Optimizing Search and Rescue Strategies for Deep-Sea Submersibles Using the Monte Carlo Method

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Abstract. Deep-sea exploration is an important field where humans challenge the limits of nature. Improving its safety and efficiency is of great significance to deep-sea scientific research and resource development. In response to this, this study established a Monte Carlo-based motion model to predict the location of the crashed submersible. To truly simulate the distress environment of the submersible, this study introduced the Monte Carlo method to randomly generate 2,000 particles to reflect the uncertainty of the initial distress location, ocean current direction, and seawater density. Finally, it was combined with a six-degree-of-freedom kinematic model to generate All possible time-varying trajectories. This study also compared the effects before and after using the corresponding equipment and found that the equipment has a good effect on reducing uncertainties. This paper expands the field focused on floating objects on the sea surface to the search for underwater targets.

Keywords: Monte Carlo, Six-degree-of-freedom, Particles, Trajectories, Submersible.

1. Introduction

When exploring unknown deep-sea environments, the safety of manned submersibles is of paramount importance. The deep sea, as one of the last underexplored frontiers on Earth, has a complex and challenging environment that places extremely high demands on submersibles and their operating teams. To ensure the safety and efficiency of deep-sea exploration, it is particularly important to research and develop targeted simulation models and optimization strategies. The systematic research and application of search theory is mainly concentrated in the field of search for floating objects on the sea surface [1,2].

For this study, this paper set out to use the Monte Carlo algorithm and six degrees of freedom kinematic equations. Tu H W et al. used Monte Carlo methods to compare and evaluate lifeboat drift prediction models [3]. Goldfrank J et al. applied the Monte Carlo method to study target tracking on offshore mobile platforms [4]. Yi J T et al. used the Monte Carlo algorithm to study the dynamic installation of self-penetrating torpedo anchors [5]. Wang J R et al. applied the six-degree-of-freedom motion equation to study the effect of ISW on submersibles [6]. Mu L et al. applied the Monte Carlo algorithm to predict the drift position of the wreckage of the crashed aircraft [7].

This paper obtained geographical environment data on the Ionian Sea's terrain, ocean currents, salinity, etc. from different authoritative websites. What's more, this paper visualized the above data and obtained a three-dimensional geographical environment map of the Ionian Sea. And then, this paper simplified the shape of the submersible into a capsule shape. Conduct force analysis on the submersible in the established coordinate system and list the six-degree-of-freedom kinematic equations of the unpowered submersible. By introducing a Monte Carlo algorithm, the uncertainty of the initial distress location, ocean current direction, and seawater density was simulated and finally combined with a six-degree-of-freedom kinematic model to generate all possible time-changing motion trajectories. To reduce uncertainty, this paper also collects the parameters of the corresponding equipment and compare the effects before and after use.

2. Seabed Topography and Ocean Dynamics Analysis

To better establish and solve the model, this paper mainly obtained the ocean current, temperature, salinity distribution, and other data of the Ionian Sea in the Poseidon System. At the same time, the seabed topographic map of this sea area was also obtained from GEBCO. The URLs of the above data sources and the URLs of the remaining data are summarized in Table 1.

Table 1: Database source

Names	Websites
Wikipedia	en.wikipedia.org/wiki/Ionian_Sea
World Cruising & Sailing Wiki	www.cruiserswiki.org/wiki/Ionian_Sea
Poseidon System	www.poseidon.hcmr.gr/
Earth NullSchool	https://earth.nullschool.net/
GEBCO	www.gebco.net

Figure 1A is a remote sensing image of the Ionian Sea, and figure 1B is a 3D map of the seabed topography of the Ionian Sea. From figure 1A, this paper can see the scope of the Ionian Sea and the depth distribution of the sea area. Figure 1B further visually shows the situation of the seabed terrain.

Specifically, the salinity of this sea area is 38‰ and most of the depth exceeds 3,000 meters. The surface currents in the Ionian Sea are roughly counterclockwise: they flow towards the north up the Greek coast and then turn west and south along the Italian coast. In general, this current is not very strong, and it rarely exceeds 1.2 knots.

