Network theory: 3
optimisation problems and networks

Liubov Tupikina (CRI)

Lecture 3

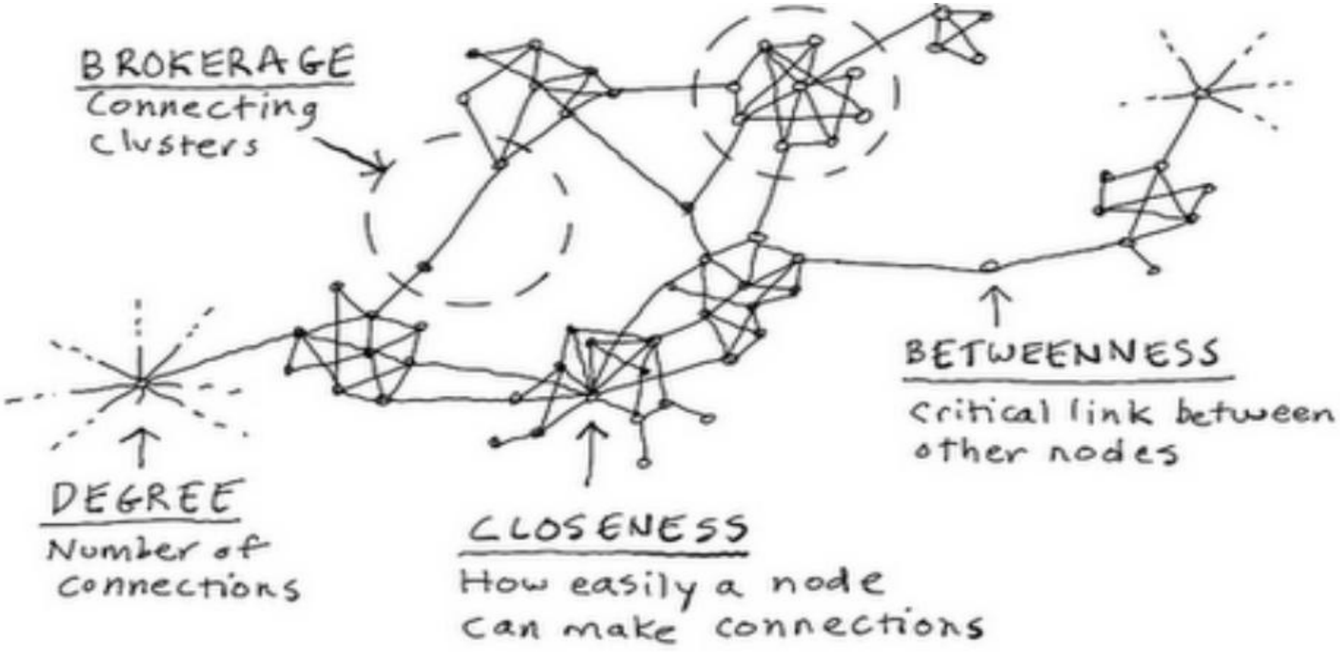
Network models and measures (reminder from lecture 2)

Optimization problems: Konigsberg problem, Steiner problem

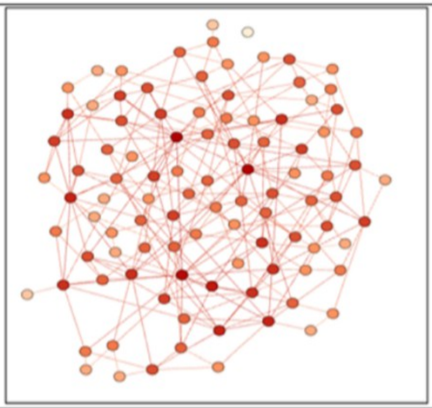
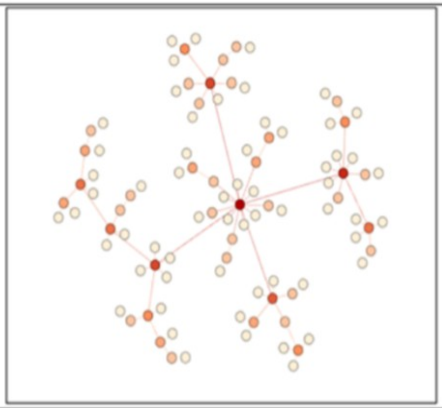
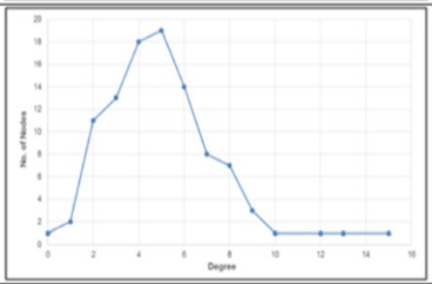
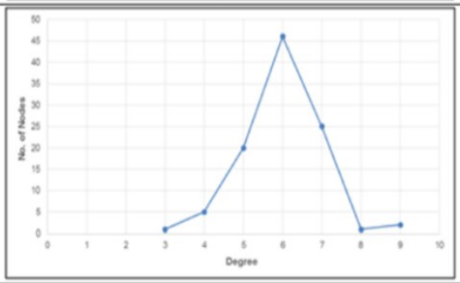
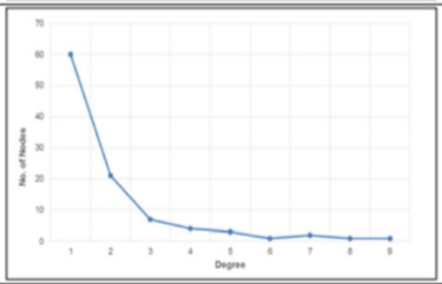
Applications to biology

