Algorithm of the maximum likelihood estimation with Newton–Raphson iteration

The log-likelihood function in the case of the Poisson Lee–Carter model:

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_{xj}^{(1)}\kappa_{tj}^{(1)}) - E_{xtj}e^{\alpha_{xj}+\beta_{xj}^{(1)}\kappa_{tj}^{(1)}}\right\} + \text{constant}$$

The log-likelihood functions in the case of the Poisson Carter–Lee model:

$$\ell(\theta) = \sum_{xt}\left\{D_{xt}\,(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)}) - E_{xt}e^{\alpha_x+\beta_x^{(1)}\kappa_t^{(1)}}\right\} + \text{constant at national level}$$

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_{xj}^{(1)}\kappa_t^{(1)}) - E_{xtj}e^{\alpha_{xj}+\beta_{xj}^{(1)}\kappa_t^{(1)}}\right\} + \text{constant at regional level}$$

The log-likelihood functions in the case of the Poisson CF model:

$$\ell(\theta) = \sum_{xt}\left\{D_{xt}\,(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)}) - E_{xt}e^{\alpha_x+\beta_x^{(1)}\kappa_t^{(1)}}\right\} + \text{constant at national level}$$

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_x^{(1)}\kappa_t^{(1)}) - E_{xtj}e^{\alpha_{xj}+\beta_x^{(1)}\kappa_t^{(1)}}\right\} + \text{constant at regional level}$$

The log-likelihood functions in the case of the Poisson CAE model:

$$\ell(\theta) = \sum_{xt}\left\{D_{xt}\,(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)}) - E_{xt}e^{\alpha_x+\beta_x^{(1)}\kappa_t^{(1)}}\right\} + \text{constant at national level}$$

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_x^{(1)}\kappa_{tj}^{(1)}) - E_{xtj}e^{\alpha_{xj}+\beta_x^{(1)}\kappa_{tj}^{(1)}}\right\} + \text{constant at regional level}$$

The log-likelihood functions in the case of the Poisson ACF model:

$$\ell(\theta) = \sum_{xt}\left\{D_{xt}\,(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}) - E_{xt}e^{\alpha_x+\beta_x^{(1)}\kappa_t^{(1)}+\beta_x^{(2)}\kappa_t^{(2)}}\right\} + \text{constant at national level}$$

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_x^{(1)}\kappa_t^{(1)} + \beta_{xj}^{(2)}\kappa_{tj}^{(2)}) - E_{xtj}e^{\alpha_{xj}+\beta_x^{(1)}\kappa_t^{(1)}+\beta_{xj}^{(2)}\kappa_{tj}^{(2)}}\right\} + \text{constant at regional level}$$

The log-likelihood functions in the case of the Poisson ACF–CAE model:

$$\ell(\theta) = \sum_{xt}\left\{D_{xt}\,(\alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}) - E_{xt}e^{\alpha_x+\beta_x^{(1)}\kappa_t^{(1)}+\beta_x^{(2)}\kappa_t^{(2)}}\right\} + \text{constant at national level}$$

$$\ell_j(\theta) = \sum_{xt}\left\{D_{xtj}\,(\alpha_{xj} + \beta_x^{(1)}\kappa_t^{(1)} + \beta_x^{(2)}\kappa_{tj}^{(2)}) - E_{xtj}e^{\alpha_{xj}+\beta_x^{(1)}\kappa_t^{(1)}+\beta_x^{(2)}\kappa_{tj}^{(2)}}\right\} + \text{constant at regional level}$$

Steps of the iteration:

1. step: choosing the initial values for the parameters $\theta = (\alpha, \beta, \kappa)$
2. step: finding the optimal (*) values for the parameter $\hat{\alpha}_x$ with constraints
3. step: finding the optimal (*) values for the parameter $\hat{\alpha}_{xj}$ with constraints
4. step: finding the optimal (*) values for the parameter $\hat{\kappa}_t^{(i)}$ with constraints
5. step: finding the optimal (*) values for the parameter $\hat{\kappa}_{tj}^{(i)}$ with constraints
6. step: finding the optimal (*) values for the parameter $\hat{\beta}_x^{(i)}$ with constraints
7. step: finding the optimal (*) values for the parameter $\hat{\beta}_{xj}^{(i)}$ with constraints

$$\theta^* = \theta - \frac{\partial \ell_{(j)} / \partial \theta}{\partial^2 \ell_{(j)} / \partial \theta^2}$$