

Optimization and Tips for Neural Network Training

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Gradient Descent

Optimization Goal

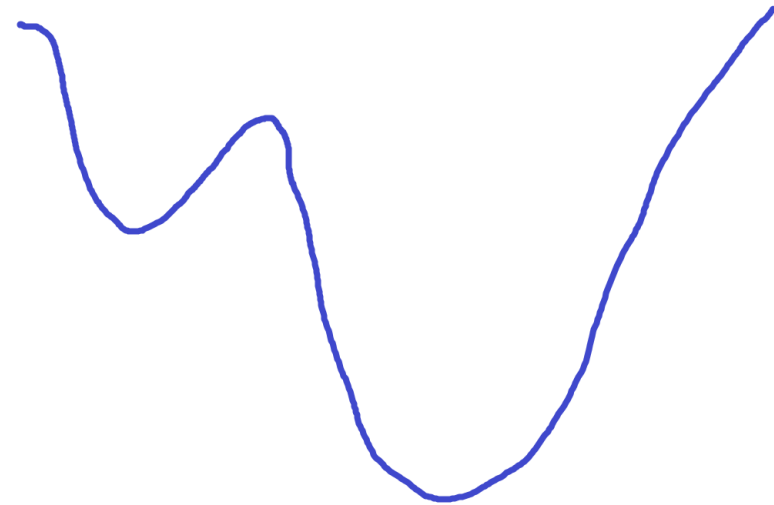
Find a set of (optimized) weights which minimize the error (or loss function) at the output

Weight update rule

$$\dot{W}_{nm}^L \leftarrow W_{nm}^L - \alpha * \delta W_{nm}^L$$


$$W_{ij} \leftarrow W_{ij} - \alpha \frac{\partial \mathcal{L}}{\partial W_{ij}}$$

Global minimum vs. local minimum



Stochastic Gradient Descent

How many training samples at a time do we include to calculate the error?


$$W_{ij} \leftarrow W_{ij} - \alpha \frac{\partial \mathcal{L}}{\partial W_{ij}}$$

Practically we use mini batches

lep

Training speed and accuracy vs. minibatch size

Stochastic Gradient Descent

With decreasing learning rate (Learning rate scheduling)

Algorithm 8.1 Stochastic gradient descent (SGD) update

Require: Learning rate schedule $\epsilon_1, \epsilon_2, \dots$

Require: Initial parameter θ

$k \leftarrow 1$

while stopping criterion not met **do**

Sample a minibatch of m examples from the training set $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m)}\}$ with corresponding targets $\mathbf{y}^{(i)}$.

Compute gradient estimate: $\hat{\mathbf{g}} \leftarrow \frac{1}{m} \nabla_{\theta} \sum_i L(f(\mathbf{x}^{(i)}; \theta), \mathbf{y}^{(i)})$

Apply update: $\theta \leftarrow \theta - \epsilon_k \hat{\mathbf{g}}$

$k \leftarrow k + 1$

end while

Stochastic Gradient Descent

(notations)

learning rate — $\alpha, \gamma, \varepsilon$

momentum : ν, \checkmark

weights : w, θ

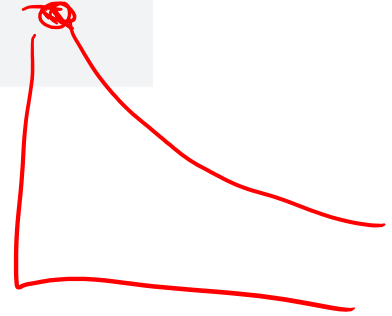
loss ft : \mathcal{L}, J, E

SGD tuning parameters

```
tf.keras.optimizers.SGD(  
    learning_rate=0.01, momentum=0.0, nesterov=False, name='SGD', **kwargs  
)
```

Popular options to tweak

- learning_rate: the base learning rate
- momentum
- decay
- nesterov
- (advanced) callback



Stochastic Gradient Descent with momentum

SGD with learning rate alone is slow to converge

Adding a momentum (moving average of a weight) can make it faster

$$\begin{aligned} \mathbf{v} &\leftarrow \alpha \mathbf{v} - \epsilon \nabla_{\theta} \left(\frac{1}{m} \sum_{i=1}^m L(\mathbf{f}(\mathbf{x}^{(i)}; \boldsymbol{\theta}), \mathbf{y}^{(i)}) \right), \\ \boldsymbol{\theta} &\leftarrow \boldsymbol{\theta} + \mathbf{v}. \end{aligned}$$

** see what happens when the gradient is 0 (on plateau)

warning- different notations used (from deeplearningbook.org)

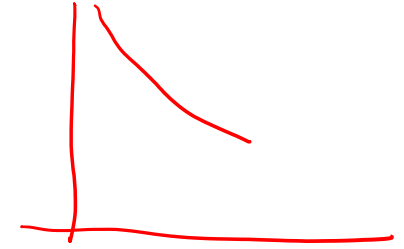
Stochastic Gradient Descent with decay

Learning rate scheduling using decay

For iteration k (epoch)