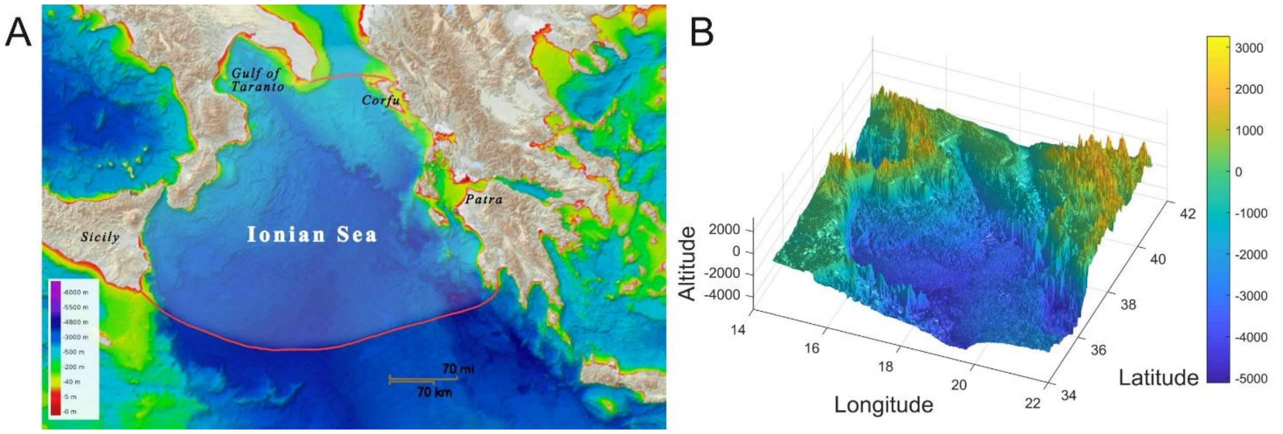

Figure 1: Data Screening A: geographical location B:3D terrain

As shown in figure 2, the shape of the submersible is simplified, and then the body coordinate system and the ground coordinate system are established. The body coordinate system: The body coordinate system is established on the center of gravity of the submersible and is fixed with the submersible body. The ground coordinate system: its origin can take the sea surface or any point in the sea. In order to establish the six-degree-of-freedom kinematic equation, the above work is necessary.

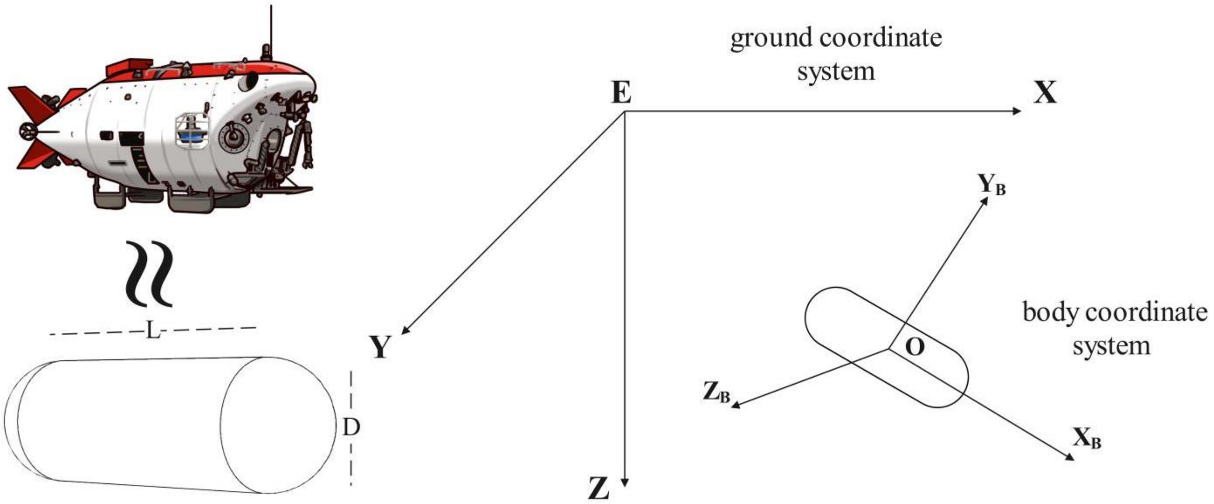

Figure 2: Kinematic coordinate system

3. Prediction of Submersible Sinking Trajectories under Ocean Current Influences

Based on the Monte Carlo method, an underwater unpowered sinking model is constructed. The basic framework of the model is shown in Figure 3 below.

