

OptFlowCam

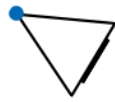
*A 3D-Image-Flow-Based Metric in Camera Space
for Camera Paths in Scenes with Extreme Scale Variations*

Lisa Piotrowski, Michael Motejat, Christian Rössl, and Holger Theisel

April 26, 2024, Limassol, Cyprus

Motivation

Given two virtual camera poses, how do we connect them?

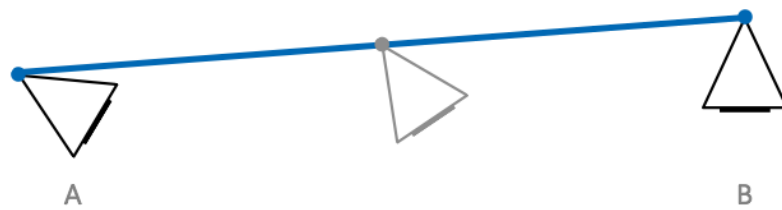


A

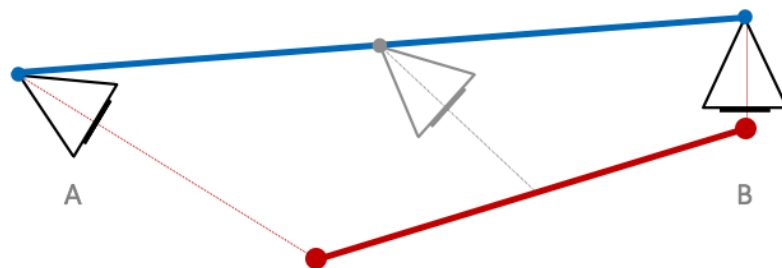


B

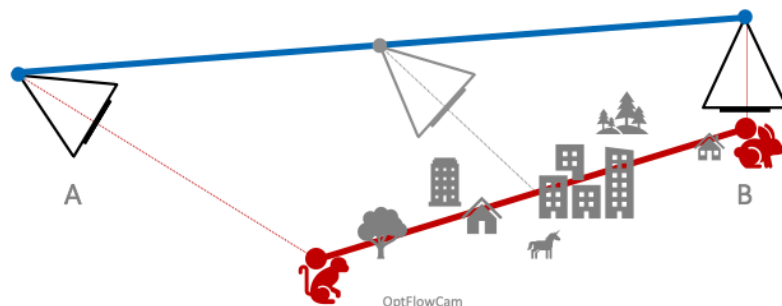
Simplest Idea: Linear Interpolation
of **eye point** and **angle of rotation**



Simplest Idea: Linear Interpolation
of **eye point** and **point of interest**



Simplest Idea: Linear Interpolation
of **eye point** and **point of interest**



Lisa Piotrowski

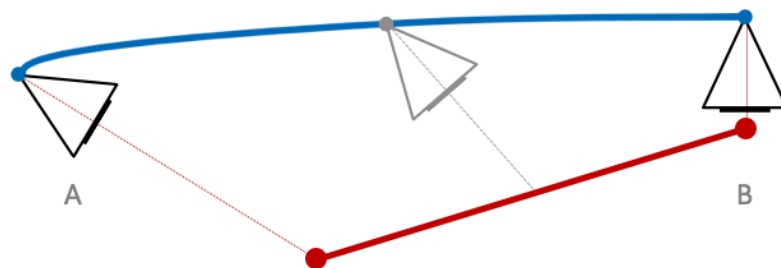
OptFlowCam

5

In the scene there can be anything or nothing between those two camera poses and points of interest. We assume nothing about the scene here.

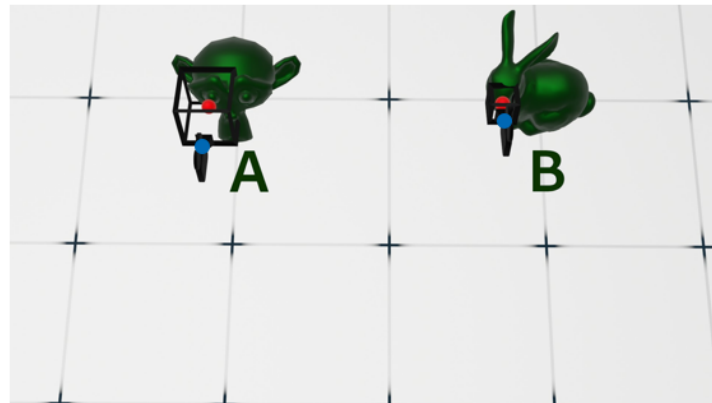
Simplest Idea: Linear Interpolation

of **point of interest**,
distance to the **point of interest**
and **angle of rotation**



Motivation: What is the problem?

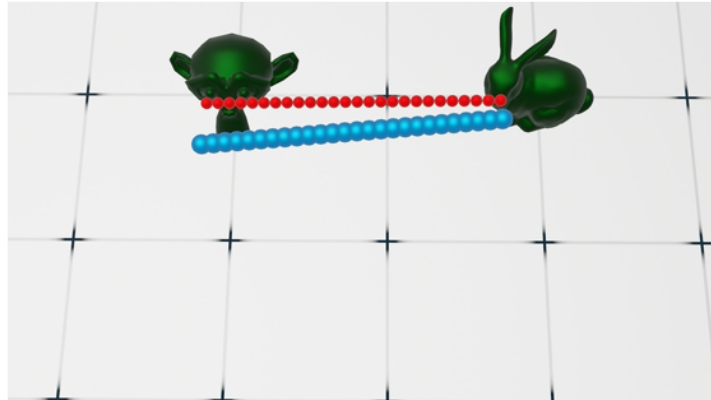
For a scene like this...



In the scene, the blue dots are the eye point of the camera and the red dots the points of interest. The cube is an approximation of the view frustum (i.e. about the extend we can see in the camera view)

Motivation: What is the problem?

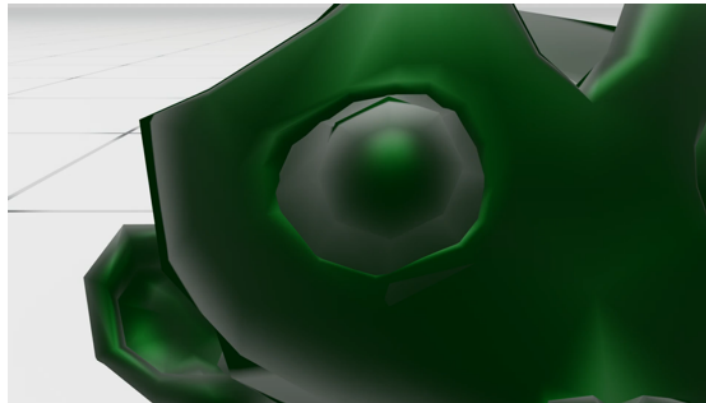
... we get a path like this ...



POI and eye point are linearly interpolated. If sampled at equidistant parameter t , the blue and red dots have the same distance to each other. This doesn't look too bad from the outside, but...

Motivation: What is the problem?

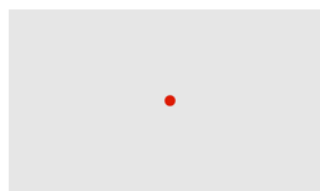
... that looks like this (in Euclidean space) ...



When look at from the camera view, the result is rather jarring. The objects slide in and out of view while being really close to them and half of the time we can't see either of them.

Motivation: What is the problem?

- **Detail-to-detail view without overview in between**
→ can be disorienting
- **Whole path has same velocity**
→ can create effects of *perceived* speeding up/slowing down (esp. when changing scale like Overview-to-detail or Detail-to-overview)



Camera View

The second point couldn't be seen that well in the first video but is very apparent in this example. We seem to speed up approaching the red dot although the camera has a constant "real world" velocity.

