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Time-varying formation tracking with distributed multi-sensor multi-target filtering

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***Abstract*— Formation tracking is used widely in targets enclosing, monitoring and striking, however, in practical scenes, the targets are always uncooperative. The problem of time-varying formation tracking for multiagent with multi-target which states are unknown is studied in this paper. In order to obtain the accurate state estimations of targets, a distributed multi-sensor multi-target filtering algorithm based on the cubature Kalman filter scheme and multiple heterogeneous sensors is proposed. Then, the state estimations obtained by the filtering algorithm are used to design a time-varying formation tracking protocol for multiagent, enabling multiagent to form a time-varying formation to track the convex combination of targets. Finally, the effectiveness of this proposed algorithm is illustrated by simulation experiments.**

I. Introduction

The issue of formation tracking has received wide attention from researchers in the past few years [1]-[3] and has played an important rule in civil and military fields [4]-[6]. Formation tracking uses the information interaction between agents and the states of targets to design a protocol to make sure that multiagent can track multi-target and keep the formation. [7] proposes a sufficient and necessary condition for achieving time-varying formation tracking for a multiagent system with multi-target, this protocol can be further applied to higher-order systems.. [8]-[9] further consider the distributed and the fully adaptive time-varying formation tracking problems.

Compared with centralized filtering, distributed multi-sensor multi-target filtering has the advantages of being real-time, fault-tolerant, flexible [10], and can be divided into two methods by using data association (DA) [11]-[14] and random finite set (RFS) [15]-[16]. The computing speed of distributed filtering with RFS is fast, but the theory of this method is not perfect enough. After associating the data with the method of DA, distributed filtering for nonlinear system can be performed through extended Kalman filter (EKF) [17], unscented Kalman filter (UKF) [18] and CKF [19]-[21]. Since the weight of each integral point in CKF is same and positive, CKF has better numerical stability than UKF. Meanwhile, EKF has lower precision compared with CKF [22]. In this case, the problem of distributed multi-sensor multi-target filtering based on the method of DA and CKF is discussed.

However, in most of the practical scenes, the states of the targets are unknown, i.e. targets are uncooperative. In previous work [23], the authors consider the problem of time-varying formation tracking with unknown disturbances in both agents and targets. In [24], the authors combine the multi-target filtering with formation tracking and propose a formation tracking protocol for heterogeneous second-order system, but they do not consider the case of heterogeneous sensors and nonlinear observation models. The design of formation tracking protocol based on multi-target filtering by multiple heterogeneous sensors with nonlinear observation models is still an important issue.

This work was supported by the Science and Technology Innovation 2030-Key Project of "New Generation Artificial Intelligence" under Grant 2020AAA0108200, the National Natural Science Foundation of China under Grants 62103016, 61922008, 62103023, 61973013, 61873011.

A distributed multi-target cubature Kalman filter (MT-CKF) algorithm is proposed in this paper by assuming that all targets can be measured by each sensor, and the results of DA between target measurements and target trajectories are investigated. In this algorithm, radar sensors and infrared sensors are used to observe the targets. The two types of sensors have different measurement capabilities, the radar sensor can measure the distance and angle between the sensor and the target, and the infrared sensor can only measure the angle between the sensor and the target. The state estimations obtained from the MT-CKF algorithm are futher applied to the problem of formation tracking for multiagent with multi-target. The main contributions proposed in this paper are as follows:

- A distributed MT-CKF algorithm with the nonlinear measuring models based on the CKF scheme and heterogeneous sensors is proposed in this paper. In [17], only linear measuring model is considered. Due to the properties of the sensors in most of the practical scenes, the measuring models are always nonlinear, the approach in [17] cannot be applied to solve this problem.
- Based on the distributed MT-CKF algorithm, a time-varying formation tracking protocol is proposed. However, in [7]-[9], only the problem that the target states are known by the agents is considered, the approach proposed in [7]-[9] cannot be directly used to solve the issue that targets are uncooperative in thos paper.

The rest of the work in this paper is organized as follows. The communication topology and the system models are designed in Section 2. A distributed MT-CKF algorithm and a time-varying formation tracking protocol are proposed in Section 3. The simulation experiments are given in Section 4. This paper is concluded in Section 5.

II. Problem Formulation

A. Communication topology

The communication structure of this network can be described by the directed graphs $G_1=(V,\varepsilon,\Pi)$ and $G_2=(D,\chi,W)$, where V and D are the node sets, $\varepsilon\in V\times V$ and $\chi\in D\times D$ are the edge sets, Π and W are the weighted adjacency matrices. The edge $(j,i)\in\varepsilon$ or $(j,i)\in\chi$ means that node i can receive data sent from node

j, and j is the in-neighbor node of i. $\mathrm{N}_i = \{j : (j,i) \in \varepsilon \text{ or } (j,i) \in \chi\}$ is the in-neighbor set of i, $i \notin \mathrm{N}_i$, the number of nodes in set N_i is $|\mathrm{N}_i|$.

Suppose there are M agents, N-M targets, M radar sensors and M_1 infrared sensors in the network, $M \geq M_1$. Let $V_1 = \{1,2,...,M\}$, $V_2 = \{M+1, M+2..., N\}$ represent the sets of multiagent and multi-target, respectively, $V = V_1 \cup V_2$. Let $D_1 = \{1,2,...,M\}$ and $D_2 = \{M+1, M+2,.., M+M_1\}$ represent the sets of radar sensors and infrared sensors, respectively, $D = D_1 \cup D_2$.

