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Content

Abstract

The report aims to design and analyses an optical communication system in both communication and phonics aspects, such as decision threshold and bit error rate etc. Decision threshold limits the bit error rate and will be deduced at the end of the report. Also, influence of noise should be analysed using signal-to-noise ratio (SNR), on the later section of report it is proved that the bit error rate decreases with increasing of the SNR.

1. Introduction

The system applies amplitude shift keying (ASK) with non-return-to-zero (NRZ) modulation. The wavelength of the optical carrier is 1550nm for modulation, and the transmitter of the system produce regular NRZ pulses with average modulated output optical power of 4 dBm and extinction ratio of 8dB. For the receiver, it receives symbols drawn from the alphabet $\{pA, A\}$, where $0 \leq p < 1$, and A is the amount of the photocurrent, generated by receiver photodiode only occur when binary '1' is received from the transmitter, also, the amount of photocurrent could be determined by extinction ratio. Between transmitter and receiver, a standard single-mode optical fibre is used for connection. Compare with other two type of optical fibre: step-index multi-mode optical fibre and graded index multi-mode optical fibre, single-mode optical fibre allows light pass through the fastest without any variation of time due to the narrower core and leads to less refractivity. Also, because of the narrower core and faster speed of propagation, optical fibre with single mode tends to have a high tolerance in order to reduce coupling loss.
The report starts with a detail discussion of the optical communication system, then design features with clear methodology will be shown and results will be stated. Finally, a conclusion will be given.

2. Discussion and Result

This session will present a detail discussion of the design feature of the optical communication system including bit error rate (BER), maximum system length, maximum bit-rate-distance product, power density spectrum, pin photodiode design and situation when symbols are transmitted with unequal probability.

2.1 Bit Error Rate (BER)

The Bit Error Rate could be identified as the probability of the bit error, P_e . It is highly link to the signal-to-noise ratio (SNR) which is defined as the ratio of signal power and noise power. As it is mentioned before that the optical communication system receives symbol drawn from alphabet $\{pA, A\}$, therefore, when 'A' is encoded for binary '1' and 'pA' is encoded for binary '0'. This could determine that the power of the received signal for binary '1' and binary '0' are A^2 and p^2A^2, respectively. It is assumed that the probability for two received signals are identical, therefore the power of the signal, P_s:

$$P_s = \frac{A^2+p^2A^2}{2} \quad (1)$$

For the noise power, it is considered that the Gaussian noise in this scenario, therefore the power of the noise, P_n:

$$P_n = \sigma^2 \quad (2)$$

where σ^2 is the variance of the probability density function of the Gaussian noise. The SNR in dB of the optical communication system could be deducted, therefore:

$$SNR\ (in\ dB) = 10\log\left(\frac{P_s}{P_n}\right) = 10\log\left(\frac{A^2+p^2A^2}{2\sigma^2}\right) \quad (3)$$

The bit error rate could be analyzed as the probability of receiving a '0' when a '1' is sent, and the probability of receiving a '1' when a '0' is sent, therefore:

$$P_e = P(0|1) * P(1) + P(1|0) * P(0) \quad (4)$$

It was assumed the probability for two received signals are identical:

$$P_e = \frac{1}{2}(P(0|1) + P(1|0)) \quad (5)$$

$$P_e = \frac{1}{2}\frac{1}{\sigma\sqrt{2\pi}}\{\int_{-\infty}^{\frac{(1-p)A}{2}} \exp\left[-\frac{(v-(1-p)A)^2}{2\sigma^2}\right] dv + \int_{\frac{(1-p)A}{2}}^{+\infty} \exp\left[-\frac{v^2}{2\sigma^2}\right] dv\} \quad (6)$$

The overall bit error rate is:

$$P_e = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) \quad (7)$$

where Q is $\frac{(1-p)A}{2\sigma}$, therefore:

$$Q^2 = \frac{(1-p)^2A^2}{4\sigma^2} \quad (8)$$

Compare Equation (3) and Equation (8), it could be deduced that:

$$Q = \sqrt{10^{\frac{SNR}{10}} * \frac{(1-p)^2}{2(1+p^2)}} \quad (9)$$

The bit error rate could be transformed as:

$$P_e = \frac{1}{2}erfc\left[\frac{1}{\sqrt{2}} * \sqrt{10^{\frac{SNR}{10}} \cdot \frac{(1-p)^2}{2(1+p^2)}}\right] \quad (10)$$

Therefore, the probability of bit error decreases when the SNR increases.