Motivation: What would be preferable?

- **Transition between detailed viewpoints with overview in between**
→ creates context
- **Adapt velocity to current scale**
→ similar perceived velocity over the path
- **Closed-form solution of interpolation scheme**
→ easier to integrate into existing systems
(and faster than performing an optimization routine)

Motivation: What can we do?

Observation: Problem is the *Euclidean space*.

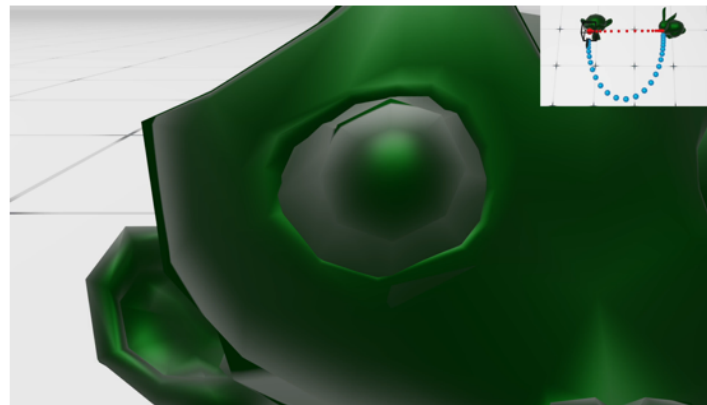
→ **Objective**

- introduce Riemannian metric in *camera space*
- do linear interpolation in *new space*

Euclidean space = world space/geometric space

Motivation: What can we do?

... and then looks like this (in **our new space**):



This is much better than the previous example in Euclidean space because it shows all of the desired properties we talked about in the previous slide.

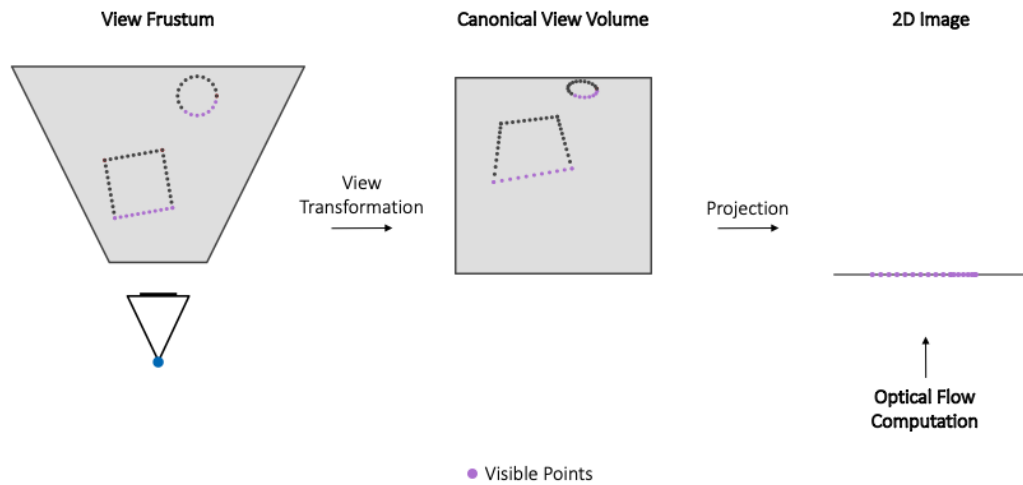
Related Work

- **Interpolation techniques** (linear interpolation of camera parameters or camera transformations [Ale02])
- **Toric Space** [LC15]
- **Zooming and panning in 2D** [vN03]
- **Camera path planning** (lots of different works)



We are firmly in the interpolation techniques section. We do NO path planning or collision avoidance or cinematographic optimization (see paper for more comparisons)

Idea: Base camera path on optical flow



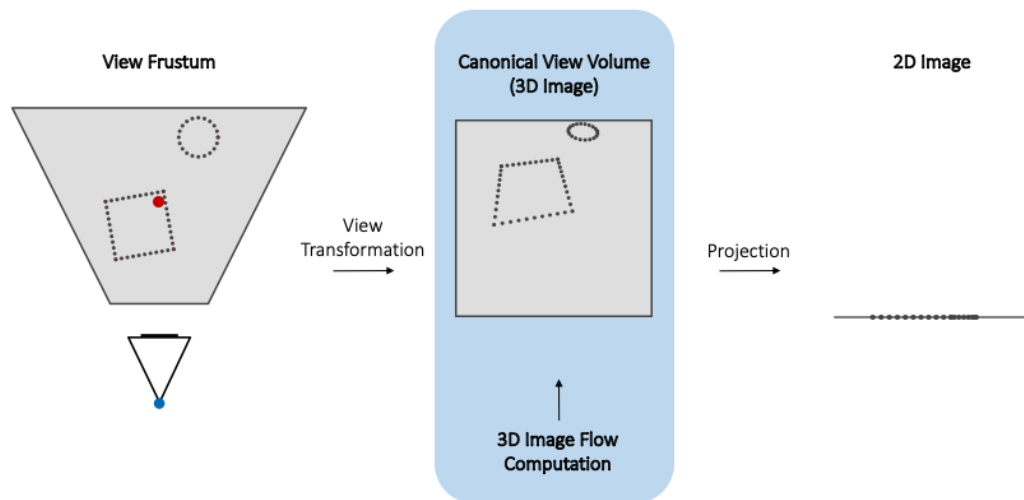
Idea: Base camera path on optical flow

Optical Flow depends on visible surfaces which means...

- ... it needs an optimization routine on a discretized path
- ... but objects entering and exiting the view create discontinuities (which makes it difficult to optimize)
- ... and collisions can derail the optimization routine.

→ need something that is mostly agnostic to the (visible) scene
for a closed-form solution

Idea: Base Camera Path on Flow in the Canonical View Volume (3D Image)



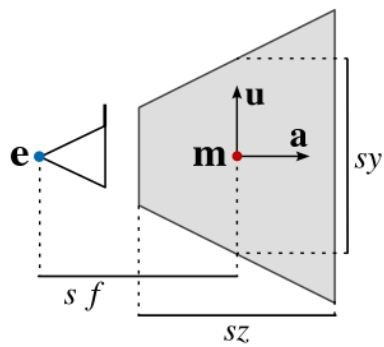
All our schematic examples are in 2D so the projection is to 1D. Of course for a 3D camera the projection is a 2D image on the screen

Idea: Base Camera Path on Flow in the Canonical View Volume (3D Image)

3D image flow ...

- ... does not depend on objects in the scene but the position of the **point of interest** in relation to the **eye point of the camera**
- ... which implicitly defines if it is a detail view or overview
- ... and can be expressed as a closed-form solution.

