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Simple electronic circuit for the demonstration of chaotic phenomena

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This paper describes an electronic circuit which is very versatile for the demonstration and examination of chaotic phenomena. A coupled logistic equation is taken as a basis and typical features of chaotic phenomena such as successive bifurcations leading to chaotic regimes and so on can easily be seen on an oscilloscope by adjusting variable resistors.

I. INTRODUCTION

In recent years much interest has been expressed with regard to chaotic phenomena of nonlinear systems. Period doubling, bifurcations, and onset of chaos have been observed in many physical and chemical systems.[1] These are fundamental properties of nonlinear systems and in most cases, they are governed by nonlinear differential equations. It is well known that the typical features of the phenomena are involved in the logistic equation[2]

$$X_{n+1} = 1 - AX_n^2, \tag{1}$$

where X_n and X_{n+1} are values of a dynamical variable X at successive times separated by the period of the system.

In this paper we describe an electronic circuit which is designed for the real-time demonstration and examination of the chaotic phenomena. The circuit represents time evolution of the variable governed by the logistic equation. Successive bifurcations leading to period-doubling and chaotic regimes can easily be seen on an oscilloscope. In order to examine the behavior of a coupled chaos[3] as well, the circuit is designed based on the following equations:

$$X_{n+1} = 1 - AX_n^2 - B(X_n - Y_n), \tag{2}$$

$$Y_{n+1} = 1 - CY_n^2 - D(Y_n - X_n), \tag{3}$$

which is reduced to two independent logistic equations when the coupling constants B and D are zero. Two-dimensional as well as one-dimensional maps can be examined for various values of parameters A, B, C, and D which can be continuously changed by variable resistors.

II. APPARATUS

The block diagram of the circuit is shown in Fig. 1. It contains two blocks X and Y connected with each other reflecting the structure of Eqs. (2) and (3). The circuit diagrams of X and Y blocks are the same and shown in Fig. 2. (People who do not need two-dimensional maps may omit one of the blocks X and Y in Fig. 1, and OA III and attached circuit in Fig. 2.) An analog multiplier IC (Analog Devices AD533H) is used as a squarer. A variable resistor (VR) of 5 kΩ connected with pin 1 is a gain controller. VR's of 20 kΩ connected with pins 7 and 9 are a balance adjustor for input polarity and an output-offset adjustor, respectively. They are adjusted so that output voltage at pin 4 becomes 0, 10, and 10 V for the input voltage of 0, 10, and − 10 V at pin 6, respectively; i.e., AD533H acts as a squarer taking 10 V as a unit. In the following, notations X_n, X_{n+1}, Y_n, and Y_{n+1} mean voltages normalized by this unit. During the adjustment the feedback loop from LF398 must be disconnected.

The output voltage proportional to X_n^2 is fed to operational amplifiers (OA:LF356) I and II which act as a subtractor and an adder, respectively. The dc level of the output of OA II is adjusted to be 10 V when pins 3 of OA's I and II are grounded. OA III acts as a subtracter and gives a voltage proportional to $(X_n - Y_n)$ at the input (pin 3) of OA II. When the coupling switch is off (or on) the output of OA II can be represented by $1 - AX_n^2$ [or $1 - AX_n^2 - B(X_n - Y_n)$]. The values of A and B can be changed from 0 to 2, and 0 to 0.4 by VR's A and B, respectively. The output voltage of OA II becomes a new input voltage X_n for AD533H after passing through two sample-and-hold (SH) circuits (LF398). The SH's are triggered by timing pulses from the clock generator and provide a delay of feedback. The delay is essential to obtain the solution of Eq. (1). The timing pulses $T1$ and $T2$, and the voltages at P, Q, and R are schematically shown in Fig. 3. The clock generator determines the fundamental period.

From the above argument it is evident that the circuit in Fig. 1 represents the time evolution governed by Eqs. (2) and (3).