Define $\Pi = [\pi_{ij}]_{N\times N}$, $\pi_{ij} \geq 0$ as the weighted adjacency matrix, which is the corresponding interaction strength of directed edge (j,i), $j,i \in V$, define π_{ij} as

$$\pi_{ij} = \begin{cases} 0, & i = j \quad or \quad (j,i) \notin \varepsilon \\ b_j > 0, j \in V_2 \; and \; (j,i) \in \varepsilon \\ a_{ij} > 0, j \in V_1 \; and \; (j,i) \in \varepsilon \end{cases} \tag{1}$$

where b_j, a_{ij} can both be taken as 1. Define the in-degree of node i $(i \in V)$ as $\deg_{in}(i) = \sum_{j=1}^{N} \pi_{ij}$, then the degree matrix of G_1 is $DEG = \mathrm{diag}\{\deg_{in}(i), i \in V\}$. Define the Laplacian matrix of G_1 as $L = DEG - \Pi$, then one gets $L = \begin{bmatrix} L_1 & L_2 \\ 0 & 0 \end{bmatrix}$ according to (1), where $L_1 \in R^{M\times M}$, $L_2 \in R^{M\times(N-M)}$.

Remark 1: The targets considered in this paper are uncooperative and cannot send their real states to the agents, but the sensors located on the agents can estimate the states of all targets, so all targets can be considered as in-neighbor nodes of the agents and the agents could receive the state estimations from the targets.

B. System model

The state dynamic of target s $(s \in V_2)$ can be modeled as

$$x_{k,s} = \Phi_s x_{k-1,s} + w_{k-1,s}, \tag{2}$$

where $x_{k,s} \in R^n$ is the state of target s at time instant k, $\Phi_s \in R^{n\times n}$ is the system matrix, and $w_{k-1,s} \in R^n$ is the driving Gaussian noise which mean is 0 and covariance matrix is $R_{k-1,s}$.

Define $x_k = [(x_{k,M+1})^T, (x_{k,M+2})^T, ..., (x_{k,N})^T]^T$ as the set of states of all targets at time instant k, then the state dynamics of multi-target is

$$x_k = \dot{\Phi} x_{k-1} + w_{k-1}, \tag{3}$$

where $\dot{\Phi} = \mathrm{diag}\{\Phi_{M+1}, \Phi_{M+2}, ..., \Phi_N\}$, $w_{k-1} = [(w_{k-1,M+1})^T, (w_{k-1,M+2})^T, ..., (w_{k-1,N})^T]^T$, w_{k-1} is the driving Gaussian noise which mean is 0 and covariance matrix is $R_{k-1} = \mathrm{diag}\{R_{k-1,M+1}, R_{k-1,M+2}, ..., R_{k-1,N}\}$.

Suppose all sensors are located in M agents, each agent carries one radar sensor and one or zero infrared sensor. The measuring model of sensor i $(i \in D)$ for target s $(s \in V_2)$ at time instant k is

$$y_{k,s}^i = h_s^i(x_{k,s}) + v_{k,s}^i, \tag{4}$$

where $y_{k,s}^i \in R^m$ is the measurement of sensor i for target s, $h_s^i(\cdot)$ is a nonlinear measurement function, $v_{k,s}^i$ is the Gaussian measurement noise which mean is 0 and covariance matrix is $Q_{k,s}^i$, $h_s^i(\cdot)$ and $Q_{k,s}^i$ can be depicted as

$$\begin{cases} h_s^i(x_{k,s}) = \begin{bmatrix} r_{k,s}^i \\ \theta_{k,s}^i \end{bmatrix}, Q_{k,s}^i = \begin{bmatrix} C_{k,s}^{i,r} & 0 \\ 0 & C_{k,s}^{i,\theta} \end{bmatrix}, i \in D_1 \\ h_s^i(x_{k,s}) = [\theta_{k,s}^i], \quad Q_{k,s}^i = C_{k,s}^{i,\theta}, \qquad i \in D_2 \end{cases}, \tag{5}$$

where $r_{k,s}^i$ and $\theta_{k,s}^i$ are the radial distance and angle between target s and sensor i at time instant k, respectively. $C_{k,s}^{i,r}$ and $C_{k,s}^{i,\theta}$ are the measurement noise covariances of the radial distance and the angle of i, respectively. The distance measurement of the radar sensor i' located in the same agent with the infrared sensor i can be used to expand the dimension of $y_{k,s}^i$, then the measuring model for target s by sensor i $(i \in D)$ at time instant k is

$$\dot{y}_{k,s}^i = \dot{h}_s^i(x_{k,s}) + \dot{v}_{k,s}^i,$$

$$\begin{cases} \dot{h}_s^i(x_{k,s}) = \begin{bmatrix} r_{k,s}^i \\ \theta_{k,s}^i \end{bmatrix}, \dot{Q}_{k,s}^i = \begin{bmatrix} C_{k,s}^{i,r} & 0 \\ 0 & C_{k,s}^{i,\theta} \end{bmatrix}, i \in D_1 \\ \dot{h}_s^i(x_{k,s}) = \begin{bmatrix} r_{k,s}^{i'} \\ \theta_{k,s}^i \end{bmatrix}, \dot{Q}_{k,s}^i = \begin{bmatrix} C_{k,s}^{i',r} & 0 \\ 0 & C_{k,s}^{i,\theta} \end{bmatrix}, i \in D_2 \end{cases}, \tag{6}$$

where $\dot{v}_{k,s}^i$ is the Gaussian measurement noise which mean is 0 and covariance matrix is $\dot{Q}_{k,s}^i$. (6) is used as the measuring model of the sensor network in the filtering algorithm.