Method: Camera Model



$$\mathbf{e} = \mathbf{m} - f s \mathbf{a}$$

$$\hat{\mathbf{c}} = \begin{pmatrix} mx \\ my \\ mz \\ sx \\ sy \\ sz \\ \phi \\ \theta \\ \psi \end{pmatrix}$$

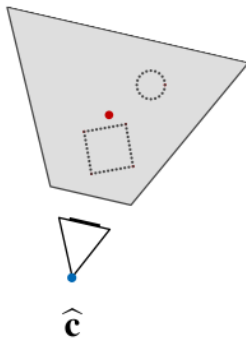
Frustum center point \mathbf{m}

Scaling factors of the frustum
 $s = (sx \cdot sy \cdot sz)^{\frac{1}{3}}$

Euler angles defining the frustum orientation

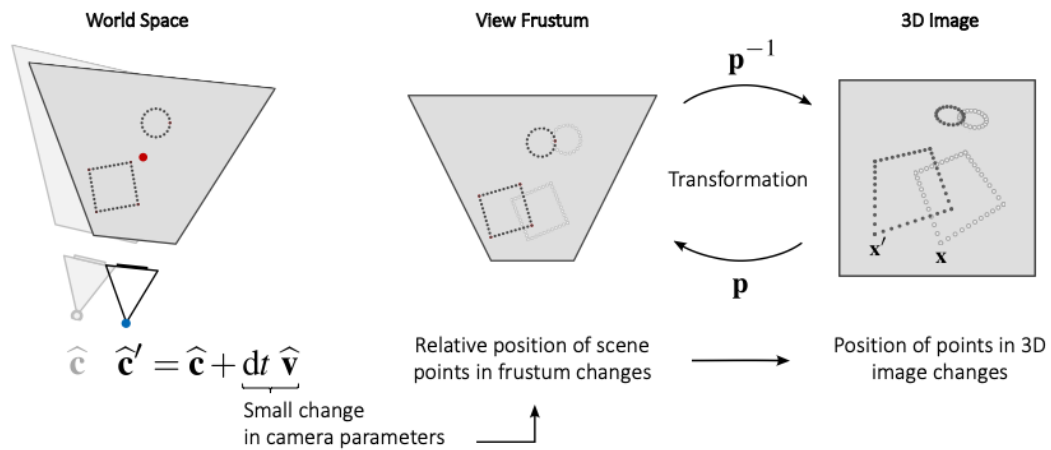
f Constant related to focal length

The eye point (blue dot) is a computed property here. It is possible for most camera models to convert it to our model and vice-versa.



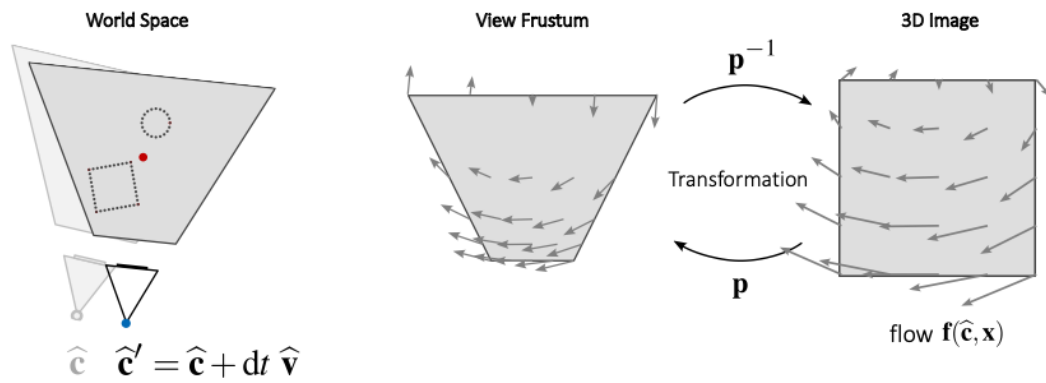
We start with a camera looking at a scene

Method: 3D Image Flow



Moving that camera a little bit changes the relative position of the scene objects in the view frustum. This change is also present in the 3D image if we transform the view frustum to it

Method: 3D Image Flow

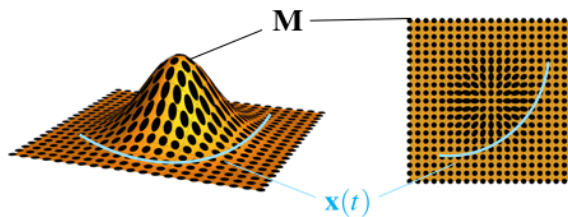


By going infinitesimally small with the camera change ($dt \rightarrow 0$) we define a flow.

Method: What do we do with the 3D image flow?

$$\int_0^1 \dot{\mathbf{x}}(t)^T \mathbf{M}(\mathbf{x}(t)) \dot{\mathbf{x}}(t) dt$$

Energy Functional



Objective: find a space other than Euclidean space for interpolation

- Energy functional describes how "expensive" a path is
→ minimizing gives geodesic equation
- Use 3D image flow to define a new metric and solve geodesic equation
→ shortest path in new space

→ need to define metric tensor \mathbf{M} to solve the geodesic equation for finding geodesic $\mathbf{x}(t)$

The flow can then be used in the energy functional. The energy functional can measure the length of a path (here $\mathbf{x}(t)$) if given a metric (here $\mathbf{M}(t)$). The metric \mathbf{M} defines at every point of a space, how the local lengths look like (here, the ellipses representing the metric in the image say that going up or down the bump make the path longer because they are longer in that direction). So to minimize the path length (or energy we need to travel along it) we minimize the functional. That gives us the geodesic equations (a set of differential equations) which we can solve to get the shortest path (the geodesic).

For us this means we have a two step process:

- Define metric tensor $\mathbf{M}(\mathbf{x}(t))$
- Solve geodesic equation for $\mathbf{x}(t)$

Method: 3D-Image-Flow-Based Metric

$$\begin{aligned}
 & \int_{\Omega} \mathbf{f}^T \mathbf{f} \, d\mathbf{x} \\
 &= \hat{\mathbf{v}}^T \underbrace{\hat{\mathbf{M}}(\hat{\mathbf{c}})}_{\substack{\text{3D-Image-Flow-Based} \\ \text{Metric}}} \hat{\mathbf{v}}
 \end{aligned}
 \left. \begin{array}{l} \text{Energy functional} \\ \text{(squared 3D image flow magnitude} \\ \text{over whole 3D image volume)} \end{array} \right\} \begin{array}{l} \text{can be written in terms of metric} \end{array}$$

→ has a (complicated) closed-form solution!

We use the flow magnitude to judge the length of the path. By expanding this, we can rearrange the terms to get out the metric which, with the current camera model, has a very complicated solution

[illegible]

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OptFlowCam

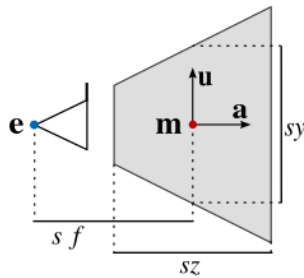
25

Geodesic equation: second order differential equation

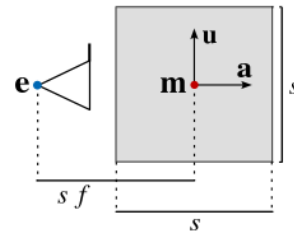
-> basically impossible to solve with this metric, so having the metric is only half the battle

Method: Metric Simplification

Replace frustum by best-fitting cube



$$\hat{\mathbf{c}} = (mx, my, mz, sx, sy, sz, \phi, \theta, \psi)^T$$



$$\bar{\mathbf{c}} = (mx, my, mz, s, \phi, \theta, \psi)^T$$

Simplifying the camera space simplifies the metric a lot. We also tested that calculating the energy functional with simplified metric gives a similar result to the original metric -> can conclude the simplification is justified

Method: Metric Simplification

Replace frustum by best-fitting cube

$$\overline{\mathbf{M}}(\vec{c}) = \begin{pmatrix} \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4s^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & -\frac{\sin \theta}{6} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sin \theta}{6} & 0 & \frac{1}{6} \end{pmatrix}$$

translation + scale
rotation

→ use metric to solve geodesic equation
translation and scale can be handled independently of rotation

Have two independent blocks which allow us to solve the geodesic equation independently for each block