$$\epsilon_k = (1 - \alpha)\epsilon_0 + \alpha\epsilon_\tau$$

$$\alpha = \frac{k}{\tau}$$



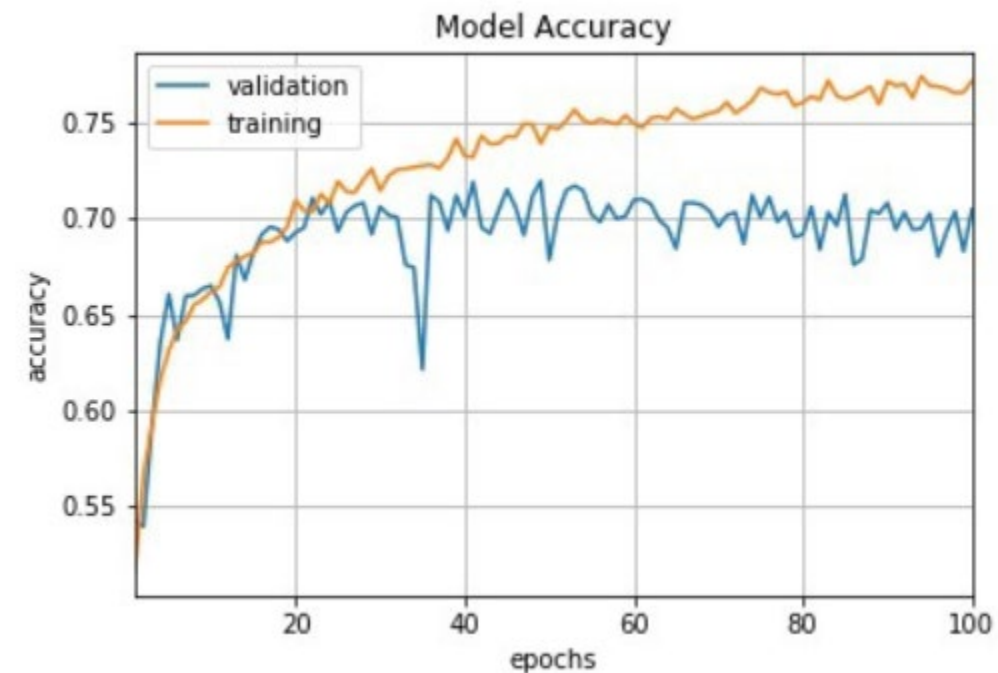
** In the algorithm pseudocode k is for step (each mini batch),
and decay learning rate by step,
but normally we decrease learning rate each epoch

warning- different notations used (from deeplearningbook.org)

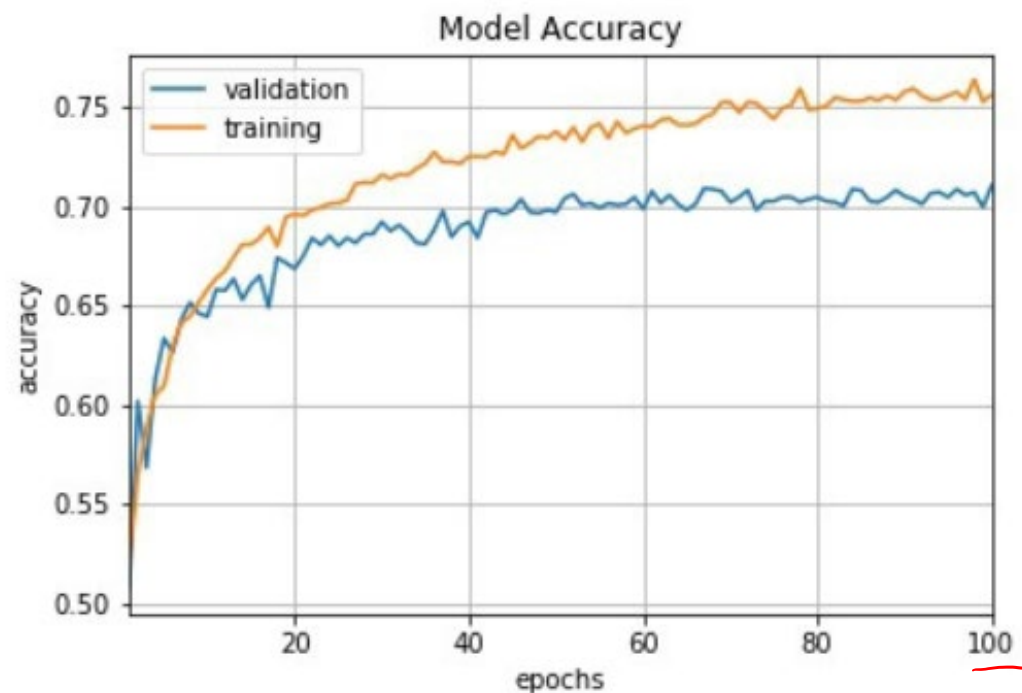
Learning rate scheduling

```
tf.keras.optimizers.SGD(  
    learning_rate=0.01, momentum=0.0, nesterov=False, name='SGD', **kwargs  
)
```

learning_rate=0.1,
momentum=0, decay=0, nesterov=False



learning_rate=0.1,
momentum=0.8, decay=~~learning_rate/epochs~~ ^{0.001}



Learning rate scheduling (custom)

```
tf.keras.callbacks.LearningRateScheduler(  
    schedule, verbose=0  
)
```

```
def step_decay(epoch):  
    initial_lrate = 0.1  
    drop = 0.5  
    epochs_drop = 10.0  
    lrate = initial_lrate * math.pow(drop,  
        math.floor((1+epoch)/epochs_drop))  
    return lrate
```

Ex2

drop lr by half every 10 epochs

```
lrate = LearningRateScheduler(step_decay)
```

```
# This function keeps the learning rate at 0.001 for the first ten epochs  
# and decreases it exponentially after that.  
def scheduler(epoch):  
    if epoch < 10:  
        return 0.001  
    else:  
        return 0.001 * tf.math.exp(0.1 * (10 - epoch))
```

Ex1

```
callback = tf.keras.callbacks.LearningRateScheduler(scheduler)  
model.fit(data, labels, epochs=100, callbacks=[callback],  
        validation_data=(val_data, val_labels))
```

Learning rate scheduling (custom)

