Longitudinal fluctuations of the fireball density in heavy-ion collisions

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Abstract

We show that fluctuations of the fireball shape in the longitudinal direction generate nontrivial rapidity correlations that depend not only on the rapidity difference, $y_1 - y_2$, but also on the rapidity sum, $y_1 + y_2$. This is explicitly demonstrated in a simple wounded nucleon model, and the general case is also discussed. We show how to extract different components of the fluctuating fireball shape from the measured two-particle rapidity correlation function. The experimental possibility of studying the longitudinal initial conditions in heavy-ion and proton-proton collisions is emphasized.

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I. INTRODUCTION

When a fireball produced in heavy-ion collisions is studied in an azimuthal angle, it is very useful to parametrize its initial shape with the help of the Fourier decomposition [1–3]. This decomposition clarifies trends in the elliptic flow, the fluctuations in elliptic flow, and the higher harmonics flows, which are associated with the triangularity and other shape parameters of the fireball [4, 5].

The harmonic analysis gives important information about the transverse initial conditions in heavy-ion collisions, and the mechanism of the subsequent evolution of the produced fireball – see *e.g.* [6]. In particular, the success of the hydrodynamic model in describing the v_n data at RHIC and LHC places new constraints on the initial conditions in the transverse direction.

A similar idea can be applied to study the shape of the fireball in the longitudinal direction. In this paper we will focus on rapidity, y, but our arguments hold for any longitudinal variable. This study was initiated in Ref. [7] (see also [8]) where it was argued that long-range rapidity correlations can be interpreted in terms of the fluctuating rapidity density of the created fireball. When applied to the STAR data [9], a significant asymmetric component in the fireball's rapidity shape was found in the most central Au+Au collisions.

In this paper we extend the discussion presented in Ref. [7]. We demonstrate that the fluctuations in the fireball rapidity density result in a nontrivial structure of the rapidity correlation function, and propose to study the additional components beyond asymmetry described above. The experimental method to extract various components is also discussed.

The structure of this paper is following. In the next section we discuss the problem in a simple model. We show that an event-by-event difference between the number of wounded nucleons in the target and the projectile results in a long-range asymmetry of the fireball. We derive the correlation function and show that it depends on both the rapidity difference and the rapidity sum. In section 3, we decompose the different components of the fireball rapidity density with Chebyshev polynomials, and show how to extract the strength of these components from the measured rapidity correlation function. The practical application of this idea is discussed in section 4, where we also include several comments. We summarize our paper with the conclusions in section 5.

II. SIMPLE MODEL

In this section we explicitly demonstrate in a simple model that an event-by-event global fluctuations of the fireball rapidity shape lead to a nontrivial two-particle rapidity correlation function.

For a given heavy ion event, we denote the number of wounded nucleons moving to the left and to the right with w_L and w_R respectively. The average over many events will be denoted by $\langle w_L \rangle$ and $\langle w_R \rangle$. In collisions characterized by $\langle w_L \rangle \neq \langle w_R \rangle$, the single particle rapidity distribution is asymmetric with respect to $y = 0$, where y represents the rapidity in the center-of-mass frame. This asymmetry is clearly evident in d+Au collisions as measured at RHIC [10], and is easily reproduced by practically all models of heavy-ion collisions – see *e.g.* [11–14].

In symmetric heavy ion collisions (Au+Au for example) we have $\langle w_L \rangle = \langle w_R \rangle$, and the single particle rapidity distribution is obviously symmetric with respect to $y = 0$ [15]. However, this distribution is symmetric only when averaged over many events. In a single event the shape (in rapidity) of the fireball may be asymmetric since $w_L \neq w_R$.[1] Indeed, in a single event the number of wounded nucleons going to the left may differ from the number of wounded nucleons going to the right. As discussed below, the asymmetry can be quantified by $\langle (w_L - w_R)^2 \rangle$, which is significantly larger than zero in Au+Au collisions.