Method: Geodesics - Translation and Scale

$$\overline{\mathbf{M}}(\bar{\mathbf{c}}) = \begin{pmatrix} \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4s^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & -\frac{\sin \theta}{6} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sin \theta}{6} & 0 & \frac{1}{6} \end{pmatrix}$$

translation + scale

Method: Geodesics - Translation and Scale

Some helpful definitions

$$\left\{ \begin{array}{l} d = \|\mathbf{m}_1 - \mathbf{m}_0\|, \quad \mathbf{d} = \frac{\mathbf{m}_1 - \mathbf{m}_0}{d} \\ b_i = \frac{s_1^2 - s_0^2 + (-1)^i 4d^2}{4s_i d} \\ r_i = \ln(-b_i + \sqrt{b_i^2 + 1}) \end{array} \right. \quad \text{for } i = 0, 1$$

Scale changes faster
if current scale is higher
(i.e. camera further away from POI)

$$\left\{ \begin{array}{l} s(t) = \frac{s_0 \cosh(r_0)}{\cosh((r_1 - r_0)t + r_0)} \end{array} \right.$$

Frustum position changes faster
if current scale is high

$$\left\{ \begin{array}{l} m(t) = \frac{s_0}{2} \cosh(r_0) \tanh((r_1 - r_0)t + r_0) - \frac{s_0}{2} \sinh(r_0) \\ \mathbf{m}(t) = \mathbf{m}_0 + m(t) \mathbf{d}. \end{array} \right.$$

→ slower movement when close to POI, faster movement when further away
zooming in/out in between viewpoints depending on distance and scale

Method: Geodesics - Translation and Scale

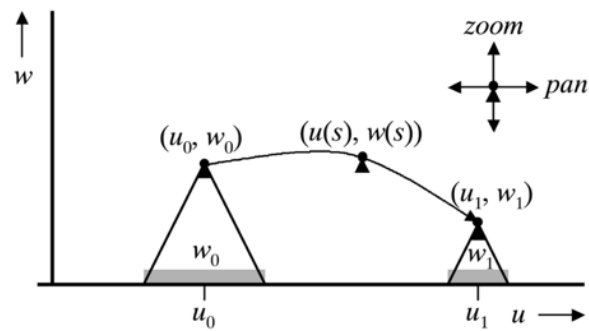


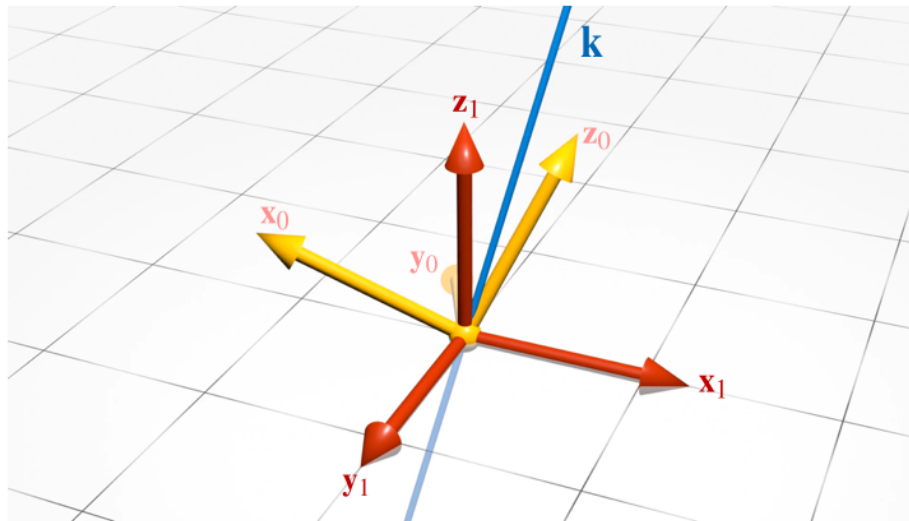
Image: VAN WIJK J. J., NUIJ W. A. A.: Smooth and efficient zooming and panning. In IEEE Symposium on Information Visualization 2003 (IEEE Cat. No.03TH8714) (Oct 2003), IEEE Computer Society, pp. 15–23. doi: 10.1109/INFVIS.2003.1249004

Very similar work to ours for 2D zooming and panning in images.

Method: Geodesics - Rotation

$$\overline{\mathbf{M}}(\bar{\mathbf{c}}) = \begin{pmatrix} \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{s^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4s^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & -\frac{\sin \theta}{6} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\sin \theta}{6} & 0 & \frac{1}{6} \end{pmatrix}$$

rotation



Rotation turns out to be a constant speed rotation around a fixed axis of rotation. This axis of rotation (k) can be calculated by solving an eigenvalue problem in 3D.

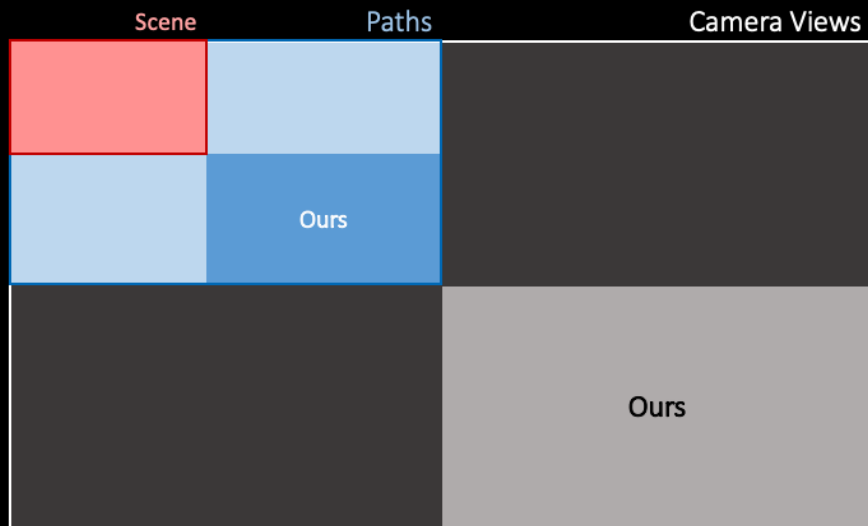
Method: Geodesics - Rotation

Rotation matrix transforming
axis of rotation \mathbf{k}
to/from the z-axis
(solution of an eigenvalue problem)

$$\underbrace{\mathbf{R}(t)}_{\text{Final orientation matrix}} = \underbrace{\mathbf{R}_{\mathbf{k}}}_{\text{Rotation matrix rotating by an angle of } t\xi \text{ around the z-axis}} \underbrace{\mathbf{R}_{\xi}(t)}_{\text{Rotation matrix rotating by an angle of } t\xi \text{ around the z-axis}} \underbrace{\mathbf{R}_{\mathbf{k}}^T}_{\text{Rotation matrix rotating by an angle of } t\xi \text{ around the z-axis}} \underbrace{\mathbf{R}_0}_{\text{Initial orientation matrix}}$$

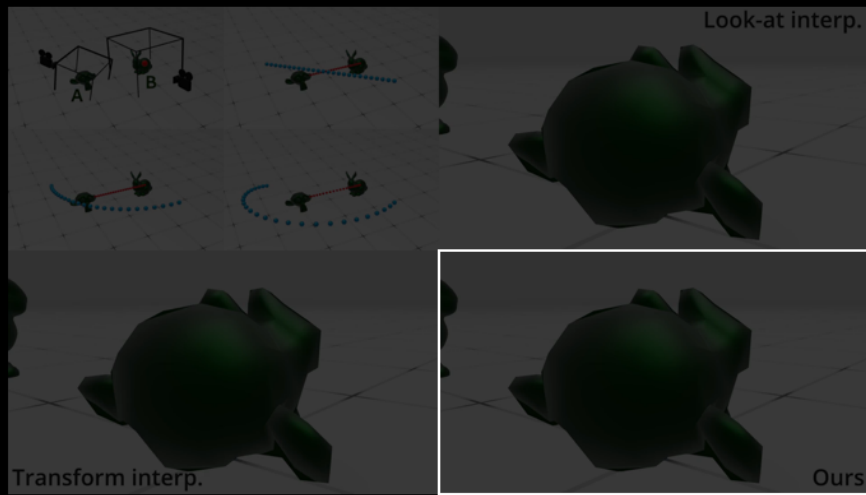
→ final rotation is around a constant axis with constant speed

Results: Video Layout



Layout of the video examples because there is so much going on in them that it's easy to get overwhelmed.