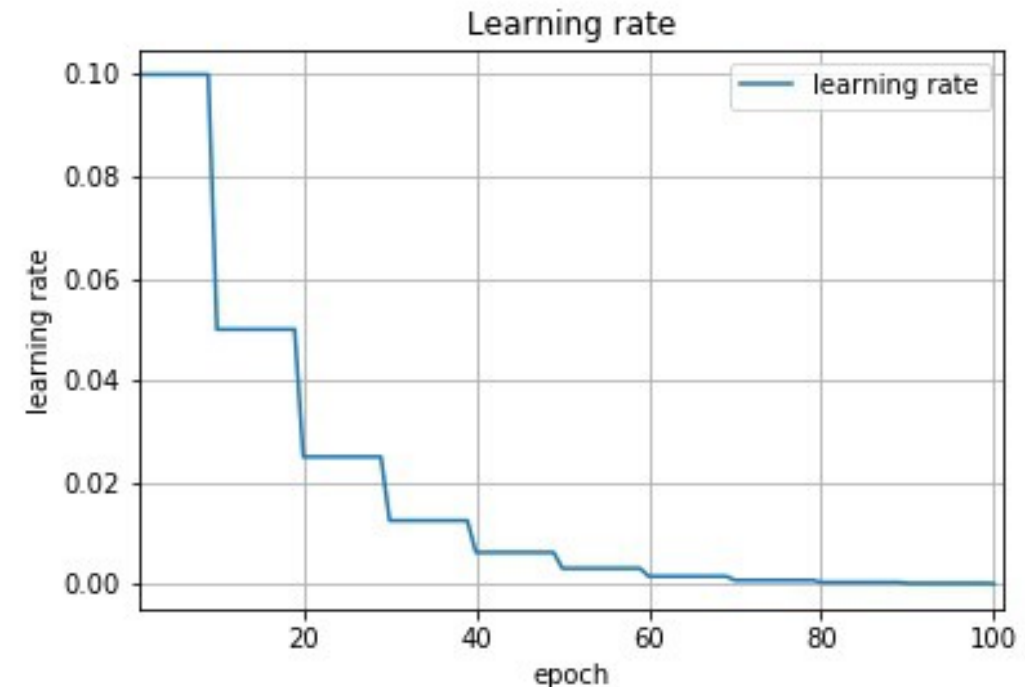
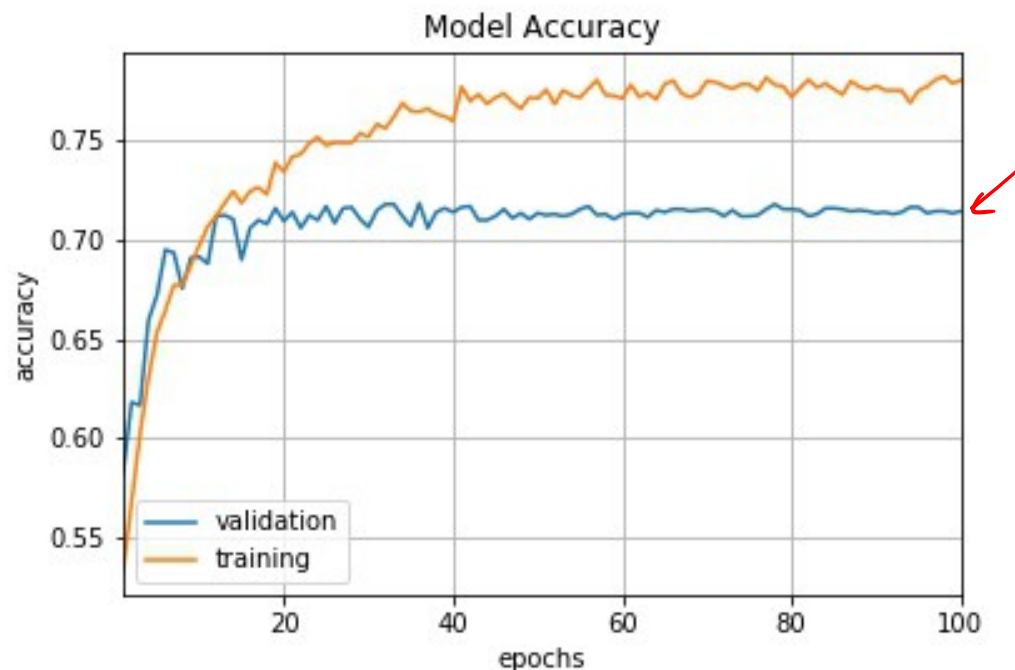
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Ex2

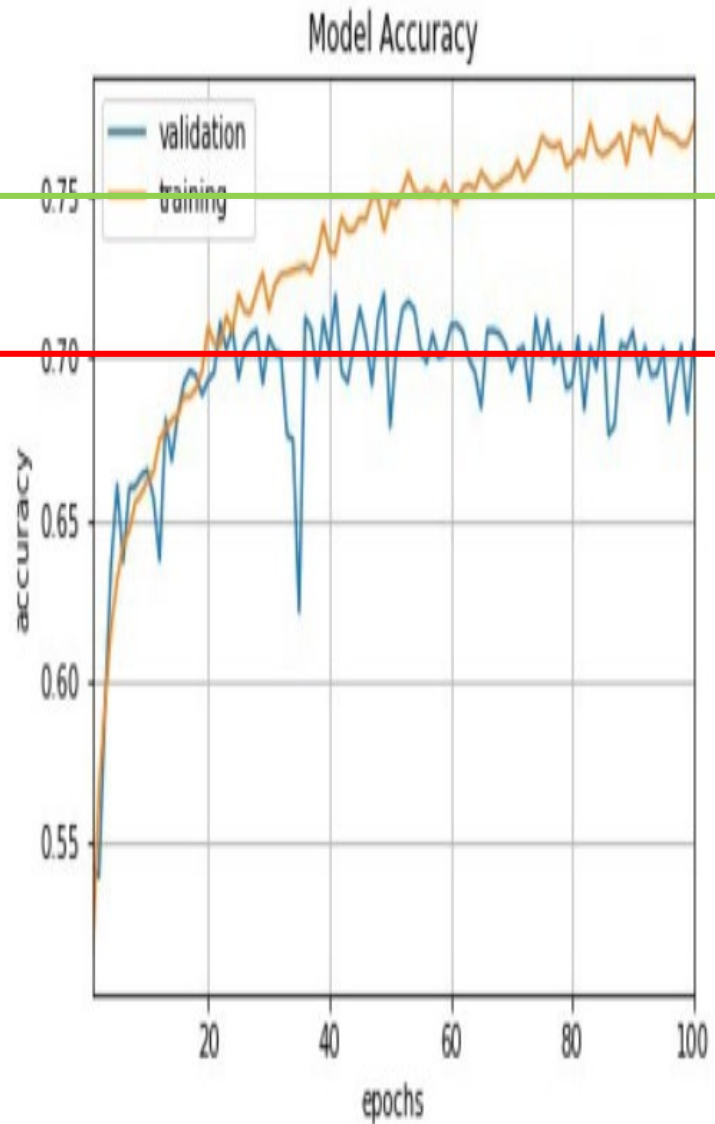
drop lr by half every 10 epochs

```
lrate = LearningRateScheduler(step_decay)
```