Define $y_k^i = [(\dot{y}_{k,M+1}^i)^T, (\dot{y}_{k,M+2}^i)^T, ..., (\dot{y}_{k,N}^i)^T]^T$ as the measurement of sensor i $(i \in D)$ for targets s $(s \in V_2)$ at time instant k. Then the measuring model of sensor i for the multi-target system is

$$y_k^i = h^i(x_k) + v_k^i, \tag{7}$$

where $h^i(x_k)=[(\dot{h}_s^i(x_{k,M+1}))^T,(\dot{h}_s^i(x_{k,M+2}))^T,...,(\dot{h}_s^i(x_{k,N}))^T]^T$, $v_k^i=[(\dot{v}_{k,M+1}^i)^T,(\dot{v}_{k,M+2}^i)^T,...,(\dot{v}_{k,N}^i)^T]^T$, v_k^i is the Gaussian measurement noise which mean is 0 and covariance matrix is $Q_k^i=\mathrm{diag}\{\dot{Q}_{k,M+1}^i,\dot{Q}_{k,M+2}^i,...,\dot{Q}_{k,N}^i\}$.

III. Formation Tracking Based on Distributed MT-CKF Aloritghm

A. Distributed MT-CKF algorithm

1) Initialize

For each sensor $i\ (i\in D)$, initialize the state estimation $\hat{x}_0^i$ and error covariance matrix P_0^i at $k=0$ according to (8) and (9),

$$\hat{x}_0^i=E(x_0), \tag{8}$$

$$P_0^i=E[(x_0-\hat{x}_0^i)(x_0-\hat{x}_0^i)^T]. \tag{9}$$

2) Predict

One gets S_{k-1}^i according to (10)

$$P_{k-1}^i=S_{k-1}^i(S_{k-1}^i)^T. \tag{10}$$

Based on the decomposition results, the cubature points can be calculated as

$$x_{k-1}^{i,e}=S_{k-1}^i[(\xi_{M+1}^e)^T,(\xi_{M+2}^e)^T,...,(\xi_N^e)^T]^T+\hat{x}_{k-1}^i, \tag{11}$$

where $e=1,2,...,2n$, $\xi_s^e\ (s\in V_2)$ is the eth element of ξ_s,

$$\xi_s=\sqrt{n}\underbrace{\left\{\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix},\begin{bmatrix}0\\1\\\vdots\\0\end{bmatrix},...,\begin{bmatrix}0\\0\\\vdots\\1\end{bmatrix},\begin{bmatrix}-1\\0\\\vdots\\0\end{bmatrix},\begin{bmatrix}0\\-1\\\vdots\\0\end{bmatrix},...,\begin{bmatrix}0\\0\\\vdots\\-1\end{bmatrix}\right\}}_{2n}.$$

The propagated cubature points can be calculated by

$$x_{k|k-1}^{i,e}=\Phi x_{k-1}^{i,e}. \tag{12}$$

The priori state estimation $x_{k|k-1}^i$ and priori estimation error covariance matrix $\hat{P}_{k|k-1}^i$ can be obtained by

$$\hat{x}_{k|k-1}^i=\frac{1}{2n}\sum_{e=1}^{2n}x_{k|k-1}^{i,e}, \tag{13}$$

$$\tilde{P}_{k|k-1}^i=\frac{1}{2n}\sum_{e=1}^{2n}x_{k|k-1}^{i,e}(x_{k|k-1}^{i,e})^T-\hat{x}_{k|k-1}^i(\hat{x}_{k|k-1}^i)^T+R_{k-1}=\begin{bmatrix}\tilde{P}_{k|k-1}^{i,[1,1]} & ,...,\tilde{P}_{k|k-1}^{i,[1,N-M]}\\ \vdots & ,\ddots,\ \vdots\\ \tilde{P}_{k|k-1}^{i,[N-M,1]} & ,...,\tilde{P}_{k|k-1}^{i,[N-M,N-M]}\end{bmatrix},$$

$$\hat{P}_{k|k-1}^i=\mathrm{diag}\{\tilde{P}_{k|k-1}^{i,[1,1]},\tilde{P}_{k|k-1}^{i,[2,2]},...,\tilde{P}_{k|k-1}^{i,[N-M,N-M]}\}. \tag{14}$$