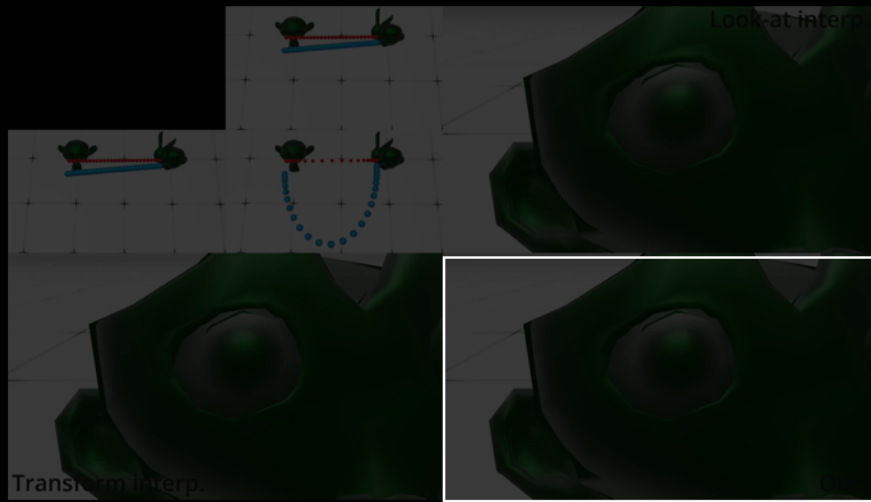
Results: Scenario 1



[View on YouTube](#)

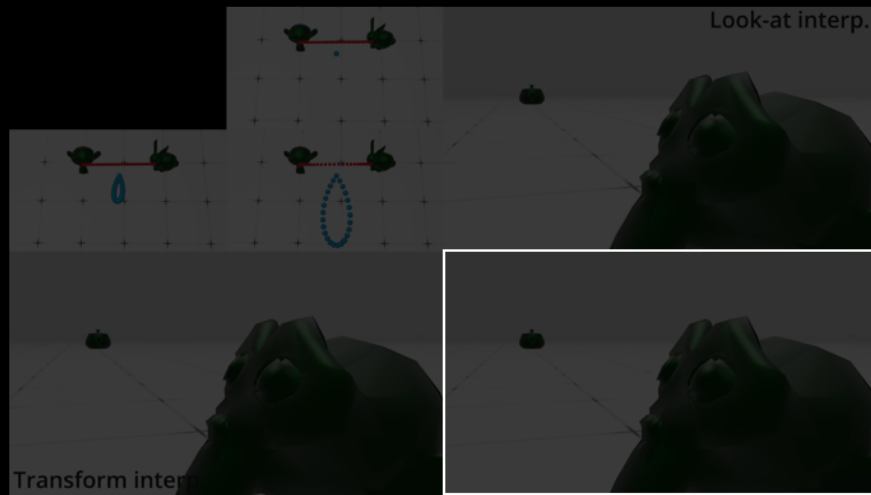
Nice zooming out/in in for out method. Alternative methods in the upper right and lower left loose sight of the scene objects

Results: Scenario 2



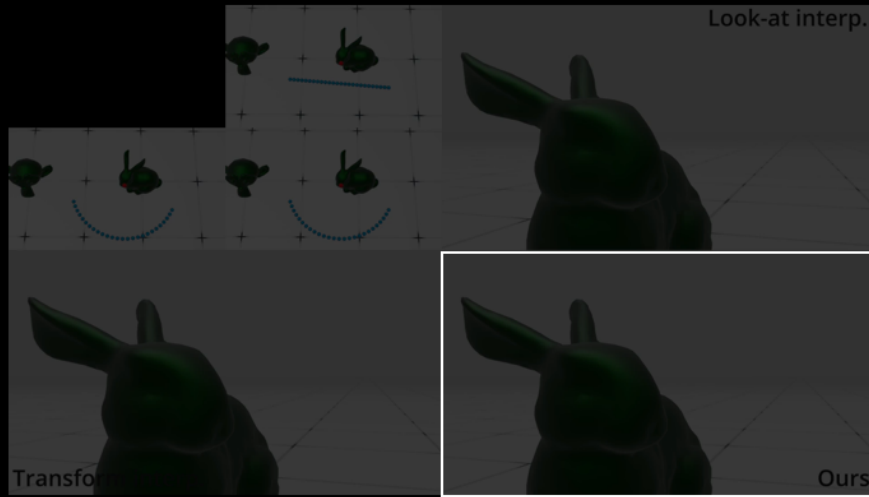
[View on YouTube](#)

Results: Scenario 3



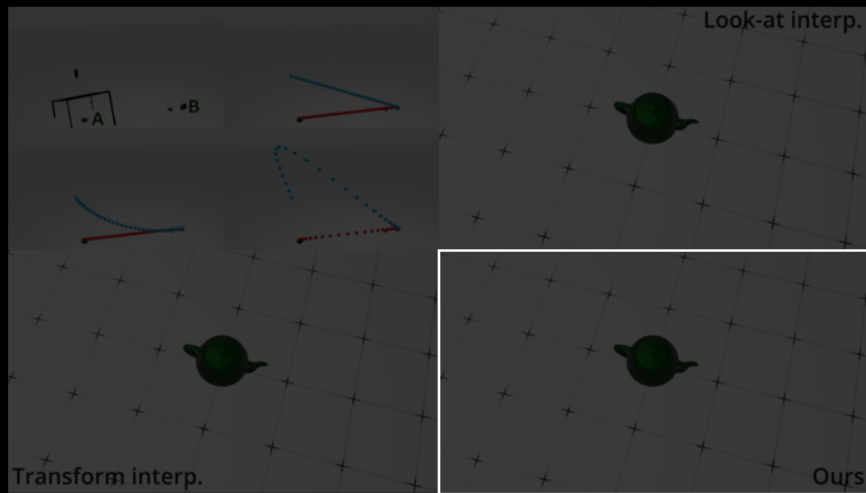
[View on YouTube](#)

Results: Scenario 4



[View on YouTube](#)

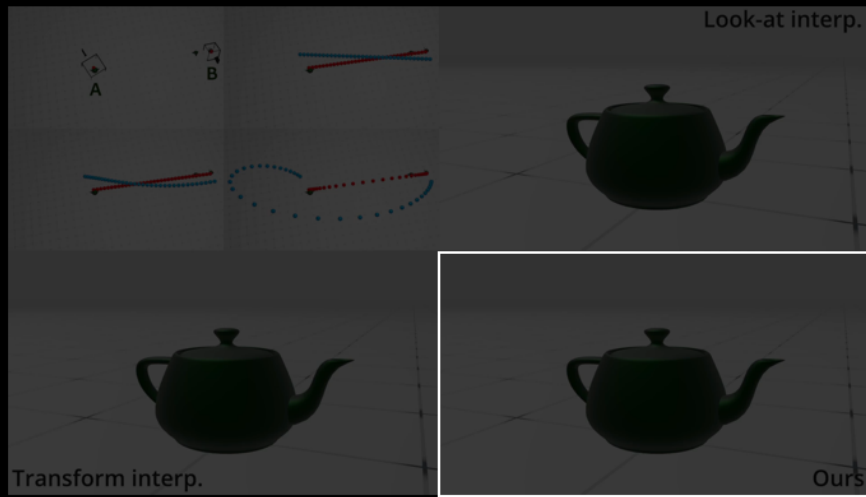
Results: Scenario 5



[View on YouTube](#)

Again nice zooming out/in for our method. We actually see the objects most of the time while the other methods just look at the floor.