Small network quiz

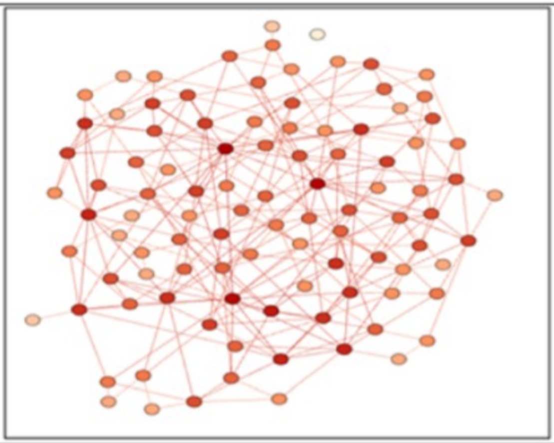
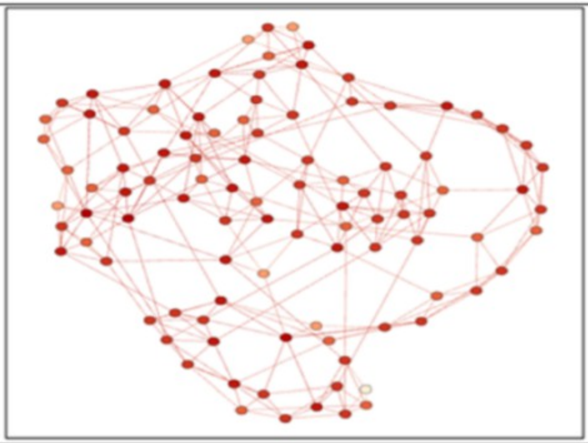
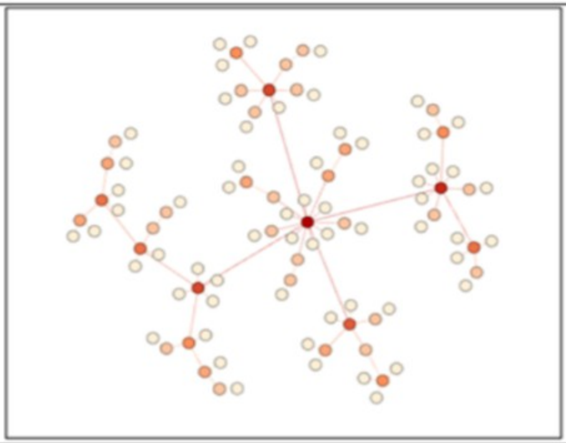
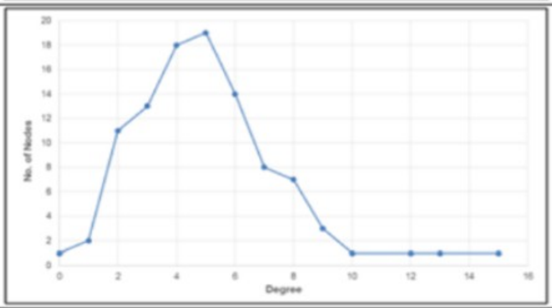
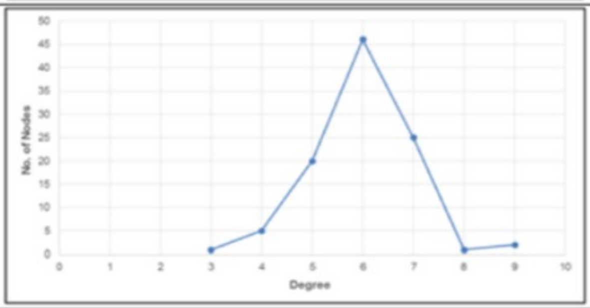
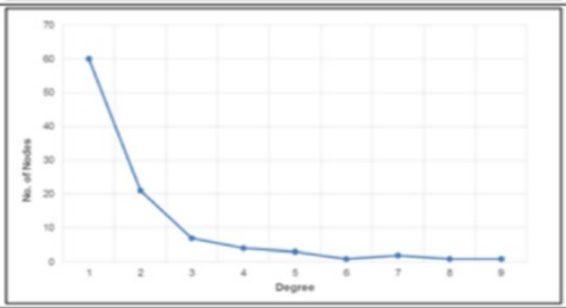
Network measures



Random network models

	Random	Small-World	Scale-Free
Visualisation of the Topology			
Degree Distribution			
Robustness Characteristics	Responds similarly to both random and targeted attacks.		Resilient against random failures but very sensitive to targeted attacks.

How to estimate properties of Erdos-Reny, Watts-Strogatz and Barabasi-Albert models?

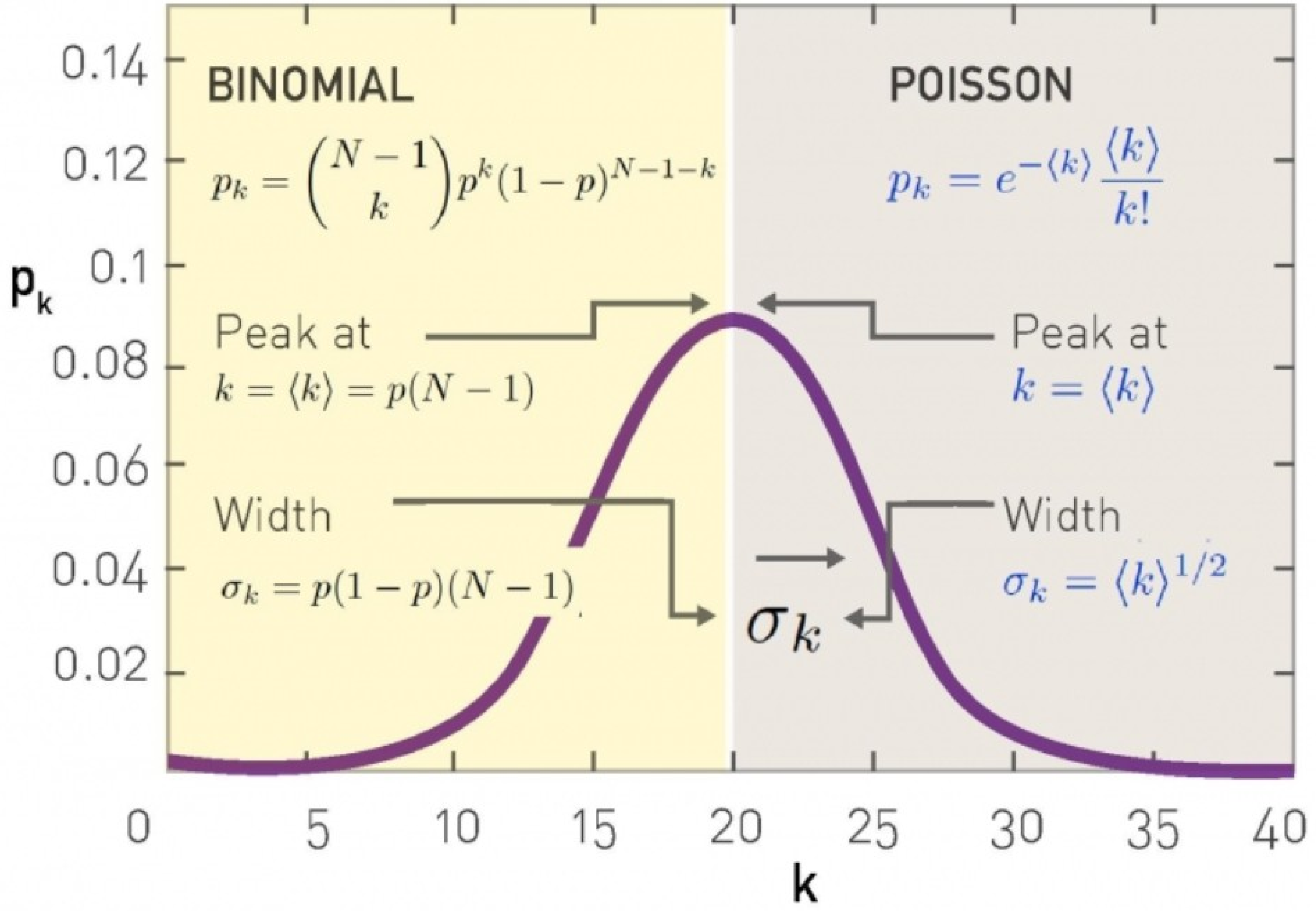
	Random	Small-World	Scale-Free
Visualisation of the Topology			
Degree Distribution			
Robustness Characteristics	Responds similarly to both random and targeted attacks.		Resilient against random failures but very sensitive to targeted attacks.