3) Update

The obtained cubature points can be transformed into

$$y_{k|k-1}^{i,e}=h^i(x_{k-1}^{i,e}),\ e=1,2,...,2n. \tag{15}$$

Then the prediction of measurement $\hat{y}_k^i$ and error covariance $P_{k|k-1,yy}^i$, $P_{k|k-1,xy}^i$ can be obtained by

$$\hat{y}_k^i=\frac{1}{2n}\sum_{e=1}^{2n}y_{k|k-1}^{i,e}, \tag{16}$$

$$\tilde{P}_{k|k-1,yy}^i=\frac{1}{2n}\sum_{e=1}^{2n}y_{k|k-1}^{i,e}(y_{k|k-1}^{i,e})^T-\hat{y}_k^i(\hat{y}_k^i)^T+Q_k^i=\begin{bmatrix}\tilde{P}_{k|k-1,yy}^{i,[1,1]} & ,...,\tilde{P}_{k|k-1,yy}^{i,[1,N-M]}\\ \vdots & ,\ddots,\ \vdots\\ \tilde{P}_{k|k-1,yy}^{i,[N-M,1]} & ,...,\tilde{P}_{k|k-1,yy}^{i,[N-M,N-M]}\end{bmatrix},$$

$$P_{k|k-1,yy}^i=\mathrm{diag}\{\tilde{P}_{k|k-1,yy}^{i,[1,1]},\tilde{P}_{k|k-1,yy}^{i,[2,2]},...,\tilde{P}_{k|k-1,yy}^{i,[N-M,N-M]}\}, \tag{17}$$

$$\tilde{P}_{k|k-1,xy}^i=\frac{1}{2n}\sum_{e=1}^{2n}x_{k-1}^{i,e}(y_{k|k-1}^{i,e})^T-\hat{x}_{k|k-1}^i(\hat{y}_k^i)^T=\begin{bmatrix}\tilde{P}_{k|k-1,xy}^{i,[1,1]} & ,...,\tilde{P}_{k|k-1,xy}^{i,[1,N-M]}\\ \vdots & ,\ddots,\ \vdots\\ \tilde{P}_{k|k-1,xy}^{i,[N-M,1]} & ,...,\tilde{P}_{k|k-1,xy}^{i,[N-M,N-M]}\end{bmatrix},$$

$$P_{k|k-1,xy}^i=\mathrm{diag}\{\tilde{P}_{k|k-1,xy}^{i,[1,1]},\tilde{P}_{k|k-1,xy}^{i,[2,2]},...,\tilde{P}_{k|k-1,xy}^{i,[N-M,N-M]}\}. \tag{18}$$

The filter gain is

$$K_k^i=P_{k|k-1,xy}^i(P_{k|k-1,yy}^i)^{-1}. \tag{19}$$

The state estimation $\hat{x}_k^i$ and error covariance P_k^i of this multi-target system obtained by sensor $i\ (i\in D)$ at time instant k can be calculated as

$$\hat{x}_k^i=\hat{x}_{k|k-1}^i+K_k^i(y_k^i-\hat{y}_k^i), \tag{20}$$

$$P_k^i=\hat{P}_{k|k-1}^i-K_k^iP_{k|k-1,yy}^i(K_k^i)^T. \tag{21}$$

4) Consensus iterative

Assumption 1: The directed interaction topology of mutiagent is strongly connected, and the sensor located on the agent can receive data from other sensors located on the same agent and the in-neighbor agents. The consensus weights w_{ij} are chosen to make sure that the consensus matrix W is doubly stochastic.

Remark 2: W is doubly stochastic if the sum of the elements in each row and column of W is 1.

In order to make sure that W is doubly stochastic, the elements of W can be chosen according to the Metropolis weights principle as

$$w_{ij} = \begin{cases} \dfrac{1}{1+\max(|N_i|,|N_j|)}, \ if\ i \in D,\ j \in N_i \\ 1-\sum_{j\in N_i} w_{ij},\ if\ i \in D, i=j \\ 0,\ if\ i, j \in D,\ j \notin N_i \end{cases} . \tag{22}$$

Suppose $l\ (l=1,2,...,L)$ is the l th consensus step, $\hat{x}_k^{i,l}$ and $P_k^{i,l}$ are the state estimation and error covariance obtained by sensor i at time instant k after the l th iteration, respectively. Initialize $\hat{x}_k^{i,l}$, $P_k^{i,l}$ as $\hat{x}_k^{i,0}=\hat{x}_k^i$, $P_k^{i,0}=P_k^i$ when $l=0$. Then $\hat{x}_k^{i,l}$ and $P_k^{i,l}$ can be calculated by

$$\hat{x}_k^{i,l} = w_{ii}\hat{x}_k^{i,l-1} + \sum_{j\in N_i} w_{ij}\hat{x}_k^{j,l-1}, \tag{23}$$

$$P_k^{i,l} = w_{ii}P_k^{i,l-1} + \sum_{j\in N_i} w_{ij}P_k^{j,l-1}. \tag{24}$$

When Assumption 1 holds, all elements in $\lim_{L\to+\infty} W^L$ converge to $1/|D|$. According to the fusion rule, after $L \to +\infty$ iterations, the state estimations and error covariances can be achieved to be converge, i.e. for $\forall i, j \in D$, one gets $\hat{x}_k = \hat{x}_k^{i,L} = \hat{x}_k^{j,L} = [(\hat{x}_{k,M+1})^T, (\hat{x}_{k,M+2})^T, ..., (\hat{x}_{k,N})^T]^T$, $P_k = P_k^{i,L} = P_k^{j,L} = diag(P_{k,M+1}, P_{k,M+2}..., P_{k,N})$ when $L \to +\infty$, where $\hat{x}_{k,s}$ and $P_{k,s}$ are the state estimation and error covariance of target $s\ (s \in V_2)$ after $L \to +\infty$ iterations at time instant k, respectively.