Figure.1 indicates the relationship between bit error rate and signal-to-noise ratio for different value of parameter p:

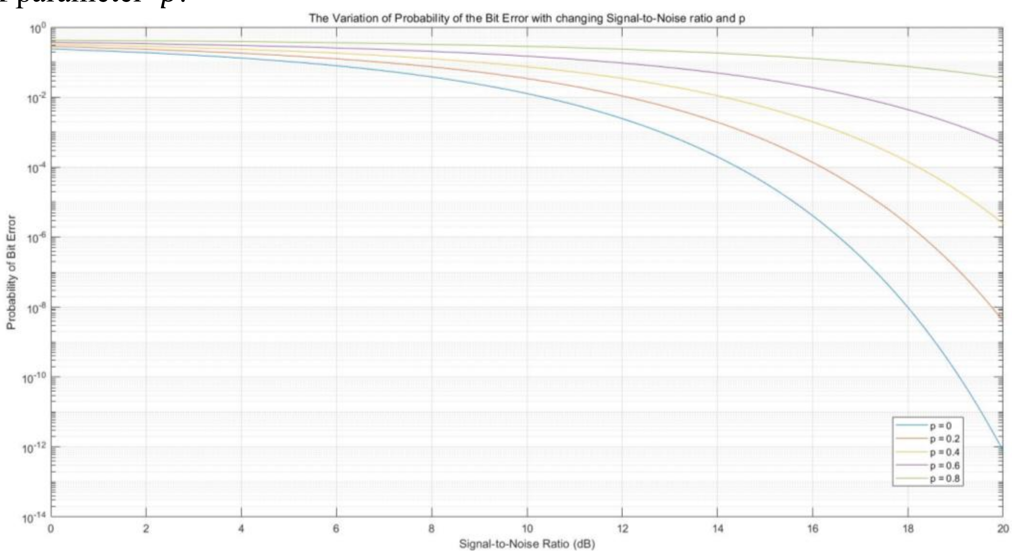

Figure.1 The Variation of Probability of the Bit Error with changing Signal-to-Noise ratio and p

As it is shown on the graph that with lower value of p, less signal-to-noise ratio would be applied for the BER to achieve less than 10^{-3}. As the system transmitter output extinction ratio, ER is 8dB, using equation:

$$ER = 10log_{10}\left(\frac{P(A)}{P(pA)}\right) = 10log_{10}\left(\frac{A}{pA}\right) \quad (11)$$

Therefore, the value of p can be determined as 0.158. Using Equation 10, the SNR would be Around 14.19dB, when BER is 10^{-3}.

2.2 Maximum System Length

It could be identified that the maximum system length is affected by both dispersion and fibre attenuation. The overall dispersion of an optical fibre is material dispersion with waveguide dispersion, and is also known as the chromatic dispersion. Group Velocity dispersion contributes to chromatic dispersion in optical fibres, which causes pulse spreading propagate through the optical fibre. The material dispersion varies with different materials which have made up the signal-node optical fibre due to different properties. This will cause different speed of wave propagation, and therefore, different frequency of the wave. The waveguide dispersion can be considered as a bias which shift the zero-dispersion wavelength to a slightly longer wavelength. Therefore, the waveguide dispersion is negative and could be neglected in shorter wavelength. In this case, the chromatic dispersion has a coefficient $17\ ps\ nm^{-1}\ km^{-1}$ approximately when the wavelength is 1550 nm.