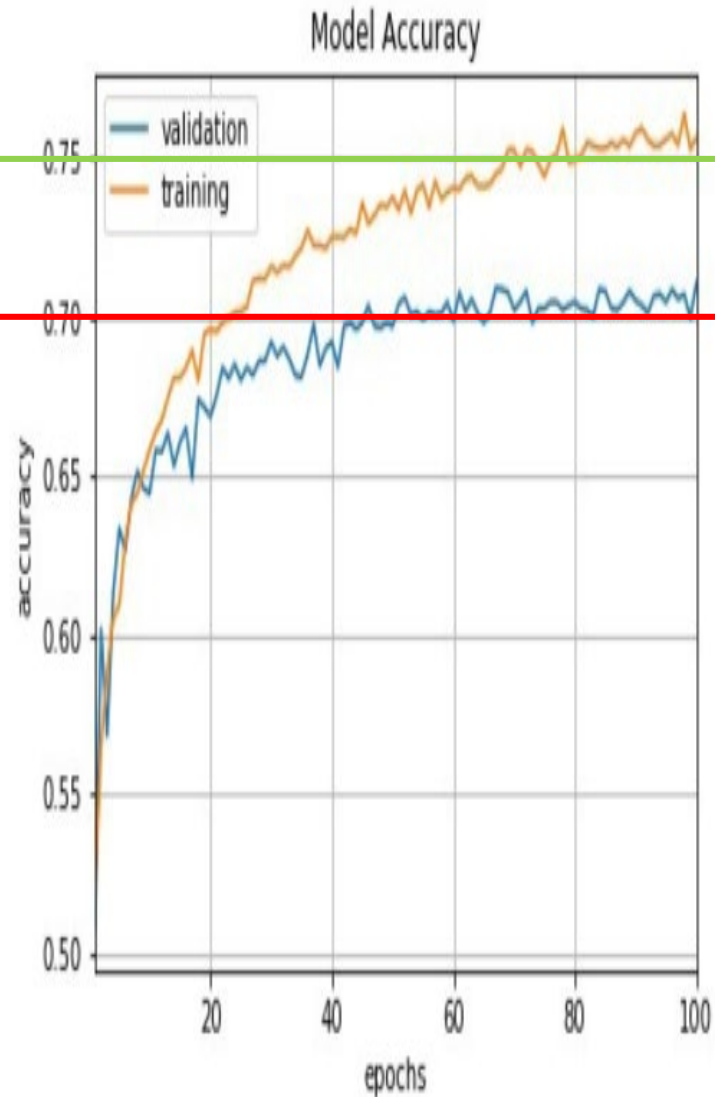


comparison

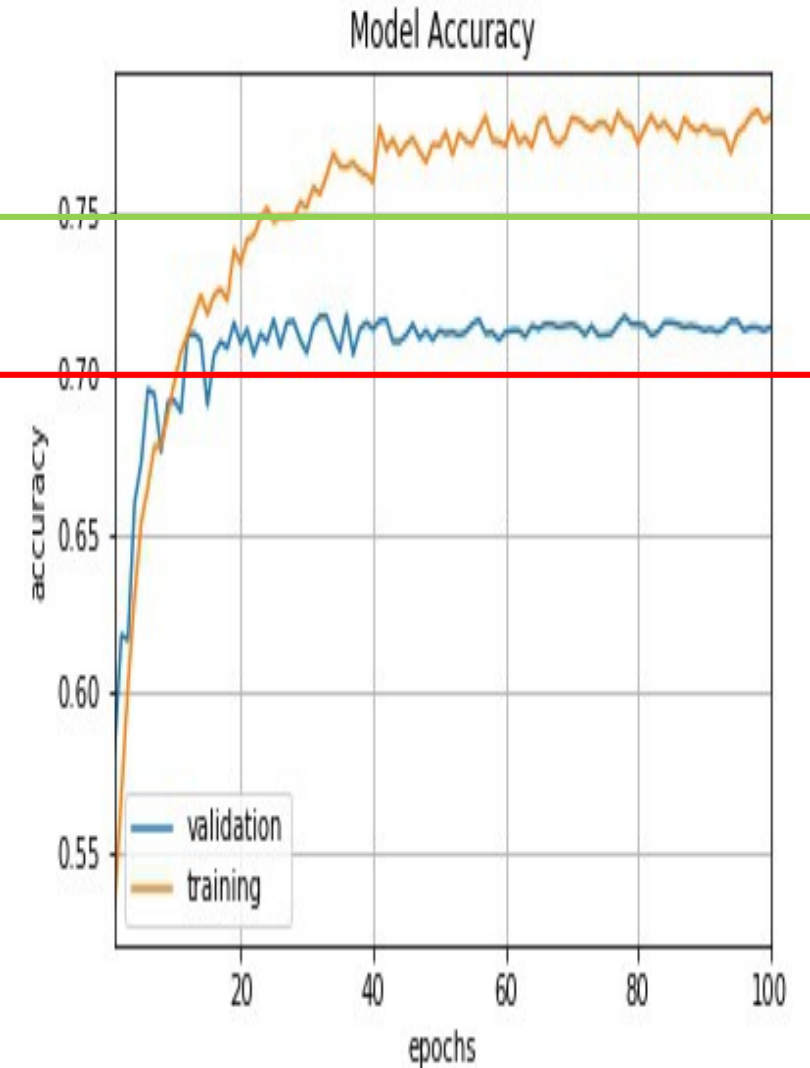
Base (fixed lr)