Algorithm 1 in Table 1 summarizes the distributed MT-CKF algorithm proposed in this paper.

TABLE I. DISTRIBUTED MT-CKF ALGORITHM

1) Initialization:

$$x_k = \dot{\Phi} x_{k-1} + w_{k-1},$$

$$y_k^i = h^i(x_k) + v_k^i.$$

2) Predict:

$$\hat{x}_{k|k-1}^i = \frac{1}{2n}\sum_{e=1}^{2n} x_{k|k-1}^{i,e},$$

$$\hat{P}_{k|k-1}^i = \mathrm{diag}\{\tilde{P}_{k|k-1}^{i,[1,1]}, \tilde{P}_{k|k-1}^{i,[2,2]}, ..., \tilde{P}_{k|k-1}^{i,[N-M,N-M]}\}.$$

3) Update:

$$P_{k|k-1,yy}^i = \mathrm{diag}\{\tilde{P}_{k|k-1,yy}^{i,[1,1]}, \tilde{P}_{k|k-1,yy}^{i,[2,2]}, ..., \tilde{P}_{k|k-1,yy}^{i,[N-M,N-M]}\},$$

$$P_{k|k-1,xy}^i = \mathrm{diag}\{\tilde{P}_{k|k-1,xy}^{i,[1,1]}, \tilde{P}_{k|k-1,xy}^{i,[2,2]}, ..., \tilde{P}_{k|k-1,xy}^{i,[N-M,N-M]}\},$$

$$K_k^i = P_{k|k-1,xy}^i (P_{k|k-1,yy}^i)^{-1},$$

$$\hat{x}_k^i = \hat{x}_{k|k-1}^i + K_k^i(y_k^i - \hat{y}_k^i),$$

$$P_k^i = \hat{P}_{k|k-1}^i - K_k^i P_{k|k-1,yy}^i (K_k^i)^T.$$

4) Consensus iterative:

A) Initialize:

$$\hat{x}_k^{i,0} = \hat{x}_k^i,\ P_k^{i,0} = P_k^i.$$

B) Choose the consensus matrix as (22).

for $l\ (l=1,2,...,L)$

Exchange information with neighbor nodes and calculate the weighted fused state estimation and error covariance:

$$\hat{x}_k^{i,l} = w_{ii}\hat{x}_k^{i,l-1} + \sum_{j\in N_i} w_{ij}\hat{x}_k^{j,l-1},$$

$$P_k^{i,l} = w_{ii}P_k^{i,l-1} + \sum_{j\in N_i} w_{ij}P_k^{j,l-1}.$$

end

5) The state estimation and error covariance of target $s\ (s \in V_2)$ can be obtained by

$$\hat{x}_k = \hat{x}_k^{i,L} = \hat{x}_k^{j,L} = [(\hat{x}_{k,M+1})^T, (\hat{x}_{k,M+2})^T, ..., (\hat{x}_{k,N})^T]^T,$$

$$P_k = P_k^{i,L} = P_k^{j,L} = diag(P_{k,M+1}, P_{k,M+2}..., P_{k,N}).$$

B. *Time-varying formation tracking based on distributed MT-CKF algorithm*

Suppose $\Phi_s = \Phi\ (s \in V_2)$ in (2), then the state dynamic of target $s\ (s \in V_2)$ are shown in (25)

$$x_{k,s} = \Phi x_{k-1,s} + w_{k-1,s}. \tag{25}$$

After using the MT-CKF algorithm with $L \to +\infty$ iterations, the state estimation for target $s \in V_2$ at time instant $k-1$ is $\hat{x}_{k-1,s}$, and the state dynamics of the agents can be depicted as

$$X_k^c = \Phi X_{k-1}^c + Bu_{k-1}^c,\ c \in V_1, \tag{26}$$

where $X_k^c \in R^n$ is the state of agent c at time instant k, $u_{k-1}^c \in R^r$ is the time-varying formation tracking protocol of c. Note that $B \in R^{n\times r}$ and $rank(B) = r$, $T = [\tilde{B}^T, \overline{B}^T]^T$ is a nonsingular matrix consisted of $\tilde{B} \in R^{r\times n}$ and $\overline{B} \in R^{(n-r)\times n}$, where $\tilde{B}B = I_r$, $\overline{B}B = 0$, I_r is an unit matrix of size r. u_{k-1}^c is designed as

$$\begin{aligned} u_{k-1}^c = {} & K\sum_{j=1}^{M} \pi_{cj}((X_{k-1}^c - g_{k-1}^c) - (X_{k-1}^j - g_{k-1}^j)) \\ & + K\sum_{s=M+1}^{N} \pi_{cs}(X_{k-1}^c - g_{k-1}^c - \hat{x}_{k-1,s}) + n_{k-1}^c \end{aligned}, \tag{27}$$

where $c \in V_1$, $n_{k-1}^c = -\tilde{B}(\Phi g_{k-1}^c - g_k^c) \in R^r$ is the formation tracking compensational signal. $g_{k-1}^c \in R^n$ is the relative state of agent c corresponding to the formation reference $\sum_{s=M+1}^{N} \alpha_s \hat{x}_{k-1,s}$ at time instant $k-1$, $\sum_{s=M+1}^{N} \alpha_s \hat{x}_{k-1,s}$ is the convex combination of the state estimations of all targets at time instant $k-1$, α_s, $s \in V_2$ is a positive constant satisfying $\sum_{s=M+1}^{N} \alpha_s = 1$.