1) Aiming at the problem of underwater motion dispersion of the target, the six-degree-of-freedom motion equation of unpowered sinking is first established, which can reasonably characterize the motion trajectory of the submersible.

2) Then, the Monte Carlo random function is introduced to reflect the randomness of the initial position in the form of coordinate (x, y, z), the randomness of the current in the form of velocity f (x, y), and the density of the seawater in the form of coordinate(z).

3) The underwater unpowered sinking model based on the Monte Carlo method is composed of a six-degree-of-freedom unpowered motion equation and Monte Carlo random function.

4) A random generation method is established for the random variable (x, y, z), f (x, y), and (z) in the model, which is generated on the computer.

5) The calculation results of the unpowered sinking model are analyzed, and the accuracy estimation of the solution is given.

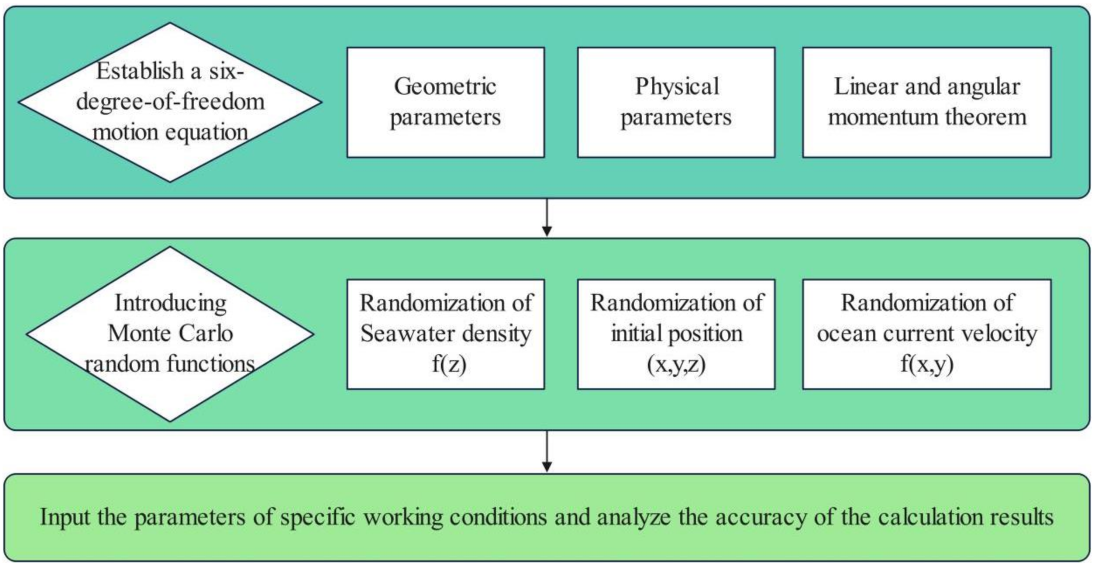

Figure 3: Monte Carlo-based motion model framework

The underwater motion model of the submersible is mainly established based on the translation equation and rotation equation of the submersible. Assume that at any time, the position of the submersible in the ground coordinate system is (x_0, y_0, z_0), the corresponding speed is $(\dot{x}_0, \dot{y}_0, \dot{z}_0)$, and the speed of the submersible in the body coordinate system is (v_x, v_y, v_z).

The model consists of 14 mathematical equations, and the loads acting on the submersible are mainly its gravity, buoyancy, water resistance, and ocean current force. According to the momentum theorem and the momentum moment theorem, the dynamic equation of the submersible is established as follows:

$$A_m \begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{pmatrix} + A_v \left(A_m \begin{pmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \right) = A_F \tag{1}$$

In the formula: A_m is the inertia matrix, and there is:

$$A_m = \begin{pmatrix} m & 0 & 0 & 0 & mz_c & -my_c \\ 0 & m & 0 & -mz_c & 0 & mx_c \\ 0 & 0 & m & my_c & -mx_c & 0 \\ 0 & -mz_c & my_c & J_{xx} & -J_{xy} & -J_{xz} \\ mz_c & 0 & -mx_c & -J_{yx} & J_{yy} & -J_{yz} \\ -my_c & mx_c & 0 & -J_{zx} & -J_{zy} & J_{zz} \end{pmatrix} \tag{2}$$

