Announcements

Data Abstraction

Data Abstraction

- Compound values combine other values together
  - A date: a year, a month, and a day
  - A geographic position: latitude and longitude
- Data abstraction lets us manipulate compound values as units
- Isolate two parts of any program that uses data:
  - How data are represented (as parts)
  - How data are manipulated (as units)
- Data abstraction: A methodology by which functions enforce an abstraction barrier between ***representation*** and ***use***

All Programmers

Great Programmers

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor

- `rational(n, d)` returns a rational number x

Selectors

- `numer(x)` returns the numerator of x
- `denom(x)` returns the denominator of x

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

General Form

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

the functions we use to access data

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

These functions implement an abstract representation for rational numbers

Pairs

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
2
```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

Element selection function

More lists next lecture

Representing Rational Numbers

```
def rational(n, d):
    """A representation of the rational number N/D."""
    return [n, d]
```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Select item from a list

(Demo)

A Problem of Specification

Our specification at the moment is ambiguous:

- "Numerator" refers to a particular way of writing a certain rational.
- For example, what is the numerator of 6/8?
  - Could say it is 6, but 6/8 = 3/4, so why not 3?

- Let's be more precise:

```
def numer(x):
    """Return the numerator of rational number X in lowest terms and having
    the same sign as X."""

def denom(x):
    """Return the denominator of rational number X in lowest terms and positive."""
```

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2} \qquad \frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from fractions import gcd
```

Greatest common divisor

```
def rational(n, d):
    """A representation of the rational number N/D."""
    g = gcd(n, d)          # Always has the sign of d
    return [n//g, d//g]
```

you just have to change the constructor because you use data abstraction

(Demo)

Abstraction Barriers

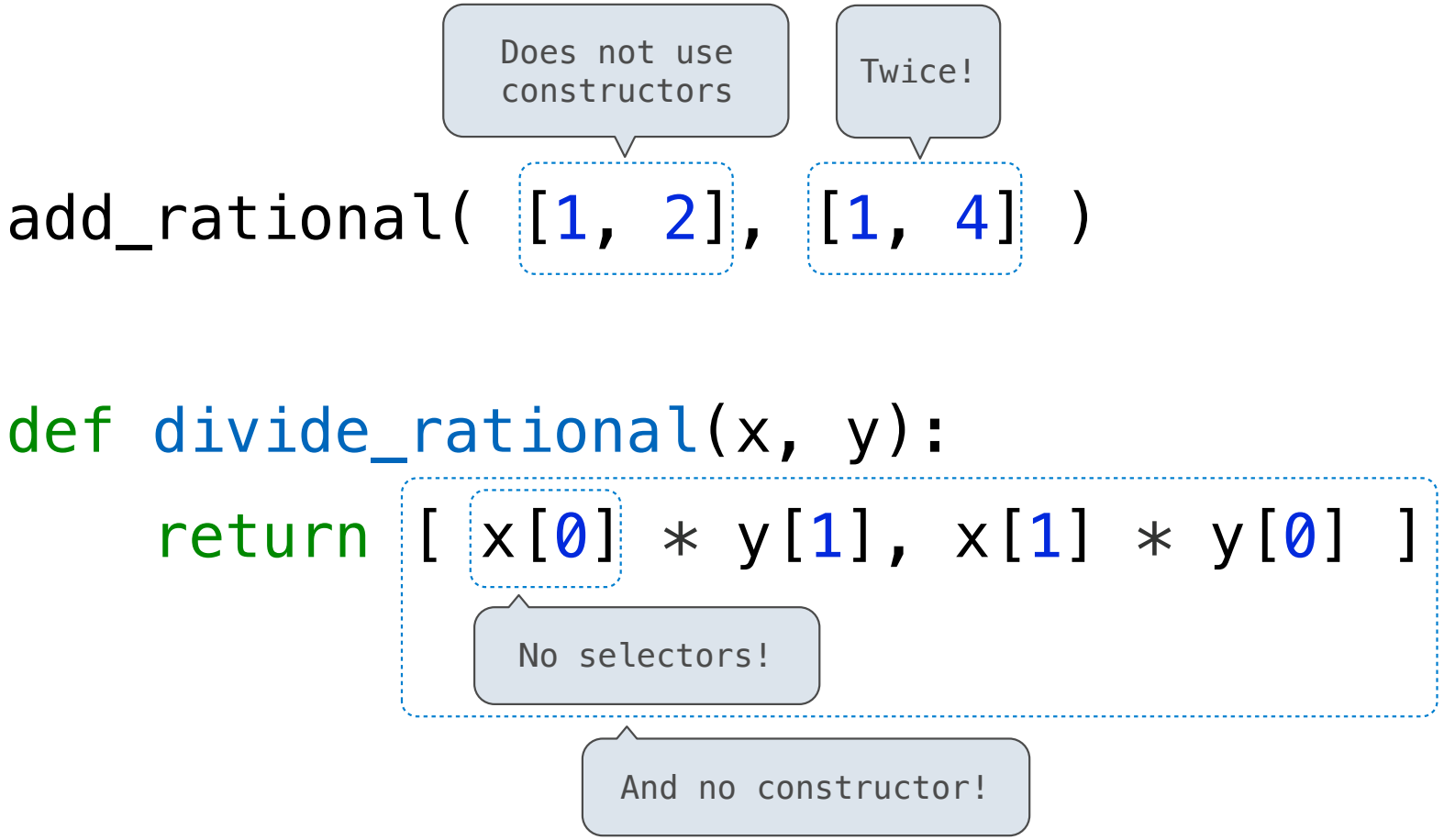
allow you to make changes to one part of your program and have other programs take advantage of such changes without breaking in any way or create inconsistencies