with momentum and decay



custom learning rate schedule (step drop)



Nestrov momentum

Nestrov momentum does early correction on gradient

It's supposed to make converge faster, but on SGD it doesn't do much

Regular momentum

$$\begin{aligned} \mathbf{v} &\leftarrow \alpha \mathbf{v} - \epsilon \nabla_{\boldsymbol{\theta}} \left(\frac{1}{m} \sum_{i=1}^m \underbrace{L(\mathbf{f}(\mathbf{x}^{(i)}; \boldsymbol{\theta}), \mathbf{y}^{(i)})}_{\text{red underline}} \right), \\ \boldsymbol{\theta} &\leftarrow \boldsymbol{\theta} + \mathbf{v}. \end{aligned}$$

Nestrov momentum

$$\begin{aligned} \mathbf{v} &\leftarrow \alpha \mathbf{v} - \epsilon \nabla_{\boldsymbol{\theta}} \left[\frac{1}{m} \sum_{i=1}^m \underbrace{L(\mathbf{f}(\mathbf{x}^{(i)}; \boldsymbol{\theta} + \alpha \mathbf{v}), \mathbf{y}^{(i)})}_{\text{red underline}} \right], \\ \boldsymbol{\theta} &\leftarrow \boldsymbol{\theta} + \mathbf{v}, \end{aligned}$$

Advanced optimization

Adagrad

learning rate is ~~normalized~~ by the sqrt of the total sum of the gradient

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}$$

An overview of gradient descent optimization algorithms
<https://arxiv.org/pdf/1609.04747.pdf>

warning- different notations used

Advanced optimization

Adadelta

learning rate is normalized by the RMS of the gradient

Weight change is proportional to the RMS ratio

$$\Delta\theta_t = -\frac{\eta}{\text{RMS}[g]_t} g_t$$

Handwritten red notes: A bracket under the denominator $\text{RMS}[g]_t$ is labeled with $\sqrt{\sum g^2 / n}$.

$$\Delta\theta_t = -\frac{\text{RMS}[\Delta\theta]_{t-1}}{\text{RMS}[g]_t} g_t$$

Handwritten red note: An arrow points to the $\text{RMS}[\Delta\theta]_{t-1}$ term in the numerator.

$$\theta_{t+1} = \theta_t + \Delta\theta_t$$

Advanced optimization

RMSprop

Variant of Adadelta

RMSprop takes a moving average when it calculate the RMS of the gradient

$$E[g^2]_t = 0.9E[g^2]_{t-1} + 0.1g_t^2$$
$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\underbrace{E[g^2]_t}_{\text{moving average}} + \underbrace{\epsilon}_{\text{small constant}}}} g_t$$

An overview of gradient descent optimization algorithms

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warning- different notations used

Advanced optimization

Adaptive Moment Estimation (Adam)

Mimics momentum for gradient and gradient-squared

m_t and v_t are estimates of the first moment (the mean) and the second moment (the uncentered variance) of the gradients

$$\begin{cases} m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t \\ v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2 \end{cases}$$

