Correlations in Heavy-ion Collisions using the R2 Variable

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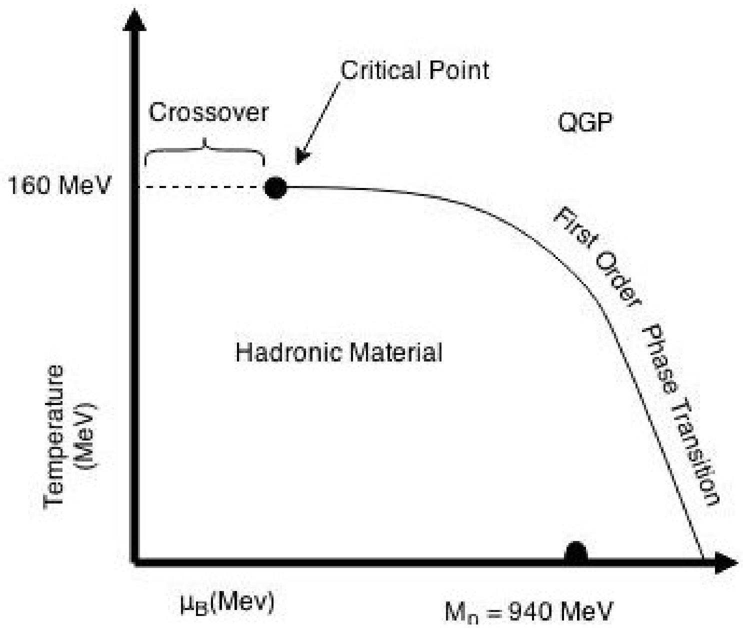
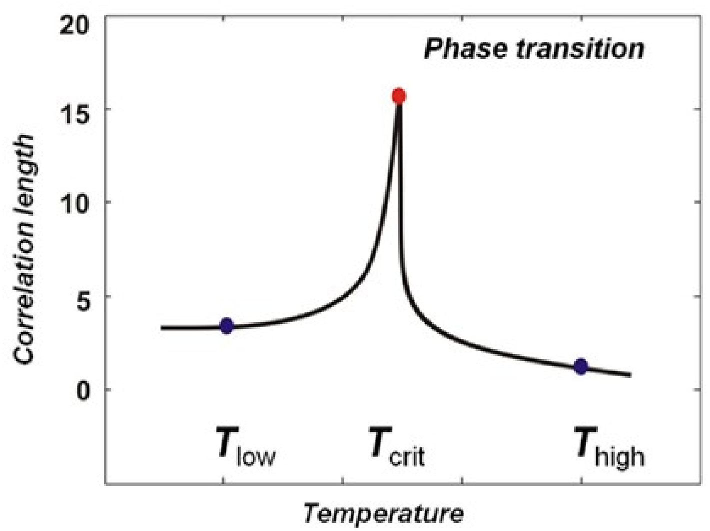
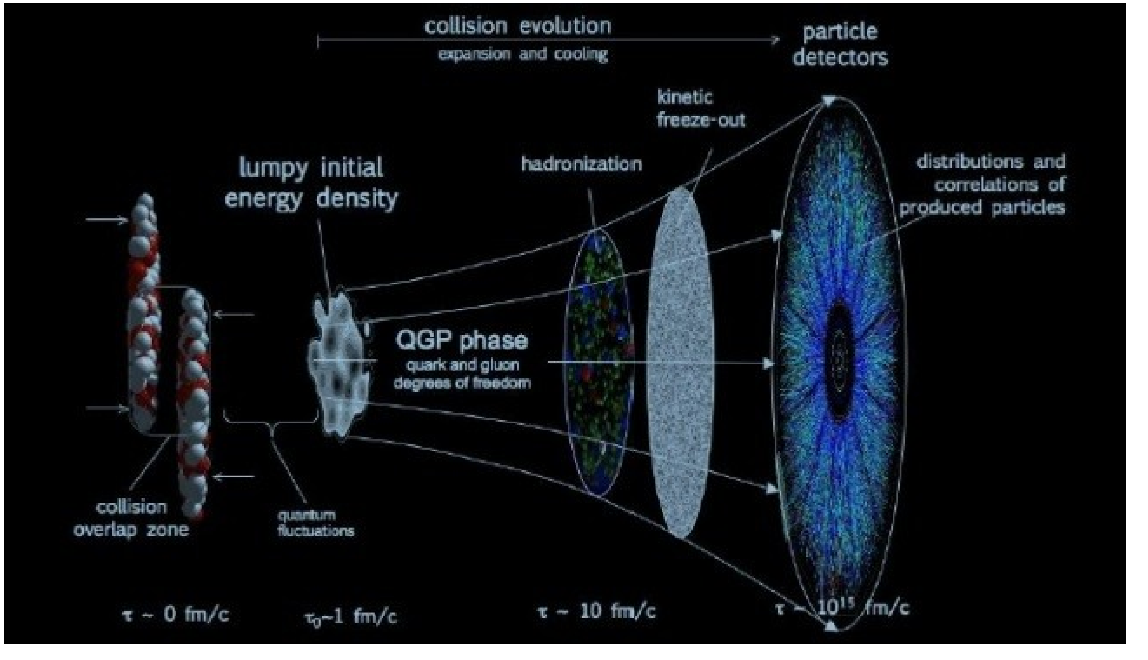
- Phases of nuclear matter are plotted on a diagram of density (μ_B) and temperature (T): namely hadronic matter and quark gluon plasma. Diagram below is mainly speculative.
- Looking for critical point by searching for increased correlations vs the beam energy (as $\sqrt{s_{NN}} \sim 1/\mu_B$)
- We explore these aspects using the R2 variable and simulated data where there is no critical point





