TAREFA: SISTEMAS LINEARES

→ REGRA DE CRAMER

1.

a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} \rightarrow 6 - 1 = \boxed{5}$

$X = \dfrac{Dx}{D} = \dfrac{3}{5}$ ou $0,6$

$Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} \rightarrow 6 - 3 = \boxed{3}$

$Y = \dfrac{Dy}{D} = \dfrac{-4}{5}$ ou $-0,8$

$Dy = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} \rightarrow -6 - (-2) = \boxed{-4}$

$V = \left\{ \left(\dfrac{3}{5}, \dfrac{-4}{5} \right) \right\}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases} \rightarrow y = 0$

$0 + 9 + 4 = 13$

$D = \begin{array}{|ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & 3 & 2 & 0 \\ 4 & 1 & 2 & 4 & 1 \end{array} \rightarrow -10 - 13 = -23$

$0 - 12 + 2 = -10$

$0 + 3 - 2 = 1$

$D_x = \begin{array}{|ccc|cc} 1 & -1 & 1 & 1 & -1 \\ -1 & 0 & 3 & -1 & 0 \\ 7 & 1 & -2 & 7 & 1 \end{array} \rightarrow -22 - 1 = -23$

$0 - 21 - 1 = -22$

$-4 + 63 - 4 = 55$

$D_y = \begin{array}{|ccc|cc} 3 & 1 & 1 & 3 & 1 \\ 2 & -1 & 3 & 2 & -1 \\ 4 & 7 & -2 & 4 & 7 \end{array} \rightarrow 32 - 55 = -23$

$6 + 12 + 14 = 32$

$0 - 3 - 14 = -17$

$D_z = \begin{array}{|ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 7 & 4 & 1 \end{array} \rightarrow 6 - (-17) = 23$

$0 + 4 + 2 = 6$

$x = \dfrac{D_x}{D} = \dfrac{-23}{-23} = 1 \qquad y = \dfrac{D_y}{D} = \dfrac{-23}{-23} = 1 \qquad z = \dfrac{D_z}{D} = \dfrac{23}{-23} = -1$

$$V = \{(1, 1, -1)\}$$

2. $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$

$-5 - 12 + 48 = 31$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \begin{matrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{matrix} \rightarrow 61 - 31 = 30$$

$45 + 8 + 8 = 61$

$-12 + 48 + 12 = 48$

$$Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{matrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{matrix} \rightarrow 78 - 48 = 30$$

$108 + 2 - 32 = 78$

$$Y = \frac{Dy}{D} = \frac{30}{30} = 1$$

$Y = 1$ (A)

3. $\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$

$2 - 33 - 6 = -37$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix} \rightarrow 36 - (-37) = 1$$

$-1 - 44 + 9 = -36$

$1 - 33 + 4 = -28$

$$Dx = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ -2 & 1 \\ 1 & 3 \end{matrix} \rightarrow -29 - (-28) = 1$$

$-1 - 22 - 6 = -29$

$$-4 - 11 - 3 = 18$$

$$-2 - 6 + 6 = 2$$

$$DY = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix} \rightarrow -11 - (-18) = \boxed{1}$$

$$D_Z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix} \rightarrow 2 - 2 = \boxed{0}$$

$$2 - 22 + 3 = -17$$

$$1 - 8 + 8 = 2$$

$$x = \frac{Dx}{D} = \frac{-1}{1} = -1 \qquad y = \frac{DY}{D} = \frac{1}{1} = 1 \qquad z = \frac{DZ}{D} = \frac{0}{1} = 0$$

$$\begin{matrix} A & B & C \\ \downarrow & \downarrow & \downarrow \\ X & Y & Z \\ \downarrow & \downarrow & \downarrow \\ -1 & + 1 & + 0 = 0 \end{matrix}$$

Ⓒ

4.
$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

$-9 - 2 - 4 = -15$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix}\begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix} \rightarrow 1 - (-15) = 16$$