It is a useful exercise to calculate in a simple model the two-particle rapidity correlation function originating from fluctuations in $w_L - w_R$. Here we consider the wounded nucleon model [16], which is a very useful model for understanding many features of heavy-ion data [11, 17, 18]. To simplify our considerations, let us assume that the single particle rapidity distribution measured in d+Au collisions can be approximated by a liner function of rapidity[2]. Consequently the distribution from a single wounded nucleon is also a linear function of rapidity. In the wounded nucleon model, the single particle distribution at a given w_L and w_R is given by [17]

$$
\begin{aligned}
\rho(y; w_L, w_R) &= w_R(a + by) + w_L(a - by) \\
&= a\,(w_L + w_R) - by\,(w_L - w_R),
\end{aligned} \tag{2.1}
$$

where $a + by$ is the rapidity distribution from a right-mowing wounded nucleon, and $a - by$ is the contribution form a left-mover. As seen from above equation we have an asymmetric component that is proportional to y. Assuming further that at a given w_L and w_R there are no correlations in the system[3], the two-particle rapidity distribution at a given w_L and w_R is

$$
\begin{aligned}
\rho_2(y_1, y_2; w_L, w_R) &= \rho(y_1; w_L, w_R)\rho(y_2; w_L, w_R) \\
&= a^2\,(w_L + w_R)^2 - ab(w_L^2 - w_R^2)(y_1 + y_2) + y_1 y_2 b^2 (w_L - w_R)^2. \tag{2.2}
\end{aligned}
$$

Summing Eq. (2.2) over w_L and w_R with an appropriate probability distribution, $P(w_L, w_R)$, we obtain the experimentally accessible two-particle rapidity distribution. Taking $\langle w_L^2 \rangle = \langle w_R^2 \rangle$, corresponding to symmetric Au+Au collisions, we obtain

$$
\rho_2(y_1, y_2) = a^2 \left\langle (w_L + w_R)^2 \right\rangle + y_1 y_2 b^2 \left\langle (w_L - w_R)^2 \right\rangle. \tag{2.3}
$$

Consequently, the two-particle rapidity correlation function reads

$$
\begin{aligned}
C(y_1, y_2) &\equiv \rho_2(y_1, y_2) - \rho(y_1)\rho(y_2), \\
&= a^2 \left[\left\langle w_+^2 \right\rangle - \left\langle w_+ \right\rangle^2 \right] + y_1 y_2 b^2 \left\langle w_-^2 \right\rangle, \tag{2.4}
\end{aligned}
$$

where $w_+ = w_L + w_R$ and $w_- = w_L - w_R$. As seen from Eq. (2.4) the fluctuations in $w_L - w_R$ result in a nontrivial rapidity structure of the two-particle correlation function. $C(y_1, y_2)$

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[1] The asymmetry due to the finite number of produced particles is not relevant to this analysis.

[2] This assumption is quite reasonable outside the fragmentation regions [10].

[3] We want to study correlations originating only from shape fluctuations and we neglect all other correlations. We will come back to this point in section 4.
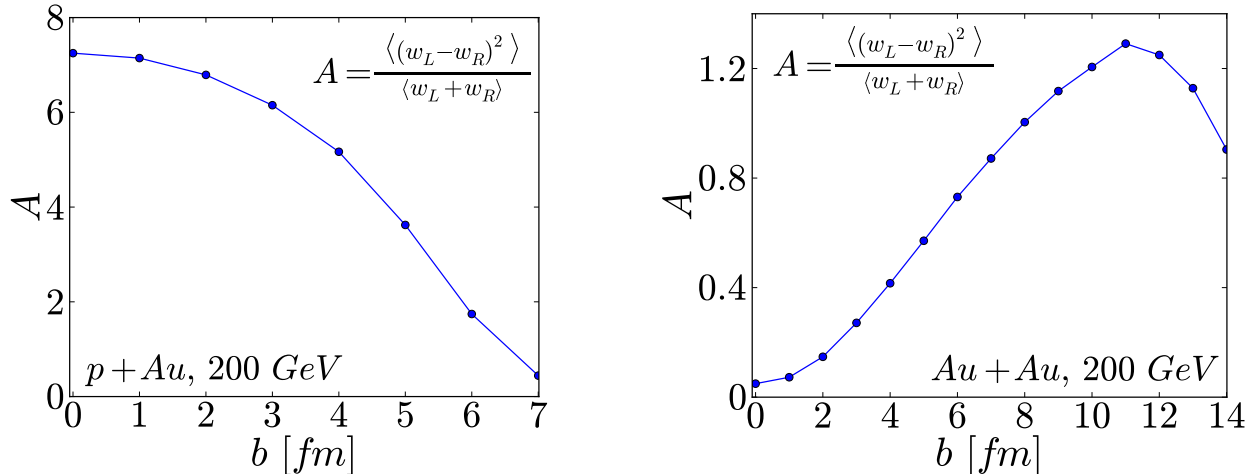
FIG. 1: Asymmetry in wounded nucleons in p+Au (left) and Au+Au (right) collisions at $\sqrt{s} = 200$ GeV as a function of the impact parameter b.