UrQMD Data

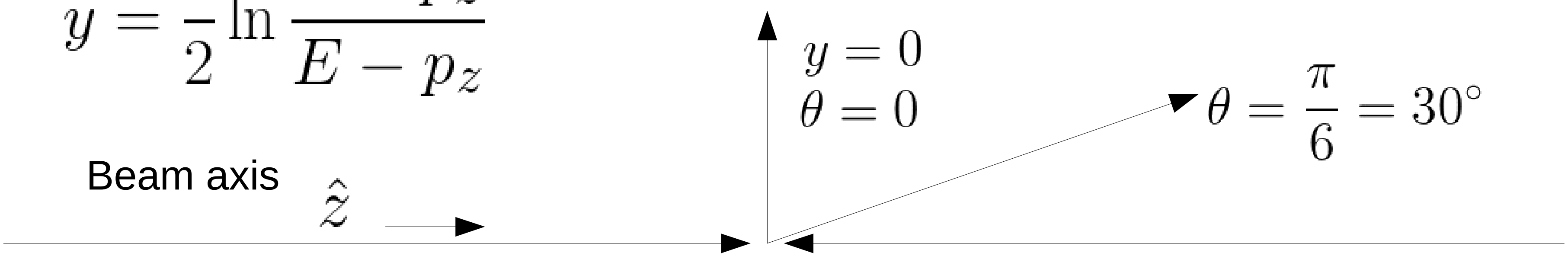
- Ultra-relativistic quantum molecular dynamics (UrQMD) model data used to simulate ultra-relativistic heavy ion collisions
- Study central (0-5%) Au+Au collisions at different beam energies 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4, 200 GeV
- Same data collected by STAR experiment at RHIC
- No critical point in UrQMD: model is used to develop code and explore the R2 variable



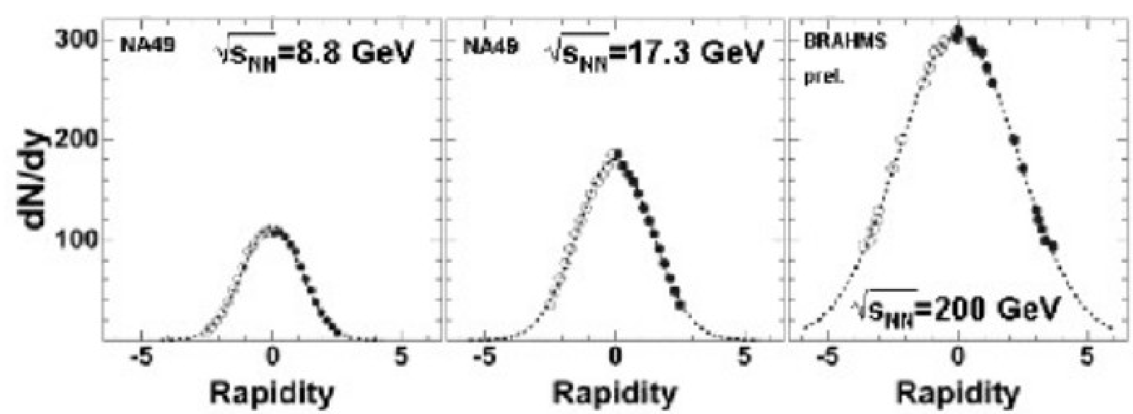
Particle Rapidity

- Calculated using momentum components

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$



- Rapidity distributions for Au+Au ion collisions at various energies



- Study intra-event rapidity correlations between pairs of protons via R2 variable vs difference in rapidity

Rapidity Dependent Two Particle Correlations

- Calculated by the two particle correlation function R2
- R2 comes from C2, the cumulant correlation function

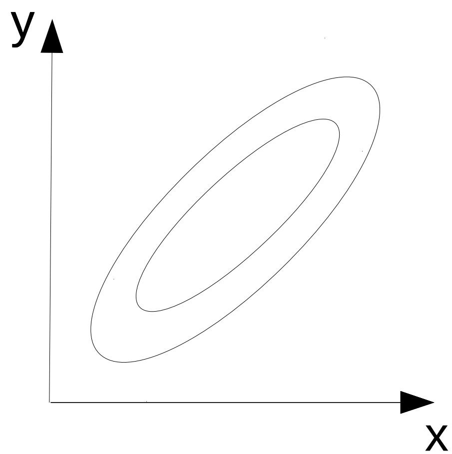
$$C_2 = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

$$R_2 = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$

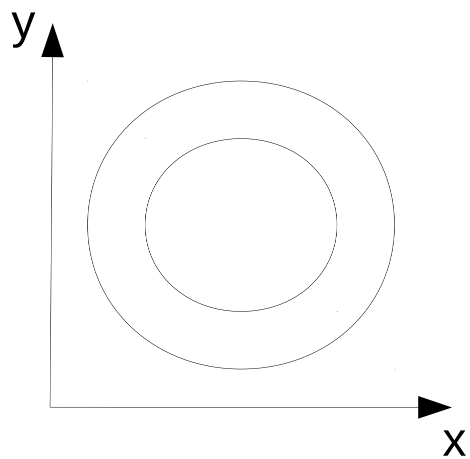
- R2 = 0 for random emission, >0 for correlated emission, <0 for anti-correlated emission
- Divergence of correlation length could be reflected by increased correlations