IV. Numerical Simulation

Consider the case that 3 targets need to be tracked. The state of target s at time instant k is $x_{k,s} = [a_{k,s}; b_{k,s}; \dot{a}_{k,s}; \dot{b}_{k,s}]$, $s = \{M+1, M+2, M+3\}$, $k \in [1,100]$, where $a_{k,s}$, $b_{k,s}$ are the position, and $\dot{a}_{k,s}$, $\dot{b}_{k,s}$ are the velocity of target s in direction X and direction Y, respectively. The state dynamics of target s is designed as (2), where

$$\Phi_s = \Phi = \begin{bmatrix} 1 & 0 & \sin(\omega^* T)/\omega & -(1-\cos(\omega^* T)/\omega \\ 0 & 1 & (1-\cos(\omega^* T)/\omega & -\sin(\omega^* T)/\omega \\ 0 & 0 & \cos(\omega^* T) & -\sin(\omega^* T) \\ 0 & 0 & \sin(\omega^* T) & \sin(\omega^* T) \end{bmatrix},$$

$\omega = -\frac{3\pi}{180}$, $T=1$ is the sensing period, $R_{k-1,s}=0.05 I_4$. The initial states of the targets are [-200;-200;10;10], [-200;-210;10;10], [-200;-220;10;10], respectively. The trajectories of the targets are shown in Fig.1.

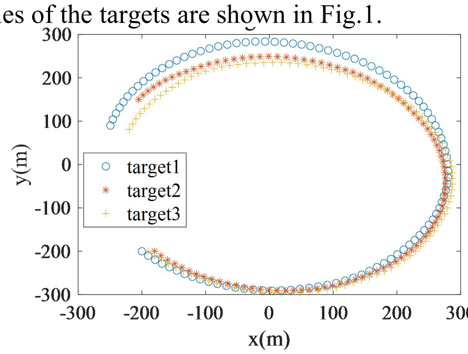

Fig 1. Trajectories of the targets.

Suppose there are 10 agents, each carries one radar sensor and one or zero infrared sensor, and there are 5 infrared sensors in the system, i.e. $V_1=\{1,\ldots,10\}$, $V_2=\{11,12,13\}$, $M_1 = 5$, $M = 10$, $D_1=\{1,\ldots,10\}$, $D_2=\{11,\ldots,15\}$. The state of the agent with sensor i at time instant k is designed as $X_k^i = [A_k^i; B_k^i; \dot{A}_k^i; \dot{B}_k^i]$, $i \in D_1 \cup D_2$. A_k^i, B_k^i and $\dot{A}_k^i$, $\dot{B}_k^i$ are the position and velocity of sensor i in the direction of X and Y, respectively. The measuring model of sensor i for target s at time instant k is depicted as

$$y_{k,s}^i = h^i(x_{k,s}) + v_k^i = \begin{cases} \begin{bmatrix} \sqrt{(A_k^i - a_{k,s})^2 + (B_k^i - b_{k,s})^2} \\ \arctan((B_k^i - b_{k,s})/(A_k^i - a_{k,s})) \end{bmatrix} + v_k^i, i \in D_1 \\ \arctan((B_k^i - b_{k,s})/(A_k^i - a_{k,s})) + v_k^i, i \in D_2 \end{cases}.$$

Since the infrared sensor is more accurate in angle measuring than radar sensor, one gets

$$Q_k^i = \begin{cases} \mathrm{diag}(1, \pi/180), & i \in D_1 \\ 0.2\pi/180, & i \in D_2 \end{cases}.$$

The state dynamics of each agent is taken by (26) with $B = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}^T$, $n_{k-1}^c = 0$, $c = 1, 2, \ldots, 10$, $g_k^c = [30\sin\frac{(c-1)\pi}{5}, 30\sin\frac{(c-1)\pi}{5}, 0, 0]^T$, $K = \begin{bmatrix} 0.0000 & 0.0550 & 0.0014 & -0.0548 \\ -0.0550 & 0.0000 & -0.0547 & -0.0043 \end{bmatrix}$ with initial states [-150;-150;10;10]. Suppose the directed interaction topology of mutiagent $c \in V_1$ and multi-target $s \in V_2$ is as shown in Fig. 2.

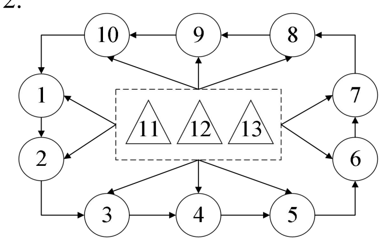

Fig 2. Directed interaction topology.

L is chosen to be 100, and $MC = 50$ independent Monte Carlo simulations are performed to show the average estimation errors $\left\{\frac{1}{MC}\sum_{\eta=1}^{MC}[\| \hat{x}_{k|\eta}^i - x_{k|\eta} \|^2]\right\}^{1/2}$ of the MT-CKF algorithm with the Metropolis weights principle, where $\hat{x}_{k|\eta}^i$ and $x_{k|\eta}$ are the estimated and true states at the η th Monte Carlo simulations, respectively. For comparison, the max-consensus MT-CKF algorithm and the centralized MT-CKF algorithm are also performed in the same scenario. The average estimation errors are shown in Fig. 3. Fig. 3 shows that when $L = 100$, the boundness of all three algorithms is guaranteed and the performance of MT-CKF algorithm with the Metropolis weights principle is superior compared with the performance of max-consensus MT-CKF algorithm.

