TAREFA: DISCUSSÃO SOBRE SISTEMAS LINEARES.

1.a)

$$\begin{cases} ax + 4y = 1 \\ x + 2y = B \end{cases} \Rightarrow m\begin{pmatrix} a & 4 & \vdots & 1 \\ 1 & 2 & \vdots & 0,5 \end{pmatrix} \sim \begin{pmatrix} a-2 & 0 & \vdots & 0 \end{pmatrix} \quad (a-2) = 0$$

$x = \frac{0}{a-2}$ (F)

b) $x = \frac{0}{a-2}$

$x = \frac{0}{2-2}$ (V)

$x = \frac{0}{0}$ indeterminado

c) Se o $a \neq 2$, qualquer valor terá uma solução única (F)

d) Em $a = 2$, tem infinitas soluções (F)

e) $A = 0$ é indeterminado (F)

1) R: (B)

2.

$$I \rightarrow -K \begin{pmatrix} 1 & K & \vdots & 1 \\ K & 1 & \vdots & 1-K \end{pmatrix} \sim \begin{pmatrix} & & \vdots & \\ 0 & 1-2K & \vdots & 1-K^2 \end{pmatrix} \rightarrow Y = \frac{1-K^2}{1-2K}$$ não é indeterminado

$II \rightarrow K = 0,5 \quad Y = \frac{1-0,5^2}{1-2.0,5} = \frac{0,75}{0}$ inconcebível

III → $K \neq 0,5$ → Determinado

R: (D)

3. $\begin{cases} x + 2y + cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$

a)

$3c + 2 + 0 = 3c + 2$

$$A = \begin{pmatrix} 1 & 2 & c \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{pmatrix} \qquad A = \left|\begin{array}{ccc|cc} 1 & 2 & c & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 3 & 2 & 2 & 3 & 2 \end{array}\right|$$

$8 - (3c + 2) = 6 - 3c$ → Det A

$2 + 6 + 0 = 8$

b)

$$\begin{pmatrix} 1 & 2 & c & \vdots & 1 \\ 0 & 1 & 1 & \vdots & 2 \\ 3 & 2 & 2 & \vdots & -1 \end{pmatrix} \begin{matrix} \\ -2 \\ -2 \end{matrix} \sim \begin{pmatrix} 1 & 0 & c-2 & \vdots & -3 \\ & & & \vdots & \\ 3 & 0 & 0 & \vdots & -5 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ 0 & 0 & -3c+6 & \vdots & 4 \end{pmatrix}$$

S.P Det $z = 1$ $-3c + 6 \neq 0$

$D \neq 0$ $-3c$ $6 \neq 3c$

$6/3 \neq c$

$2 \neq c$

R: $c \in \mathbb{R} - \{2\}$

4. $\begin{pmatrix} 1 & -1 & 0 & \vdots & k \\ 12 & -k & 1 & \vdots & 1 \\ 36 & 0 & k & \vdots & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & -k+12 & 1 & \vdots & 1-12k \\ 0 & 36 & 1 & \vdots & 2-36k \end{pmatrix} \sim$

$\begin{pmatrix} 0 & k^2-12 & k+36 & \vdots & 2-12k^2-37k \end{pmatrix}$

$y = 12k^2 - 37k + 2$

$k^2 - 12k + 36 \neq 0$

$6 + 6 = 12 \rightarrow k \neq 6$

$6 \times 6 = 36$

R: (E)

5. $\begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ x + 2y - z = -5 \end{cases} = \begin{pmatrix} 1 & -1 & 1 & 6 \\ 2 & 1 & -1 & -3 \\ 1 & 2 & -1 & -5 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ 0 & 3 & -3 & \vdots & -15 \\ 0 & 3 & -2 & \vdots & -11 \end{pmatrix}$

$\begin{pmatrix} & & & \vdots & \\ & & & \vdots & \\ 0 & 0 & 1 & \vdots & 4 \end{pmatrix} \rightarrow z = 4$

$3y - 3.4 = -15$

$3y - 12 = -15$

$3y = 15 + 12$

$3y = -3$

$y = -1$

$x + 1 + 4 = 6$

$x + 5 = 6$

$x = 6 - 5$

$x = 1$

S.P indeterminado

$x \cdot y \cdot z$

$1 \cdot -1 \cdot 4$

-4 // (B)

6. $\begin{pmatrix} 1 & 1 & 1 & \vdots & k \\ k & 1 & 1 & \vdots & 1 \\ 1 & 1 & -1 & \vdots & k \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ 0 & 1-k & 1-k & \vdots & 1-k^2 \\ 0 & 0 & -2 & \vdots & 0 \end{pmatrix}$ $\rightarrow -2z = 0$

$z = 0/-2$

$z = 0$

$(1-k)y + (1+k).0 = 1-k^2$

$(1-k)y = 1-k^2$

$y = \frac{1-k^2}{1-k}$

$1-k^2 = 0$ $\quad 1-k=0$ $\quad k=1 \rightarrow$ S.P. indeterminado.