Three Types of Correlations

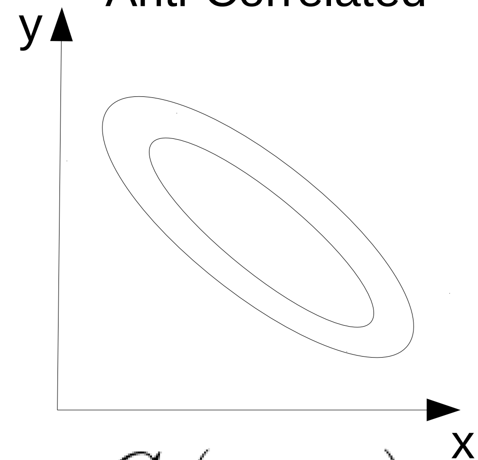
Correlated



Uncorrelated



Anti-Correlated

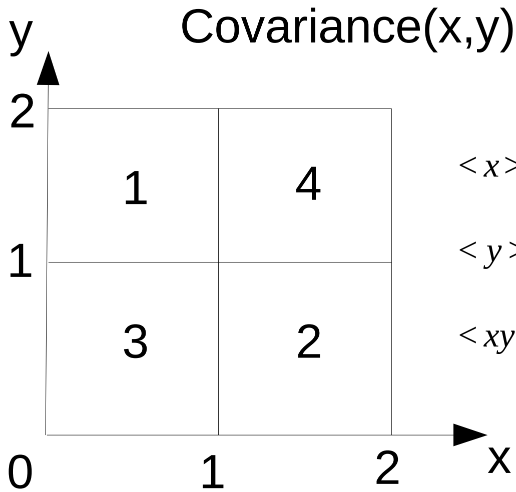


$$C_2 = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

$$R_2 = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)}$$

R2 is like a **Covariance**

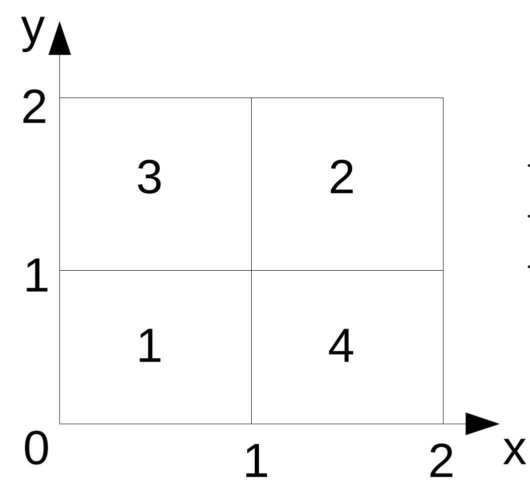
Covariance(x,y) = <xy> - <x><y>



$$<x> = \frac{3*1+1*1+2*2+4*2}{10} = \frac{16}{10}$$

$$<y> = \frac{3*1+2*1+1*2+4*2}{10} = \frac{15}{10}$$

$$<xy> = \frac{3*1*1+2*1*2+1*2*1+4*2*2}{10} = \frac{25}{10}$$



$$<x> = 1.6$$
$$<y> = 1.5$$
$$<xy> = 2.3$$

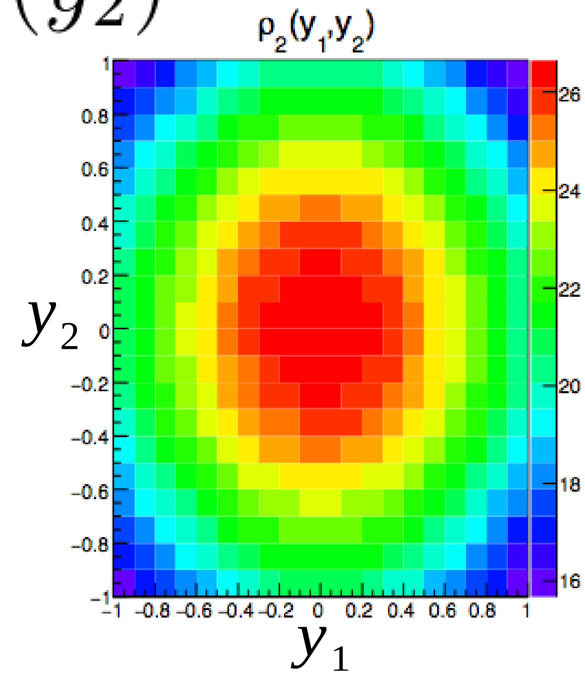
Covariance = 2.5 – (1.6)(1.5) = +0.1

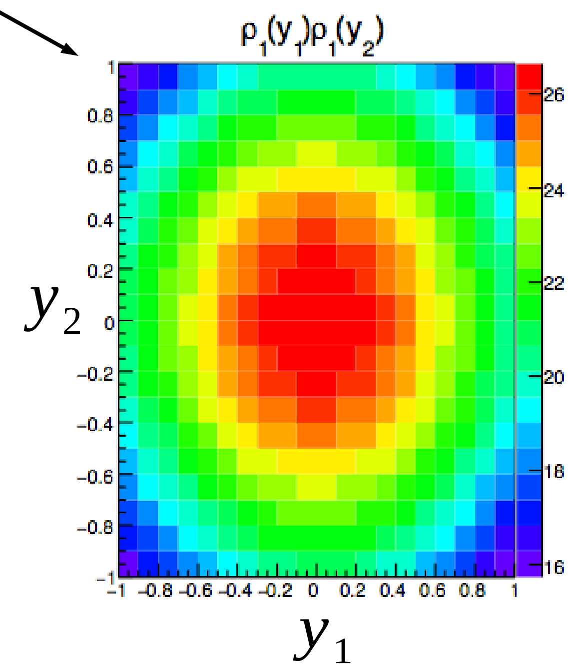
Covariance = 2.3 – (1.6)(1.5) = -0.1

$$\rho_2(y_1, y_2) \quad \& \quad \rho_1(y_1)\rho_1(y_2)$$

- For each pair except the self-pair, add entry into 2D plot of y1 vs y2
- Form tensor product of single particle distribution
- Normalize both to number of events
- Divide them
- Subtract 1 from every bin

