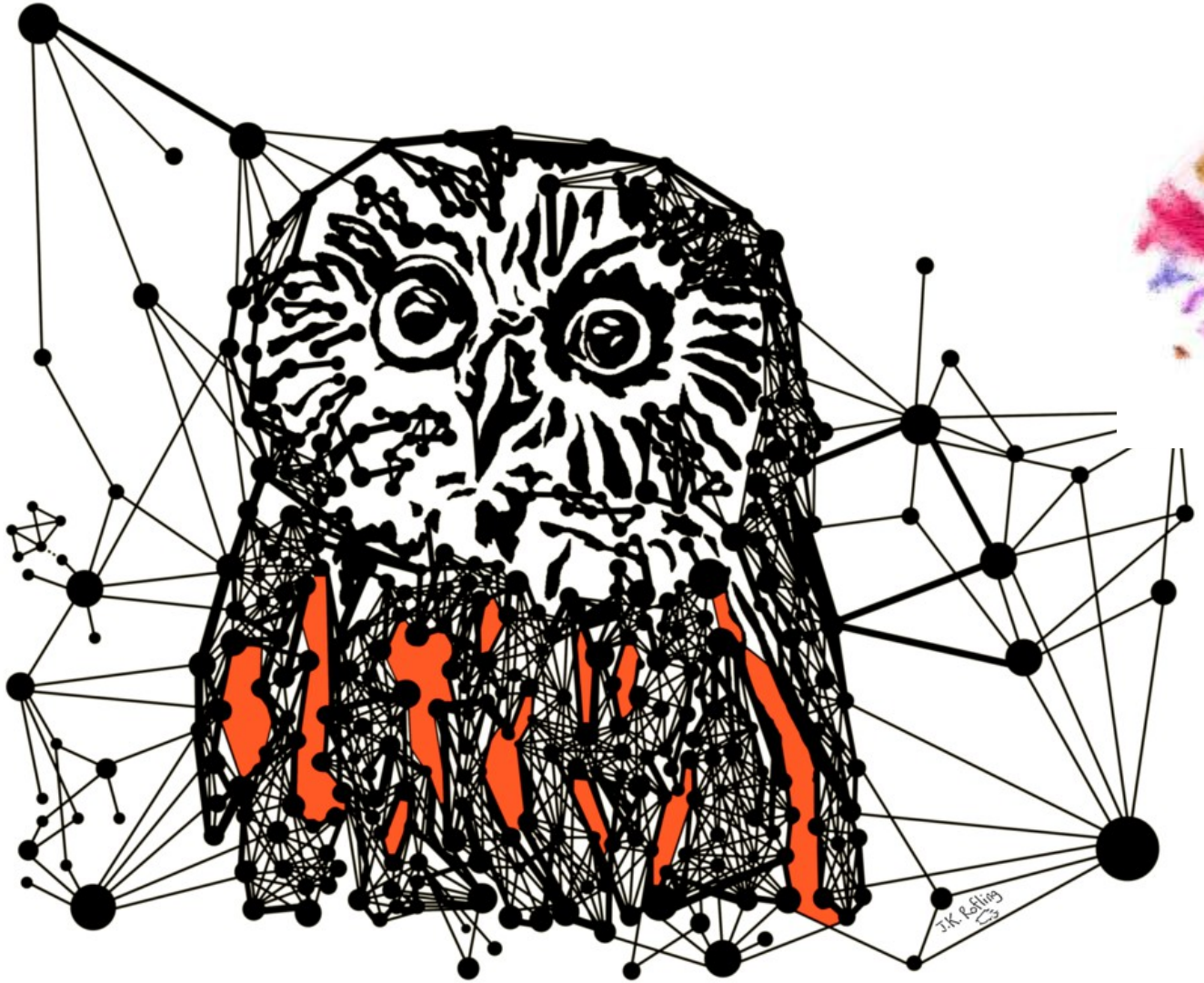
Networks theory: 2
methods for network analysis

Liubov Tupikina (CRI)