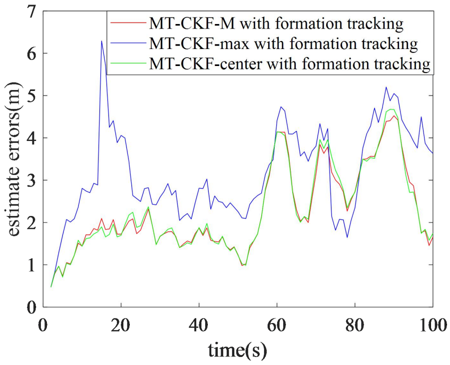

Fig 3. State estimation errors with formation tracking

Fig. 4 shows the state estimations and state estimation errors obtained by the proposed algorithm within one sensing period after several iterations, respectively. Obviously, multi-sensor in the formation can estimate the states of multi-target with the bounded estimation errors, and the state estimations and the state estimation errors can achieve consistent at the current accuracy after about 90 iterations, respectively.

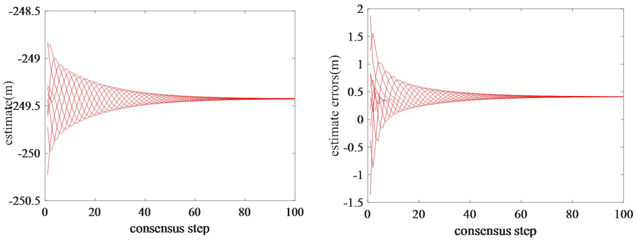

Fig 4. State estimation and state estimation errors for different number of iterations in one sensing period.

The states of multiagent, the state estimations of multi-target and the convex combination of the real states and the state estimations of multi-target are shown in Fig. 5 at $t = 20s, 40s, 60s, 80s$, which are marked as '□', '○', '*', '+', respectively. Obviously, the convex combination of state estimations is close to that of the real states of multi-target, and agents make up a time-varying formation surrounding multi-target.

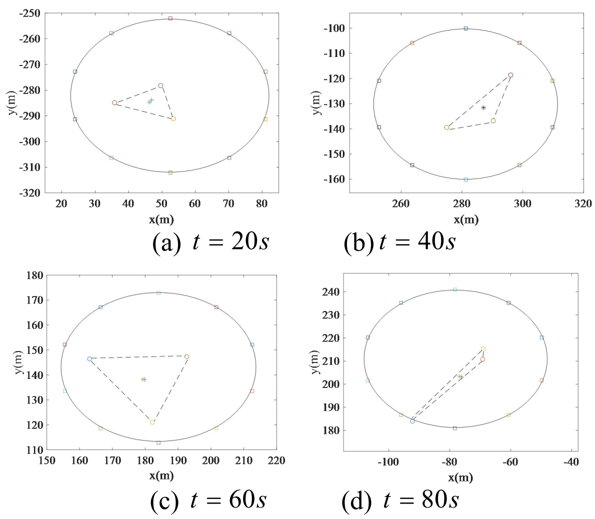

Fig 5. Four time instants of multiagent and multi-target

The formation tracking errors $\| X_k^c - g_k^c - \sum_{s=M+1}^{N} (\frac{b_s}{\sum_{j=M+1}^{N} b_j}\hat{x}_{k,s})\|^2$, $c \in V_1$, $s \in V_2$ are shown in Fig. 6. Obviously, the boundedness of $\lim_{k\to+\infty} E(\| X_k^c - g_k^c - \sum_{s=M+1}^{N} (\frac{b_s}{\sum_{j=M+1}^{N} b_j}\hat{x}_{k,s})\|^2)$ is guaranteed.

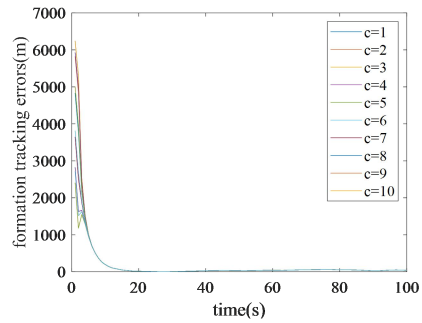

Fig 6. The formation tracking errors.

V. Conclusion

In this paper, a distributed MT-CKF algorithm based on the CKF scheme and heterogeneous sensors for multi-sensor multi-target filtering systems is designed. Then based on the distributed MT-CKF algorithm, a time-varying formation tracking protocol for multiagent with multi-target is proposed. How to design a protocol for multiagent system to form a time-varying formation and track multiple nonlinear targets which states are unknown and have unknown input is our interest in future work.

References

[1] L Wang, H He, Z Zeng, M F Ge, "Model-independent formation tracking of multiple Euler-Lagrange systems via bounded inputs," IEEE Transactions on Cybernetics, vol. 51, no. 5, pp. 2813-2823, May. 2021.

