Cálculo Diferencial
Regras de Derivação

Derivadas das principais funções elementares

$f(x)$	$f'(x)$	**Observações**
c	0	$c \in \mathbb{R}$
x^n	$n\,x^{n-1}$	
$\ln x$	$\frac{1}{x}$	$x > 0$
a^x	$a^x \ln a$	$a > 0 \; e \; a \neq 1$
$sen\,x$	$\cos x$	
$cos\,x$	$-\text{sen}\,x$	
$tg\,x$	$\sec^2 x$	

Derivadas das principais funções elementares

Exemplos:

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\mathrm{sen}\, x$
$tg\ x$	$\sec^2 x$

$$f'(x) = -5x^{-5-1} = -5x^{-6}$$

7. $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\mathrm{sen}\, x$
$tg\ x$	$\sec^2 x$

12. $f(x) = \mathrm{tg}\, x$

$$f'(x) = \sec^2 x$$

Regras de derivação

Regras de derivação	
$(u+v)'$	$u'+v'$
$(u-v)'$	$u'-\ v'$
$(kf)'$	kf'
$(u\ v)'$	$u'v+u\ v'$
$\left(\dfrac{u}{v}\right)'$	$\dfrac{u'v-u\ v'}{v^2}$
Regra da cadeia	$f'(x)=f'(u).u'$

Regras de derivação

1. $f(x) = 3x^2 + x^4$

$f'(x) = 2 \cdot 3x^{2-1} + 4x^{4-1} = 6x + 4x^3$

2. $f(x) = \frac{1}{x^5}$

$f'(x) = -5x^{-5-1} = -5x^{-6}$

3. $f(x) = \frac{4}{x^3} - \ln x$

$f'(x) = -3 \cdot 4x^{-3-1} - \frac{1}{x} = -12x^{-4} - \frac{1}{x}$

4. $f(x) = (3x+2)(x^2-1)$

$f'(x) = (u\,v)' = u'v + u\,v'$

$u = 3x + 2$

$v = x^2 - 1$

$u' = 3$

$v' = 2x$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$

$f'(x) = 3(x^2-1) + (3x+2)2x = 3x^2 - 3 + 6x^2 + 4x = 9x^2 + 4x - 3$

Regras de derivação

5. $f(x) = (x^2 - 1)\ln x$

$f'(x) = (u\,v)' = u'v + u\,v'$

$u = x^2 - 1$

$v = \ln x$

$u' = 2x$

$v' = \dfrac{1}{x}$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\dfrac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\,x$
$tg\ x$	$\sec^2 x$

$$f'(x) = 2x\ln x + (x^2 - 1)\frac{1}{x} = 2x\ln x - \frac{(x^2 - 1)}{x} = 2x\ln x - x + \frac{1}{x}$$

Regras de derivação	
$(u\,v)'$	$u'v + u\,v'$
$\left(\dfrac{u}{v}\right)'$	$\dfrac{u'v - u\,v'}{v^2}$

Regras de derivação

6.

$$f(x) = \frac{\ln x}{x}$$

$$f'(x) = \left(\frac{u}{v}\right)' = \frac{u'v - u\,v'}{v^2}$$

$$u = \ln x$$

$$v = x$$

$$u' = \frac{1}{x}$$

$$v' = 1$$

$$f'(x) = \frac{\frac{1}{x}x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\mathrm{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$

Regras de derivação

7.

$$f(x) = \frac{x^2 - 1}{x + 3}$$

$$f'(x) = \left(\frac{u}{v}\right)' = \frac{u'v - u\,v'}{v^2}$$

$$u = x^2 - 1$$

$$v = x + 3$$

$$u' = 2x$$

$$v' = 1$$

$$f'(x) = \frac{2x\,(x+3) - (x^2 - 1)1}{(x+3)^2} = \frac{2x^2 + 6x - x^2 + 1}{(x+3)^2} = \frac{x^2 + 6x + 1}{(x+3)^2}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$

Regra da cadeia

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Dada a função $f(x) = \ln(x^2 + 1)$, como calcular $f'(x)$?