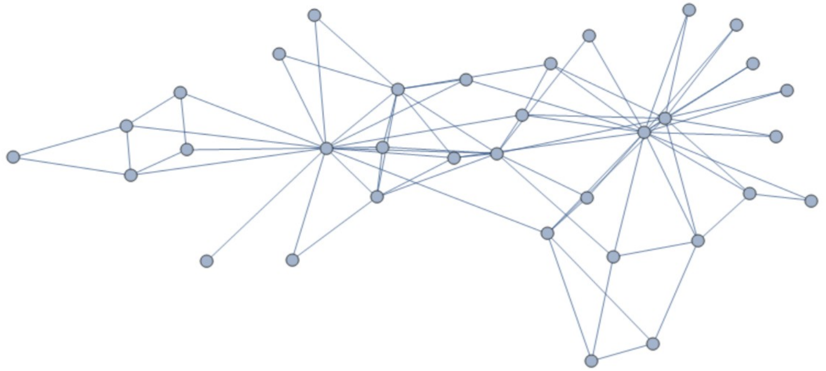
$$P(\deg(v)=k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$$

$$P(k) = \sum_{n=0}^{f(k,K)} \binom{K/2}{n} (1-\beta)^n \beta^{K/2-n} \frac{(\beta K/2)^{k-K/2-n}}{(k-K/2-n)!} e^{-\beta K/2}$$

$$P(k) \sim k^{-\gamma}$$

Probability theory
to characterize random networks

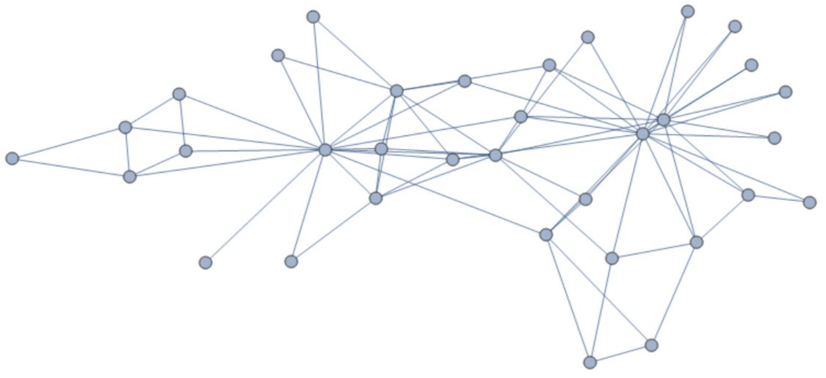


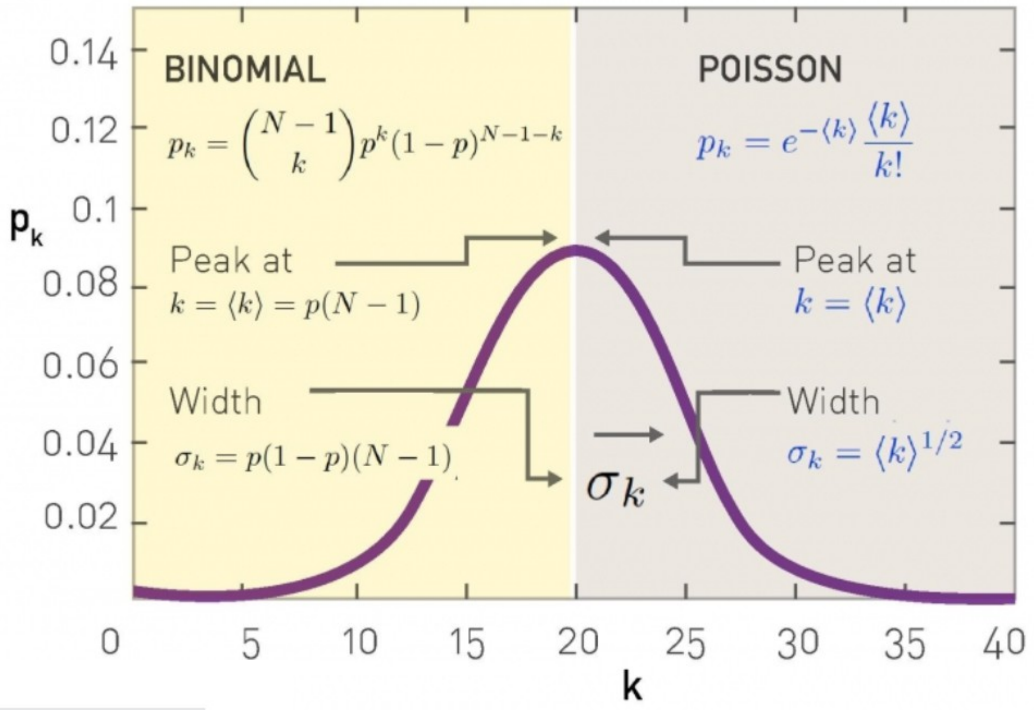


Erdos-Renyi random network

Barabasi book on network science

Probability theory and statistics





BRIEF STATISTICS REVIEW

Four key quantities characterize a sample of N values $x_1, \ldots, x_N$:

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The n^{th} *moment*:

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \ldots + x_N^n}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i^n$$

Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(x_i - \langle x \rangle \right)^2}$$

Distribution of x:

$$p_x = \frac{1}{N} \sum_i \delta_{x, x_i}$$

where p_x follows

$$\sum_i p_x = 1 \left(\int p_x \, dx = 1 \right)$$

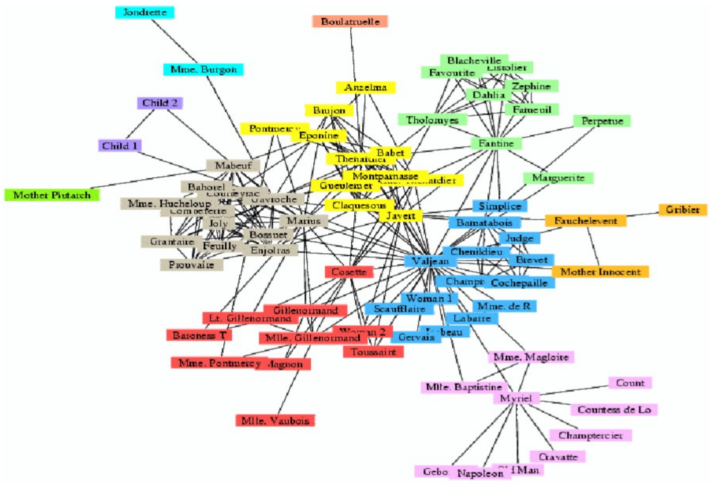
Barabasi book on network science

Compare table for random graphs and real-world networks

Network	Nodes	Links	Directed / Undirected	N	L	‹K›
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile-Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorships	Undirected	23,133	93,437	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Papers	Citations	Directed	449,673	4,689,479	10.43

Barabasibook online

Types of networks

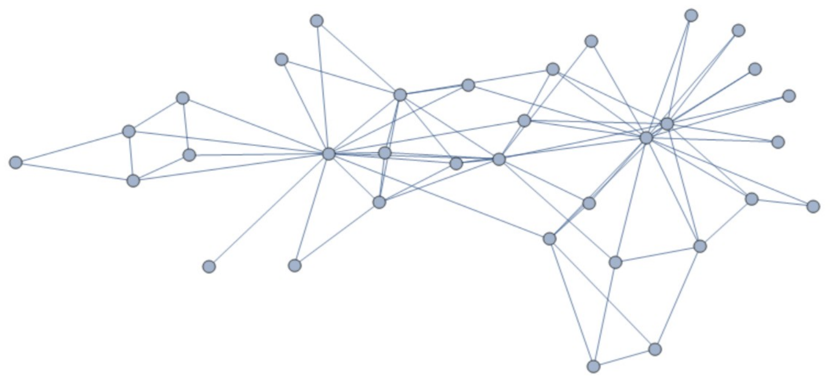


Example: Social Network in Les Miserables

M. E. J. Newman and M. Girvan, Finding and evaluating community structure in

Types of networks

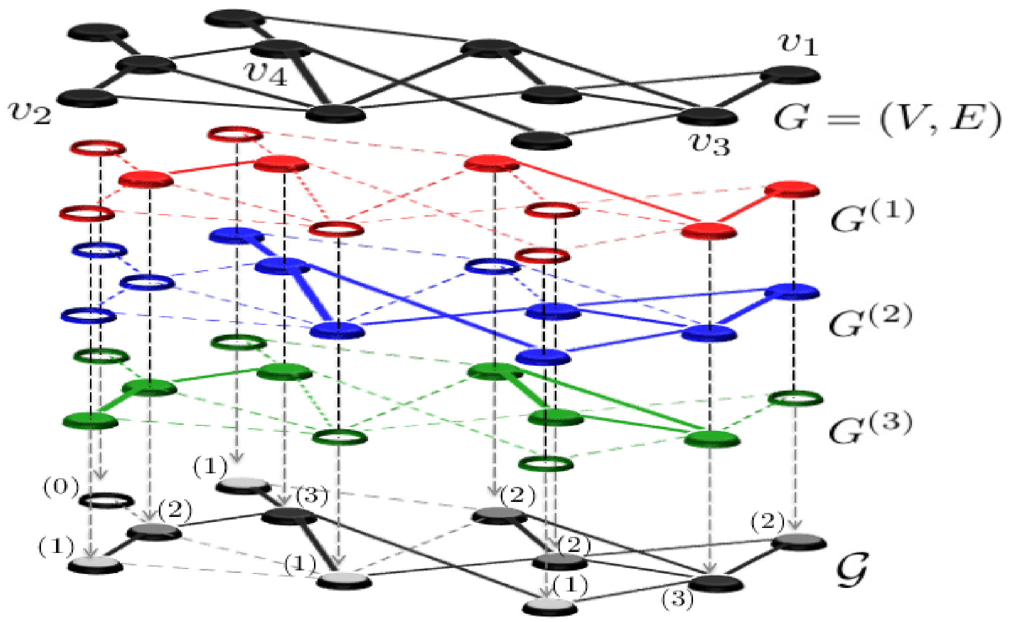
Multilayer networks
Temporal networks
Networks with attributes



