

## ① Multi-Modal Representation

Problem Semantics

Task Description: "..."  
Var: {"type", "desc.", ...}  
Con: {"type", "expr.", ...}  
Edge: {...}

Abstract Model

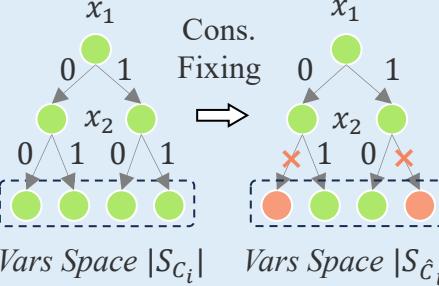
$$\begin{aligned} \min c^T x \\ s.t. Ax \leq b \\ l \leq x \leq u \\ x \in \mathbb{Z}^p \times \mathbb{R}^{n-p} \end{aligned}$$

Instantiation

$$\begin{aligned} \text{Instance } I_k &= \\ &< c_k, A_k, b_k \\ &l_k, u_k > \end{aligned}$$

## ② Tight Constraints Priority Module for Training Labels

Constraint Type  $C_i: x_1 + x_2 \leq 1$



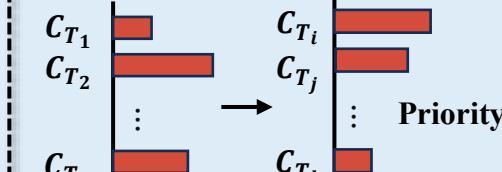
$$\begin{aligned} \text{Fixing } C_i: \\ x_1 + x_2 \leq 1 \Rightarrow x_1 + x_2 = 1 \\ \rho = |S_{\hat{C}_i}| / |S_{C_i}| = 1/2 \end{aligned}$$

$$\text{Information Gain of } C_i: \Delta H_{C_i} = -\log \rho$$

Fixed Constraint Strength  $\rho$

Type Strength  $\rho$

Set Packing	$n/(n+1)$
Set Covering	$n/(2^n - 1)$
Bin Packing	$O(1/A\sqrt{n})$
...	...
Knapsack	$O(1/A\sqrt{n})$



Ranking Types by  $\rho$

Detailed in  
Training

Predicted Constraints      Predicted Variables

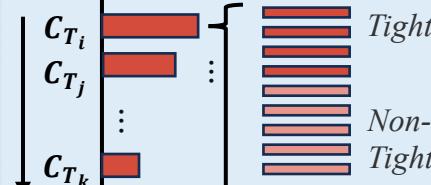
Constraint Search

$$\begin{aligned} z_0 = 0 \\ z_1 = 1 \\ z_2 = 0 \\ \|z\|_0 \leq \Delta_c \end{aligned}$$

$$\begin{aligned} Z_{v_1} &: \\ Z_{v_2} &: \\ \vdots &: \\ Z_{v_n} &: \\ Z_{c_1} &: \\ Z_{c_m} &: \end{aligned}$$

③ Model Reduction and Solving

Critical Tight Constraints (CTC)  
Identification by Type Priority



Select Top  $- k_c$  within prioritized types

JUDGE Module

CTC Set

[0,1,1,0,0,1,0,1,...,1]

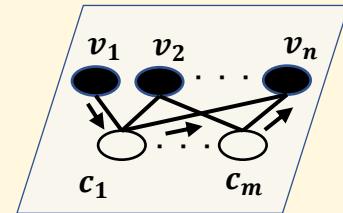
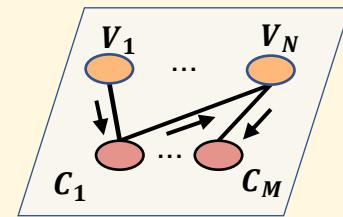
CTC Set Identification

Intra-layer Message-Passing

Pre-trained LM

Graph Construction

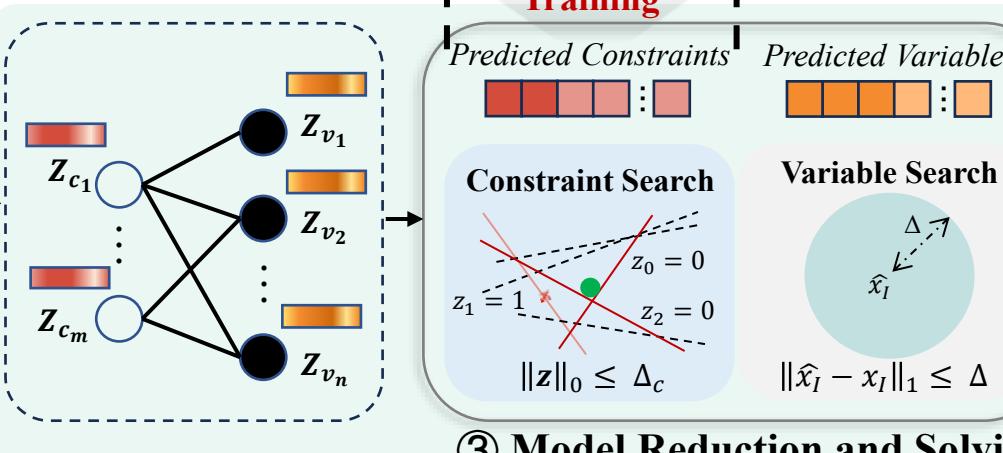
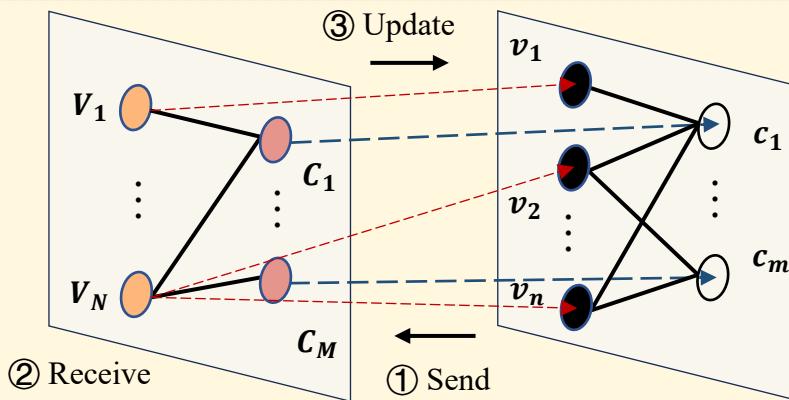
Instantiate



Higher (Abstract-level) GNN

Lower (Instance-level) GNN

Inter-layer Message Passing



$$\begin{aligned} \min c^T x \\ s.t. Ax \leq b \\ A'x = b', A' \subseteq A, b' \subseteq b \\ x_p = x_p^{pred}, x_p \in \mathcal{X}_0 \cup \mathcal{X}_1 \\ l \leq x \leq u \\ x \in \mathbb{Z}^p \times \mathbb{R}^{n-p} \end{aligned}$$

Reduced-model solving

Solver SCIP