Clock rates of 5 kHz (fast) and 5 Hz (slow) can be selected. When the slow rate is used the slow motion of light spot

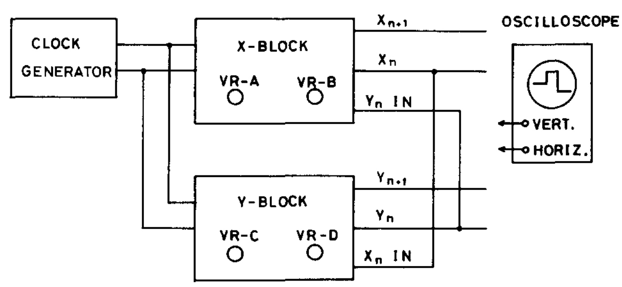

Fig. 1. Block diagram of the circuit.

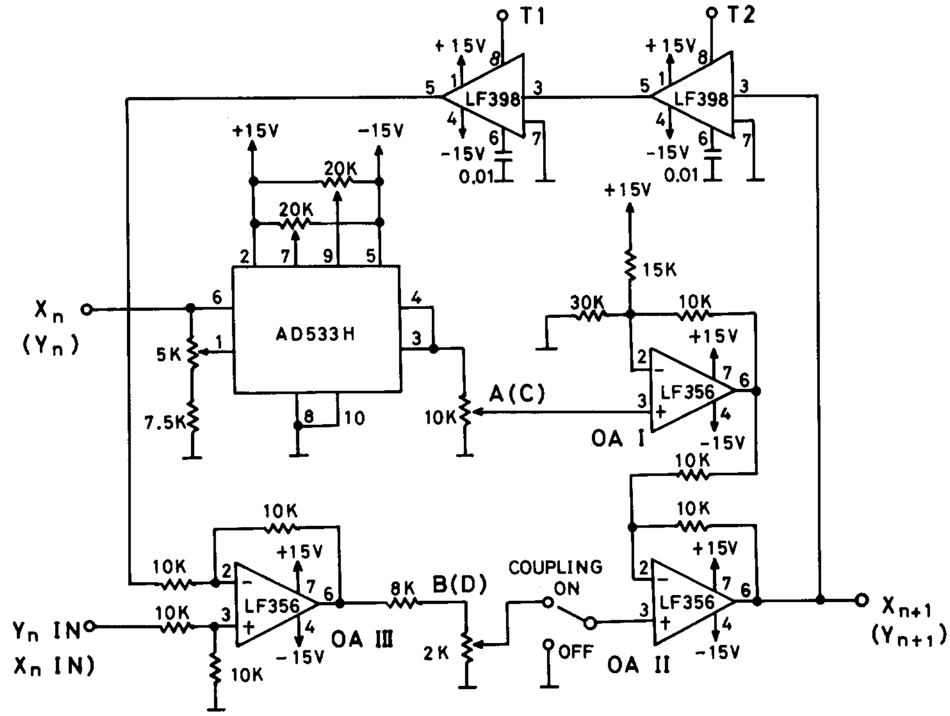

Fig. 2. Circuit diagram of X (and Y) block: All resistors are in ohms $\pm 5\%$ and $\frac{1}{4}$ W. The 0.01-μF capacitors connected with LF398 are polystyrene type and guarding technique is recommended in assembling. By-pass capacitors are omitted in the figure.

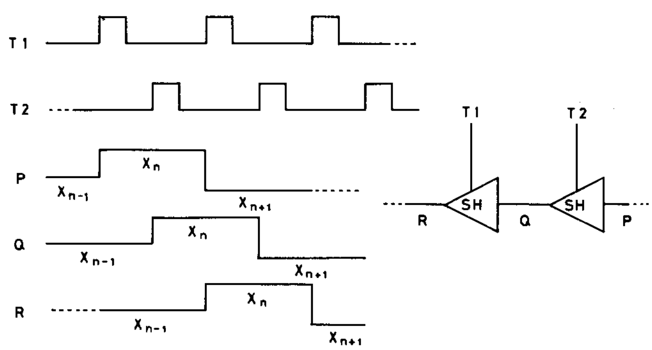

Fig. 3. Timing chart of the feedback circuit: The output voltage of OA II is fed to the input of AD533H with a delay determined by two sample-and-hold (SH) circuits triggered by clock pulses $T1$ and $T2$.

can be observed. The circuit diagram of the clock generator is given in Fig. 4. The system requires regulated power supply of ± 15 V for analog circuits and 5 V for digital circuits.