Types of networks

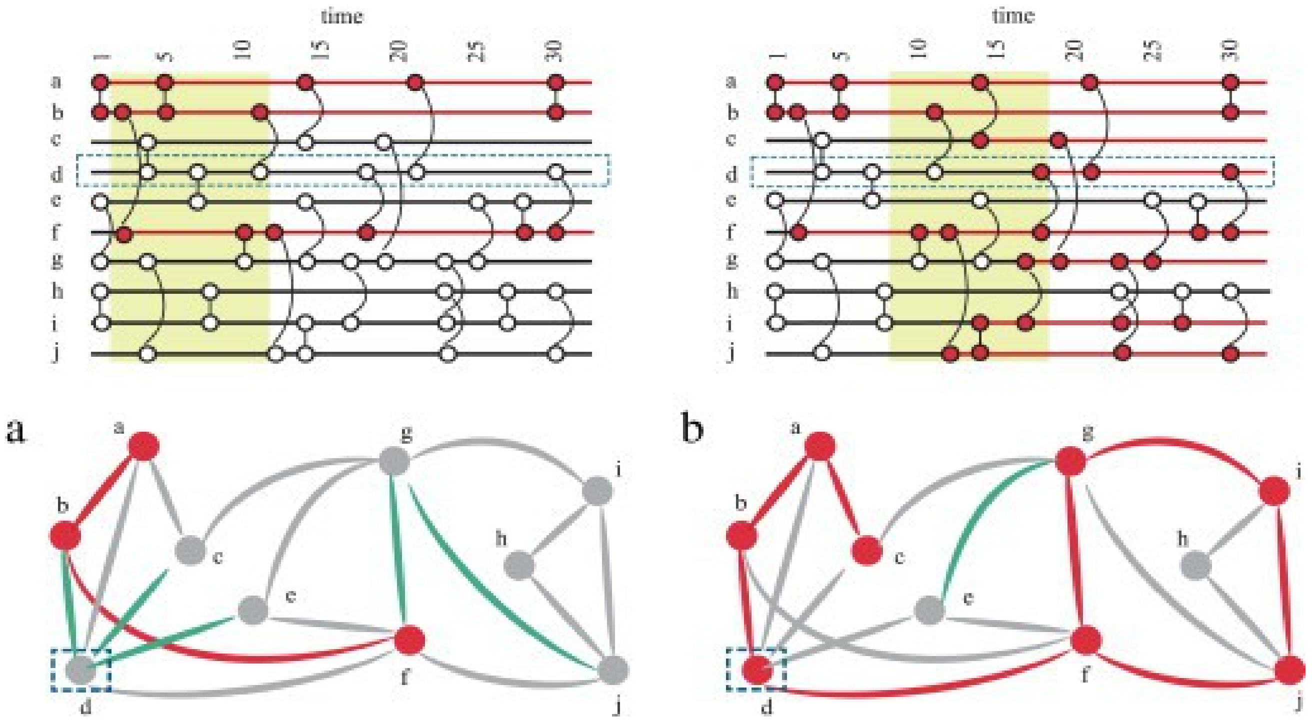
Multilayer networks

Layers.



Types of networks

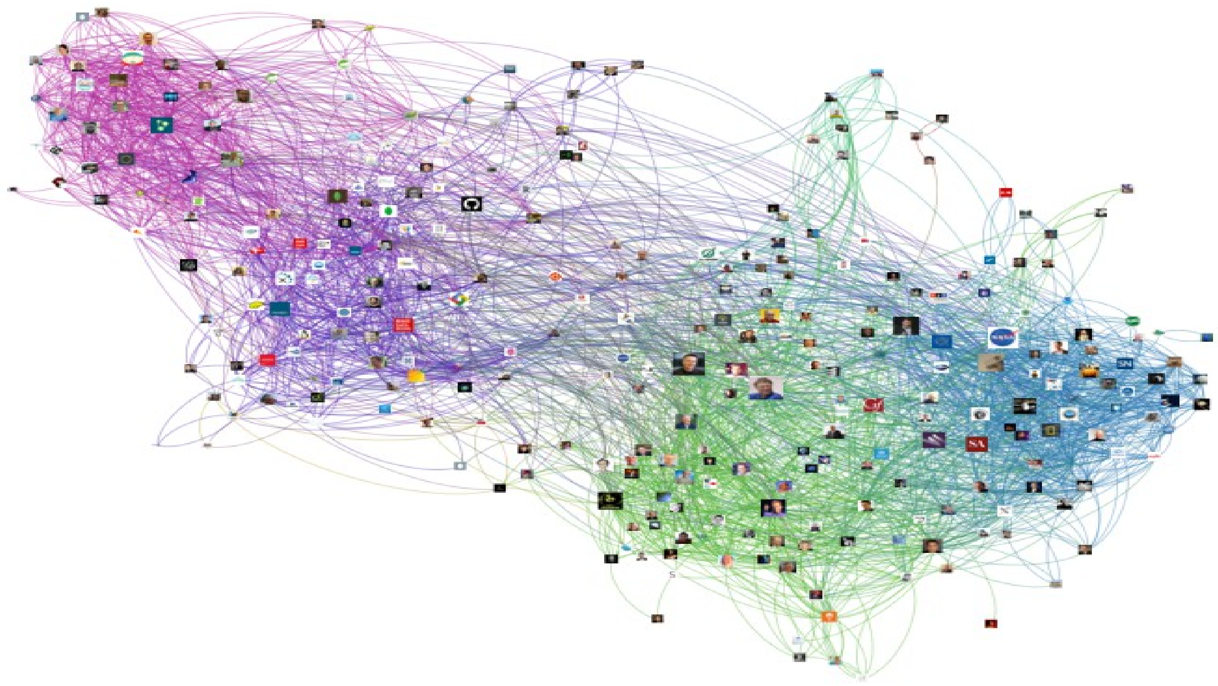
Temporal networks



P.Holme et al. Temporal network Rev.