depends not only on the rapidity difference, $y_- = y_1 - y_2$, but also on the rapidity sum, $y_+ = y_1 + y_2$. Indeed, the correlation function

$$C(y_1, y_2) \sim y_1 y_2 b^2 \left\langle w_-^2 \right\rangle = \frac{1}{4} b^2 (y_+^2 - y_-^2) \left\langle w_-^2 \right\rangle, \qquad (2.5)$$

decreases as a function of rapidity difference, y_-, and increases as a function of rapidity sum, y_+. This dependence on y_+ can distinguish fluctuations of the fireball shape from well known sources of correlations (such as resonance decays) that depend mainly on $y_1 - y_2$.

Eq. (2.4) should be taken as an illustration of the problem we would like to adress in this paper. Despite its simplicity, the model result shows quite convincingly that the event-by-event asymmetry of the fireball shape that is present in symmetric heavy-ion collisions can lead to interesting rapidity correlations [7]. Obviously there can be more complicated sources of this asymmetry in more realistic models, *e.g.* the difference in the number of flux-tubes in the CGC/Glasma approach [19, 20].

In Fig. 1 we present $\left\langle (w_L - w_R)^2 \right\rangle$ divided by the total number of wounded nucleons $\left\langle w_L + w_R \right\rangle$. We performed our calculations at $\sqrt{s} = 200$ GeV in Au+Au and p+Au collisions in the Monte-Carlo Glauber model. It is interesting to notice that $\left\langle (w_L - w_R)^2 \right\rangle$ in Au+Au collisions is quite large and comparable in magnitude to the total number of wounded nucleons. Thus, event-by-event rapidity fluctuations are of order $\sim 1/(w_L + w_R)^{1/2}$ compared to the average, and this is large compared to normal statistical fluctuations of order $\sim 1/N^{1/2}$, where N is the number of particles.

In the next section we generalize Eq. (2.4) to analyze arbitrary rapidity fluctuations of the fireball shape.

III. GENERAL SHAPE FLUCTUATIONS

In the previous section we discussed the asymmetric component of the fireball rapidity density, originating from a non-zero values of $w_L - w_R$. As seen in Eq. (2.1) for the simple

model of the previous section, the single particle rapidity distribution at a given $w_L - w_R$ is proportional to rapidity y, *i.e.* the fireball is denser on one side of the rapidity window than on the other. There may be different sources of this asymmetry such as the left-right difference in the number of collisions, or the difference in the number of asymmetric long-range flux-tubes in the CGC/Glasma approach [19, 20]. Let us denote the parameter that characterizes this asymmetry by a_1. In our simple model, $a_1 \propto w_L - w_R$.

Equation (2.1) also contains a term that is proportional to the total number of wounded nucleons. Fluctuations of this quantity lead to symmetric, rapidity independent, fluctuations of the *whole* fireball. This can naturally originate from impact parameter fluctuations, which are always present in heavy-ion collisions. Let a_0 denote the parameter that characterizes this source of fluctuation. In our simple model, $a_0 \propto w_L + w_R$.

The natural question arises if there are more components in the fluctuating shape of the fireball. For example, a "butterfly" component would characterize a symmetric fireball with higer (or lower) density on both sides of the midrapidity region, and lower (or higher) density at mid-rapidity.[4] The single particle rapidity distribution affected by this component would be proportional to y^2. Let us denote by a_2 the parameter that characterizes the strength of this effect. We will parametrize the fireball asymmetry and the butterfly component with the two Chebyshev polynomials, $T_1(y/Y)$ and $T_2(y/Y)$, which are shown in Fig. 2.

Similarly, we can introduce additional components to fully parametrize shape fluctuations in rapidity. Thus, it is tempting to expand the single particle rapidity distribution at a given
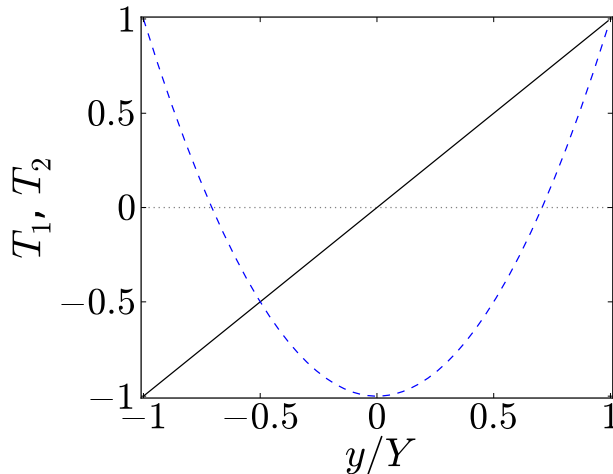

FIG. 2: Two components of the fluctuating fireball rapidity density: An asymmetry (solid black line) and a butterfly (dashed blue line).