Results: Scenario 6



Results: 3D Fractal

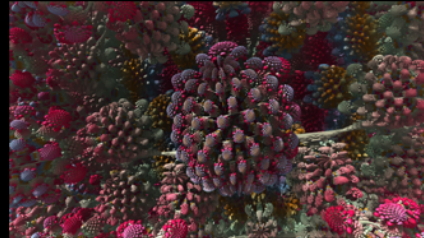
Keyframe Start



Camera distance to POI:
2 units

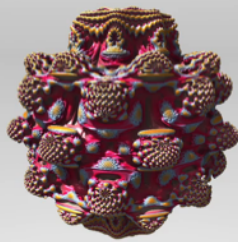


Keyframe End

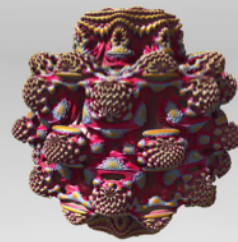


Camera distance to POI:
 1.4×10^{-12} units

Linear Interpolation (Euclidean Space)



Linear Interpolation (Our Metric Space)

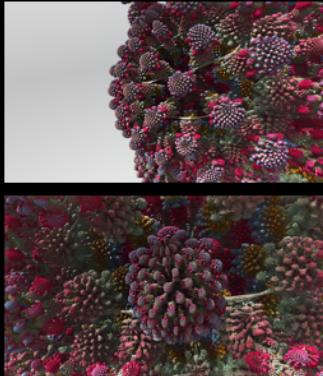


[View on YouTube](#)

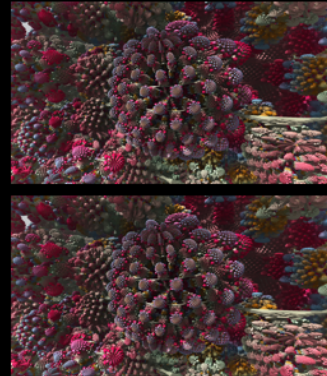
Euclidean space interpolation looks like its not moving at all in the beginning and then “crashes” into the fractal. Our method gives an apparent constant speed impression

Results: 3D Fractal

Linear Interpolation (Euclidean Space)



Linear Interpolation (Our Metric Space)



Frame 499

Frame 500

Comparison between the second to last and last frame of both methods. Euclidean method clearly has issues.

Method: Spline Interpolation

We can also chain multiple camera poses into one path.

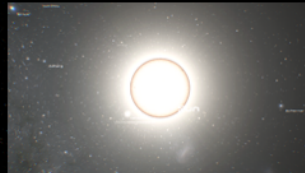
- In any spline evaluation scheme that is based on repeated linear interpolation we could substitute linear interpolation for our geodesics.
- We used Catmull-Rom curves in our examples.

Keyframe 1



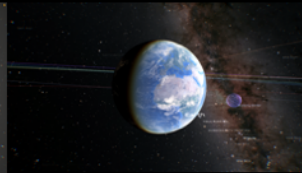
Sun (far away)
 1.21×10^{14} km

Keyframe 2



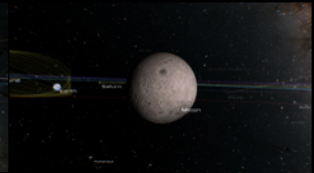
Sun (close)
 3.22×10^6 km

Keyframe 3



Earth
 2.19×10^4 km

Keyframe 4



Moon
 8.39×10^3 km

4 keyframe spline in Gaia Sky with varying distances

Catmull-Rom (Euclidean Space)

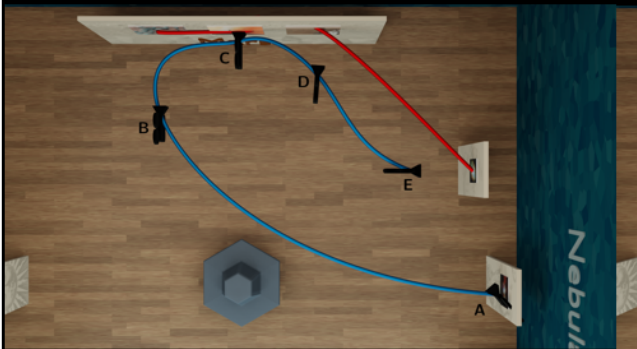


Catmull-Rom (Our Metric Space)

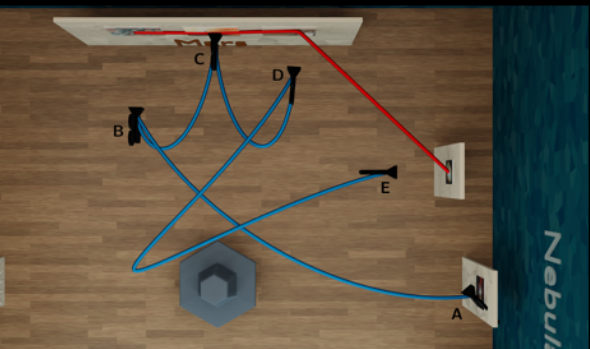
[View on YouTube](#)

Euclidean method looks like it's not moving at the beginning then overshoots all the keyframe positions, so it's basically not possible to see anything of the interesting parts. Our method gives a nice path.

Catmull-Rom (Euclidean Space)



Catmull-Rom (Our Metric Space)



5 keyframe path in a gallery. Euclidean space path slides along the pictures in the gallery so it's basically impossible to see anything. Our path looks like it would not be smooth but because the camera moves very slowly at the keyframe positions, it still looks smooth



Results: Gallery

Catmull-Rom (Euclidean Space)

Catmull-Rom (Our Metric Space)



Images: Courtesy of NASA/JPL-Caltech.

[View on YouTube](#)

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Results: Mars Rover



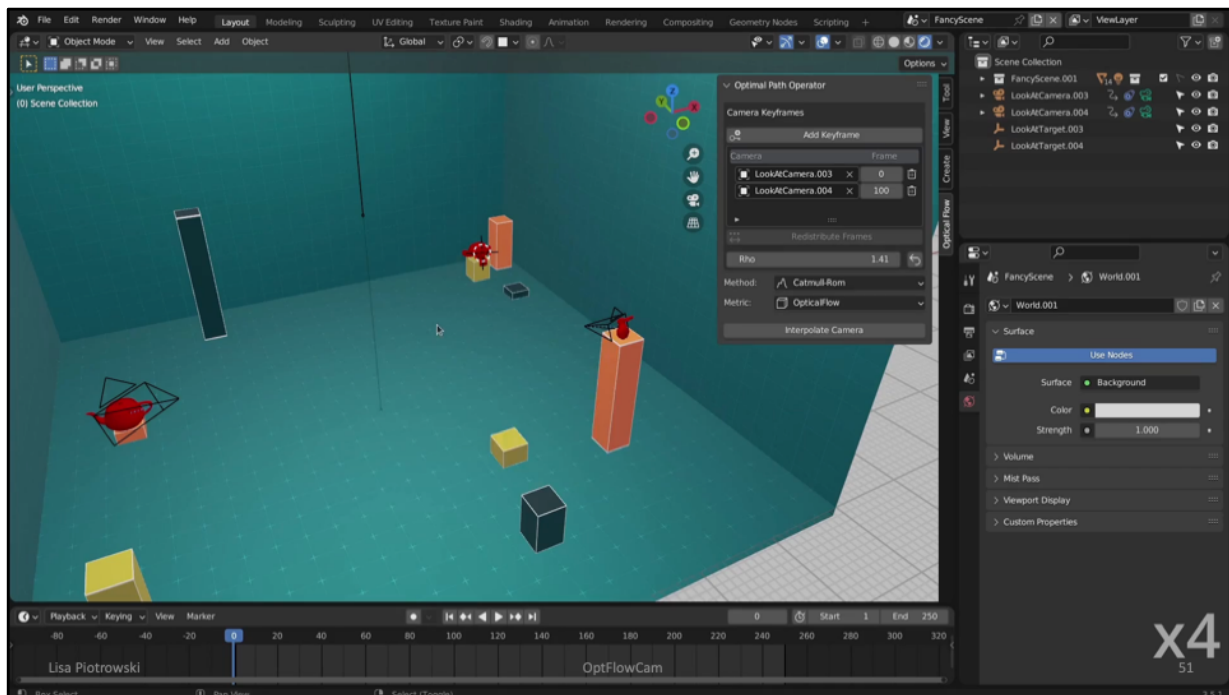
[View on YouTube](#)

Conclusion

We introduced a new camera metric space that...