III. OBSERVATION OF CHAOTIC PHENOMENA

A. One-dimensional map

One-dimensional maps can be drawn by using only one block (X or Y) of the circuit. Turn off the coupling switches ($B = D = 0$) and connect outputs X_{n+1} and X_n with vertical and horizontal inputs of the oscilloscope. It is convenient to adjust the axis gains of the oscilloscope so that full scales correspond to input voltages of ± 15 V. Set VR A at zero ($A = 0$) then a stable spot on the oscilloscope screen

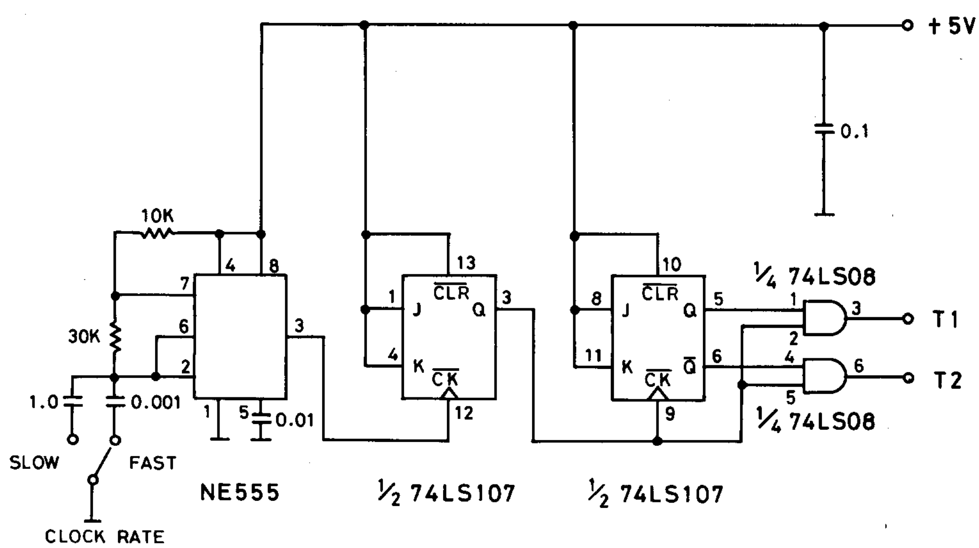

Fig. 4. Circuit diagram of the clock generator: All resistors are in ohms $\pm 10\%$ and are $\frac{1}{4}$ W. Capacitors are in microfarads. Pins 14 and 7 of 74LS08 and 74LS107 are wired to 5 V and ground, respectively.

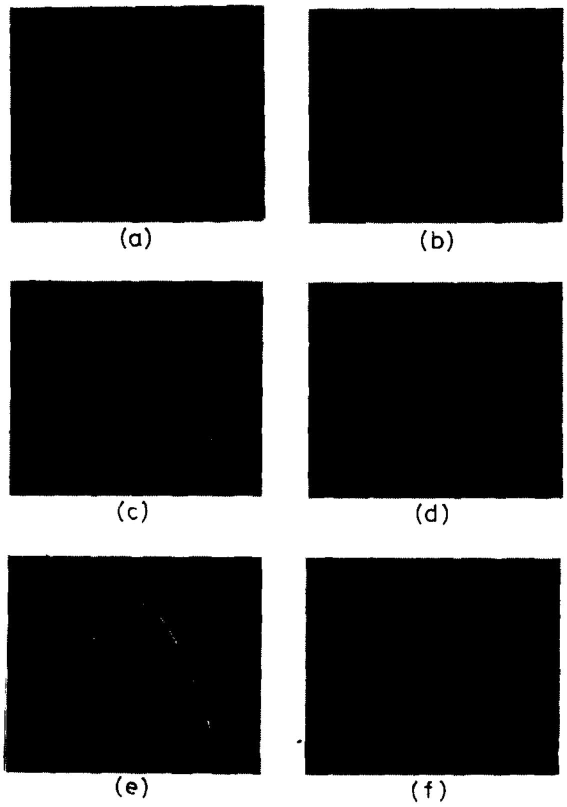

Fig. 5. Typical chaotic phenomena in one-dimensional map. Ordinate and abscissa represent values of X_{n+1} and X_n obtained by Eq. (1). As the value of A is increased figures (a) to (f) appear successively. (a) A stable spot when $A = 0$; (b)–(d) period-2, -4, and -8 states; (e) chaotic state where spots appear randomly; (f) periodic-3 state.