A_v is the velocity matrix, and there is:

$$A_v = \begin{pmatrix} 0 & -\omega_z & \omega_y & 0 & 0 & 0 \\ \omega_z & 0 & -\omega_x & 0 & 0 & 0 \\ -\omega_y & \omega_x & 0 & 0 & 0 & 0 \\ 0 & -v_z & v_y & 0 & -\omega_z & \omega_y \\ v_z & 0 & -v_r & \omega_z & 0 & -\omega_x \\ -v_y & v_x & 0 & -\omega_1 & \omega_x & 0 \end{pmatrix} \tag{3}$$

A_F is the force matrix, and has:

$$A_F = \begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix} \tag{4}$$

In the formula, $F_x, F_y, F_z, M_x, M_y, M_z$ are the total external force and the component of the total external moment in the body coordinate system respectively.

Build a kinematic model:

$$\begin{bmatrix}\dot{x}_0\\ \dot{y}_0\\ \dot{z}_0\end{bmatrix}=\begin{bmatrix}v_x\cos\theta\cos\psi+v_y(\sin\varphi\sin\psi-\sin\theta\cos\varphi\cos\psi)+v_z(\sin\psi\cos\varphi+\sin\theta\sin\varphi\cos\psi)\\ v_x\sin\theta+v_y\cos\theta\cos\varphi-v_z\cos\theta\sin\varphi\\ v_x\cos\theta\sin\psi+v_y(\sin\varphi\cos\psi+\sin\theta\cos\varphi\sin\psi)+v_z(\cos\varphi\cos\psi-\sin\theta\sin\varphi\sin\psi)\end{bmatrix} \tag{5}$$

K is the transformation matrix from the body coordinate system to the ground coordinate system:

$$K=\begin{bmatrix}\cos\theta\cos\psi & \sin\varphi\sin\psi-\sin\theta\cos\varphi\cos\psi & \sin\psi\cos\varphi+\sin\theta\sin\varphi\cos\psi\\ \sin\theta & \cos\theta\cos\varphi & \cos\theta\sin\varphi\\ \cos\theta\sin\psi & \sin\varphi\cos\psi+\sin\theta\cos\varphi\sin\psi & \cos\varphi\cos\psi-\sin\theta\sin\varphi\sin\psi\end{bmatrix} \tag{6}$$

First, random numbers are generated based on the linear congruence method, using the following formula:

$$x_{n+1}=\lambda x_n+\mathrm{c}(\mathrm{mod}M)\quad(\mathrm{n}=0,1,2,\cdots) \tag{7}$$

In the formula, x_0 is the initial value; λ is the multiplication operator; c is the increment; M is the modulus, these are all positive integers.

Then use the transformation method to generate random variables. The essence of the transformation method is to establish a functional relationship between the random variable X under study and another random variable U and use the one-to-one correspondence criterion of the functional relationship to obtain x from u.

The movement trajectory of the submersible underwater is uncertain. The idea of the random particle simulation method is used to simulate it to determine the probability distribution of the target to be searched. Assuming that there are Q particles in total, the motion vector of the q th particle can be expressed $(X_q,Y_q,U_q,V_q,T_q,\omega_q,n_q)$.

Among them, the longitude and latitude of the q th particle are X_q,Y_q; the north-south velocity components and east-west velocity components of the q th particle are U_q,V_q; the occurrence time of the q th particle is T_q; and the probability of the q th particle occurrence is ω_q; The number to which the q th particle occurs is n_q.

Each search target corresponds to a random particle. Suppose there are m scenes. The proportion of the m scene is q_m. There is $\sum_m^M q_m=1$. The number of m the particles is $q_m Q$. $\omega_q=1/Q$ is each the probability of a particle occurring.