- ... is **simple** to use
- ... gives us **closed-form geodesics**
- ... can handle **extreme scale variations** in scenes
- ... can be used in **interactive contexts**

Of course we also have limitations which we discuss in the paper



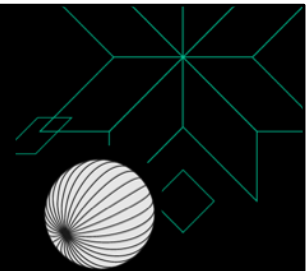
Blender plugin demonstration



OptFlowCam

A 3D-Image-Flow-Based Metric
in Camera Space
for Camera Paths in Scenes
with Extreme Scale Variations

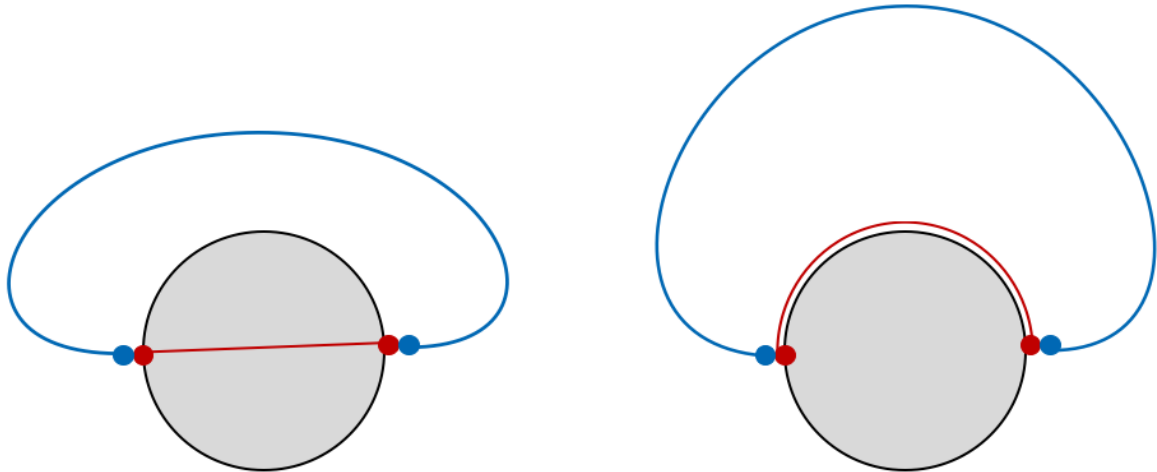
*Lisa Piotrowski, Michael Motejat
Christian Rössl, and Holger Theisel*



Method: Length of Geodesic Path

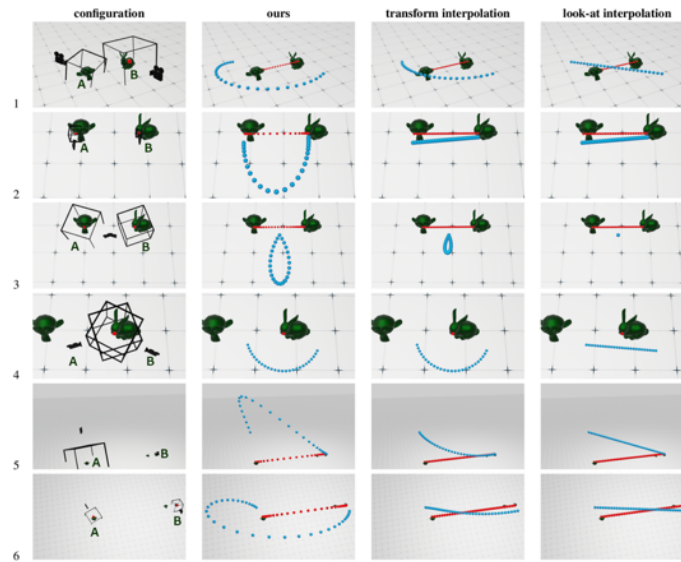
$$\begin{aligned}\text{dist}(\bar{\mathbf{c}}_0, \bar{\mathbf{c}}_1) &= \int_0^1 \sqrt{\dot{\bar{\mathbf{c}}}(t)^T \mathbf{M}(\bar{\mathbf{c}}(t)) \dot{\bar{\mathbf{c}}}(t)} \, dt \\ &= \sqrt{\frac{(r_1 - r_0)^2}{4} + \frac{\xi^2}{6}}.\end{aligned}$$

Interpolation of Opposite Points

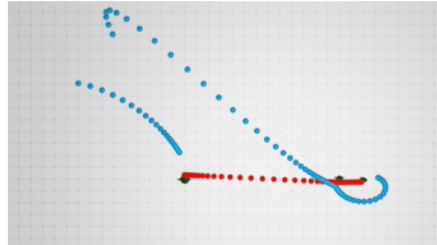
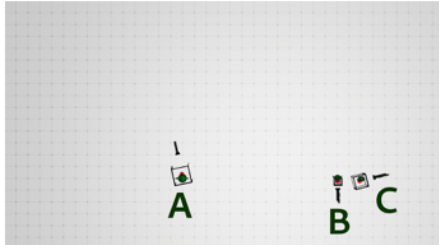


One limitation is that we do not consider any shape of an object. So if we have opposite points of the sphere, the path might not "zoom out" enough to be pleasant. We would rather want something as on the right

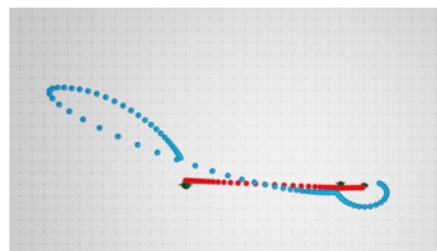
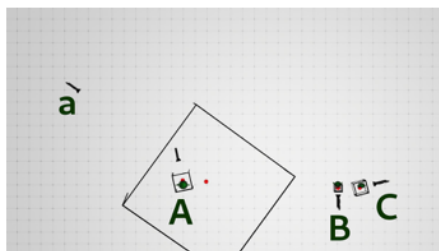
Results: Simple Scenes