$\frac{d}{dx}\ln(x^2 + 1)$ **NÃO** é igual a $\frac{1}{x^2 + 1}$

Ou, ainda, se $f(x) = (2x + 1)^{100}$, como calcular $\frac{df}{dx}$?

$\frac{d}{dx}(2x + 1)^{100}$ **NÃO** é igual a $100(2x + 1)^{99}$

Regra da cadeia

Exemplo 1: Calcular a derivada de $f(x) = \ln\underbrace{(x^2 + 1)}_{\boxplus}$.

Solução:

$$u = x^2 + 1$$

$$f(u) = \ln u$$

$$f'(x) = f'(u)\, u'$$

$$f'(u) = \frac{1}{u}$$

$$u' = 2x$$

$$f'(x) = \frac{1}{u} 2x = \frac{2x}{u} = \frac{2x}{x^2+1}$$

$$\boxed{f'(x) = \frac{2x}{x^2+1}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\,x$	$\cos x$
$cos\,x$	$-\text{sen}\,x$
$tg\,x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia

Exemplo 2: Calcular a derivada de $f(x) = \underbrace{(2x+1)}_{\boxplus}{}^{100}$

Solução:

$$u = 2x + 1$$

$$f(u) = u^{100}$$

$$f'(x) = f'(u)\, u'$$

$$f'(u) = 100\, u^{99}$$

$$u' = 2$$

$$f'(x) = 100\, u^{99} \cdot 2 = 200\, u^{99} = 200(2x+1)^{99}$$

$$\boxed{f'(x) = 200(2x+1)^{99}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\, x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\, x$	$\cos x$
$cos\, x$	$-\text{sen}\, x$
$tg\, x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\, v)'$	$u'v + u\, v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\, v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia

Exemplo 3: Calcular a derivada de $f(x) = \cos\left(\underbrace{\frac{x}{x+1}}_{\boxplus}\right)$

Solução:

$$u = \frac{x}{x+1}$$

$$f(u) = \cos u$$

$$f'(x) = f'(u)\, u'$$

$$f'(u) = -\operatorname{sen} u$$

$$u' = \left(\frac{x}{x+1}\right)' = \frac{w'v - wv'}{v^2} = \frac{1(x+1) - x\cdot 1}{(x+1)^2} = \frac{x+1-x}{(x+1)^2}$$

$w = x$

$v = x+1$

$w' = 1$

$v' = 1$

$$u' = \frac{1}{(x+1)^2}$$

$$f'(x) = \underbrace{-\operatorname{sen} u}_{f'(u)} \cdot \underbrace{\frac{1}{(x+1)^2}}_{u'} = -\operatorname{sen}\left(\frac{x}{x+1}\right)\frac{1}{(x+1)^2} = \boxed{-\frac{1}{(x+1)^2}\operatorname{sen}\left(\frac{x}{x+1}\right)}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\operatorname{sen} x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Processo rápido

Exemplo 4: Calcular a derivada de $f(x) = \ln(\underbrace{x^2 + 1}_{\boxplus})$.

$\ln \boxplus$

Solução:

$$(\ln \boxplus)' = \frac{1}{\boxplus} \cdot \boxplus'$$

$$f'(x) = \frac{1}{x^2+1}(x^2+1)' = \frac{1}{x^2+1}2x$$

$$\boxed{f'(x) = \frac{2x}{x^2+1}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Processo rápido

Exemplo 5: Calcular a derivada de $f(x) = \underbrace{(2x+1)}_{\boxplus}{}^{100}$

Solução:

$$(\boxplus^{100})' = 100\,\boxplus^{99} \cdot \boxplus'$$

$\boxplus^{100}$

$$f'(x) = 100(2x+1)^{99}(2x+1)' = 100(2x+1)^{99} \cdot 2$$

$$\boxed{f'(x) = 200(2x+1)^{99}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\,x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Processo rápido

