energy balance model: EBM $F^{\downarrow} - F^{\uparrow} = C\frac{dT}{dt}$ → heat capacity

⇒ averaged over the earth, rate of change of heat content per unit horizontal area of the climate system is given by the net heat flux into the system which is the difference between incoming & outgoing radiative power per unit area.

neglecting absorption of solar radiation above the tropopause we can take

$F^{\downarrow} = F_0 = \frac{1}{4}(1-A)F_s$ → solar irradiance

$\approx 240\ Wm^{-2}$

$F^{\downarrow}$ does not depend on conc or T of greenhouse gases, unlike $F^{\uparrow}$ which does. therefore we can write that the net heat flux $F^{\downarrow} - F^{\uparrow}$ into climate system as $F^{\downarrow} - F^{\uparrow} = Q(T, U)$

steady state climate at $T = T_{ss}$ & conc of greenhouse gases $U = U_{ss}$ $C\frac{dT}{dt} = 0$

& $Q(T_{ss}, U_{ss}) = 0$

If the steady state is slightly perturbed so that $T = T_{ss} + T'(t)$ & $U = U_{ss} + U'(t)$ $T' \ll T_{ss}$ $U' \ll U_{ss}$

$$C\frac{dT'}{dt} \approx \frac{\partial Q}{\partial T}T' + \frac{\partial Q}{\partial U}U' \quad (1)$$

define $\alpha = -\frac{\partial Q}{\partial T}$ = climate feedback

$\alpha$ quantifies how the net downward heat flux $Q$ varies with temperature

We can also define the radiative forcing $F = \frac{\partial Q}{\partial U}U'$ → how net downwards flux changes with concentration

s.t. (1) can be written as $C\frac{dT'}{dt} + \alpha T' = F(t)$ (2)

impossible to give precise value for $\alpha$ & $C$ but estimates are

$\alpha \approx 1\ Wm^{-2}K^{-1}$

$C = 1\ GJ\ K^{-1}m^{-2}$

## Solutions to the linearised EBM:

(2) is a first order DE

⇒ define $\tau = \frac{C}{\alpha}$ feedback response time

→ solving (2) gives

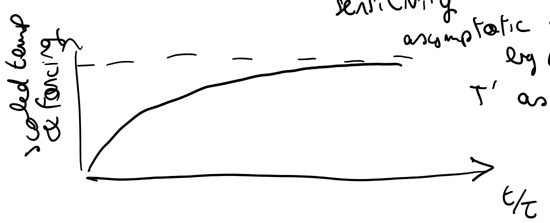
$$T'(t) = \frac{e^{-t/\tau}}{C}\int_0^t F(u)e^{u/\tau}du \quad (3)$$

→ increase in temperature

look at response for different forcings:

a) step function $F(t) = \begin{cases} 0 & t \le 0 \\ F_1 & t > 0 \end{cases}$

thus $T'(t) = S(1 - e^{-t/\tau})$ $t > 0$ where $S = F_1/\alpha$ is called the climate sensitivity




asymptotic value approached by temperature perturbation $T'$ as $t \to \infty$

ramp forcing:

$f(t) = \begin{cases} 0 & t \le 0 \\ \gamma t & t > 0 \end{cases}$

$T'(t) = \frac{\gamma\tau}{\alpha}\left(\frac{t}{\tau} - 1 + e^{-t/\tau}\right)$ $t > 0$

$$T'f(t) = \frac{\gamma\tau}{\alpha}\left(\frac{t}{\tau} - 1 + e^{-t/\tau}\right) \quad t>0$$

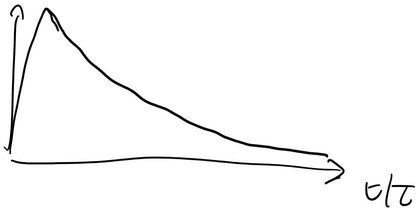


$\gamma$ is a constant $\Rightarrow$ good representation of current situation as $CO_2$ currently increases by 0.5% every year ie linear

**exponentially decaying Forcing:**

$$f(t) = \begin{cases} 0 & t \leq 0 \\ F_1 e^{-t/t_0} & t>0 \end{cases} \qquad F_1 \ \& \ t_0 \text{ are cstts}$$

might represent forcing due to injection of aerosols from volcanic eruption.





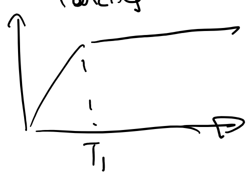
**initially linearly increasing, then cstt:**

$$T'(t) = \frac{\gamma\tau}{\alpha}\left(\frac{t}{\tau} - 1 + e^{-t/\tau}\right) \quad 0 < t \leq t_1$$

$$= \frac{\gamma\tau}{\alpha}\left(\frac{t_1}{\tau} - e^{-(t-t_1)/\tau} + e^{-t/\tau}\right) \quad t \geq t_1$$



$\Rightarrow$





climate feedback:

$$\alpha = -\frac{\partial Q}{\partial T_{ss}} = \varepsilon^{1/4}\alpha_{BB} \quad \text{derivation p 205}$$

**Radiative forcing due an increase in $CO_2$:**

calculations show that radiative forcing is ~~a~~ proportional to the log of the fractional change in $CO_2$
$\Rightarrow$ $F_{\beta CO_2} = 5.3 \ln\beta \ \ Wm^{-2}$ where the $CO_2$ increases increases by a factor $\beta$

derivation p 207 $\rightarrow$ 211
but essentially don't need to know how it is derived.