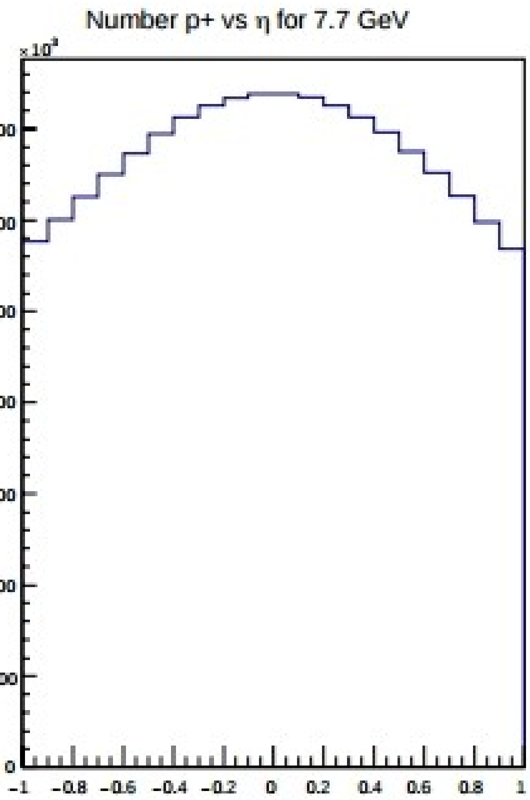
$$R_2 = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$





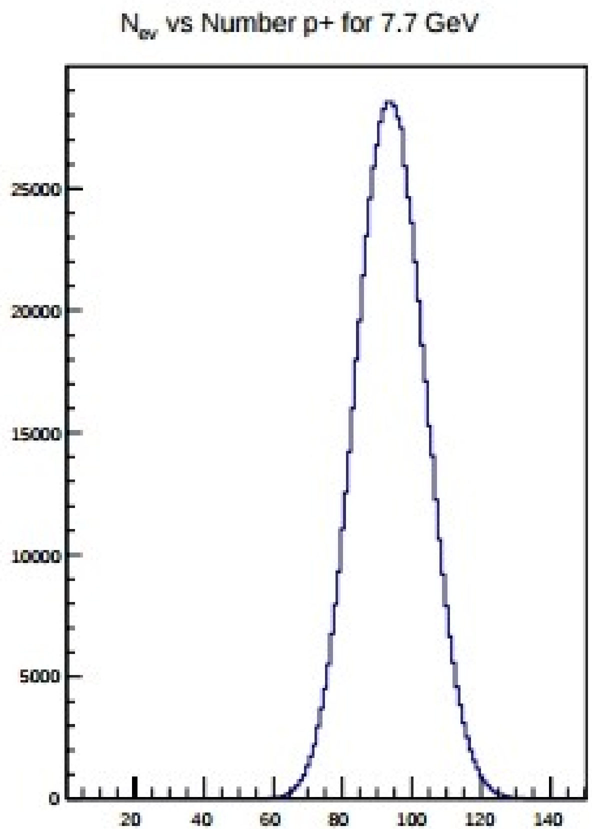
Rapidity Distribution and Proton Multiplicity for $\sqrt{s_{NN}} = 7.7$ GeV

