Chi-square, RNG hypothesis testing

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1 Example

This supplementary online material will describe a method to determine if a random number generator distributes the random numbers in a fair manner. Using a statistical method, we can prove it. However even if you use a good die or a random number generator, each possible sequence is equally likely to appear. That means that a good random number generator might also produce sequences that look nonrandom and still fail any statistical test, but actually are random. It depends on the number of samples (or die rolls), because it is possible that the random number generator produces series of multiple same numbers in a row.

One of the possible methods to statistical determine if the distributed random numbers are really random, is the hypothesis chi-square (χ^2) method [2] [3]. This method follows some steps:

1. State null (H_0) and alternative (H_1) hypothesis

2. Choose level of significance (α)

3. Find critical values

4. Find test statistic

5. Draw your conclusion

First, we will start with an example, in this example we will use the chi-square method to determine if a die from a random casino is a fair die. Since we assume that the die is fair, we can state our $\mathbf{H_0}$ hypothesis: the used die is a fair die. On the other hand, the alternative hypothesis, $\mathbf{H_1}$, will be: the used die is not a fair die.

The next step is to choose the level of significance. The level of significance, the Greek letter α, is the probability of rejecting the null hypothesis, H_0. A significance level of 0,05 means a 5% risk of concluding that a difference exists when there is not an actual difference. The level of 0,05 is quite often chosen for many applications and we will use this number for our example.

After choosing the level of significance, it is time to determine the critical value (abbreviated as c.v.). The critical value can be determined by using a chi-square table. Before we can look up the value, we need to calculate the degrees of freedom. The number of degrees of freedom is the number of values in the final calculation of a statistic that are free to vary. Most of the time it is abbreviated as d.f. To calculate the value, we use equation 1.

$$d.f. = n - 1 \tag{1}$$

For a die, this would result in $n = 6$ and

$d.f. = 6 - 1 = 5$. Since all the necessary values are now known, we can use table 1 to find our critical value. Which will be **11.07**.

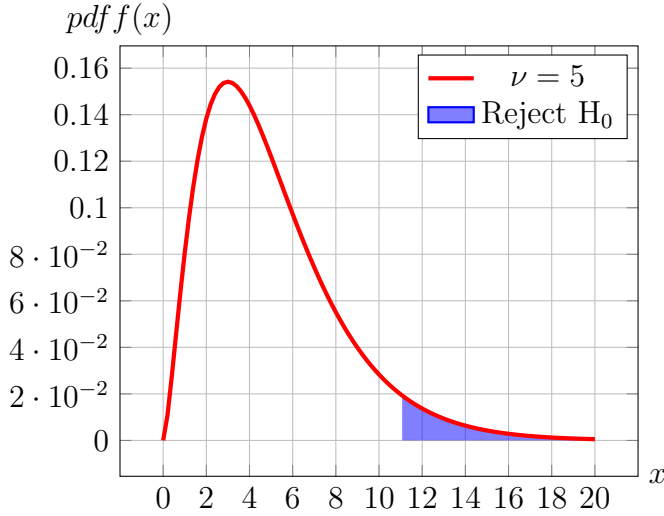| d.f.	$\chi^2_{0.99}$	$\chi^2_{0.1}$	$\chi^2_{0.05}$	$\chi^2_{0.001}$
1	0.000	2.706	3.841	6.635
2	0.020	4.605	5.991	9.210
3	0.115	6.251	7.815	11.34
4	0.297	7.779	9.488	13.28
5	0.554	9.236	11.07	15.09
6	0.872	10.64	12.59	16.81
7	1.239	12.02	14.07	18.48
8	1.646	13.36	15.51	20.09
9	2.088	14.68	16.92	21.67
10	2.558	15.99	18.31	23.21

Table 1: Chi-square critical values. [1]

Figure 1: The χ^2 (chi-square) distribution.

In figure 1 we have drawn the line of degrees of freedom and have marked the rejection region, H_0, based on the critical value.

Now we can calculate the chi-square. The formula for calculating the chi-square is given in equation 2, where **O** is the observed value and **E** the expected value. By summing all values, we get our test statistic value.

$$\chi^2 = \sum \frac{(O - E)^2}{E} \tag{2}$$

The last step in the process is drawing a conclusion. When the test statistic value is larger than the critical value, we can safely reject H_0 and accept H_1.

Nr.	Expected	Observed	χ^2
1	34	44	2.9412
2	34	24	2.9412
3	34	38	0.4706
4	34	30	0.4706
5	34	46	4.2353
6	34	22	4.2353

Table 2: Die rolls, 204 times.