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[4] Such "butterfly" fluctuations are suggested by the measured forward-backward rapidity correlations at RHIC [9]. For a fixed number of particles at midrapidity, it was observed that the particle yields in pair of narrow rapidity bins located symmetrically about midrapidity strongly fluctuate. Surprisingly, these fluctuations are also strongly correlated. Thus, even if the density is approximately fixed in the middle of the fireball, both sides of the fireball fluctuate together. The physical origin of this correlation is currently under investigation – see Refs. [21, 22].

$a_0, a_1, ...$ in terms of the orthogonal polynomials

$$\rho(y; a_0, a_1, ...) = \rho(y) \left[1 + \sum_{i=0} a_i T_i (y/Y) \right], \tag{3.1}$$

where $\rho(y)$ is the single particle distribution averaged over $a_0, a_1, ...$. Here we have expanded the distribution in Chebyshev polynomials[5], but other choices are certainly possible. The parameter Y characterizes the scale of long-range rapidity fluctuations in the system. We will discuss reasonable values of Y in the next section. Averaging both sides of Eq. (3.1) over $a_0, a_1, ...$ with an appropriate probability distribution, $P(a_0, a_1, ...)$, we obtain $\langle a_i \rangle = 0$ for all $i \geq 0$.

Assuming that at a given $a_0, a_1, ...$ there are no other large sources of long-range rapidity correlations, the two-particle rapidity distribution is

$$\rho_2(y_1, y_2; a_0, a_1, ...) = \rho(y_1; a_0, a_1, ...)\rho(y_2; a_0, a_1, ...). \tag{3.2}$$

Taking an average over a_i and subtracting $\rho(y_1)\rho(y_2)$, we obtain the two-particle rapidity correlation function

$$C(y_1, y_2) = \rho(y_1)\rho(y_2) \left[\sum_{i,k=0} \langle a_i a_k \rangle T_i (y_1/Y) T_k (y_2/Y) \right]. \tag{3.3}$$

It is useful to recall the physical meaning of the first few terms in Eq. (3.3): $\langle a_0^2 \rangle$ represents the rapidity independent fluctuations of the fireball as a whole, $\langle a_0 a_1 \rangle y_2$ describes the correlation between rapidity independent fluctuations of the fireball and its asymmetry, $\langle a_1^2 \rangle y_1 y_2$ is the asymmetric component discussed in the previous section, and $\langle a_2^2 \rangle [2(y_1/Y)^2 - 1][2(y_2/Y)^2 - 1]$ represents the butterfly contribution, *etc.*.

From the previous section, we know that the asymmetric component, $\langle a_1^2 \rangle$, introduces a long-range rapidity correlation that is a decreasing function of the rapidity difference, $y_- = y_1 - y_2$, and an increasing function of the rapidity sum, $y_+ = y_1 + y_2$. It is a straightforward to verify that the rapidity structure originating from the butterfly component leads to a correlation function that is decreasing both in y_- and y_+.

To conclude this section, we point out that the values of $\langle a_i a_k \rangle$ can be extracted directly from the correlation function $C(y_1, y_2)$. Using the orthogonality of Chebyshev polynomials

$$\int_{-1}^{1} T_i(x)T_k(x) \left(1 - x^2 \right)^{-1/2} dx = c_i \delta_{i,k}, \tag{3.4}$$

where $c_0 = \pi$ and $c_i = \pi/2$ for $i > 0$, we obtain

$$\langle a_i a_k \rangle = \frac{1}{c_i c_k} \int_{-Y}^{Y} \frac{C(y_1, y_2)}{\rho(y_1)\rho(y_2)} \frac{T_i(y_1/Y)T_k(y_2/Y)}{[1 - (y_1/Y)^2]^{1/2} [1 - (y_2/Y)^2]^{1/2}} \frac{dy_1 dy_2}{Y^2}. \tag{3.5}$$

In the next section we will discuss how Eq. (3.5) can be used in practice.