Results: Spline

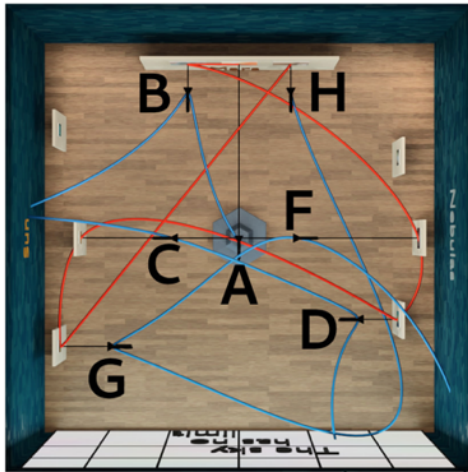


*Spline with discontinuity
caused by the Catmull-
Rom evaluation*



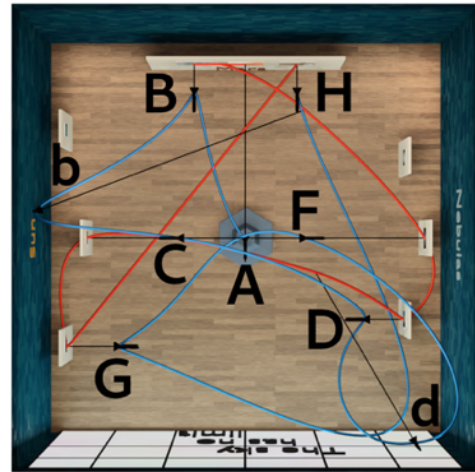
*Spline with discontinuity
resolved by using an
additional keyframe (a)*

Another limitation with Catmull-Rom curves creating gaps



With collision

Lisa Piotrowski

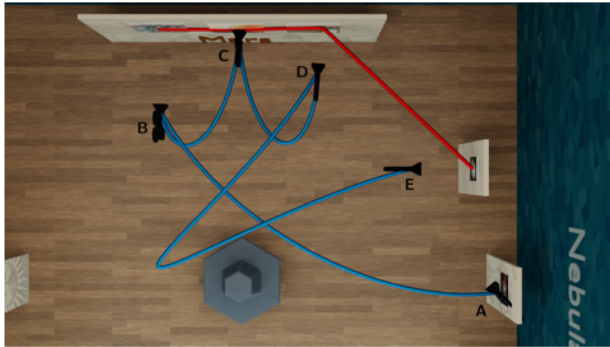


Without collision (and additional keyframes b + d)

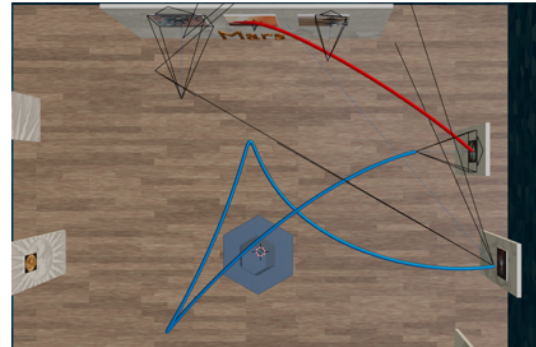
OptFlowCam

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Results: Different Spline Curves

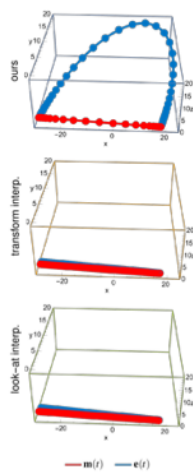


Catmull-Rom

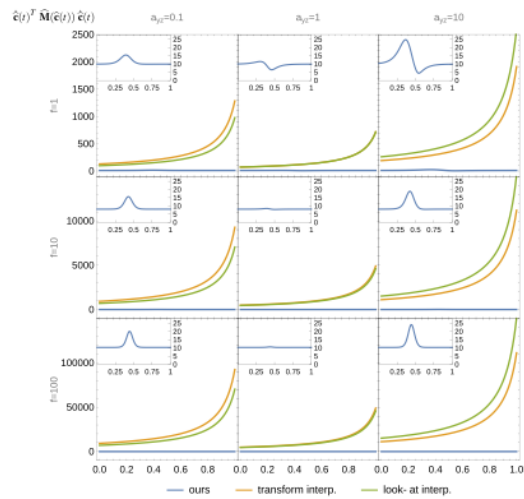


Bézier Curve

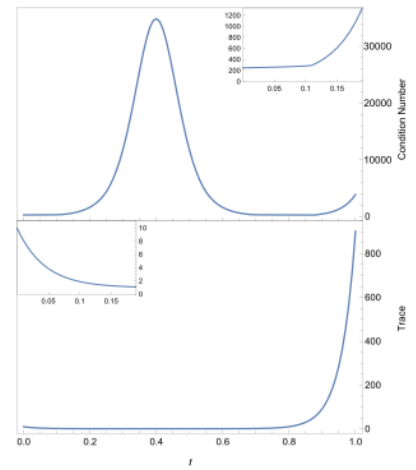
Results: Metric and Velocity Along Path



Paths



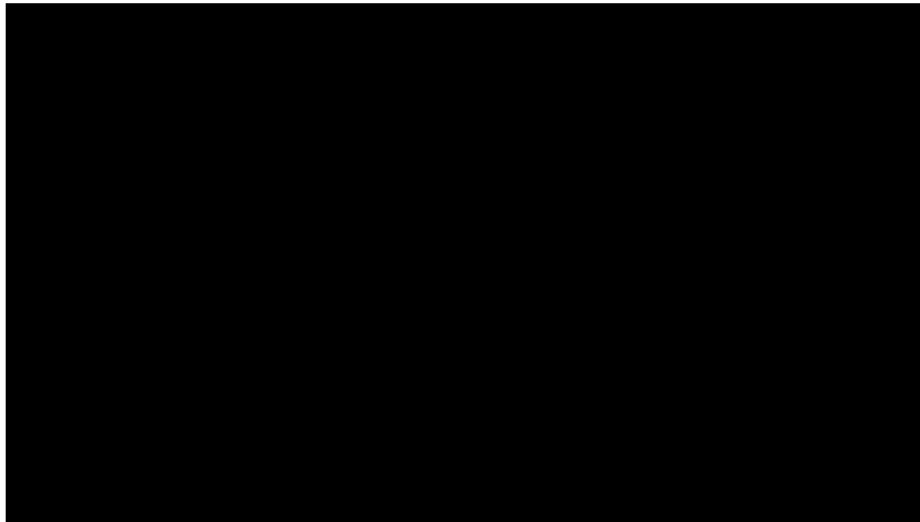
Average Squared Flow Magnitude



Condition Number and Trace



Results: Scenes



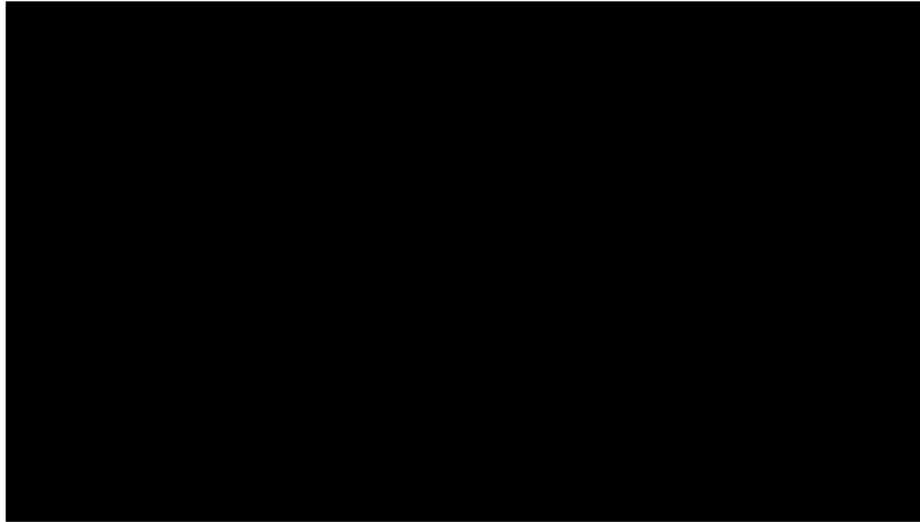
[View on YouTube](#)

OptFlowCam

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Results: Interactive Blender Session



[View on YouTube](#)
OptFlowCam