Number of protons



y

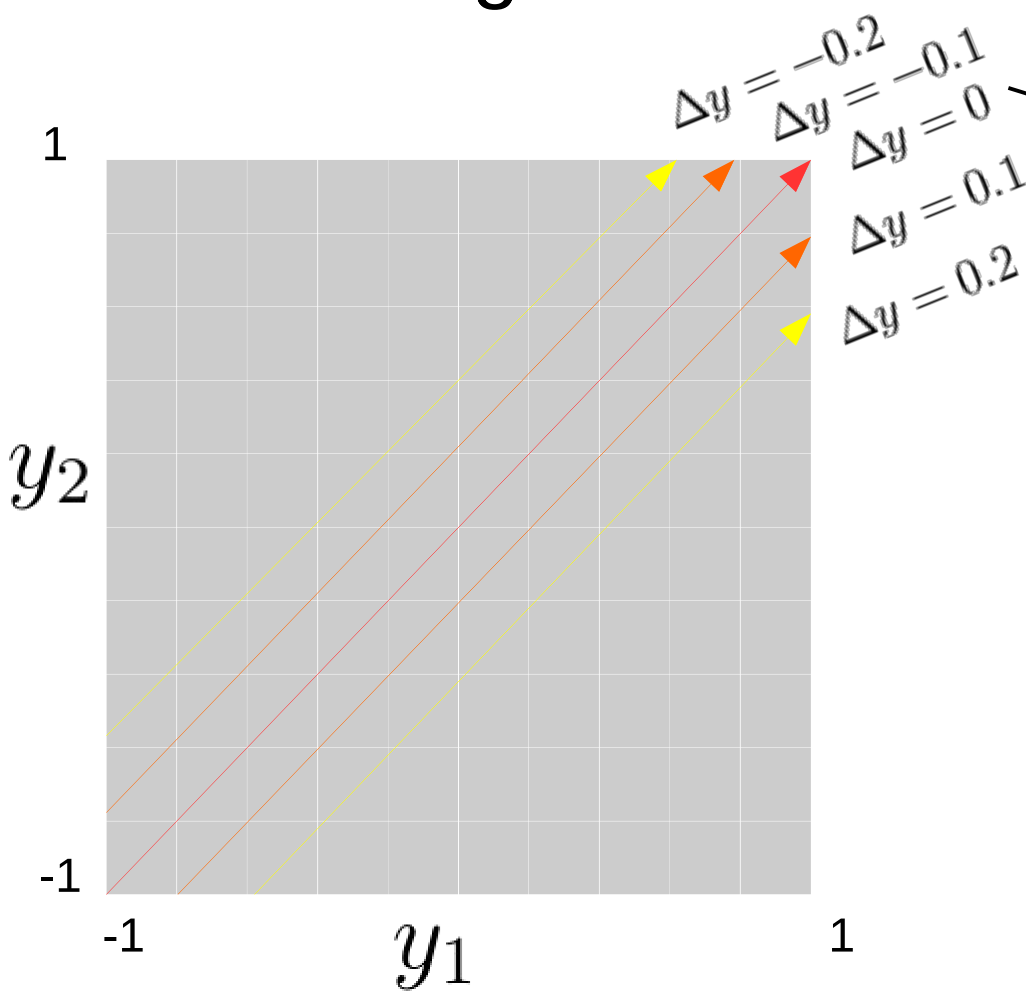
Number of Events

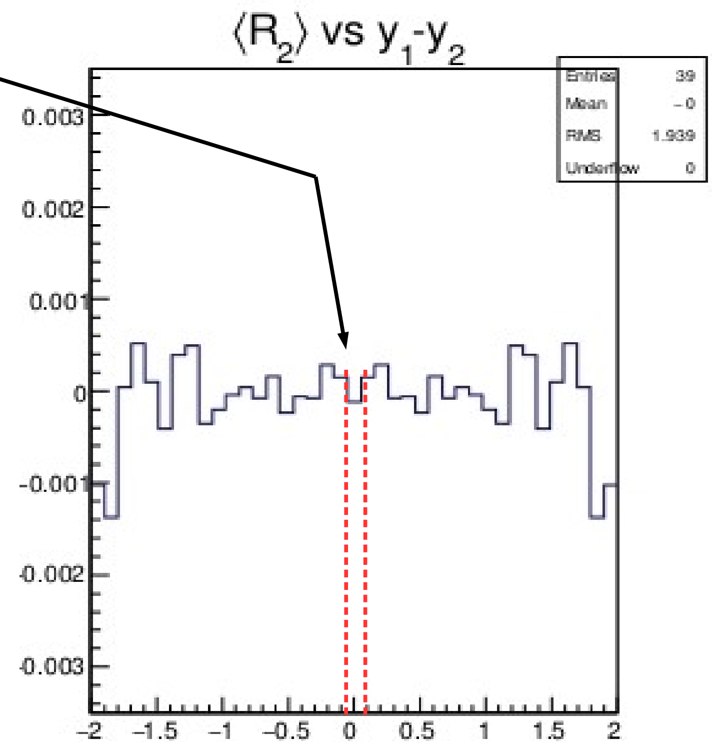


Number of protons/event

<R2> vs dy

- $$R_2 = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$
- Form average of R2 over all bins in a diagonal. Each diagonal has a fixed value of dy





By construction, <R2> vs dy is symmetric about dy = 0

Finite Counting Statistics

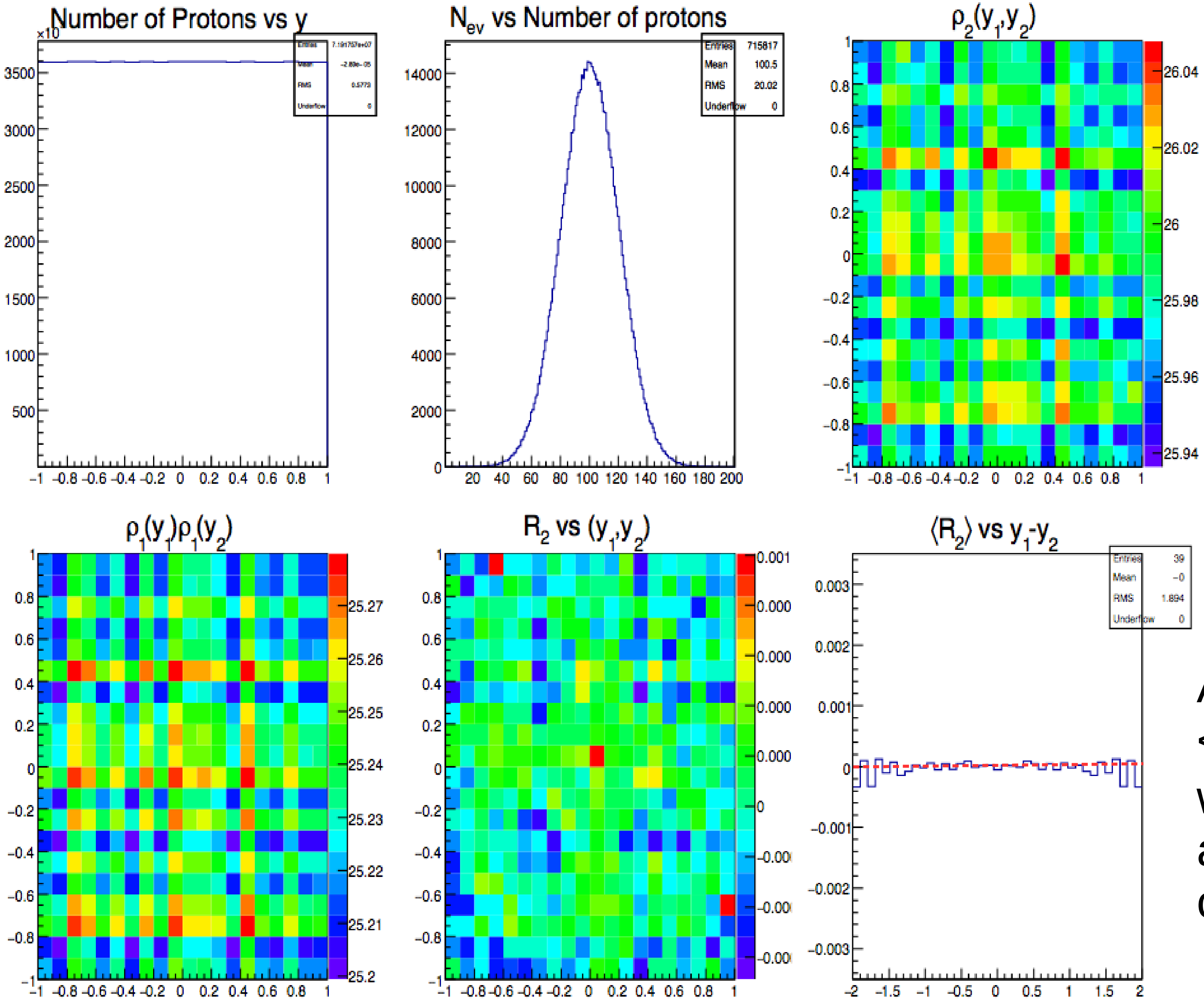
- When number of particles/evt, *n*, is small, <R2> is non-zero even if there are no correlations.
- Analytical value for fixed *n* is:

$$R_2^{bs} = \frac{\langle n(n-1) \rangle}{< n >^2} - 1$$

- Average R_2^{bs} over actual multiplicity distribution to correct for finite counting offset

N_{part}	$\langle R_2 \rangle$	N_{part}	$\langle R_2 \rangle$
2,3	-1/3	2,6	0
3,4	-1/4	4,6	-1/6
4,5	-1/5	3,6	-1/9
5	-1/5	1,6	+ 0.225
2-6	-1/6	10-14	-1/14

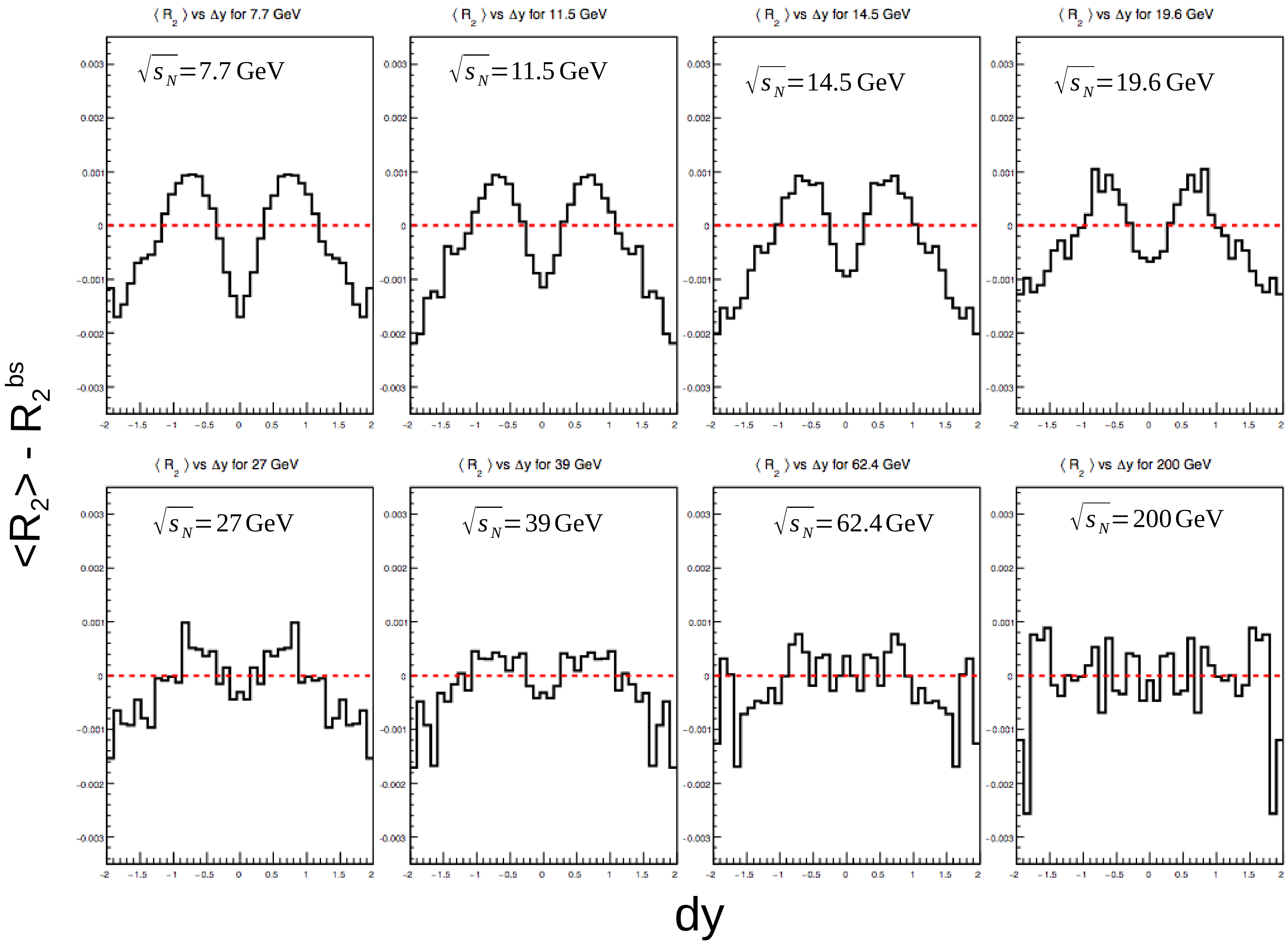
Test: rapidities randomly sampled from flat distribution and Npart/event randomly sampled from a Gaussian