Based on the observed and expected data as shown in table 2, we can calulate the chi-square and will find a sum of **15.29**. This value is larger than the **11.07**, so we can reject our H_0 hypothesis and accept H_1. Therefore the used die is not a fair die.

2 Our test case

Now we are going to test the random number generator used in our simulation model. We use the build in random number generator function to generate random numbers. To test the random number generator, we will use it as die.

The null and alternative hypothesis:

- H_0: The random number generator is a good generator

- H_1: The random number generator is a bad generator

Then we will continue and find the other necessary values:

- For the level of significance we will use the same value as used in the example, α of 0.05.
- The $d.f. = 6 - 1 = 5$.
- The critical value, found in table 1, is **11.07**.

Using the default random number generator. We have written an application that generates 50,000 times a number between 1 and 6. This way we can simulate a die roll, just like our example. The results of the die rolls are shown in table 3. The sum of the values is **6.4047** and that is our χ^2, or test statistic value.

Nr.	Expected	Observed	χ^2
1	8333.5	8316	0.03675
2	8333.5	8397	0.48386
3	8333.5	8501	3.36668
4	8333.5	8247	0.89785
5	8333.5	8218	1.60080
6	8333.5	8321	0.01875

Table 3: Die rolls, 50,000 times.

Drawing the χ^2 value in figure 1 shows, that the value is lower than the critical value. Therefore we can accept H_0, our random number generator truly generates random numbers.

3 Acceleration and brake calculations

Example calculation, results in a negative number in case of braking:

$$v = \frac{100kph}{3.6} = 27.77773ms$$

$$a = \frac{\Delta v}{\Delta t} = \frac{27.77773}{2.6} = 10.68372ms^2$$

$$s = \frac{1}{2}at^2 = \frac{1}{2} * 10.68372 * 2.6^2 = 36.11108m$$

$$36.11108m/2.6s = 13.88884m/s$$

kph	time (s)	interval (s)	a (ms^2)	distance (1s)
0-100	2.6	2.6	10.68372	36.11108
0-200	5.2	2.6	10.68372	36.11108
0-300	10.6	5.4	5.14401	74.99951

name	time (s)	a (ms^2)
Ferrari	2.48	11.20071
Mercedes	2.48	11.20071
Mercedes	2.68	10.36484
McLaren	2.8	9.92063
RedBull	2.84	9.78090

source: https://www.youtube.com/watch?v=plMtLdCjAI

4 Incidents

References

[1] Plant & Soil Sciences eLibrary. *Chi-Square Distribution Table*. Oct. 2017. URL: http : / / passel . unl . edu / pages / informationmodule . php ? idinformationmodule = 1130447119 & topicorder=8&maxto=16&minto=1.

[2] FIA. *Lap-chart Austria*. Oct. 2017. URL: https : / / www . fia . com / events / fia - formula - one - world - championship / season-2017/lap-chart-31.

[3] FIA. *Lap-chart Azerbaijan*. Oct. 2017. URL: https : / / www . fia . com / events / fia -

formula – one – world – championship /
season-2017/lap-chart-30.

[4] FIA. *Lap-chart Canada*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-29.

[5] FIA. *Lap-chart Hungary*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-32.

[6] FIA. *Lap-chart Italy*. Oct. 2017. URL: https :
/ / www . fia . com / events / fia – formula –
one – world – championship / season – 2017 /
lap-chart-25.

[7] FIA. *Lap-chart Japan*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-26.

[8] FIA. *Lap-chart Malaysia*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-34.

[9] FIA. *Lap-chart Singapore*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-33.

[10] FIA. *Lap-chart Spain*. Oct. 2017. URL:
https : / / www . fia . com / events / fia –
formula – one – world – championship /
season-2017/lap-chart-21.

[11] RANDOM.ORG. *Statistical Analysis*. Oct.
2017. URL: https : / / www . random . org /
analysis/.

[12] The Pennsylvania State University. *Chi-
Square Tests*. Oct. 2017. URL: https :
/ / onlinecourses . science . psu . edu /
statprogram/node/158.

[10] [4] [3] [2] [5] [6] [9] [8] [7]

References

[1] Plant & Soil Sciences eLibrary. *Chi-
Square Distribution Table*. Oct. 2017.
URL: http : / / passel . unl . edu /
pages / informationmodule . php ?
idinformationmodule = 1130447119 &
topicorder=8&maxto=16&minto=1.

[2] RANDOM.ORG. *Statistical Analysis*. Oct.
2017. URL: https : / / www . random . org /
analysis/.

[3] The Pennsylvania State University. *Chi-
Square Tests*. Oct. 2017. URL: https :
/ / onlinecourses . science . psu . edu /
statprogram/node/158.