4. Results and discussion

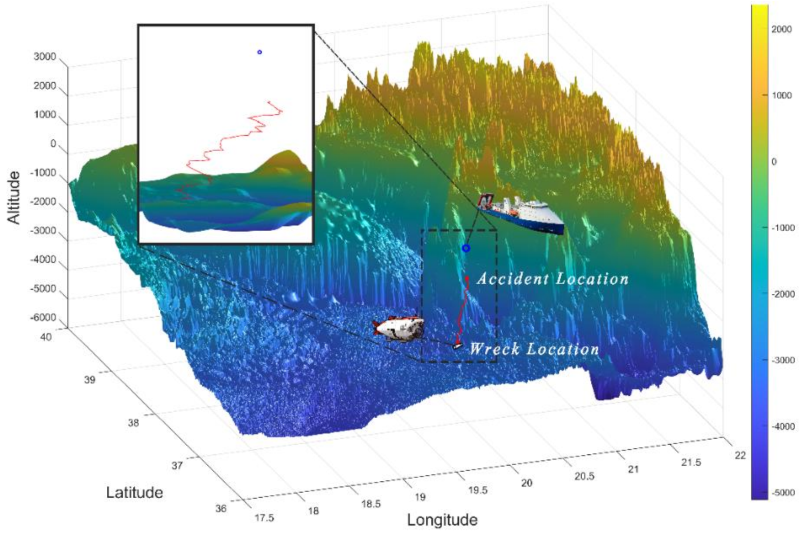

Figure 4: Wrecked submersible

Figure 4 is a schematic diagram of the submersible from its initial crash to its sinking to the bottom of the sea. Above the submersible is the main ship, and partially enlarged is the predicted trajectory of the crashed submersible's sinking.

The probability and magnitude of ocean current speed in each direction are random, and the maximum ocean current speed is 5m/s. Based on the measured ocean current data, the influence of random ocean currents is introduced, and the random ocean current velocity amplitude is(-5m/s~5m/s). The current velocity distribution is shown in Figure 5 below:

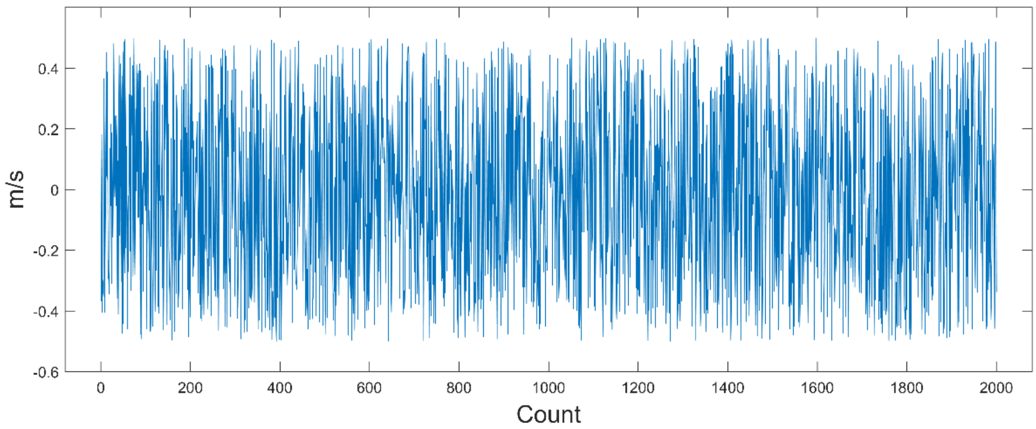

Figure 5: Random current velocity distribution

This paper took 2000 particles for simulation, and the resulting drop point distribution diagram and drop point distribution histogram are shown in Figure 6 below. The local enlargement area in figure 6A represents the random distribution of the submersible's distress points, and the blue scattered points represent the distribution of impact points. Under the influence of ocean currents, the distribution of particles has changed from the original diameter range of about 100m to 3000m. In figure 6B, the probability distribution gradually decreases outward in a ring around the highest probability point. The distribution points are mainly concentrated in the east-north direction, and there is the possibility of bottom sinking on both the west and south sides.

