Deadlines:

For all Groups: May 19 2023

Grading system:

For every problem you solve you get a score. Your score is your mark for this laboratory.

- 1 problem – 3p
- 2 problem – 3p
- 3 problem – 3p
- 4 problem – 1p

1. Secret path

You find yourself in a dense forest, standing before a great river. Legend has it that a hidden treasure lies on the other side, but a series of challenges must be overcome to reach it. You notice a set of stepping stones in the river, each labelled with a mathematical equation. These equations hold the key to unlocking the path across the river. Solve the system of linear equations represented by the stepping stones to determine the order in which you should step on them to reach the treasure. Each equation represents the coordinates of a stepping stone in terms of x and y. By solving the equations, you will unlock the correct path and successfully cross the river. You encounter five stepping stones labelled with mathematical equations. Solve the system of linear equations to determine the correct order in which to step on them:

Stone 1: $2x - y = 4$

Stone 2: $3x + 2y = 7$

Stone 3: $x + y = 3$

Stone 4: $4x - y = 1$

Stone 5: $x - 3y = -2$

Restrictions:

You don't have to use any libraries for this exercise. Exceptions are Numpy and Math libraries

2. Influence network

A team of sociologists is conducting a social network analysis to understand the dynamics of influence within a community. They have gathered data on the relationships between individuals and represented it using a connectivity matrix. Your task is to calculate the dominant eigenvalue and its corresponding eigenvector of this connectivity matrix(in the email). By unravelling the network of influence, you can identify the most influential members shaping the community's dynamics. Analyse the eigenvalues and eigenvectors to uncover the key individuals who act as opinion leaders and exert a significant impact on others within the community.

Restrictions:
You don't have to use any libraries for this exercise. Exceptions are Numpy and Math libraries

3. Cloud computing

As a software engineer focusing on cloud computing infrastructure, your goal is to optimise resource allocation for efficient task scheduling. This involves dynamically allocating computational resources among tasks or virtual machines to maximise resource utilisation and minimise response time. To accomplish this, you decide to use ordinary differential equations (ODEs) to model the utilisation of computational resources and their dynamic allocation over time. The system of ODEs that represents the resource allocation in cloud computing is as follows:

Variables:

R(t): Total available resources at time t

U(t): Utilisation of resources at time t

A(t): Arrival rate of tasks at time t

D(t): Departure rate of completed tasks at time t

ODEs:

Resource Utilisation ODE:

dU(t)/dt = A(t) - D(t)

Resource Allocation ODE:

dR(t)/dt = A(t) - D(t)

In this system, the rate of change of resource utilisation ($dU(t)/dt$) depends on the arrival rate of tasks ($A(t)$) and the departure rate of completed tasks ($D(t)$). It quantifies the net change in resource utilisation over time. Similarly, the rate of change of total available resources ($dR(t)/dt$) is determined by the arrival rate of tasks ($A(t)$) and the departure rate of completed tasks ($D(t)$). It represents the net change in the total available resources over time.

Your task is to numerically solve these coupled ODEs using appropriate numerical methods (e.g., Euler's method, Runge-Kutta methods) and analyse the resource utilisation patterns over time. Investigate how variations in the arrival rate, departure rate, and allocation policies impact the overall efficiency of resource allocation in cloud computing.

4. Traffic simulation

You have been tasked with developing a traffic simulation tool to study the flow of vehicles on a road network. The objective is to analyse the effects of different traffic conditions, road geometries, and traffic management strategies on congestion, travel times, and overall system performance. To accomplish this, you decide to utilise the Lighthill-Whitham-Richards (LWA) traffic flow model. The LWA model represents the dynamics of traffic flow by considering the traffic density and traffic velocity within each road segment. The road network is divided into segments, each characterised by a specific position. Your task is to implement the LWA traffic flow model and simulate the traffic flow over a specified duration.

The LWA model represents the dynamics of traffic flow by considering the traffic density $\rho(x, t)$ and traffic velocity $v(x, t)$. In the context of your simulation, the road network is divided into segments, each characterised by a specific position x.

The first equation to model traffic density is the conservation law:

$$\partial\rho/\partial t + \partial(\rho v)/\partial x = 0$$

This equation states that the rate of change of traffic density over time is equal to the net flow of vehicles in or out of a road segment. The traffic density $\rho(x, t)$ represents the number of vehicles per unit distance at position x and time t.

Next, the traffic velocity $v(x, t)$ is determined using the LWA equation, which considers the desired velocity V_{max}, traffic density $\rho(x, t)$, and the congestion effect caused by traffic interactions:

$$v(x, t) = V_{max} * (1 - \rho(x, t) / \rho_{max})$$

In this equation, V_{max} represents the maximum velocity that can be achieved when the road segment is free of congestion. ρ_{max} is the maximum traffic density at which the road segment is considered fully congested.

To obtain the traffic flow $q(x, t)$ passing through a given point per unit time, we calculate the product of traffic density and traffic velocity:

$$q(x, t) = \rho(x, t) * v(x, t)$$

By solving these equations numerically over time and space, you can simulate the traffic flow, observe congestion patterns, and analyse the effects of different road geometries, traffic conditions, and traffic management strategies on travel times and overall system performance.