Exemplo 6: Calcular a derivada de $f(x) = \cos\underbrace{\left(\frac{x}{x+1}\right)}_{\boxplus}$

Solução:

$$(\cos \boxplus)' = -\mathrm{sen}(\boxplus)\cdot\boxplus'$$

$\cos(\boxplus)$

$$f'(x) = -\,\mathrm{sen}\left(\frac{x}{x+1}\right)\left(\frac{x}{x+1}\right)'$$

$$\left(\frac{x}{x+1}\right)' = \frac{u'v - uv'}{v^2} = \frac{1(x+1) - x\cdot 1}{(x+1)^2} = \frac{x+1-x}{(x+1)^2}$$

$u = x$

$v = x + 1$

$u' = 1$

$v' = 1$

$$= \frac{1}{(x+1)^2}$$

$$f'(x) = -\mathrm{sen}\left(\frac{x}{x+1}\right)\cdot\frac{1}{(x+1)^2} = \boxed{-\frac{1}{(x+1)^2}\,\mathrm{sen}\left(\frac{x}{x+1}\right)}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\,x$	$\cos x$
$cos\,x$	$-\mathrm{sen}\,x$
$tg\,x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u'+v'$
$(u-v)'$	$u'-v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Exercícios

1) Calcular a derivada de $f(x) = \underbrace{(1-3x)}_{\boxplus}{}^4$

$(\boxplus^4)' = 4\,\boxplus^3 \cdot \boxplus'$

$\boxplus^4$

$f'(x) = 4(1-3x)^3\,(-3) = -12(1-3x)^3$

2) Calcular a derivada de $f(x) = \ln\underbrace{(3x^2-1)}_{\boxplus}$

$(\ln \boxplus)' = \dfrac{1}{\boxplus} \cdot \boxplus'$

$\ln(\boxplus)$

$f'(x) = \dfrac{1}{3x^2-1}\,6x = \dfrac{6x}{3x^2-1}$

3) Calcular a derivada de $f(x) = \mathrm{sen}\underbrace{(5x^3)}_{\boxplus}$

$(\mathrm{sen}\,\boxplus)' = \cos(\boxplus) \cdot \boxplus'$

$\mathrm{sen}(\boxplus)$

$f'(x) = \cos(5x^3)\,15x^2 = 15x^2\cos(5x^3)$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\dfrac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\mathrm{sen}\,x$
$tg\ x$	$\sec^2 x$

Regra da cadeia – Exercícios

4) Calcular a derivada de $f(x) = \left(\dfrac{1-x^2}{1+x^2}\right)^{10}$

$$f'(x) = 10\left(\frac{1-x^2}{1+x^2}\right)^9\left(\frac{1-x^2}{1+x^2}\right)'$$

$$\left(\frac{1-x^2}{1+x^2}\right)' = \frac{u'v - uv'}{v\,v^2} = \frac{-2x(1+x^2)-(1-x^2)2x}{(1+x^2)^2} =$$

$u = 1 - x^2$

$v = 1 + x^2$

$u' = -2x$

$v' = 2x$

$$= \frac{-2x - \cancel{2x^3} - 2x + \cancel{2x^3}}{(1+x^2)^2} =$$

$$= \frac{-4x}{(1+x^2)^2}$$

$$f'(x) = 10\left(\frac{1-x^2}{1+x^2}\right)^9 \frac{-4x}{(1+x^2)^2} = \frac{-40x}{(1+x^2)^2}\left(\frac{1-x^2}{1+x^2}\right)^9 = \frac{-40x}{(1+x^2)^2}\,\frac{(1-x^2)^9}{(1+x^2)^9}$$

$$\boxed{f'(x) = \frac{-40x(1-x^2)^9}{(1+x^2)^{11}}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\dfrac{1}{x}$
a^x	$a^x \ln a$
$sen\,x$	$\cos x$
$cos\,x$	$-\text{sen}\,x$
$tg\,x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\dfrac{u}{v}\right)'$	$\dfrac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Exercícios

