Fatec Guaratinguetá

Centro Paula Souza — GOVERNO DO ESTADO SÃO PAULO

**Cálculo - Análise e Desenvolvimento de Sistemas**

**Prova 02**

Aluno: ______________________________________________ Nota: ____________

Data: 23/06/2020

Calcule, pela definição de derivadas (fórmula do limite), as derivadas das funções abaixo:

1. $f(x) = -4x^2 + x + 2$ (**1,5**)
2. $f(x) = \frac{x+2}{x-3}$ (**2,5**)
3. $f(x) = \sqrt{x^2 - x}$ (**3,0**)
4. $f(x) = 4 - \sqrt{2x - 1}$ (**3,0**)

**Obs.: todos os desenvolvimentos deverão ser demonstrados.**

**Cálculo de derivada pela definição**

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

**Alguns produtos notáveis**

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Boa prova!

1) $f(x) = -4x^2 + x + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = -4(x+\Delta x)^2 + (x+\Delta x) + 2$$
$$= -4(x^2 + 2x\Delta x + \Delta x^2) + x + \Delta x + 2$$
$$= -4x^2 - 8x\Delta x - 4\Delta x^2 + x + \Delta x + 2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-4x^2 - 8x\Delta x - 4\Delta x^2 + x + \Delta x + 2 - (-4x^2 + x + 2)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{-4x^2} - 8x\Delta x - 4\Delta x^2 \cancel{+x} + \Delta x \cancel{+2} \cancel{+4x^2} \cancel{-x} \cancel{-2}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-8x\Delta x - 4\Delta x^2 + \Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-8x - 4\Delta x + 1)}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0} (-8x - 4\Delta x + 1) = -8x - 4 \cdot 0 + 1 = \boxed{-8x + 1}$$

2) $f(x) = \dfrac{x+2}{x-3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = \frac{(x+\Delta x)+2}{(x+\Delta x)-3} = \frac{x+\Delta x+2}{x+\Delta x-3}$$

$$f(x+\Delta x) - f(x) = \frac{x+\Delta x+2}{x+\Delta x-3} - \frac{x+2}{x-3} = \frac{(x-3)(x+\Delta x+2) - (x+2)(x+\Delta x-3)}{(x+\Delta x-3)(x-3)}$$

$$f(x+\Delta x) - f(x) = \frac{x^2 + x\Delta x + 2x - 3x - 3\Delta x - 6 - (x^2 + x\Delta x - 3x + 2x + 2\Delta x - 6)}{(x+\Delta x-3)(x-3)}$$

$$f(x+\Delta x) - f(x) = \frac{\cancel{x^2} + \cancel{x\Delta x} + \cancel{2x} - \cancel{3x} - 3\Delta x - \cancel{6} - \cancel{x^2} - \cancel{x\Delta x} + \cancel{3x} - \cancel{2x} - 2\Delta x + \cancel{6}}{(x+\Delta x-3)(x-3)}$$

$$f(x+\Delta x) - f(x) = \frac{-5\Delta x}{(x+\Delta x-3)(x-3)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\dfrac{-5\Delta x}{(x+\Delta x-3)(x-3)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-5\cancel{\Delta x}}{\cancel{\Delta x}(x+\Delta x-3)(x-3)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-5}{(x+\Delta x-3)(x-3)} = \frac{-5}{(x+0-3)(x-3)} = \boxed{\frac{-5}{(x-3)^2}}$$

3) $f(x) = \sqrt{x^2 - x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) - f(x) = \sqrt{(x+\Delta x)^2 - (x+\Delta x)} - \sqrt{x^2 - x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} - \sqrt{x^2-x}\right)}{\Delta x} \cdot \frac{\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2-x}\right)}{\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2-x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\cancel{\sqrt{}}(x+\Delta x)^2 - (x+\Delta x)\right)^{\cancel{2}} - \left(\cancel{\sqrt{}}x^2 - x\right)^{\cancel{2}}}{\Delta x\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - (x+\Delta x) - (x^2 - x)}{\Delta x\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x} - \Delta x - \cancel{x^2} + \cancel{x}}{\Delta x\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - \Delta x}{\Delta x\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x - 1)}{\cancel{\Delta x}\left(\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x + \Delta x - 1}{\sqrt{(x+\Delta x)^2 - (x+\Delta x)} + \sqrt{x^2+x}}$$

$$f'(x) = \frac{2x+0-1}{\sqrt{(x+0)^2 - (x+0)} + \sqrt{x^2+x}} = \frac{2x-1}{\sqrt{x^2-x} + \sqrt{x^2-x}}$$

$$\boxed{f'(x) = \frac{2x-1}{2\sqrt{x^2-x}}}$$

4) $f(x) = 4 - \sqrt{2x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) - f(x) = 4 - \sqrt{2(x+\Delta x)-1} - \left(4 - \sqrt{2x-1}\right)$$

$$= \cancel{4} - \sqrt{2(x+\Delta x)-1} - \cancel{4} + \sqrt{2x-1}$$

$$= \sqrt{2x-1} - \sqrt{2(x+\Delta x)-1}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\sqrt{2x-1} - \sqrt{2(x+\Delta x)-1}\right)}{\Delta x} \cdot \frac{\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}{\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\left(\cancel{\sqrt{2x-1}}\right)^{\cancel{2}} - \left(\cancel{\sqrt{2(x+\Delta x)-1}}\right)^{\cancel{2}}}{\Delta x\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x-1-\left(2(x+\Delta x)-1\right)}{\Delta x\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{2x}\cancel{-1} - \cancel{2x} - 2\Delta x \cancel{+1}}{\Delta x\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-2\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}\right)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-2}{\sqrt{2x-1} + \sqrt{2(x+\Delta x)-1}}$$

$$f'(x) = \frac{-2}{\sqrt{2x-1} + \sqrt{2(x+0)-1}} = \frac{-2}{\sqrt{2x-1} + \sqrt{2x-1}}$$

$$f'(x) = \frac{-\cancel{2}}{\cancel{2}\sqrt{2x-1}} \Rightarrow \boxed{f'(x) = \frac{-1}{\sqrt{2x-1}}}$$