Z-axis scale has small range

As expected, $\langle R_2\rangle - R_2^{bs} = 0$ when there are no correlations

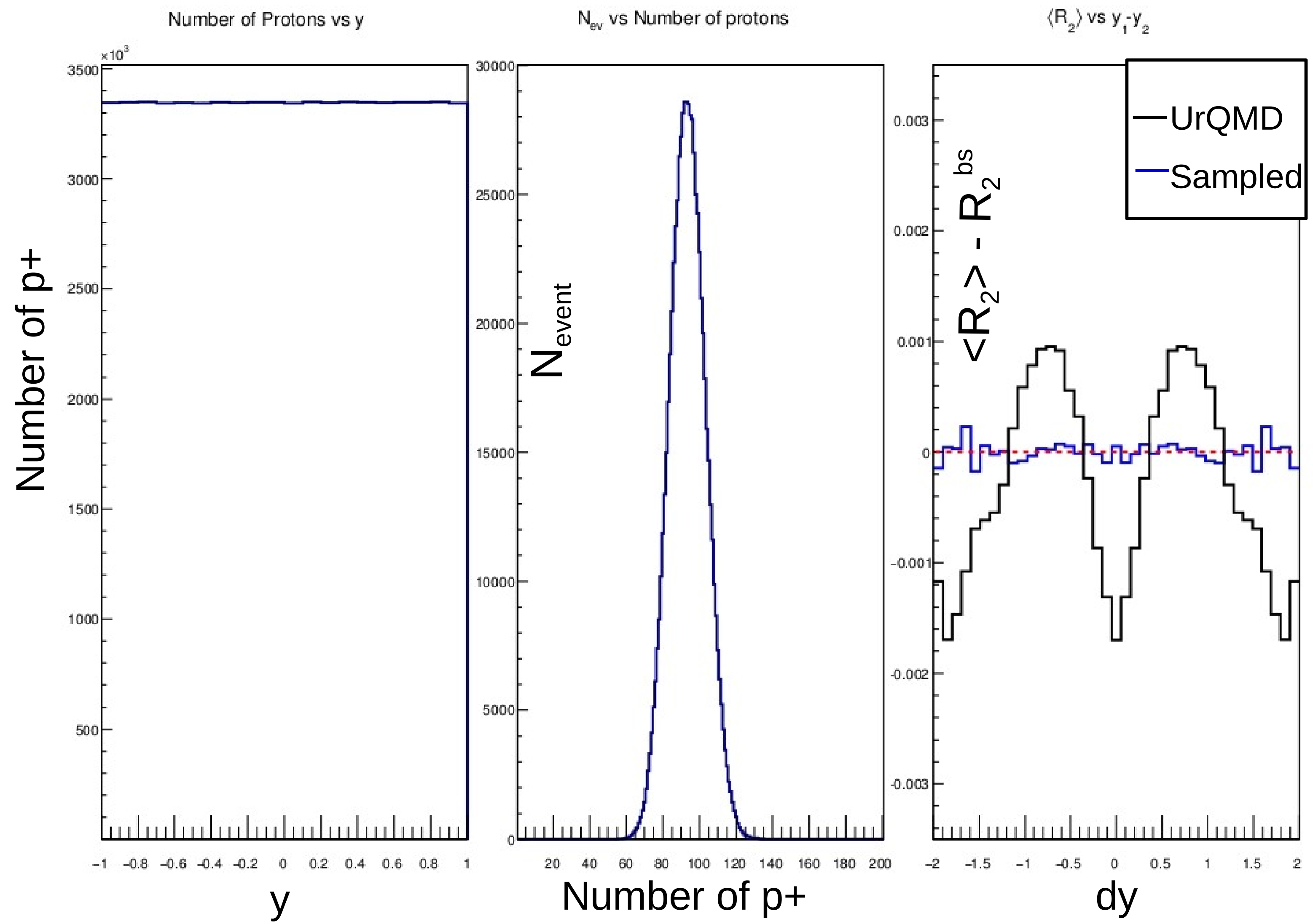
$\langle R_2 \rangle - R_2^{bs}$ values for central UrQMD events



At lowest beam energies, slightly enhanced probability for a proton pair with dy of 0.8