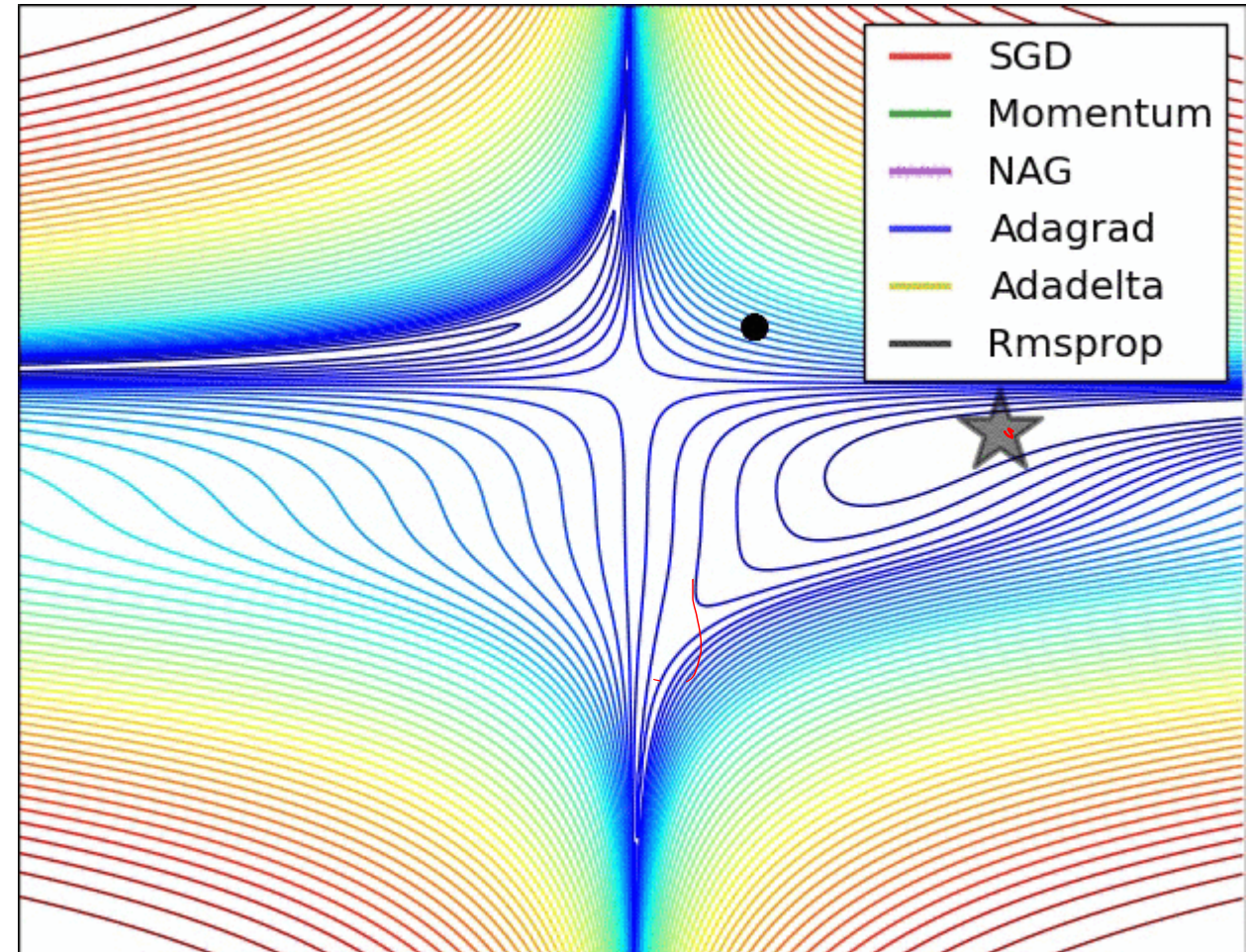
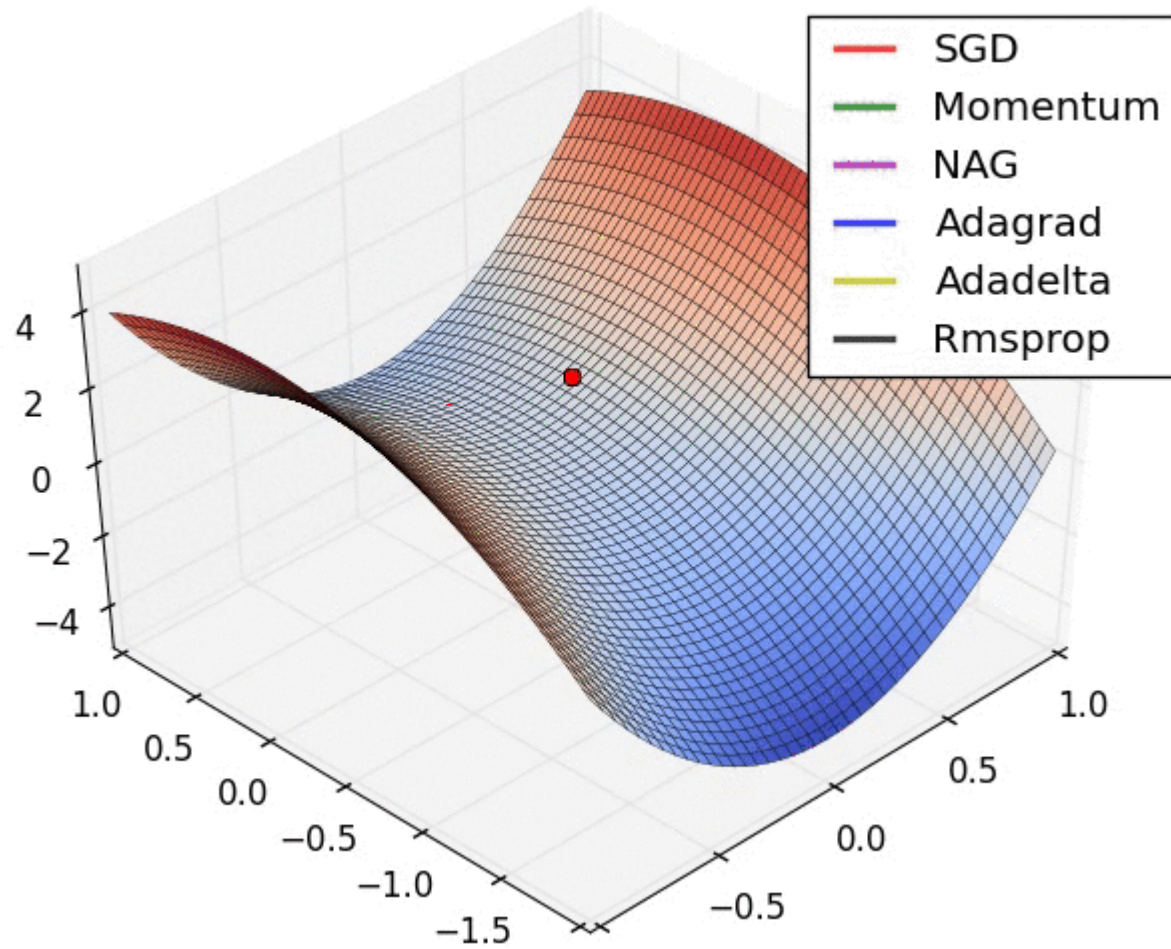
$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t$$