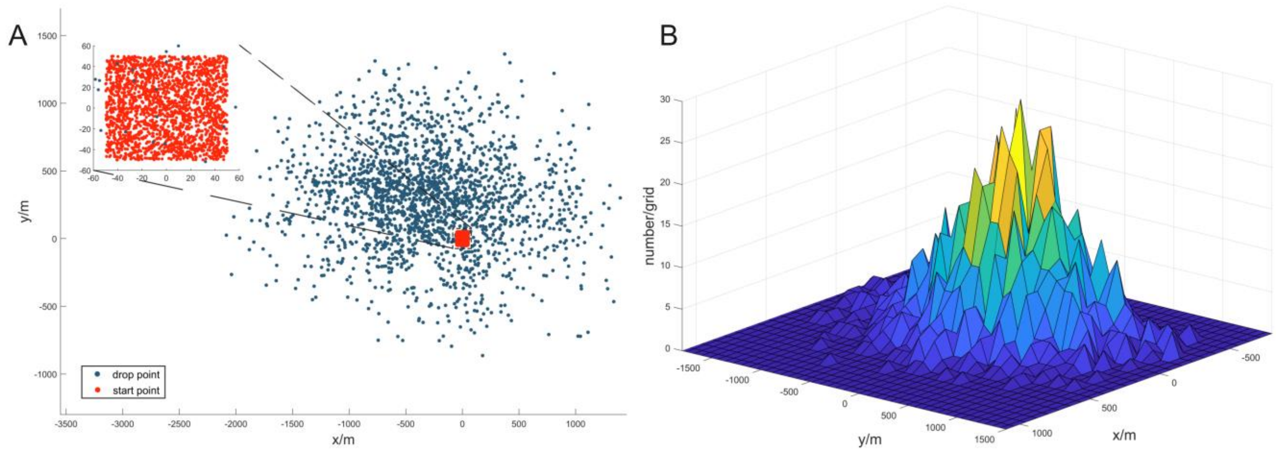

Figure 6: Drop points distribution A: Drop points distribution B: Drop point probability distribution.

Figure 7A: Divide the time it takes for the wrecked submersible to sink to the seabed into 5 segments and express the Euclidean distance from 2,000 random particles in each time segment to its cluster center using a box plot. This paper can see that the dispersion of particles increases over time. Figure 7B: To express the relationship between three-dimensional position and time, this paper converts the horizontal coordinate value into the Euclidean distances. The partially enlarged view of Figure 4 is the current prediction map of the submersible's sinking path.

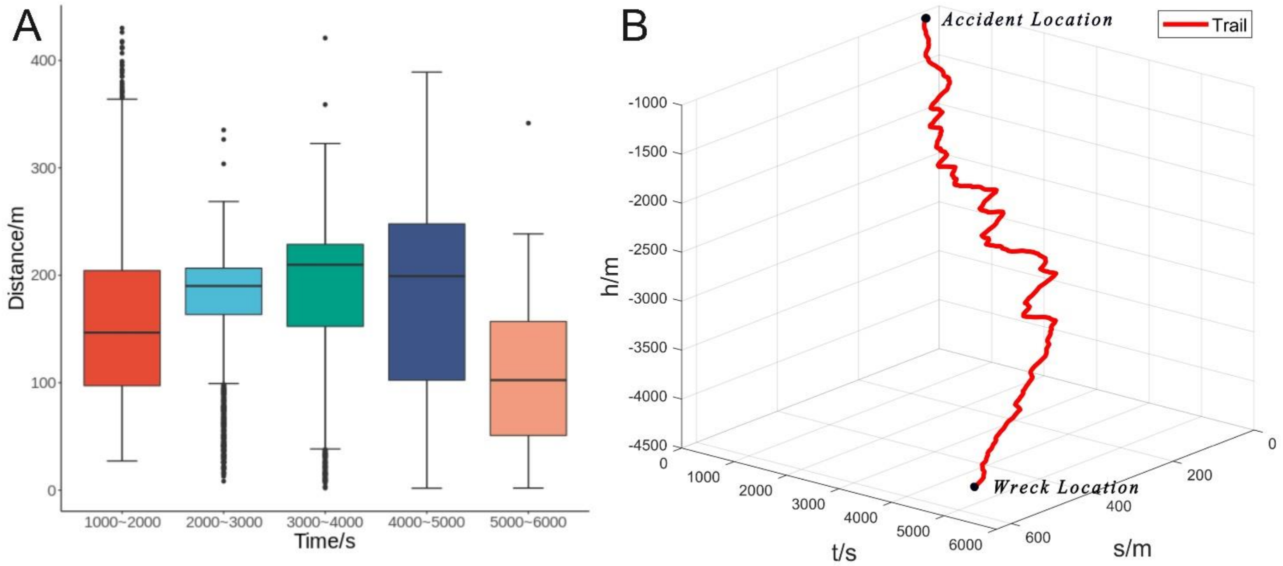

Figure 7: The relationship between submersible sinking trajectory and time A: dispersion of particles B: trajectory

This paper finds some devices that reduce the three uncertainties (position, current velocity, density) in Figure 8[8-11]. The sonar Doppler current meter can measure the velocity of the ocean current, the CTD sensor can measure the depth density and the depth of the submarine, the inertial navigation system can measure the longitude and latitude of the submarine underwater, and the sonar depth sounder can measure the depth of the submarine.