$-6 + 4 + 3 = 1$

$-72 - 58 - 16 = -146$

$$Dx = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix}\begin{matrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{matrix} \rightarrow -130 - (-146) = 16$$

$-174 + 32 + 12 = -130$

$-12 - 16 - 58 = -54$

$$DY = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix}\begin{matrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{matrix} \rightarrow 26 - (-54) = 80$$

$-8 + 58 - 24 = 26$

$87 - 4 + 16 = 99$

$$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix}\begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix} \rightarrow 3 - 99 = -96$$

$24 + 8 - 29 = 3$

$x = \dfrac{Dx}{D} = \dfrac{16}{16} = 1 \qquad y = \dfrac{DY}{D} = \dfrac{80}{16} = 5 \qquad z = \dfrac{Dz}{D} = \dfrac{-96}{16} = -6$

$$\begin{array}{ccc} A & B & C \\ \downarrow & \downarrow & \downarrow \\ X & Y & X \\ \downarrow & \downarrow & \downarrow \\ 1 + & 5 & -6 = 0 \end{array}$$

(A)

5.
$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$z=0$

$x=0$

$$0+4+0=4$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} \rightarrow 7-4=3$$

$$4+3+0=7$$

$$0+10+3=13$$

$$Dx = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{matrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{matrix} \quad 17-13=4$$

$$10+7+0=17$$

$$0+14+0=14$$

$$Dy = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} \begin{matrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{matrix} \rightarrow 21-14=7$$

$$6+15+0=21$$

$$30+12+0=42$$

$$Dz = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} \rightarrow 37-42=-5$$

$$28+9+0=37$$

$$x = \frac{Dx}{D} = \frac{4}{3} \qquad y = \frac{Dy}{D} = \frac{7}{3} \qquad Dz = \frac{Dz}{D} = \frac{-5}{3}$$

$$V = \left\{ \left(\frac{4}{3}, \frac{7}{3}, \frac{-5}{3} \right) \right\} \qquad \textcircled{D}$$

6.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{array}{ccc} 1.x & 0.Y & 0.Z \\ 2.x & 1.Y & 0.Z \\ 3.x & 2.Y & 2.Z \end{array} \rightarrow \begin{cases} x = 3 \\ 2x + 1y = 7 \\ -x + 2y + 2z = 1 \end{cases}$$

$$0+0+0=0$$
$$D = \begin{array}{|ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{array} \rightarrow 2-0 = (2)$$
$$2+0+0=2$$

$$0+0+0=0$$
$$Dx = \begin{array}{|ccc|cc} 3 & 0 & 0 & 3 & 0 \\ 7 & 1 & 0 & 7 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{array} \rightarrow 6-0 = (6)$$
$$6+0+0=6$$

$$0+0+0=0$$
$$Dy = \begin{array}{|ccc|cc} 1 & 3 & 0 & 1 & 3 \\ 2 & 7 & 0 & 2 & 7 \\ -1 & -1 & 2 & -1 & -1 \end{array} \rightarrow (14)$$
$$14+0+0=14$$

$$-3+11+0=11$$
$$Dz = \begin{array}{|ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 7 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{array} \rightarrow 11-11 = (0)$$
$$-1+0+12=11$$

$$x = \frac{Dx}{D} = \frac{6}{2} = (3) \qquad Y = \frac{Dy}{D} = \frac{14}{2} = (7) \qquad z = \frac{Dz}{D} = \frac{0}{2} = (0)$$

(E)

→ ESCALONAMENTO : GAUSS

1.

$$S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$\cdot 3$

$$\begin{pmatrix} 2 & -1 & -3 & \vdots & -5 \\ 1 & 3 & -1 & \vdots & 11 \\ 1 & 0 & -5 & \vdots & 3 \end{pmatrix}$$

$\rightarrow y = 0$

$1 \rightarrow$
-7
$$\begin{pmatrix} 7 & 0 & -10 & \vdots & -4 \\ 1 & 0 & -5 & \vdots & 3 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ & & & \vdots & \\ 0 & 0 & 25 & \vdots & -25 \end{pmatrix}$$