An overview of gradient descent optimization algorithms

<https://arxiv.org/pdf/1609.04747.pdf>

warning- different notations used

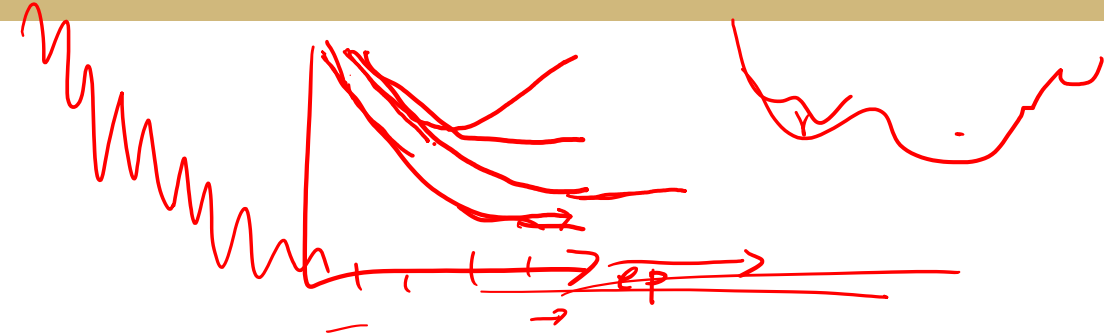
Advanced optimization



animated image source: <https://imgur.com/a/Hqolp>

Tips for training NN

Monitor overfitting as epoch goes



Train hyperparameter tuning : learning rate and other hyperparams

Architecture hyperparameter tuning : NN architecture, # layers, # neurons, activation ft, etc