As it is assumed that the symbol rate varies from 1 to 20 Gbaud, the relationship between symbol rate, S and bit rate, B is:

$$S = B * log_2\,(N) \quad (12)$$

The figure N in this case is 2, then therefore, $S = B$

The wavelength difference is:

$$\Delta\lambda = \frac{B}{\Delta f} \quad (13)$$

The distance could be deducted using the equation:

$$L = \frac{1}{\Delta\lambda * B * |D|} \quad (14)$$

where $|D|$ is the chromatic dispersion.

Therefore, from the equation 13, it can be concluded that the maximum system length decreases when the symbol (bit) rate increases. For the largest amount of dispersion, shortest distance should be obtained, thus, largest symbol rate is used, 20 Gbaud. Then it could be deduced that the distance is around 18.3 km.

The fibre attenuation is considered as there will be loss during transmission in fibre. The minimum SNR that received is assumed as 14dB in this case. The SNR could be determined as:

$$10^{\frac{SNR}{10}} = \frac{(R_S P_{in})^2}{\frac{4kTB}{R} + 2eR_S P_{in} B} \quad (15)$$

where R is the resistance of the photodiode, P_{in} is the input power, R_s is the photodiode responsibility. When the bit rate is 20Gbits/s, the P_{in} could be deduced as $1.62 * 10^{-5}$ W.

The fibre attenuation coefficient α_{dB} follows the equation:

$$\alpha_{dB} = -\frac{10}{L} log_{10}\left(\frac{P(L)}{P(0)}\right) \quad (16)$$

Term $P(L)$ is P_{in} in equation (14), and from script, it is noticed that α_{dB} is $0.25\ dBkm^{-1}$, $P(0)$ is the average output optical power, 4dBm. Thus the distance, L could be determined as 215.7 km.
Comparing the distance between dispersion and fibre attenuation, dispersion tends to have shorter distance, therefore, much stronger decaying for signal power. This leads to the fibre attenuation become trivial; thus the maximum system length will be 18.3km.

2.3 Maximum Bit-Rate Distance Product

The Bit-rate Distance product classifies the maximum bit-rate that can be transmitted from transmitter to receiver, passing though an optical fibre with dispersion. The pulse spreading of the chromatic dispersion is:

$$\Delta t = L * |D| * \Delta\lambda \tag{17}$$

For the maximum bit-rate distance product,

$$\Delta t = \frac{1}{B} \tag{18}$$

As this means that large amount of power spread from one bit slot to the adjacent bit slot, therefore, the equation transfer to:

$$B * L = \frac{1}{\Delta\lambda * |D|} \tag{19}$$

The maximum bit-rate distance product therefore is around $3.68 * 10^{11}\ bits\ s^{-1}\ m.$

2.4 Power Density Spectrum

The power density spectrum contains data of the bandwidth and bandwidth efficiency. The power density spectrum of the passband signal $S_x(f)$ could be related to the power density spectrum of the equivalent baseband signal $S_{xl}(f)$:

$$S_x(f) = \frac{1}{4} * (S_{xl}(f - f_c) + S_{xl}(f + f_c)) \tag{20}$$

In terms of $S_x(f)$:

$$S_{xl}(f) = \frac{1}{T_s} * S_{Xl}(f) * |P(f)^2| \tag{21}$$

where

$$P(f) = \int_{-\infty}^{\infty} p(t) * e^{-j\pi ft} dt \tag{22}$$

$$S_{Xl}(f) = \sum_{k=-\infty}^{\infty} R_{X_l}(k) * e^{-j2\pi kfT_s} \tag{23}$$

$$R_{X_l}(k) = E\{X_{i+k} * X_i^*\} \tag{24}$$

Therefore,

$$P(f) = \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} e^{-j\pi ft}\, dt = T_s * sinc(fT_s) \tag{25}$$

$$R_{X_l}(k) \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(pA)^2 & k = 0 \\ (\frac{1}{2}A + \frac{1}{2}pA)^2 & k \neq 0 \end{cases} \tag{26}$$

$$S_{Xl}(f) = \frac{1}{2}A^2 + \frac{1}{2}(pA)^2 + \left(\frac{1}{2}A + \frac{1}{2}pA\right)^2 (\sum_{k=-\infty}^{\infty} e^{-j2\pi kfT_s} - 1) \quad (27)$$