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[5] For example: $T_0(x) = 1$, $T_1(x) = x$, $T_2(x) = 2x^2 - 1$, $T_3(x) = 4x^3 - 3x$ etc.

IV. COMMENTS

In this section we list several comments to clarify the analysis presented in this paper. In deriving Eq. (3.3) and Eq. (2.4), we assumed that at a given $a_0, a_1, ...$ there are no correlations in the system. In other words, the only sources of correlations are fluctuations in the fireball rapidity density. Unfortunately, short-range correlations may contribute to the left-hand side of Eq. (3.3), and contaminate the signal coming from the shape fluctuations. Particularly problematic may be the correlations from resonance decays and local local charge conservation [23, 24]. These problems can be mitigated by studying Eq. (3.3) for positive and negative particles separately, which significantly reduces these unwanted backrounds. Moreover, the dependence of the correlation function on the rapidity sum, y_+, can be used to distinguish between rapidity density fluctuations and the short-range correlations of the background.

One could also worry that at a given $w_L + w_R$ the distribution of final particles is given approximately by a negative binomial distribution (NBD) [25, 26] which introduces long-range rapidity correlations into the system. This concern is unfounded, however, because the NBD leads to the following two-particle rapidity distribution[6]

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)\left(1 + 1/k\right), \qquad (4.1)$$

where k measures deviation from Poisson distribution. As seen from Eq. (3.3), the NBD ρ_2 influences only $\langle a_0^2 \rangle$. In fact, this is the expected result, since the NBD can be viewed as a rapidity independent fluctuation of the whole fireball.

As pointed out in the previous section, it is not obvious what is the appropriate value of Y in the preceding formulas. Clearly, Y parametrizes the range of global rapidity fluctuations in the fireball density. For instance, at $\sqrt{s} = 200$ GeV the single particle distribution in d+Au collisions [10] is approximately linear as a function of y for $|y| < 2$. Thus, for the asymmetric component parametrized by a_1, $Y \approx 2$ is a reasonable choice. For higher and lower energies, this parameter can be rescaled by the ratio of beam rapidities. This value of Y roughly corresponds to the size of the thermal fireball, and it is plausible that higher components, if they exist, are present in this region. If the measurement is performed in the smaller window than $[-Y, Y]$, fitting the measured correlation function with Eq. (3.3) can determine the parameters of interest, $\langle a_i a_k \rangle$.

Another choice is to assign Y to the rapidity interval of the measurement, and to investigate how the components $\langle a_i a_k \rangle$ change when this rapidity scale is varied. It is possible that the different fluctuating components are visible at different rapidity scales, and a systematic study of this sort can sort out these differences.

It would be very interesting to compare the strengths of different components, $\langle a_i a_k \rangle$, between heavy-ion and proton-proton collisions. This could reveal interesting differences in the longitudinal initial conditions between these two systems. For instance, in Ref. [7] it was shown that the asymmetric component is significantly stronger in p+p collisions than in central Au+Au collisions.

Further, the ideas presented in this paper could be extended by incorporating the multi-bin analysis proposed in Refs. [27–29]. This multi-bin analysis can be used to investigate

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[6] We sample particles from NBD and distribute them randomly in rapidity according to $\rho(y)$.

the different sources of particles production, providing a detailed picuture of the fireball in the longitudinal direction. Finally, we point out that the results obtained in this paper can be easily generalized to three- and many-particle correlation functions.

V. CONCLUSIONS

In conclusion, we showed that event-by-event fluctuations of the fireball rapidity density introduce interesting rapidity correlations that depend both on the rapidity difference, $y_1 - y_2$, and the rapidity sum, $y_1 + y_2$. We demonstrated this explicitly in the wounded nucleon model, where an event-by-event difference between the number of wounded nucleons in a target and a projectile, $w_L - w_R$, leads to the long-range asymmetry of the fireball. The resulting correlation function in symmetric A+A collisions is given in Eq. (2.4).

We further proposed to expand the measured two-particle rapidity correlation function in a series of the Chebyshev polynomials (see Eq. (3.3)), where each polynomial represents a different component of the fireball's fluctuating rapidity density. The quadratic polynomial in this expansion describes the "butterfly" fluctuations described above, which are suggested by recent measurements at RHIC. The coefficients of this expansion, $\langle a_i a_k \rangle$, characterize the strength of various components, and we propose to extract these coefficients from the measured correlation function. This can reveal nontrivial information about the structure of the fireball in the longitudinal direction, and can test various models of particle production in hadronic collisions.

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