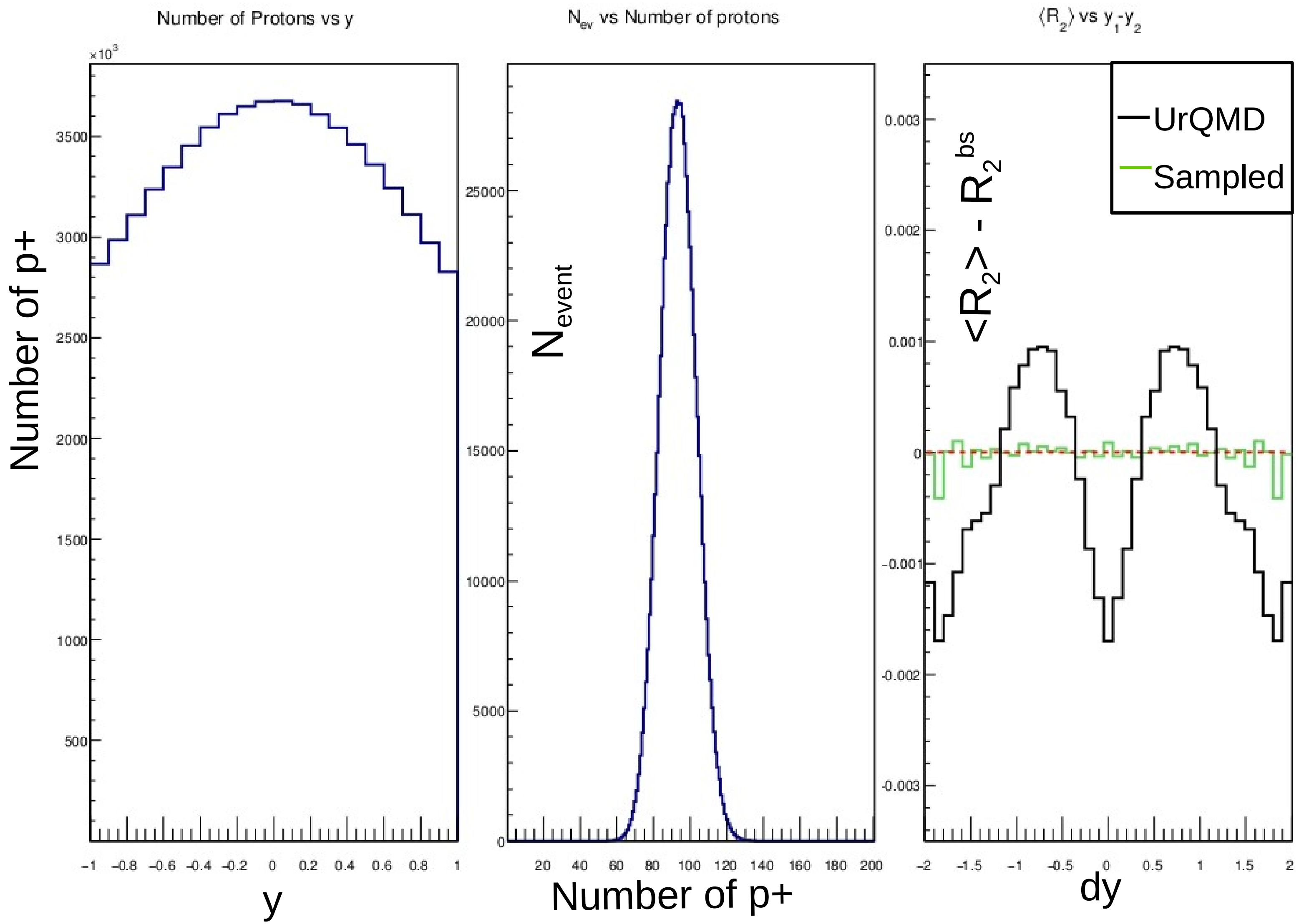
Randomly sample N/ev from UrQMD distribution at 7.7 GeV

Randomly sample particle rapidities from flat distribution



Random rapidities and UrQMD multiplicity distribution result in <R2> = 0

Randomly sample N/ev and particle rapidities from UrQMD distributions at 7.7 GeV



This test also does not reproduce correlations seen in UrQMD events.

Conclusion

- Studied two-particle rapidity correlations using the R2 variable and UrQMD events
- Multiplicities per event low enough that a correction for finite counting statistics was needed
- UrQMD data shows rapidity dependent correlations for low energies that diminish as beam energy increases
- Sampling randomly from the UrQMD Npart/evt and/or rapidity distributions destroys the observed correlations
- Future directions for this analysis include studying other event generator models and applying these techniques to experimental data

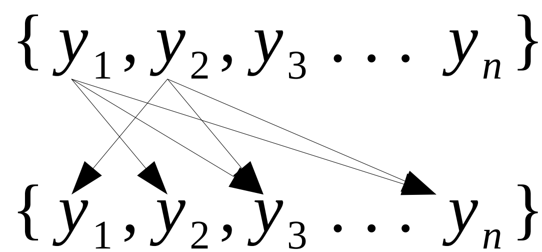
References

- Laurence Tarini. *Centrality Dependence of two-particle number and Transverse Momentum Correlations in* $\sqrt{s_{NN}}$ *= 200 GeV Au+Au collisions at RHIC.* Thesis. Wayne State University, 2011.
- Chaudhuri, Asis Kumar. *A Short Course on Relativistic Heavy Ion Collisions.* Bristol: IOP, 2014.

Difference Between $\rho_2(y_1,y_2)$ and $\rho_1(y_1)\rho_1(y_2)$

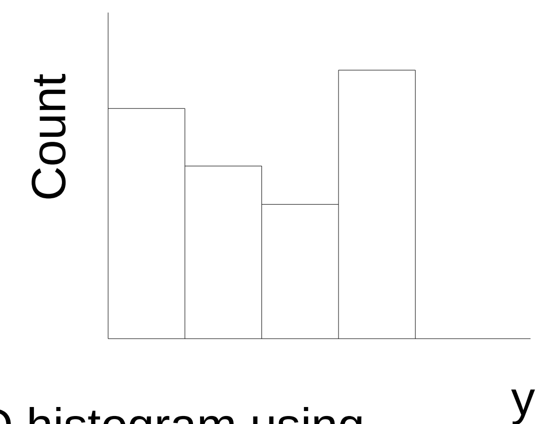
- Code used create the pair distribution:

```
for (int i=0;i<NTRK;i++){
    heta1D->Fill(Aeta[i]);
    for (int j=0;j<NTRK;j++){
        if (i!=j) { heta2D->Fill(Aeta[i],Aeta[j]); }
    }
}
```



- Code used to create the tensor product:

```
int nbin = heta1D->GetNbinsX();
for (int ibin=1;ibin<=nbin;ibin++){
    float valx1 = heta1D->GetBinCenter(ibin);
    float valn1 = heta1D->GetBinContent(ibin);
    for (int jbin=1;jbin<=nbin;jbin++){
        float valx2 = heta1D->GetBinCenter(jbin);
        float valn2 = heta1D->GetBinContent(jbin);
        hetaT->Fill(valx1,valx2,valn1*valn2);
```



Fill 2D histogram using weight of entries in compared bins