|||

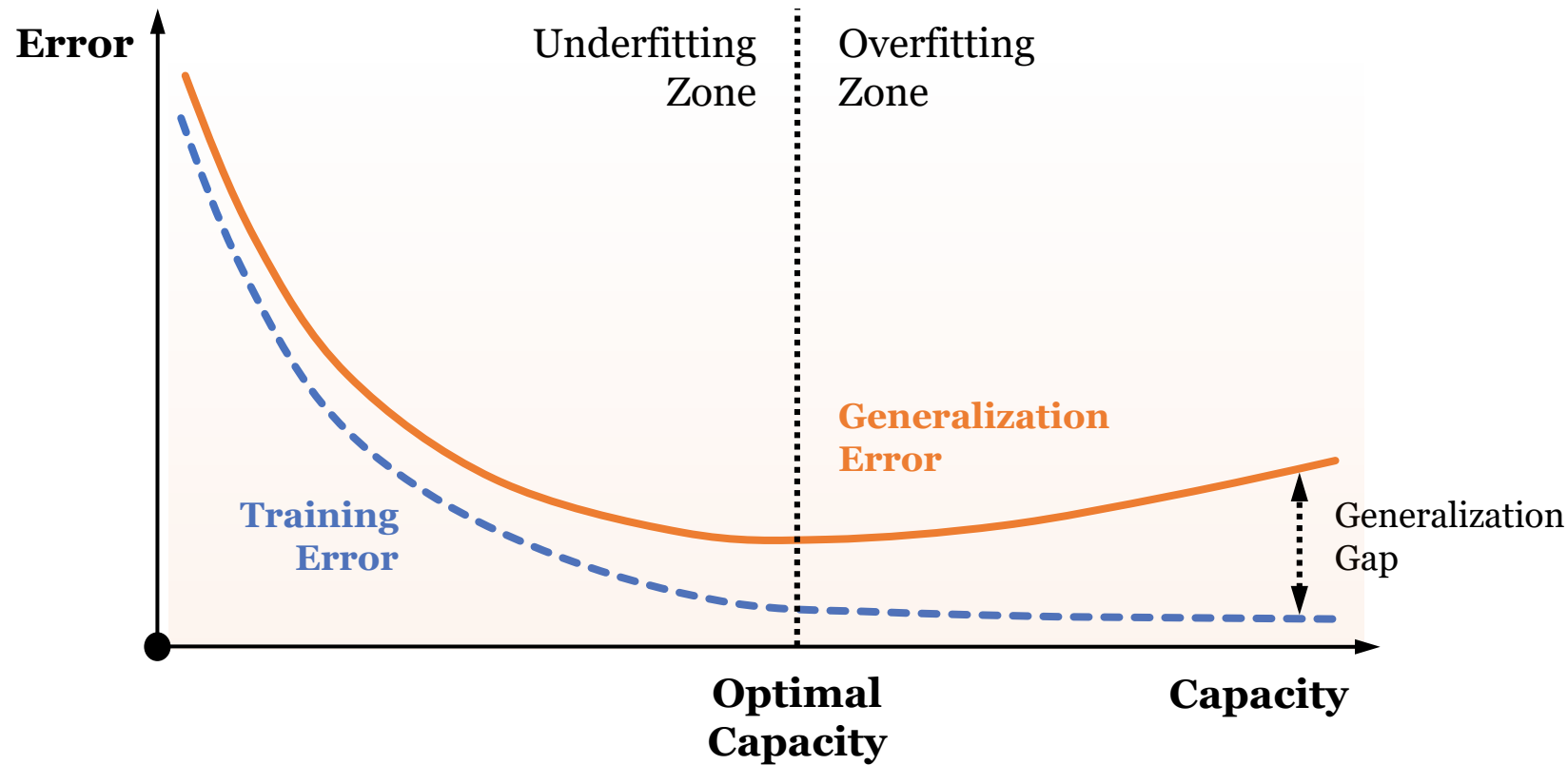
Try different optimization methods

||

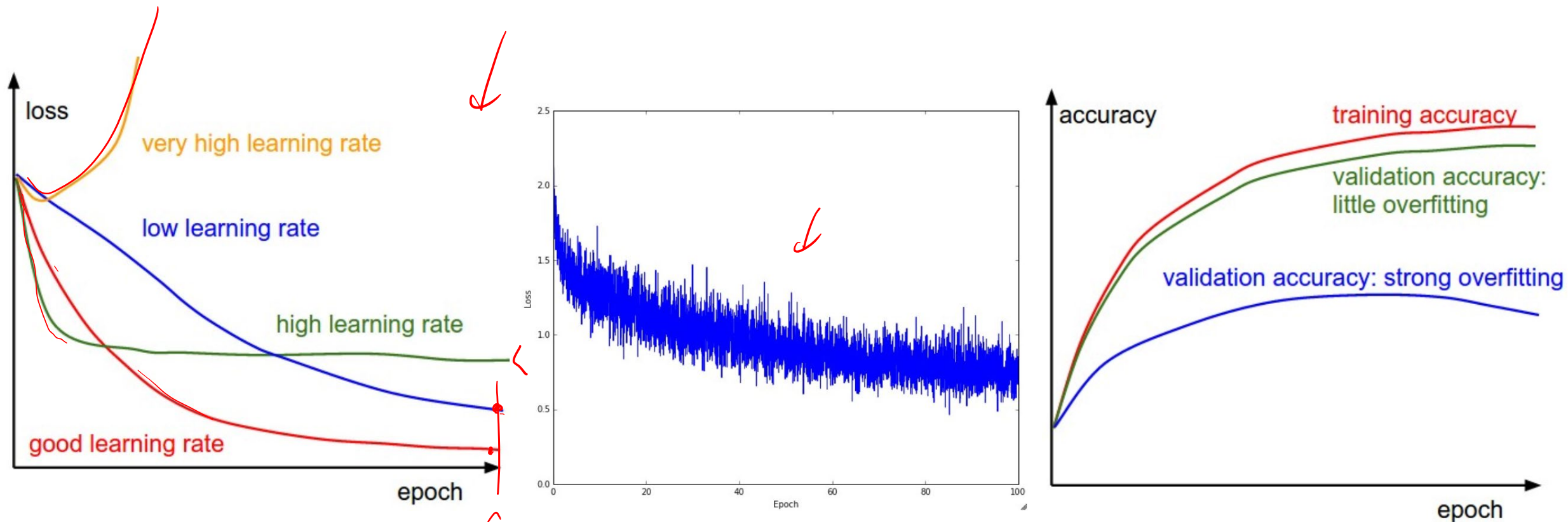
Regularization: Dropout and Batch Normalization,
or add L1/L2 reg on the loss

Monitoring Overfitting in Training

Dataset split
Train / Validation / Test

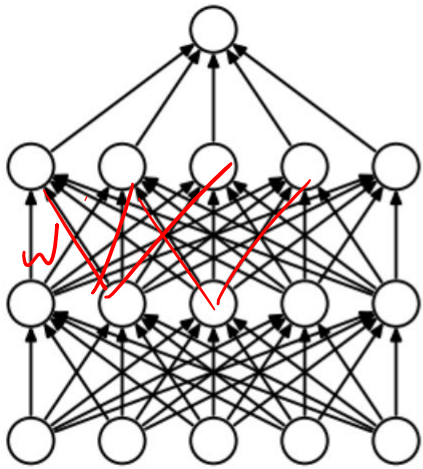


Monitoring Overfitting in Training

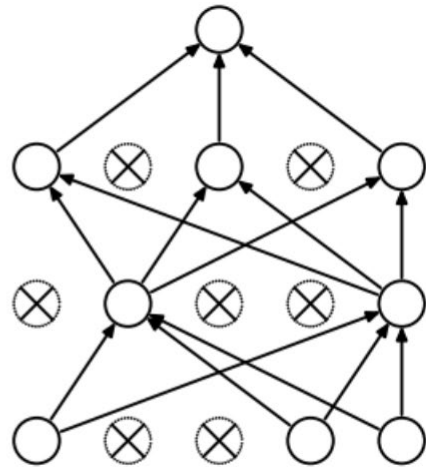


Ways to reduce overfitting

Dropout

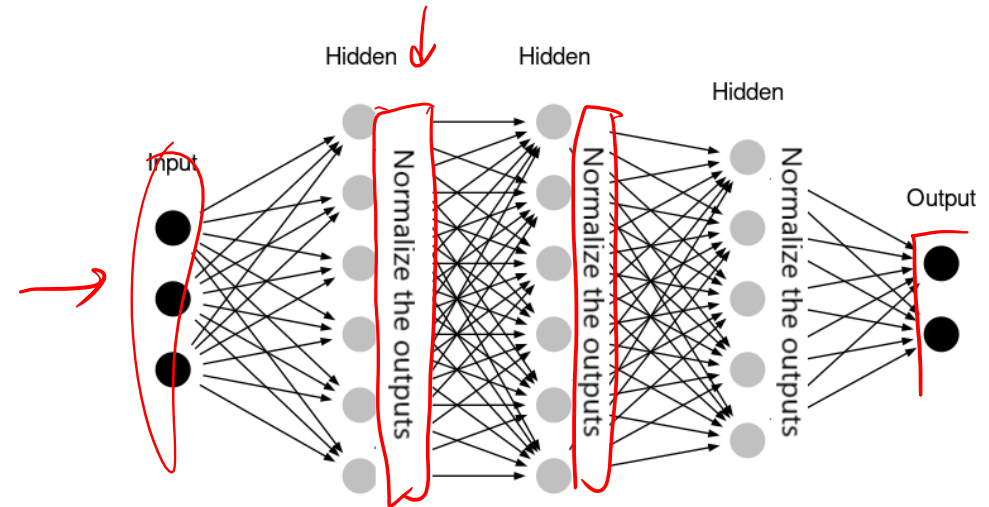


(a) Standard Neural Net



(b) After applying dropout.

Batch normalization



<https://www.kaggle.com/c/cub-csci-4622-kaggle-2-2020/overview>

To participate please check the Piazza post for the invitation link.