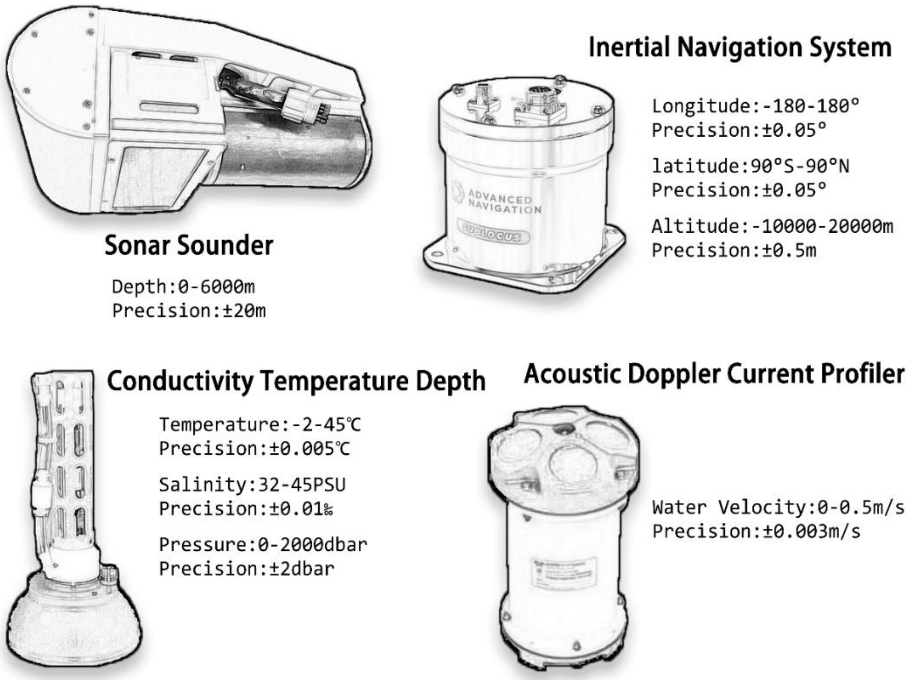

Figure 8: Device parameters

Our calculation device uses the ratio S% of the impact area of 75% of the particles before and after to measure the degree of uncertainty reduction.

$$S\% = \frac{S_{ues,\ 75\%}}{S_{not\ ues,\ 75\%}} \tag{8}$$

The changes in the scatter area before and after using the device are shown in Table 2. Among them, the change in the sensor is the most obvious, with an accuracy increase of 25%. The sonar has the worst effect, with an accuracy increase of 15%.

Table 2: Comparison of effects

	Sonar	INS	TDC	ADCP
S% (use)	0.85	0.8	0.75	0.82
S% (not use)	1	1	1	1

5. Conclusions

To truly simulate the disaster situation of the submersible, this study introduced the Monte Carlo method to randomly generate 2000 particles to reflect the uncertainty of the disaster point, ocean current direction, and seawater density. The range of the distressing point is roughly a circle with a radius of 50 meters, the velocity range of the ocean current is -5m/s~5m/s and the salinity of the seawater is about 38‰. Finally, combined with the six-degree-of-freedom kinematic model of the unpowered submersible, all possible time-varying motion trajectories were generated. The predicted bottom subsidence range is a circle with a radius of approximately 1000m. For the situation where the wrecked submersible sank from -1000m to -4500m, its possible sinking time is 1.4h. To display the research results more intuitively, this study also visualized the 3D terrain of the Ionian Sea's seafloor and put the predicted trajectory of the crashed submersible into the 3D terrain map. To reduce uncertainty, this paper also collects the parameters of the corresponding equipment and compare the effects before and after use. The accuracy can be improved by more than 15%. This article provides a research idea and framework that can be applied to marine search and rescue-related fields and proves the feasibility of using the Monte Carlo method for deep-sea search.

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