can be seen as shown in Fig. 5 (a). As VR A is increased there is a threshold value of A for which X_{n+1} becomes unstable and takes two alternating values, namely, bifurcation into the period-2 state (period doubling) occurs. Correspondingly the spot splits into two as shown in Fig. 5 (b). When the slow rate is used it can be observed that the spots appear alternatively. The threshold value of A should be 0.75. Further increase of A results in a cascade of period doubling. Figures 5 (c) and 5 (d) show the states of period 4 and 8. Near $A = 1.40$ a transition to chaotic state occurs [Fig. 5 (e)] where the system becomes unperiodic. Further increase of A gives odd-number period states, a typical example (period-3 state) is shown in Fig. 5 (f). It is noted that the spots are always on a parabola represented by Eq. (1). When A is too large the analog circuit is saturated and a light spot is fixed at the edge of oscilloscope screen. In this case, VR should be decreased until the saturation is removed.

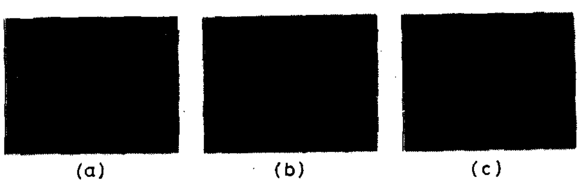

Fig. 6. One-dimensional maps in the time domain. Ordinate and abscissa are X_n and the time. (a), (b), and (c) are period-2, -4, and -3 states corresponding to Figs. 5(b), 5(c), and 5(f), respectively.

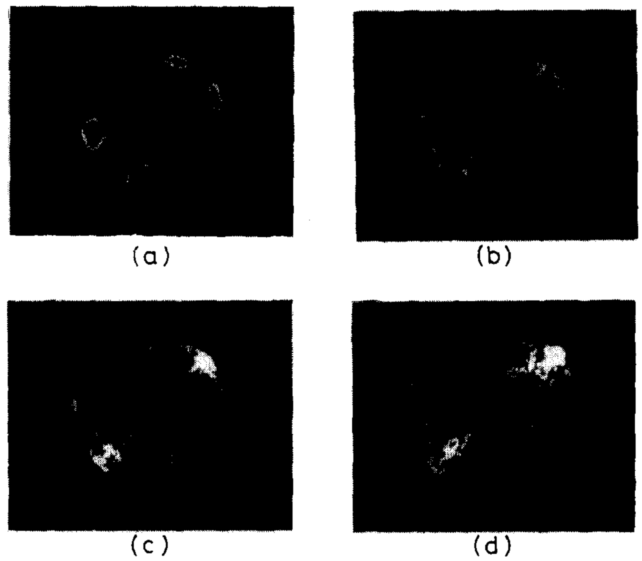

Fig. 7. Examples of two-dimensional maps. Ordinate and abscissa are X_{n+1} and Y_{n+1}. (a) represents a torus with four leaves, (b) and (c) are its deformations, and (d) is a chaotic state.

Successive bifurcations and chaotic state thus observed are representatives of chaotic phenomena. These can also be seen in the time domain as shown in Fig. 6 when the time sweep synchronized to the clock pulse is used.

B. Two-dimensional map

Plots of X_{n+1} vs Y_{n+1} show interesting two-dimensional maps associated with coupled logistic equations (2) and (3). A variety of patterns can be observed according to the values of A, B, C, and D. Only several examples are shown here. Turn on the coupling switches and set VR's B and D at large values and VR's A and C at zero. Then by increasing A and C a pattern like that in Fig. 7(a), which may be called four leaves, can be observed. This represents a stable torus. As the parameters are gradually changed, smooth curves in Fig. 7(a) become sharpened [Fig. 7(b)] and the leaves join in pairs [Fig. 7(c)], and a chaotic state is formed [Fig. 7(d)]. It is interesting to note that even when two constituent systems are chaotic, i.e., the values of A and C are larger than 1.4 the coupled system shows periodic behavior.[3]

There are many other interesting and instructive applications of the present apparatus. It is certainly worthwhile to examine the phenomena near critical regions since we can follow continuous changes of the patterns by varying the parameters. The qualitative nature of the chaotic phenomena can be understood much easier by this method than the use of computer calculations for discrete values of parameters.

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