

Transforming a RegEx to FSA

LING 570

Thompson's construction

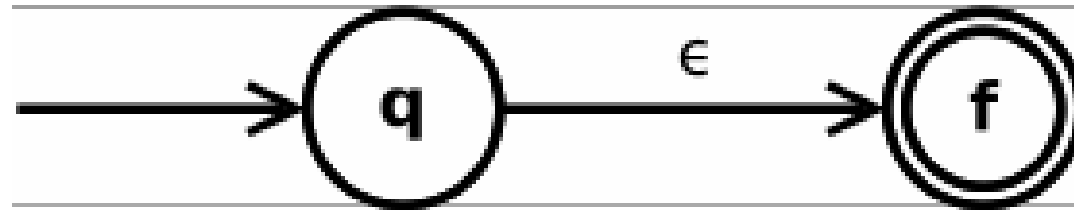
- https://en.wikipedia.org/wiki/Thompson%27s_construction
- RegEx: recursive definition (see the next slide)
 - ➔ Apply rules recursively

Definition of Regular Expression (as in formal language theory)

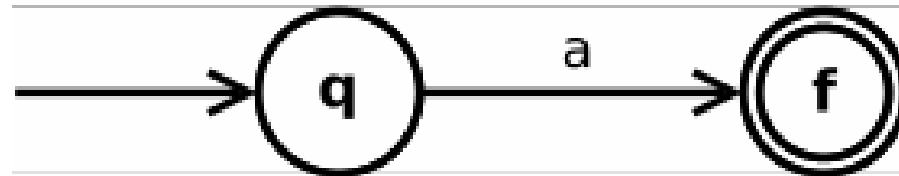
- The set of regular expressions is defined as follows:
 - (1) Every symbol of Σ is a regular expression
 - (2) ϵ is a regular expression
 - (3) If r_1 and r_2 are regular expressions, so are (r_1) , $r_1 r_2$, $r_1 \mid r_2$, r_1^*
 - (4) Nothing else is a regular expression.

