Calculating P(A), P(B), and P(A|B)

1. ***You work for a risk analysis insurer. You have read that this year, out of all drivers on the road, 5% have had accidents under the age of 25. You have also read that 10% of all drivers are under the age of 25. A new client approaches you and states that their age is 22. You want to calculate the chance that this driver has had an accident this year based on their age.***

   A: The event that a driver under the age of 25 has had an accident.
   B: The event that a driver is under the age of 25.

   P(A) = 0.05

   P(B) = 0.10

   P(A|B) = P(A) = 0.05

   Since P(A) already describes the probability of an accident for drivers under 25, the chance that this driver has had an accident this year is **5%**.

2. ***Your friend told you that they would buy you lunch if you can flip a coin and have it land on heads twice. You flip it the first time, and it lands on heads. What are your chances now of it landing on heads again?***

   A: The event of landing on heads on the first flip.
   B: The event of landing on heads on the second flip.

   P(A) = 0.5

   P(B) = 0.5

   P(B|A) = P(B) = 0.5

   Therefore, the chance of the coin landing on heads again is **50%**.

3. ***You were always told that knowing Maths helps you to achieve 80% in Computer Science. You read some statistics showing that 30% of all Computer Science graduates took Maths and achieved 80%. Overall, 60% of all Computer Science graduates took Maths. Considering you took Maths, what are your chances of achieving 80%?***

   A: The event of achieving 80% in Computer Science.
   B: The event of taking Maths.

   P(A|B) = 0.30

   P(B) = 0.60

   P(A and B) = 0.30 * 0.60 = 0.18

   P(A|B) = $\frac{\text{P(A and B)}}{\text{P(B)}} = \frac{0.18}{0.60}$ = 0.30

   Therefore, the chance of achieving 80% in Computer Science given that you took Maths is **30%**.

Bayes' Theorem

H (Hypothesis): The participant has Covid.
E (Evidence): The participant tested positive for Covid.

- **P(H):** The probability that a participant has Covid:

$$\boxed{P(H) = \frac{\text{Number of participants with Covid}}{\text{Total number of participants}} = \frac{70}{100} = 0.7}$$

- **P(E|H):** The probability of testing positive given that the participant has Covid:

$$P(E|H) = \frac{\text{Number of participants with Covid who tested positive}}{\text{Number of participants with Covid}} = \frac{63}{70} = 0.9$$

- **P(E):** The total probability of testing positive:

$$P(E) = \frac{\text{Total number of participants who tested positive}}{\text{Total number of participants}} = \frac{64}{100} = 0.64$$

- **P(H|E):** The posterior probability that a participant has Covid given that they tested positive. This is what we calculate using Bayes' Theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)} = \frac{0.9 * 0.7}{0.64} = \frac{0.63}{0.64} = 0.984$$

Therefore, the probability that a participant has Covid given that they tested positive is approximately **98.4%**.