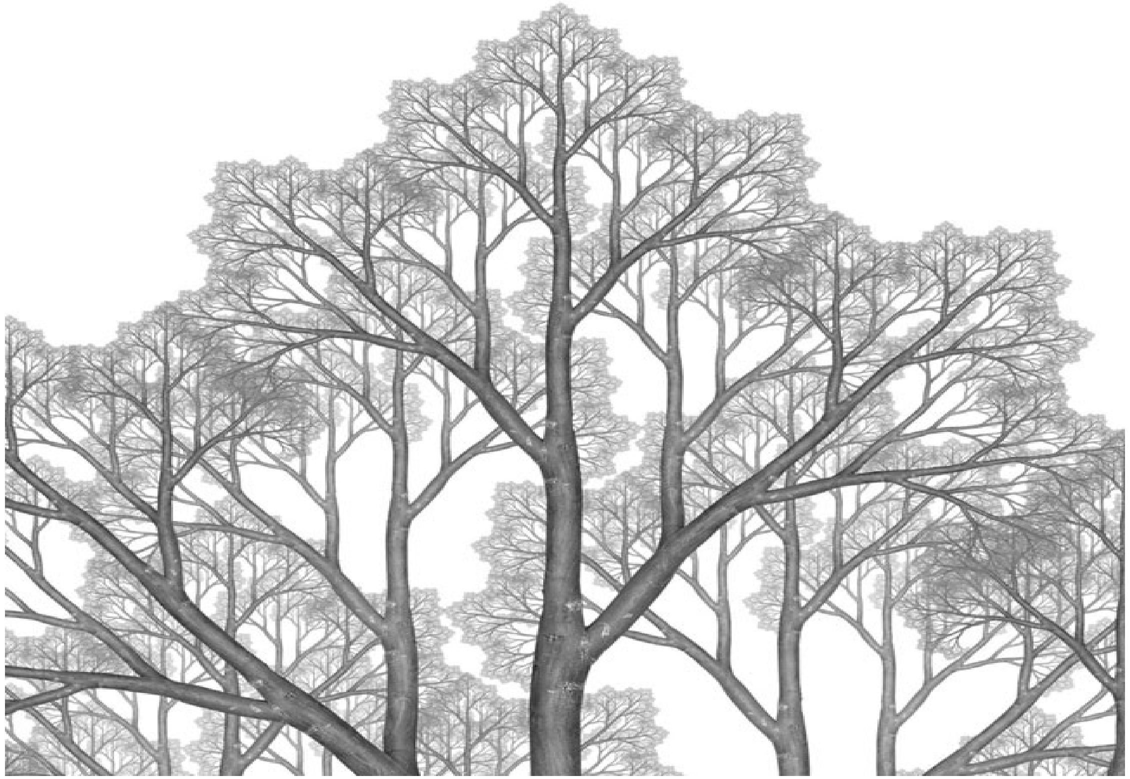
Types of networks

Networks with attributes



mainesciencefestival.

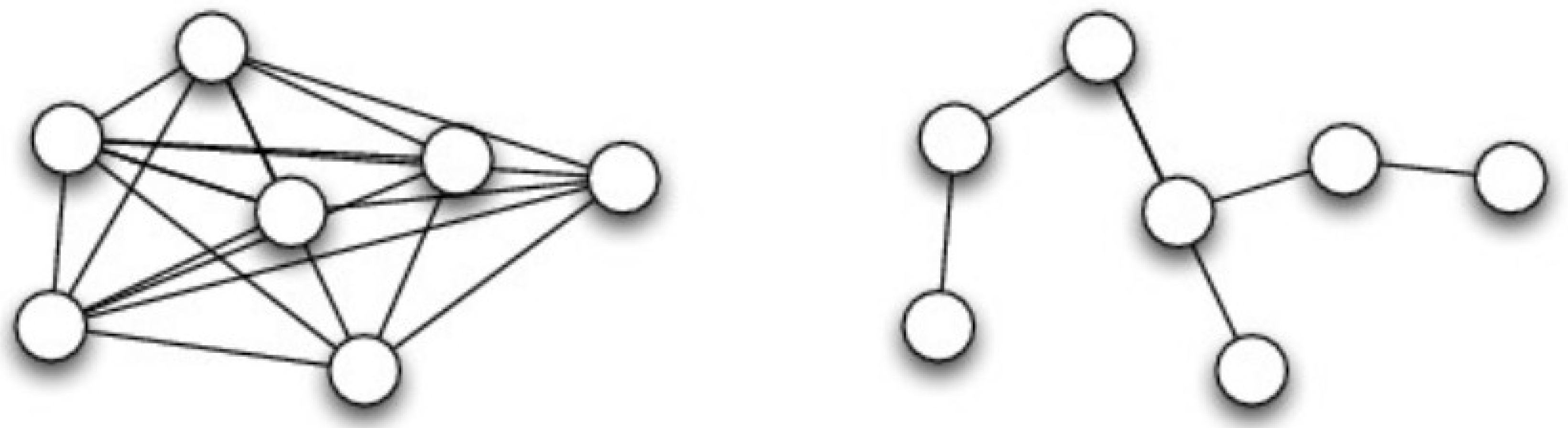
Optimisation problems



Optimisation problems

Can you identify network topology which is built for

A. Optimisation for users
B. Construction-oriented design



Optimisation problems

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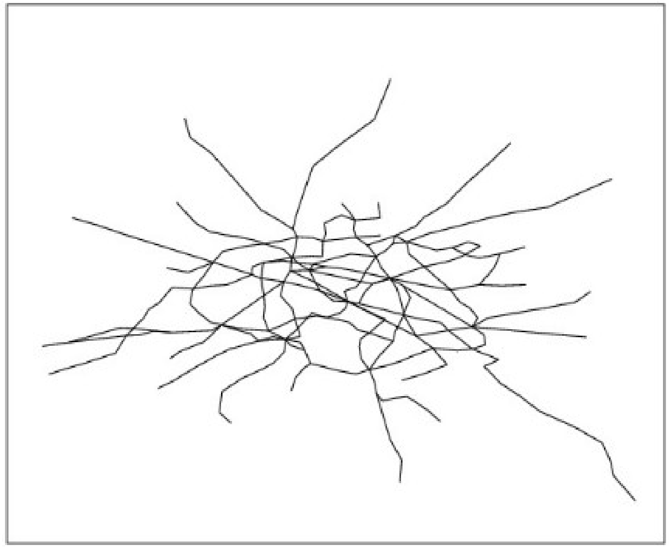
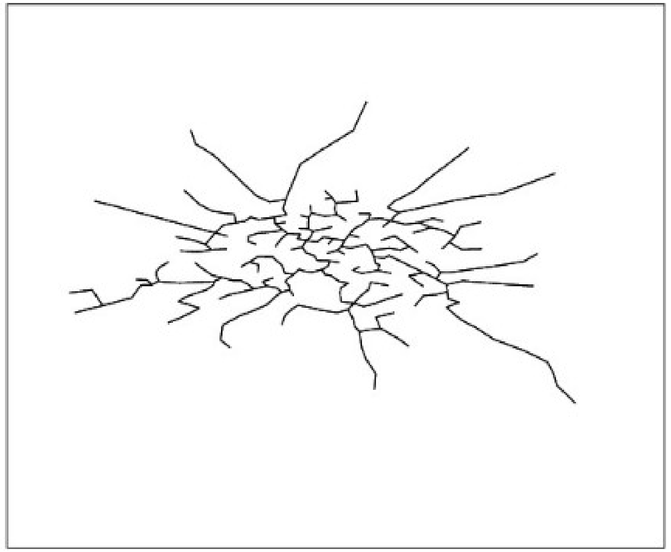
How to solve optimisation problems?