$k^2 = 1$ $\quad k = 1$

$k = 1$

R: (D)

7. $\begin{cases} x+y+z = 1 \\ mx-2y+4z = 5 \\ m^2x+4y+16z = 25 \end{cases}$

$\begin{pmatrix} 1 & 1 & 1 & = & 1 \\ m & -2 & 4 & = & 5 \\ m^2 & 4 & 16 & = & 25 \end{pmatrix} \sim \begin{pmatrix} m+2 & 0 & 6 & \vdots & 7 \\ m^2+4 & 0 & 12 & \vdots & 21 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ m^2-2m-8 & & & \vdots & 7 \end{pmatrix}$

$x = 7$

m^2-2m-8

$-2 + 4 = 2$

$-2 \times 4 = -8.$

$x_1 + x_2$

$-2+4 = 2$

(B)

SISTEMAS LINEARES HOMOGÊNEOS

1. $\begin{bmatrix} 1 & 7 \\ 7 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = k \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{pmatrix} 1 & 7 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} x+7y \\ 7y+y \end{pmatrix} = \begin{pmatrix} yk \\ yk \end{pmatrix} \rightarrow \begin{cases} x+7y = kx \\ 7x+x = ky \end{cases}$

$\cdot(-7)\left(\begin{array}{cc|c} 1 & 7 & k \\ 7 & 1 & k \end{array}\right) \sim \left(\begin{array}{cc|c} & & \\ 0 & -48 & -6k \end{array}\right) \rightarrow$

$-48y = -6k$

$y = \frac{-6k}{-48} \quad :6$

$y = \frac{-k}{-8} \cdot(-1)$

$y = \frac{k}{8} \rightarrow k = 8y$

$k = 8.1$

$k = 8$

$x + 7y = kx$

$x + 7y = 8yx$

$\frac{x}{x} = 8y - 7y$

$1 = y$

R: (E)

2. $\begin{cases} 3x + 4y - z = 0 \\ 2x - y + 3z = 0 \\ x + y + 0 = 0 \end{cases}$

$\cdot(3)\left(\begin{array}{ccc|c} 3 & 4 & -1 & 0 \\ 2 & -1 & 3 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 11 & 11 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \div 11 \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right)$ ↳ iguais

indeterminado

(D)

$x + y = 0$ $\quad 2x - y + 3z = 0$

$x + \alpha = 0$ $\quad -2\alpha - \alpha + 3z = 0$

$x = -\alpha$ $\quad -3\alpha + 3z = 0$ $\quad V = \{(-\alpha, \alpha, \alpha)\}$

$3z = 3\alpha$

$z = \alpha$

3. $\begin{cases} x + y + z = 0 \\ kx + 3y + 4z = 0 \\ x + ky + 3z = 0 \end{cases}$

$3 + 4k + 3k = 3 + 7k$

$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ k & 3 & 4 & k & 3 \\ 1 & k & 3 & 1 & k \end{vmatrix}$ $\quad 9 - 4 + k^2 - (3 + 7k)$

$k^2 - 7k + 10 = 0$

$2 + 5 = 7$

$2 \times 5 = 10$ $\quad k' + k''$

$2 + 5 = 7$ (D)

4. $\begin{cases} x + 0 + k = 0 \\ kx + y + 0 = 0 \\ x + ky + 0 = 0 \end{cases}$

$\begin{pmatrix} 1 & 0 & k & 0 \\ k & 1 & 0 & 0 \\ 1 & k & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & k & 0 \\ & & & \\ 1-k^2 & 0 & 0 & 0 \end{pmatrix}$

$x = 0$

$1 - k^2 \neq 0 \rightarrow 1 - k^2 \neq 0$

$1 \neq k^2$

$\pm 1 \neq k$

solução única

$V = \{k \in \mathbb{R} / k \neq 0, k \neq 1, k \neq -1\}$

(A)

5. $\begin{cases} -x + 2y - 3 = 0 \\ 3x - y + 3 = 0 \\ 2x - 4y + 6 = 0 \end{cases}$

$$\begin{pmatrix} -1 & 2 & \vdots & 3 \\ 3 & -1 & \vdots & -3 \\ 2 & -4 & \vdots & -6 \end{pmatrix} \sim \begin{pmatrix} & & \vdots & \\ 0 & 5 & \vdots & 6 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \rightarrow 5y = 6$$

$y = 6/5$, Determinado

$$-x + \frac{2 \cdot 6}{5} = 3$$

$$-x + \frac{12}{5} = 3$$

$$-x = 3 - \frac{12}{5}$$

$$-x = \frac{3 \cdot (-1)}{5}$$

$x = \frac{-3}{5}$, Determinado

R: (B)