5) Calcular a derivada de $f(x) = \dfrac{\ln(2+x^2)}{2+x^2}$ $\begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$$u = \ln(2+x^2)$$
$$v = 2+x^2$$
$$u' = \frac{1}{2+x^2}2x = \frac{2x}{2+x^2}$$
$$v' = 2x$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{2x}{\cancel{2+x^2}}\cancel{(2+x^2)} - \ln(2+x^2)\,2x}{(2+x^2)^2}$$

$$f'(x) = \frac{2x - 2x\ln(2+x^2)}{(2+x^2)^2}$$

$$\boxed{f'(x) = \frac{2x(1-\ln(2+x^2))}{(2+x^2)^2}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u'+v'$
$(u-v)'$	$u'-v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v+u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v-u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Exercícios

6) Calcular a derivada de $f(x) = \ln\left(\dfrac{x^3 - x}{4 - x^2}\right)$

$$f'(x) = \frac{1}{\dfrac{x^3 - x}{4 - x^2}}\left(\frac{x^3 - x}{4 - x^2}\right)' = \frac{4 - x^2}{x^3 - x}\left(\frac{x^3 - x}{4 - x^2}\right)'$$

$$\left(\frac{x^3 - x}{4 - x^2}\right)' = \frac{u'v - uv'}{v^2} = \frac{(3x^2 - 1)(4 - x^2) - (x^3 - x)(-2x)}{(4 - x^2)^2}$$

$$= \frac{12x^2 - 3x^4 - 4 + x^2 - (-2x^4 + 2x^2)}{(4 - x^2)^2}$$

$$= \frac{12x^2 - 3x^4 - 4 + x^2 + 2x^4 - 2x^2}{(4 - x^2)^2}$$

$$= \frac{-x^4 + 11x^2 - 4}{(4 - x^2)^2}$$

$u = x^3 - x$
$v = 4 - x^2$
$u' = 3x^2 - 1$
$v' = -2x$

$$f'(x) = \frac{\cancel{4 - x^2}}{x^3 - x}\,\frac{-x^4 + 11x^2 - 4}{(4 - x^2)^{\cancel{2}}} = \boxed{\frac{-x^4 + 11x^2 - 4}{(x^3 - x)(4 - x^2)}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\dfrac{1}{x}$
a^x	$a^x \ln a$
$sen\ x$	$\cos x$
$cos\ x$	$-\text{sen}\, x$
$tg\ x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u' + v'$
$(u-v)'$	$u' - v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v + u\,v'$
$\left(\dfrac{u}{v}\right)'$	$\dfrac{u'v - u\,v'}{v^2}$
Regra da cadeia	$f'(x) = f'(u).u'$

Regra da cadeia – Exercícios

7) Calcular a derivada de $f(x) = \ln(\ln x)$

$$f'(x) = \frac{1}{\ln x}(\ln x)' = \frac{1}{\ln x}\frac{1}{x} = \frac{1}{x \ln x}$$

$$\boxed{f'(x) = \frac{1}{x \ln x}}$$

$f(x)$	$f'(x)$
c	0
x^n	$n\,x^{n-1}$
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$sen\,x$	$\cos x$
$cos\,x$	$-\text{sen}\,x$
$tg\,x$	$\sec^2 x$

Regras de derivação	
$(u+v)'$	$u'+v'$
$(u-v)'$	$u'-v'$
$(kf)'$	kf'
$(u\,v)'$	$u'v+u\,v'$
$\left(\frac{u}{v}\right)'$	$\frac{u'v-u\,v'}{v^2}$
Regra da cadeia	$f'(x)=f'(u).u'$