Base case

$r = \epsilon$

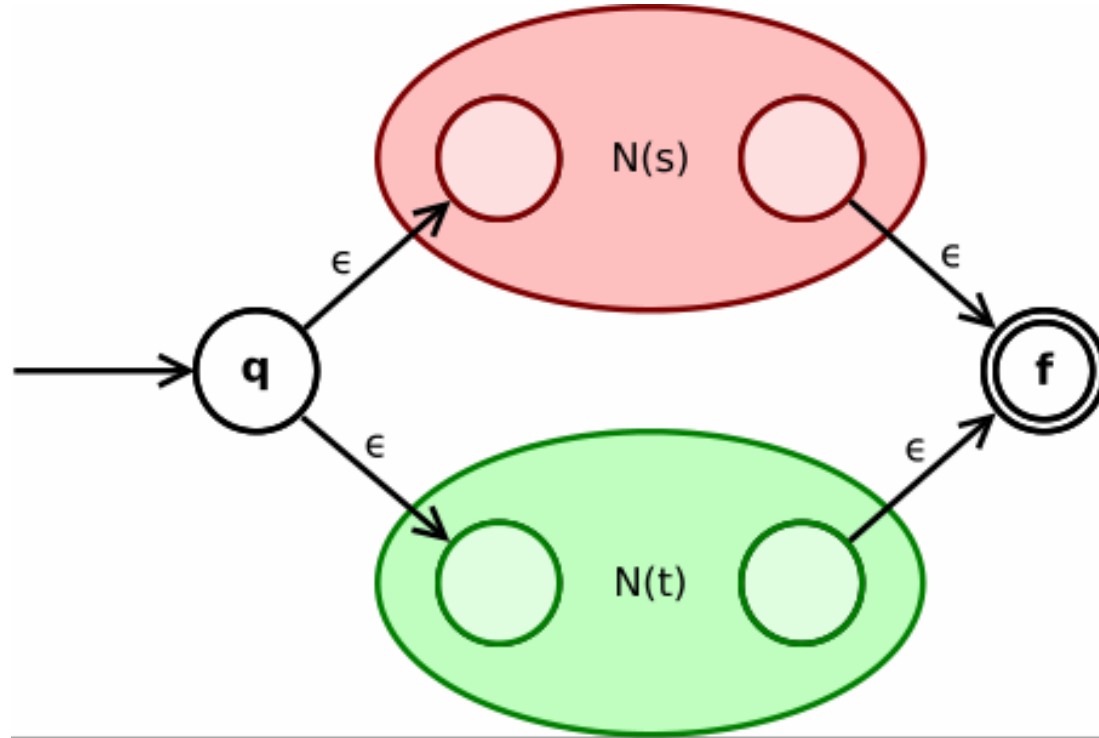


$r = a$



Union expression

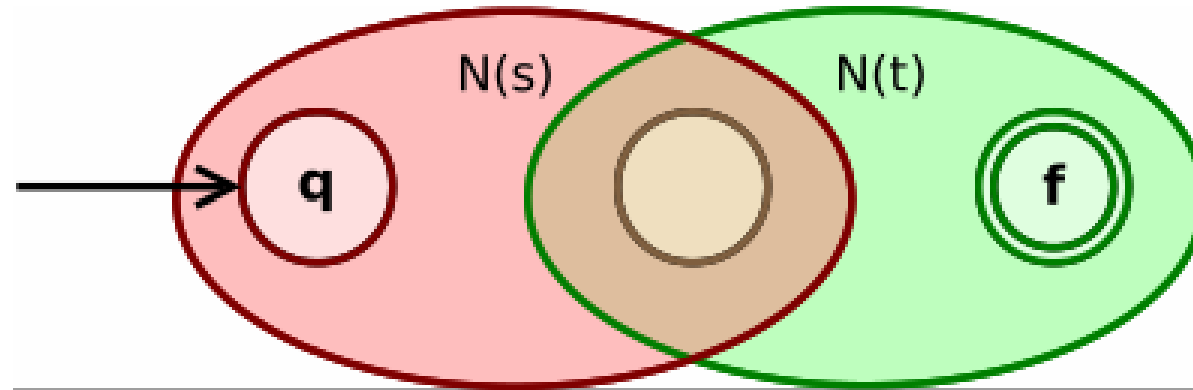
$$r = s \mid t$$



$N(s)$ and $N(t)$ are FSAs for s and t , respectively.

Concatenation expression

$r = s t$

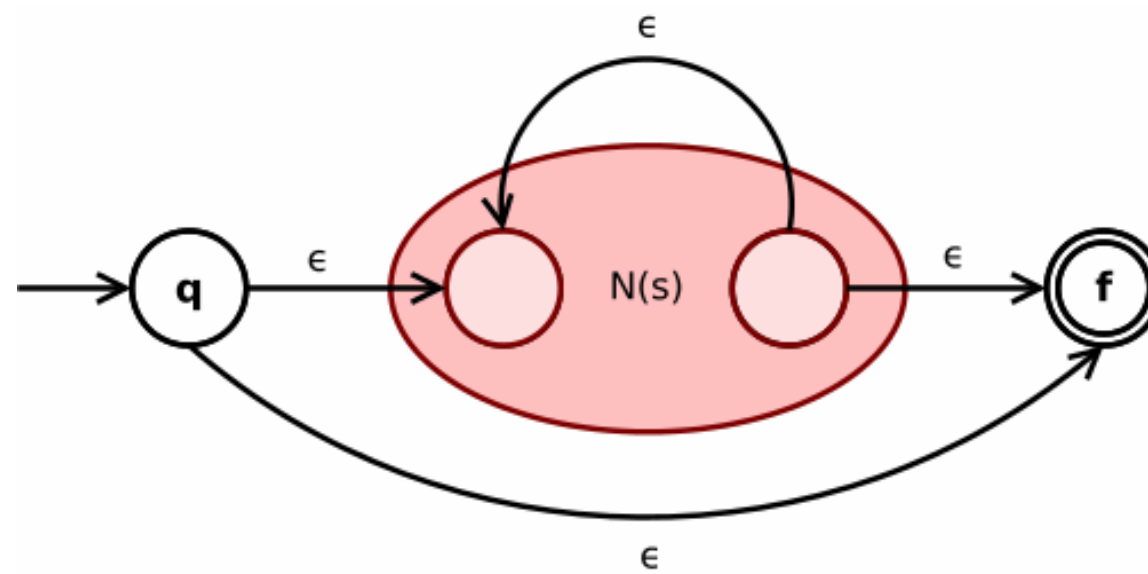


The initial state of $N(\textcolor{red}{s})$ is the initial state of the whole NFA. The final state of $N(\textcolor{red}{s})$ becomes the initial state of $N(\textcolor{green}{t})$. The final state of $N(\textcolor{green}{t})$ is the final state of the whole NFA.

Here we assume each FSA has only one initial state. If that's not the case, you can always create a new initial state that goes to each of the original states with ϵ -transition. The same can be done to have a single final state.

Kleene star expression

$$r = s^*$$



Create a new start state q and a new final state f .

An ϵ -transition connects new initial and final state of the NFA with the sub-NFA $N(s)$ in between.

Another ϵ -transition from the final state to the initial state of $N(s)$ allows for repetition of expression s according to the star operator.