Using the Poisson summation formula as shown in equation (28):

$$\sum_{n=-\infty}^{\infty} e^{-2j\pi nfD} = \frac{1}{D}\sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{D}) \quad (28)$$

Therefore:

$$S_{Xl}(f) = \frac{1}{2}(1+p^2)A^2 + \frac{1}{4}(1+p)^2A^2(\sum_{k=-\infty}^{\infty} e^{-j2\pi kfT_s}) - [\frac{1}{4}(1+p)^2A^2]$$

$$S_{Xl}(f) = \frac{1}{4}(1-p^2)A^2 + \frac{1}{4T_s}(1+p)^2A^2[\sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T_s})] \quad (29)$$

According to equation (21) and (29):

$$S_{xl}(f) = \left\{\frac{1}{4}T_s(1-p^2)A^2 + \frac{1}{4}(1+p)^2A^2\left[\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_s}\right)\right]\right\} * [sinc(fT_s)^2] \quad (30)$$

Because the sum of delta function in terms of delta function occur as null in sinc function, thus:

$$\left[\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_s}\right)\right] * [sinc(fT_s)^2] = \delta(f) \quad (31)$$

Therefore:

$$S_{xl}(f) = \left\{\frac{1}{4}T_s(1-p^2)A^2[sinc(fT_s)^2] + \frac{1}{4}(1+p)^2A^2\delta(f)\right\} \quad (32)$$

Using equation (20) and (32), the power density is:

$$S_x(f) = \left\{\frac{1}{16}T_s(1-p^2)A^2[sinc((f-f_c)T_s)^2] + \frac{1}{16}(1+p)^2A^2\delta(f-f_c)\right\} +$$
$$\left\{\frac{1}{16}T_s(1-p^2)A^2[sinc((-f-f_c)T_s)^2] + \frac{1}{16}(1+p)^2A^2\delta(-f-f_c)\right\} \quad (33)$$

Because the symbol rate is the same as bit rate which is 5 Gbits/s, therefore T_s is:

$$T_s = \frac{1}{B} = 2 * 10^{-10} \quad (34)$$

Thus,

$$S_x(f) = \left\{1.25 * 10^{-11}(1-p^2)A^2[sinc(2 * 10^{-10} * (f-f_c)^2] + \frac{1}{16}(1+p)^2A^2\delta(f-f_c)\right\}$$
$$+ \left\{1.25 * 10^{-11}(1-p^2)A^2[sinc(2 * 10^{-10} * (-f-f_c)^2] + \frac{1}{16}(1+p)^2A^2\delta(-f-f_c)\right\}$$

(35)

2.5 Pin Photodiode Design

The pin photodiode is similar to photodiode p-n junction but contains an depletion region made by ‘intrinsic’ material between the p and n regions as shown in Figure.2. To compare the pin photodiode with pn photodiode, the pin photodiode allows the width of the depletion region which generates carriers, to be controlled independently due to the region of intrinsic semiconductor between p and n region. Also, with the little space charge formed at the edge of the p and n region, the intrinsic semiconductor causes an uniform field in depletion region, this provides a better external quantum efficiency by sweeping out the photogenerated carriers of the depletion region.

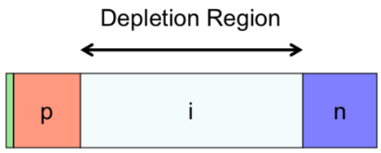

Figure.2 Pin Photodiode

The relationship between the electric field and the charge density could be described using Poisson's Equation:

$$\frac{dE}{dx} = \frac{eN_D}{\varepsilon_r \varepsilon_0} \tag{36}$$

where N_D is the charge density. The uniform electric field influent the electron-hole pairs by separate them and drift them across the depletion region.

