

Deadlines:

For all Groups: December 6 2022

Grading system:

One problem is one point.

- 1 problem - 1
- 2 problems - 2
- 3 problems - 3
- 4 problems - 4
- 5 problems - 5
- 6 problems - 6
- 7 problems - 7
- 8 problems - 8
- 9 problems - 9
- 10 problems - 10

1. Interesting observation

One student during a probability lecture noticed that although the number of faces of dice with integers from 1 to 6 with total 9 is like the number of such triples with value 10, a 9 seemed to come up less frequently than a 10 when three dice were rolled.

Write a program to verify if the student was correct. For better proof, plot a graph with results.

2. Densities

By selecting the break points at random, cut a stick that is one unit long into three pieces. It is believed that the break points will be selected simultaneously. What is the probability that the three pieces can be used to create a triangle?

3. Densities

Take a stick of unit length and break it into two pieces, choosing the breakpoint at random. Now break the longer of the two pieces at a random point. What is the probability that the three pieces can be used to form a triangle?

Write a computer program that would simulate the experiment. Explain your results.

4. Densities

Three points are chosen at random on a circle of unit circumference. What is the probability that the triangle defined by these points as vertices has three acute angles?

5. Densities

Choose independently two numbers B and C at random from the interval $[-1, 1]$ with uniform distribution, and consider the quadratic equation $x^2 + Bx + C = 0$. Find the probability that the roots of this equation

- are both real
- are both positive

6. Densities

At the Ziua Vinului, a coin toss game works as follows. Coins of 25 bani are tossed onto a checkerboard. The management keeps all the coins, but for each coin landing entirely within one square of the checkerboard the management pays 1 leu. Assume that the edge of each square is twice the diameter of a coin, and that the outcomes are described by coordinates chosen at random. Is this a fair game?

7. Densities

Write a program to carry out the following experiment. A coin is tossed 100 times and the number of heads that turn up is recorded. This experiment is then repeated 1000 times. Have your program plot a bar graph for the proportion of the 1000 experiments in which the number of heads is n, for each n in the interval $[35, 65]$. Does the bar graph look as though it can be fit with a normal curve?

8. Densities

At a mathematical conference, 10 participants are randomly seated around a circular table for meals. Using simulation, estimate the probability that no two people sit next to each other at both lunch and dinner. Can you make an intelligent conjecture for the case of n participants when n is large?

9. Random Variables

A game is played as follows: A random number X is chosen uniformly from $[0, 1]$. Then a sequence Y_1, Y_2, \dots of random numbers is chosen independently and uniformly from $[0, 1]$. The game ends the first time that $Y_i > X$. You are then paid $(i - 1)$ dollars. What is a fair entrance fee for this game?

10. Important Distributions

Jora Petrovici never pays the *troleibuz* (cost = 6 lei in Chisinau). He assumes that there is a probability of 0.05 that he will be caught by the *controlor*. The first offense costs 50 lei, the second costs 150 lei, and subsequent offenses cost 300 lei. There's also an empirically proven probability of 0.02 that the *taxatoarea* is a hairy muscular guy, which means that he'll have to pay the 6 lei anyway. How does the expected cost of riding the *troleibuz* for a year (two times a day) without paying compare to the cost that is paid by us, law-abiding students?