Optimisation problems

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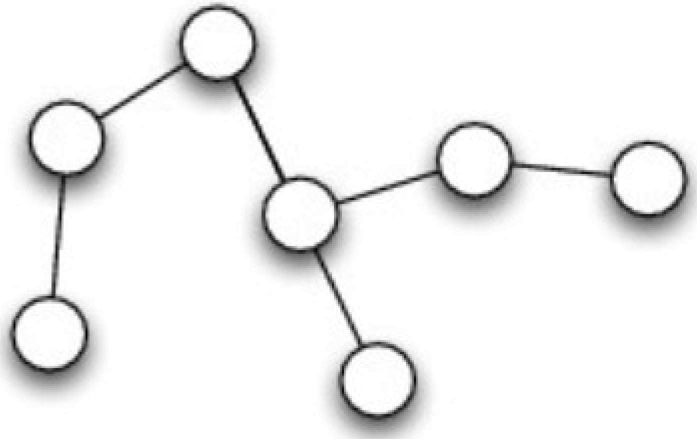
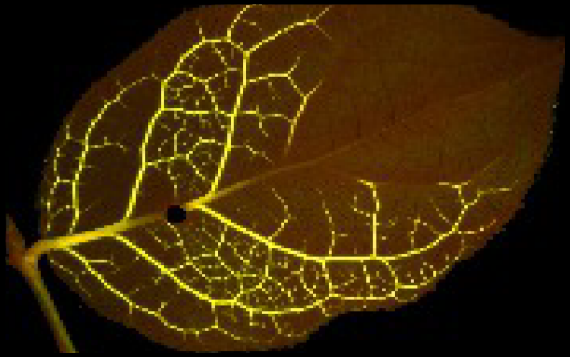
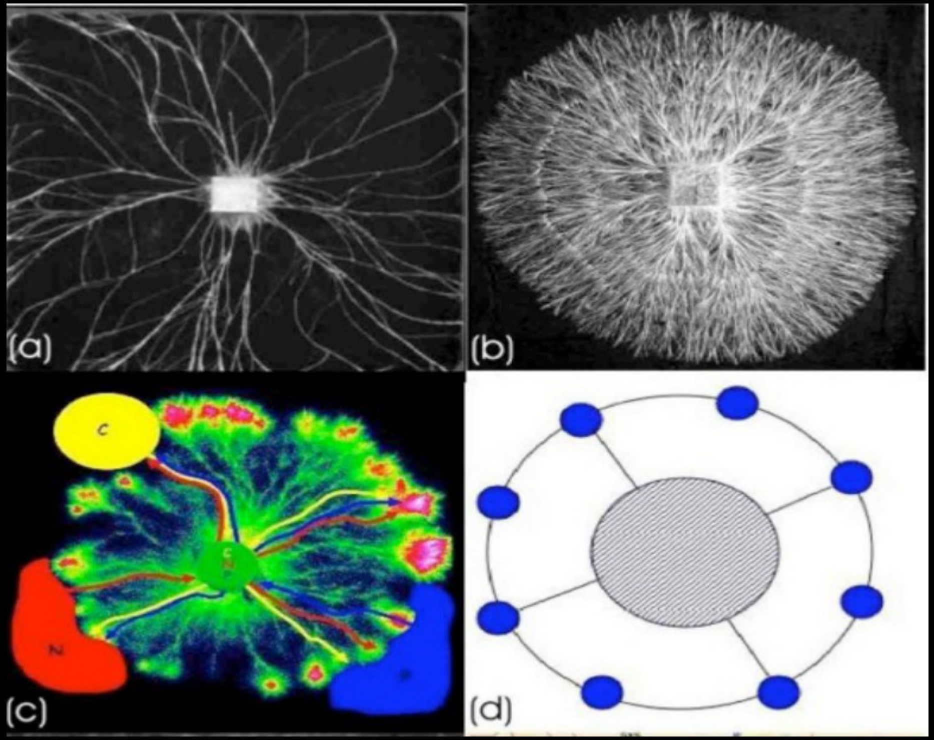
How to solve optimisation problems?
To identify variables of the system: $x_1, x_2, x_3 \ldots x_n$
To simplify the system
To minimize the cost-function for the problem: $F(x_1, x_2, x_3 \ldots x_n)$