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	`add_rational`, `mul_rational` `rationals_are_equal`, `print_rational`
Create rationals or implement rational operations	numerators and denominators	`rational`, `numer`, `denom`
Implement selectors and constructor for rationals	two-element lists	list literals and element selection

Implementation of lists

Violating Abstraction Barriers



Data Representations

What is Data?

- We need to guarantee that constructor and selector functions work together to specify the right behavior
- Behavior condition: If we construct rational number x from numerator n and denominator d, then numer(x)/denom(x) must equal n/d
- Data abstraction uses selectors and constructors to define behavior
- If behavior conditions are met, then the representation is valid

You can recognize an abstract data representation by its behavior

(Demo)

Rationals Implemented as Functions

by defining a function in another function, we are able to access N and D in the enclosing scope (parent frame)

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
```

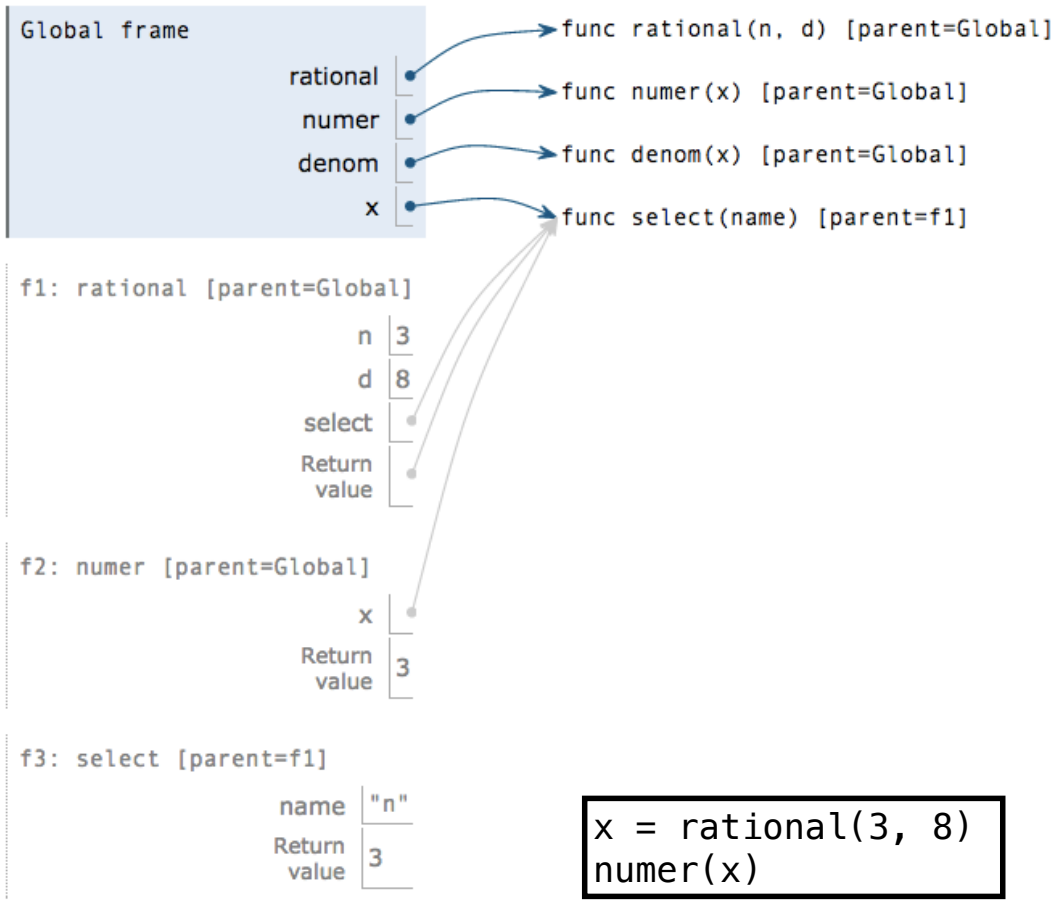
This function represents a rational number

Constructor is a higher-order function

```
def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

Selector calls x




```
x = rational(3, 8)
numer(x)
```

Interactive Diagram