the rejection region, H_0, based on the critical value.
Now we can calculate the Chi-Square. The formula for calculating the Chi-Square is given in equation 2, where **O** is the observed value and **E** the expected value. By summing all values, we get the statistic value.

$$\chi^2 = \sum \frac{(O-E)^2}{E} \tag{2}$$

The last step in the process is drawing a conclusion. When the test statistic value is larger than the critical value, we can safely reject H_0 and accept H_1.

Nr.	Expected	Observed	χ^2
1	34	44	2.9412
2	34	24	2.9412
3	34	38	0.4706
4	34	30	0.4706
5	34	46	4.2353
6	34	22	4.2353

Table 2: Die rolls, 204 times.

Based on the observed and expected data as shown in table 2, we can calulate the Chi-Square and will find a sum of **15.29**. This value is larger than the **11.07**, so we can reject our H_0 hypothesis and accept H_1. Therefore the used die is not a fair die.

2 Ruby random number generator

In our simulation we used Ruby's pseudo-random number generator[1] which is a modified Mersenne Twister[14] with a period of 2**19937-1. To test the distribution of the random number generator, we will use it as die.
The null and alternative hypothesis:

- H_0: The random number generator is a good generator
- H_1: The random number generator is a bad generator

Other necessary values:

- the level of significance 0.05α (same as example-value);
- the $d.f. = 6 - 1 = 5$;
- and the critical value, found in table 1, **11.07**.

The results of the 'die rolls' are shown in table 3, with a quantity of 50,000. The sum of the values in the χ^2 column is **1.2834**, which is our χ^2.

Nr.	**Expected**	**Observed**	χ^2
1	8333.5	8316	0.03675
2	8333.5	8397	0.48386
3	8333.5	8501	3.36668
4	8333.5	8247	0.89785
5	8333.5	8218	1.60080
6	8333.5	8321	0.01875

Table 3: Die rolls, 50,000 times with seed: 32614225125840008605808656793674267044.

Drawing the χ^2 value in figure 1 shows, that the value is lower than the critical value. Therefore we can accept H_0, our random number generator truly generates random numbers.

3 Acceleration and brake calculations

The model in the simulation makes use of various accelerations. Braking can be seen as acceleration as well, in that case the outcome is negative. The used numbers are calculated as following and based on various observations [12][13]:

v $= \frac{100kph}{3.6} = 27.77773ms$

a $= \frac{\Delta v}{\Delta t} = \frac{27.77773}{2.6} = 10.68372ms^2$

s $= \frac{1}{2}at^2 = \frac{1}{2} * 10.68372 * 2.6^2 = 36.11108m$

$36.11108m/2.6s = 13.88884m/s$

kph	time (s)	interval (s)	a (ms^2)	distance (1s)
0-100	2.6	2.6	10.68372	36.11108
0-200	5.2	2.6	10.68372	36.11108
0-300	10.6	5.4	5.14401	74.99951

name	time (s)	a (ms^2)
Ferrari	2.48	11.20071
Mercedes	2.48	11.20071
Mercedes	2.68	10.36484
McLaren	2.8	9.92063
RedBull	2.84	9.78090

4 Incidents

In nine of the currently been 16 races in season 2017 events, an incident has taken place in the first section including the first turn.
[11] [5] [4] [3] [6] [7] [10] [9] [8]

References

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