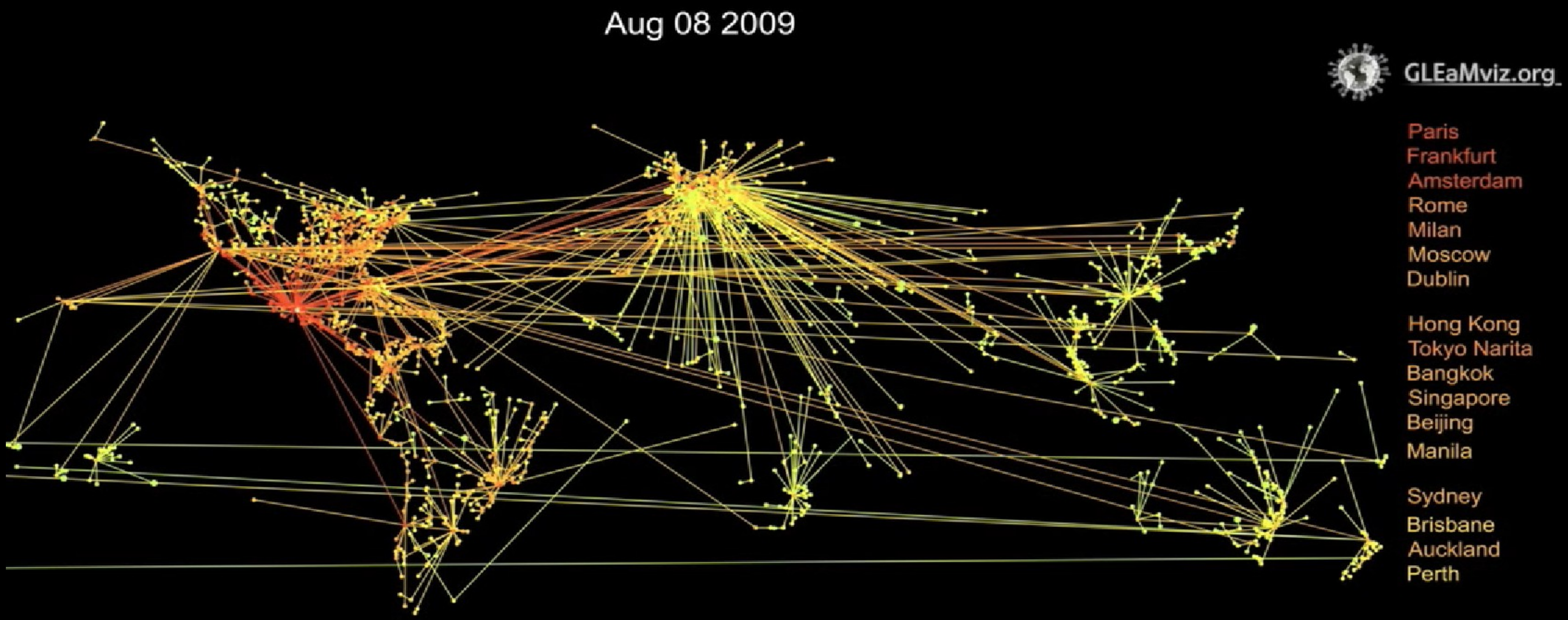


Def. Tree network
is a graph in which any two vertices are connected by exactly one path.

Every acyclic connected graph is a tree, and vice versa. A forest is a disjoint union of trees, or equivalently an acyclic graph that is not necessarily connected.

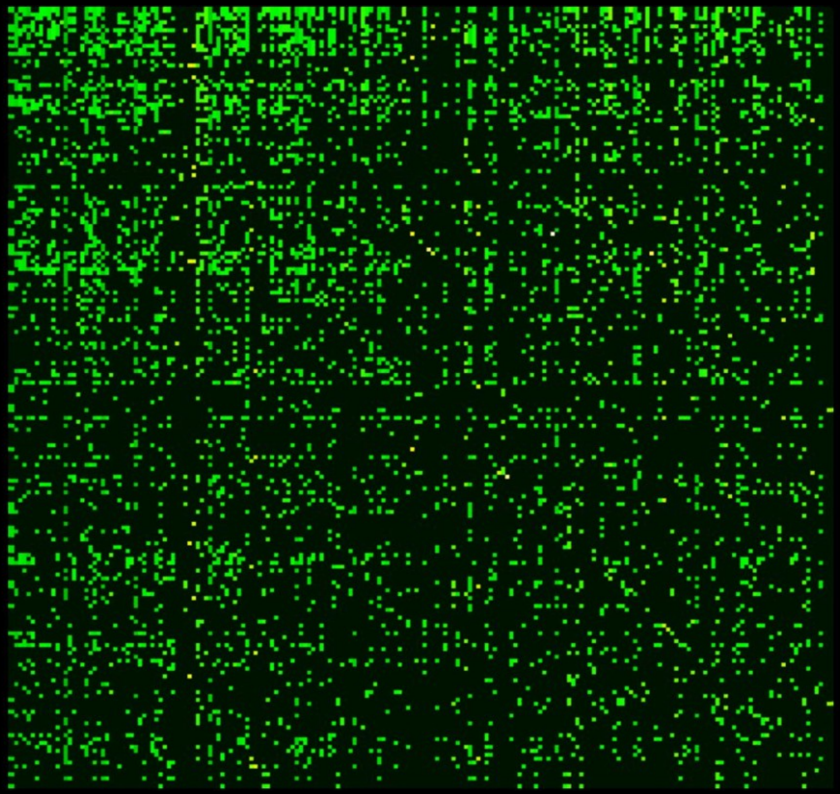


Thank you!!!
If you have any questions, ideas how to make the course better, write to me:
liubov.tupikina@cri-paris.org

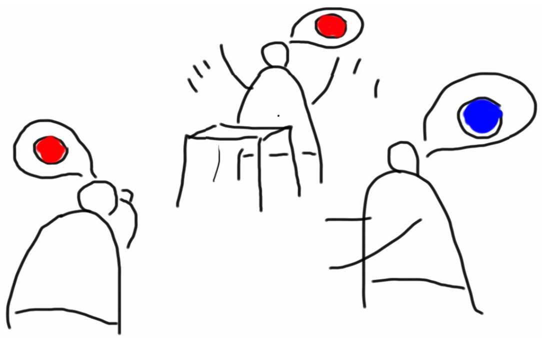


Network quiz:

1. it is for understanding of our understanding
2. it is for 10 mins
3. we can discuss about it after (if you want)



What I am really working on...



Spreading dynamics:

$$P_{x_0x_a}(t) = \sum_{n=1}^{\infty}(Q^n)_{x_0x_{abs}}P_n(t) = \frac{e^{-t/\tau}}{\tau}\sum_{n=1}^{\infty}(Q^n)_{x_0x_a}\frac{(t/\tau)^{n-1}}{(n-1)!}$$

Modelling crowded dynamics:

$$\dot{P}_{x_0\bar{x}}(t) = \frac{d}{dt}\int_0^t \big(\frac{1}{2}\sum_{x'=\bar{x}\pm 1} M_{x'}(t-t')P_{x_0x'}(t') - \\ -M_{\bar{x}}(t-t')P_{x_0\bar{x}}(t')\big)dt'.$$

Data analysis:

If you want to know more...

2.16

D.Grenbenkov, LT. PRE (2017)
P. Holme, LT, New Jour. Physics (2018)
M.Lindner, N.Molkenthin, LT, R.Donner Springer (2019)

How would you analyze this network?

Where networks are useful?
Where are not?