Fatec Guaratinguetá

Centro Paula Souza — Governo do Estado São Paulo

Cálculo - Análise e Desenvolvimento de Sistemas

Prova 02

Aluno: ____ Nota: ____________

Data: 23/06/2020

Calcule, pela definição de derivadas (fórmula do limite), as derivadas das funções abaixo:

1. $f(x) = 2x^2 - x + 5$ (**1,5**)
2. $f(x) = \frac{x-2}{x+3}$ (**2,5**)
3. $f(x) = \sqrt{x^2 + 2x}$ (**3,0**)
4. $f(x) = 2 - \sqrt{x+3}$ (**3,0**)

Obs.: todos os desenvolvimentos deverão ser demonstrados.

Cálculo de derivada pela definição

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Alguns produtos notáveis

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Boa prova!

Cálculo ADS - P2 a

1) $f(x) = 2x^2 - x + 5$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = 2(x+\Delta x)^2 - (x+\Delta x) + 5$$
$$= 2(x^2 + 2x\Delta x + \Delta x^2) - x - \Delta x + 5$$
$$= 2x^2 + 4x\Delta x + 2\Delta x^2 - x - \Delta x + 5$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 - x - \Delta x + 5 - (2x^2 - x + 5)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + 2\Delta x^2 \cancel{- x} - \Delta x \cancel{+5} \cancel{-2x^2} \cancel{+x} \cancel{-5}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 - \Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(4x + 2\Delta x - 1)}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0} (4x + 2\Delta x - 1) = 4x + 2 \cdot 0 - 1 = \boxed{4x - 1}$$

2) $f(x) = \dfrac{x-2}{x+3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = \frac{(x+\Delta x)-2}{(x+\Delta x)+3} = \frac{x+\Delta x - 2}{x+\Delta x+3}$$

$$f(x+\Delta x) - f(x) = \frac{x+\Delta x-2}{x+\Delta x+3} - \frac{x-2}{x+3} = \frac{(x+3)(x+\Delta x-2) - (x-2)(x+\Delta x+3)}{(x+\Delta x+3)(x+3)}$$

$$f(x+\Delta x) - f(x) = \frac{x^2 + x\Delta x - 2x + 3x + 3\Delta x - 6 - (x^2 + x\Delta x + 3x - 2x - 2\Delta x}{(x+\Delta x+3)(x+3)}$$

$$f(x+\Delta x) - f(x) = \frac{\cancel{x^2} + \cancel{x\Delta x} - \cancel{2x} + \cancel{3x} + 3\Delta x - \cancel{6} - \cancel{x^2} - \cancel{x\Delta x} - \cancel{3x} + \cancel{2x} + 2\Delta x + \cancel{6}}{(x+\Delta x+3)(x+3)}$$

$$f(x+\Delta x) - f(x) = \frac{5\Delta x}{(x+\Delta x+3)(x+3)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\dfrac{5\Delta x}{(x+\Delta x+3)(x+3)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{5\cancel{\Delta x}}{\cancel{\Delta x}(x+\Delta x+3)(x+3)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{5}{(x+\Delta x+3)(x+3)} = \frac{5}{(x+0+3)(x+3)} = \boxed{\frac{5}{(x+3)^2}}$$

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3) $f(x)=\sqrt{x^2+2x}$

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}-\sqrt{x^2+2x}\right)}{\Delta x}\cdot\frac{\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}{\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\left(\cancel{\sqrt{}}(x+\Delta x)^2+2(x+\Delta x)\right)^{\cancel{2}}-\left(\cancel{\sqrt{}}x^2+2x\right)^{\cancel{2}}}{\Delta x\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2+2(x+\Delta x)-(x^2+2x)}{\Delta x\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{x^2}+2x\Delta x+\Delta x^2+\cancel{2x}+2\Delta x-\cancel{x^2}-\cancel{2x}}{\Delta x\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2x\Delta x+\Delta x^2+2\Delta x}{\Delta x\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(2x+\Delta x+2)}{\cancel{\Delta x}\left(\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}\right)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2x+\Delta x+2}{\sqrt{(x+\Delta x)^2+2(x+\Delta x)}+\sqrt{x^2+2x}}$$

$$f'(x)=\frac{2x+0+2}{\sqrt{(x+0)^2+2(x+0)}+\sqrt{x^2+2x}}$$

$$f'(x)=\frac{2x+2}{\sqrt{x^2+2x}+\sqrt{x^2+2x}}=\frac{\cancel{2}(x+1)}{\cancel{2}\sqrt{x^2+2x}}=\boxed{\frac{x+1}{\sqrt{x^2+2x}}}$$

4) $f(x) = 2 - \sqrt{x+3}$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) - f(x) = 2 - \sqrt{(x+\Delta x)+3} - \left(2 - \sqrt{x+3}\right)$$

$$f(x+\Delta x) - f(x) = \cancel{2} - \sqrt{(x+\Delta x)+3} - \cancel{2} + \sqrt{x+3} = \sqrt{x+3} - \sqrt{(x+\Delta x)+3}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\sqrt{x+3} - \sqrt{(x+\Delta x)+3}\right)}{\Delta x} \cdot \frac{\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}{\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\cancel{\sqrt{x+3}}\right)^{\cancel{2}} - \left(\cancel{\sqrt{(x+\Delta x)+3}}\right)^{\cancel{2}}}{\Delta x\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x+3 - \left((x+\Delta x)+3\right)}{\Delta x\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{x}+\cancel{3}-\cancel{x}-\Delta x-\cancel{3}}{\Delta x\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}^{\,-1}}{\cancel{\Delta x}\left(\sqrt{x+3} + \sqrt{(x+\Delta x)+3}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x+3} + \sqrt{(x+\Delta x)+3}} = \frac{-1}{\sqrt{x+3} + \sqrt{x+0+3}}$$

$$f'(x) = \frac{-1}{\sqrt{x+3} + \sqrt{x+3}} = \boxed{\frac{-1}{2\sqrt{x+3}}}$$