[2] Z Liu, M Wang, J Qi, C Wu, Z Ma, "Time-varying formation tracking control for multi-UAV systems with communication delays and switching topologies," 2022 41st Chinese Control Conference, pp 4802-4807, Jul. 2022.

[3] Y Yuan, Y Wang, L Guo, "Sliding-mode-observer-based time-varying formation tracking for multispacecrafts subjected to switching topologies and time-delays," IEEE Transactions on Automatic Control, vol. 66, no. 8, pp. 3848-3855, Aug. 2021.

[4] X W Dong, "Formation and containment control for high-order linear swarm systems," Berlin, Germany: Springer-Verlag, pp. 36-43, 2015.

[5] R H Zheng, Y H Liu, D Sun, "Enclosing a target by nonholonomic mobile robots with bearing-only measurements,", Automatica, vol. 53, pp. 400-407, Mar. 2015.

[6] B Pratama, A Muis, A Subiantoro, "Improved distributed formation control and trajectory tracking of multi quadrotor in leader-follower formation," 2019 5th International Conference on New Media Studies, pp. 129-134, Oct. 2019.

[7] X W Dong, G Q Hu. "Time-varying formation tracking for linear multiagent systems with multiple leaders," IEEE Transactions on Automatic Control, vol. 62, no. 7, pp. 3658-3664, Jul. 2017.

[8] J Y Hu, P Bhowmick, A Lanzon. "Distributed adaptive time-varying group formation tracking for multiagent systems with multiple leaders on directed graphs," IEEE Transactions on Control of Network Systems, vol. 7, no. 1, pp. 140-150, Mar. 2020.

[9] J L Yu, X W Dong, Z Ren, "Fully adaptive practical time-varying output formation tracking for high-order nonlinear stochastic multiagent system with multiple leaders," IEEE Transactions on Cybernetics, vol. 51, no. 4, pp. 2265-2277, Apr. 2021.

[10] S He, H S Shin, A Tsourdos, "Distributed joint probabilistic data association filter with hybrid fusion strategy," IEEE Transactions on Instrumentation and Measurement, vol. 69, no. 1, pp. 286-300, Jan. 2020.
[11] R Su, J Tang, J Yuan, Y Bi, "Nearest Neighbor Data Association Algorithm Based on Robust Kalman Filtering," 2021 2nd International Symposium on Computer Engineering and Intelligent Communications, pp. 177-181, Aug. 2021.
[12] S Zhao, Y Huang, K Wang, T Chen, "Multi-source data fusion method based on nearest neighbor plot and track data association," 2021 IEEE Sensors, pp. 1-4, Oct. 2021.
[13] Z Duan, P P K Chen, Q Wu, X Hu, "Data association method based on scenario analysis using weak tracking data for multi-target tracking," 2017 International Conference on Machine Learning and Cybernetics, pp. 692-699, Jul. 2017.
[14] M Sun, Z Ma, S Cui, H Lv, "Noise-tuning-based hysteretic noisy chaotic neural network for data association in multi-target tracking," 2014 IEEE Workshop on Electronics, Computer and Applications, pp. 930-933, May. 2014.
[15] R P Mahler, "Random set theory for target tracking and identification", BocaRaton FL: CRC press, 2001.
[16] B T Vo, B N Vo and A Cantoni, "Bayesian filtering with random finite set observations," IEEE Transactions on Signal Processing, vol. 56, no. 4, pp. 1313-1326, Apr. 2008.
[17] D C Dalwadi, H B Soni, "A novel channel estimation technique of MIMO-OFDM system based on Extended Kalman filter," 2017 4th International Conference on Electronics and Communication Systems, pp. 158-163, Feb. 2017.
[18] G Zhang, "An improved UKF algorithm for extracting weak signals based on RBF neural network," IEEE Transactions on Instrumentation and Measurement, vol. 71, pp. 1-14, Jul. 2022.
[19] Z Wang, Y Huang, M Wang, J Wu, Y Zhang, "A computationally efficient outlier-robust cubature Kalman filter for underwater gravity matching navigation," IEEE Transactions on Instrumentation and Measurement, vol. 71, pp. 1-18, Jan. 2022.
[20] M Kooshkbaghi, H J Marquez, "Event-triggered discrete-time cubature Kalman filter for nonlinear dynamical systems with packet dropout," IEEE Transactions on Automatic Control, vol. 65, no. 5, pp. 2278-2285, Oct. 2019.
[21] B Manavoğlu, "Target tracking with passive sensors using CKF," 2018 26th Signal Processing and Communications Applications Conference, pp. 1-4, May. 2018.
[22] Z Zhang, Q Li, L Han, X Dong, Y Liang , Z Ren, "Consensus Based Strong Tracking Adaptive Cubature Kalman Filtering for Nonlinear System Distributed Estimation," IEEE Access, vol. 7, pp. 98820-98831, Jul. 2019.
[23] L Tian, X W Dong, Q L Zhao, Q D Li, J H Lv, Z Ren, "Distributed adaptive time varying output formation tracking for heterogeneous swarm systems," Acta Automatica Sinica, vol. 47, no. 10, pp. 2386−2401, Oct. 2021.
[24] Y Zhang, L Sun, G Hu, "Distributed consensus-based multitarget filtering and its application in formation-containment control," IEEE Transactions on Control of Network Systems, vol. 7, no. 1, pp. 503-515, Mar. 2020.