$25z = -25$

$z = \dfrac{-25}{25}$

$\boxed{z = -1}$

$7x - 10z = -4$

$7x - 10 \cdot (-1) = -4$

$7x = -4 - 10$

$7x = -14$

$\boxed{z = -2}$

$2x - y - 3z = -5$

$2 \cdot (-2) - y - 3 \cdot (-1) = -5$

$-4 - y + 3 = -5$

$-5 + 5 + 3 = y$

$\boxed{4 = x}$

2.
$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$$

$$x = 2y$$
$$z = \frac{2y}{3}$$

$$x + y + z = 11$$
$$\frac{2y + y + 2y}{3} = 1(.3)$$

$$6y + 3y + 2y = 33$$
$$11y = 33$$
$$y = 3$$

$$x = 2 \cdot y$$
$$x = 2 \cdot 3$$
$$x = 6$$

$$2y = 3 \cdot z$$
$$2 \cdot 3 = 3 \cdot z$$
$$6 = 3z$$
$$2 = z$$

$$x + 2y + 3z = ?$$
$$6 + 2 \cdot 3 + 3 \cdot 2 = 18$$

(B)

3.
$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$

$\begin{array}{c} -2 \\ \hookleftarrow \end{array}\left(\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right)$

$\begin{array}{c} .2 \\ .1 \end{array}\left(\begin{array}{ccc:c} 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc:c} & & & \\ 0 & 0 & -5 & 10 \end{array} \right)$

$$-5z = -10$$
$$z = \frac{-10}{-5}$$
$$z = 2 \quad \textcircled{D}$$

4.

A → Ali
B → Bia   → $A + B + C = 68$ I
C → Caco

$B + 0.2c = A$ II $C + 0.2a = 3b$ III

$a + b + c = 68$
$(b + 0.2c) + b + c = 68$ IV
$b + 0.2c + b + c = 68$
$2b + 1.2c = 68$

$2b + 1.2c = 68$ IV $C + 0.2a = 3b$ III
$2b = 68 - 1.2c$ $c + 0.2a = 3(34 - 0.6c)$
$b = 34 - 0.6c$ $c + 0.2a = 102 - 1.8c$
                              $2.8c + 0.2a = 102$

                              $2.8c + 0.2(b + 0.2c) = 102$
$2.84c + 0.2(34 - 0.6c) = 102$ ← $2.84c + 0.2b = 102$
$2.84c + 6.8 - 0.12c = 102$
$2.72c = 102 - 6.8$
$2.72c = 95.2$
$c = 35$

$B = 34 - 0,6c$

$b = 34 - 0,6 \cdot 35$

$b = 34 - 21$

$b = 13$

$C - A =$

$35 - 20 = 15$

(A)

$a + b + C = 68$

$a + 13 + 35 = 68$

$a + 48 = 68$

$a = 68 - 48$

$a = 20$

5.

x : alfeu $= 134$
y : bento $= 115$
z : cíntia $= 48$

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ ou } \begin{bmatrix} 134 \\ 115 \\ 48 \end{bmatrix}$$

$$\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$2. \hookrightarrow \left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ \textcircled{1} & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ & & & \\ 0 & 1 & -10 & -182 \end{array} \right) \sim \left(\begin{array}{ccc|c} & & & \vdots \\ & & & \vdots \\ 0 & 0 & 34 & 680 \end{array} \right)$$

-3

$34z = 680$
$z = 20$

$x + 5z = 115$
$x + 5 \cdot 20 = 115$
$x + 100 = 115$
$x = 115 - 100$
$x = 15$

$3y + 4z = 134$
$3y + 4 \cdot 20 = 134$
$3y + 80 = 134$
$3y = 134 - 80$
$3y = 54$
$y = 18$

$15 + 18 + 20 = 53$

$\textcircled{A}$