By increasing the length of the depletion region, the external quantum efficiency can be grown up to almost 100%. However, this will cause both longer response time and drop of capacitance. For longer response time, the bandwidth will be decreasing, while reducing the capacitance will cause increasing of the bandwidth, therefore, a trade-off should be made when designing.

To design the photodiode, bandwidth, drift, diffusion, and RC time constant are crucial to be considered. It is suggested to apply thin p-layer to decrease diffusion time, thus $t_d \ll \frac{1}{B}$, then it is also suggested to choose intrinsic layer width such that the drift time, $t_c < \frac{1}{2B}$. Moreover, it is recommended to choose junction capacitance so that $t_{RC} < \frac{1}{2\pi B}$ for required bandwidth.

Other than the above elements, material of the photodiode, thickness of the layer and device area should be determined to allow better performance. The optical communication system is processing at a symbol rate of 5 Gbaud with binary amplitude shift keying and the wavelength of the optical signal is at 1550 nm. Therefore, Indium Gallium Arsenide is an appropriate material for the intrinsic layer of photodiode as it has a wavelength sensitivity between 800nm to 2600nm. The p-layer and n-layer could be made by aluminium gallium arsenide and gallium arsenide, respectively as usual. The thickness of the intrinsic layer could be determined using t_c, as:

$$t_c < \frac{1}{2B} \text{ and } t_c = \frac{x}{v} \tag{37}$$

where x is the thickness and v is the saturated drift velocity of photodiode semiconductor material which is $10^5\ ms^{-1}$. As it is mentioned before, in the system, the symbol rate equals to the bit rate which is also equivalent to the bandwidth. Therefore, the thickness could be calculated and is around $10\ \mu m$. Also, it was assumed that the resistance of the load resistor is 50Ω. Therefore, the capacitance could be determined as 0.637pF using equation 26:

$$B = \frac{1}{2\pi RC} \tag{38}$$

The relative permittivity of the indium gallium arsenide is 13 approximately.

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} \tag{39}$$

By using equation (27), the device area could be determined as $5.53 * 10^{-8} m^2$.

2.6 Situation when Symbols are Transmitted with Unequal Probability

As it was assumed for the Bit Error Rate (BER) that the probabilities for each symbol is identical, however, in reality, the probabilities may be unequal. At the end of the passband digital receiver, decision device is set up to compare the output with decision threshold. Two types of decision rules could be applied in the decision device, the MAP decision rule and the ML decision rule. The MAP decision rules is used minimizes the error probability, while the ML decision rule transmit symbol that is closest to the receive symbol which may not minimize the error probability during process. Therefore, the MAP decision rule is used in this situation.
Because of the unequal probability, it is assumed that:

$$P(X_k = pA) = r \ and \ P(X_k = A) = 1 - r \quad (40)$$

The output of the matched filter at time k can be written as:

$$Y_k = X_k + N_k \quad (41)$$

where N_k is a Gaussian random variable with mean zero and variance $\frac{N_0}{2}$, therefore, the probability density function are:

$$P_{N_k}(N_k) = \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{{N_k}^2}{N_0}} \quad (42)$$

$$P_{Y_k|X_k}(Y_k|X_k = A) = \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{(y_k - A)^2}{N_0}} \quad (43)$$

The decision region D_1^{MAP} such that $Y_k \in D_1^{MAP}$ leads to $\hat{X}_k = pA$:

$$D_1^{MAP} = Y_k: P(X_k = pA) * P_{Y_k|X_k}(Y_k|X_k = pA) > P(X_k = A) * P_{Y_k|X_k}(Y_k|X_k = A) \quad (44)$$

Therefore, by substitution:

$$D_1^{MAP} = \left\{ Y_k: r * \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{(y_k - pA)^2}{N_0}} > (1 - r) * \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{(y_k - A)^2}{N_0}} \right\}$$

$$= \left\{ Y_k: Y_k > \frac{N_0 \ln\left(\frac{1-r}{r}\right) - A^2(1-p)}{2pA - 2A} \right\} \quad (45)$$

The region D_2^{MAP} such that $Y_k \in R_2^{MAP}$ leads to $\hat{X}_k = A$:

$$D_2^{MAP} = Y_k: P(X_k = A) * P_{Y_k|X_k}(Y_k|X_k = A) > P(X_k = pA) * P_{Y_k|X_k}(Y_k|X_k = pA) \quad (46)$$

Therefore, by substitution:

$$D_2^{MAP} = \left\{ Y_k: (1 - r) * \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{(y_k - A)^2}{N_0}} > r * \frac{1}{\sqrt{\pi N_0}} * e^{-\frac{(y_k - pA)^2}{N_0}} \right\}$$

$$= \left\{ Y_k: Y_k < \frac{N_0 \ln\left(\frac{1-r}{r}\right) - A^2(1-p)}{2pA - 2A} \right\} \quad (47)$$

Therefore, the general equation for a decision threshold that minimises BER is :

$$\text{Decision Threshold} = \frac{N_0 \ln\left(\frac{1-r}{r}\right) - A^2(1-p)}{2pA - 2A} \tag{48}$$

For the decision region, when r is greater than 0.5:

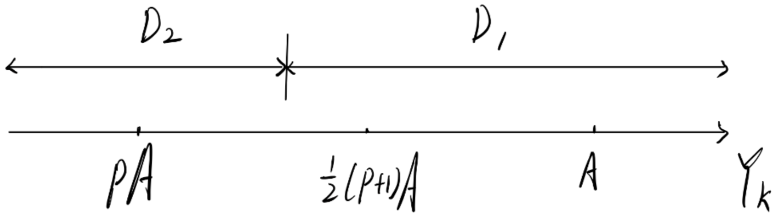

Figure.3 Decision Region when r > 0.5

when r is less than 0.5:

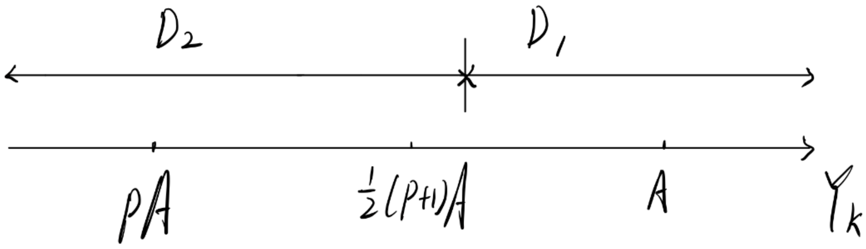

Figure.4 Decision Region when r < 0.5

When p tends to 0, the decision threshold will become:

$$\text{Decision Threshold} = \frac{N_0 \ln\left(\frac{1-r}{r}\right) - A^2}{-2A} \tag{49}$$

Also, the decision region will have the boundaries between D_2 and D_1 shift to the left when p tends to 0. Moreover, the symbols of the system will be 0 and A if p becomes 0. This would lead to on-off keying of the system, thus reduces the Bit-Error Rate.

3. Conclusion

For sum up, the report has proved that the probability of bit error decreases when the signal-to-noise ratio increases. Also, the value of p is determined as 0.158 by the extinction ratio and when BER is 10^{-3}, the SNR is 14.19dB. Moreover, it was analysed that the maximum system length decreases when the symbol rate increases, the maximum system length is 18.3km in the above situation and the maximum bit-rate distance product is $3.68 * 10^{11}\ bits\ s^{-1}\ m$ for the system. What is more is that the power density spectrum of the received signal is deducted as equation (35), and in order to design the photodiode in the receiver, pin design is used and aluminium gallium arsenide is used for semiconductor material with $10\ \mu m$ thickness and $5.53 * 10^{-8} m^2$ device area. Finally, the general equation for decision threshold is deduced as shown in equation (48), the decision region graph has been sketched as shown in Figure.3 and Figure.4. The situation when p approaches to 0 is considered, which may lead to on-off keying of the system and reduce the Bit-Error Rate.

In conclude, the report has discussed and analysed all the required topics.