Network quiz

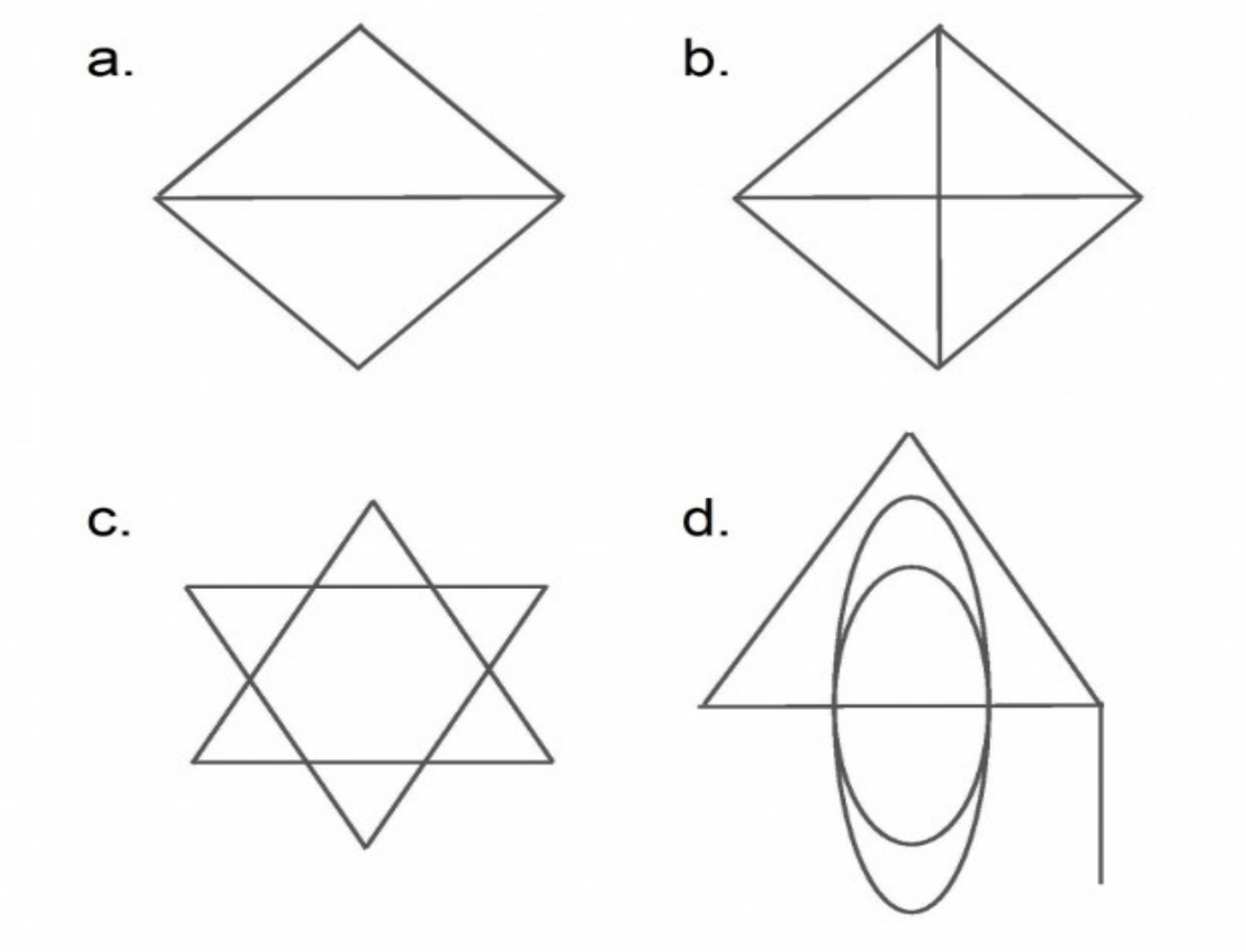


Brendan network artist
Barabasi book on network science

Network quiz

Brendan network artist
Barabasi book on network science

Network quiz



Brendan network artist
Barabasi book on network science

Lecture 2

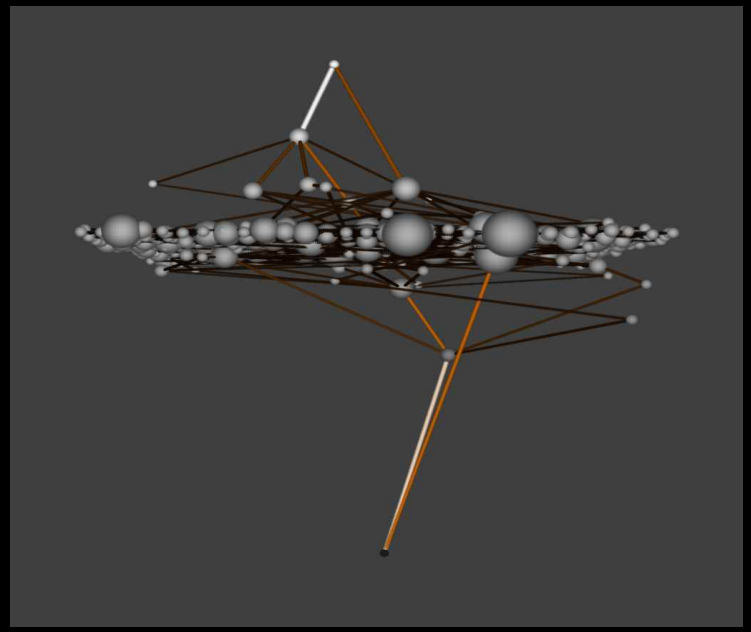
Network games

Network theory and graph theory (random graph theory)

Network measures

Networks and statistical mechanics

Study networks in groups



Network theory and other fields

There are methods, which people brought to network science from other fields

probability theory

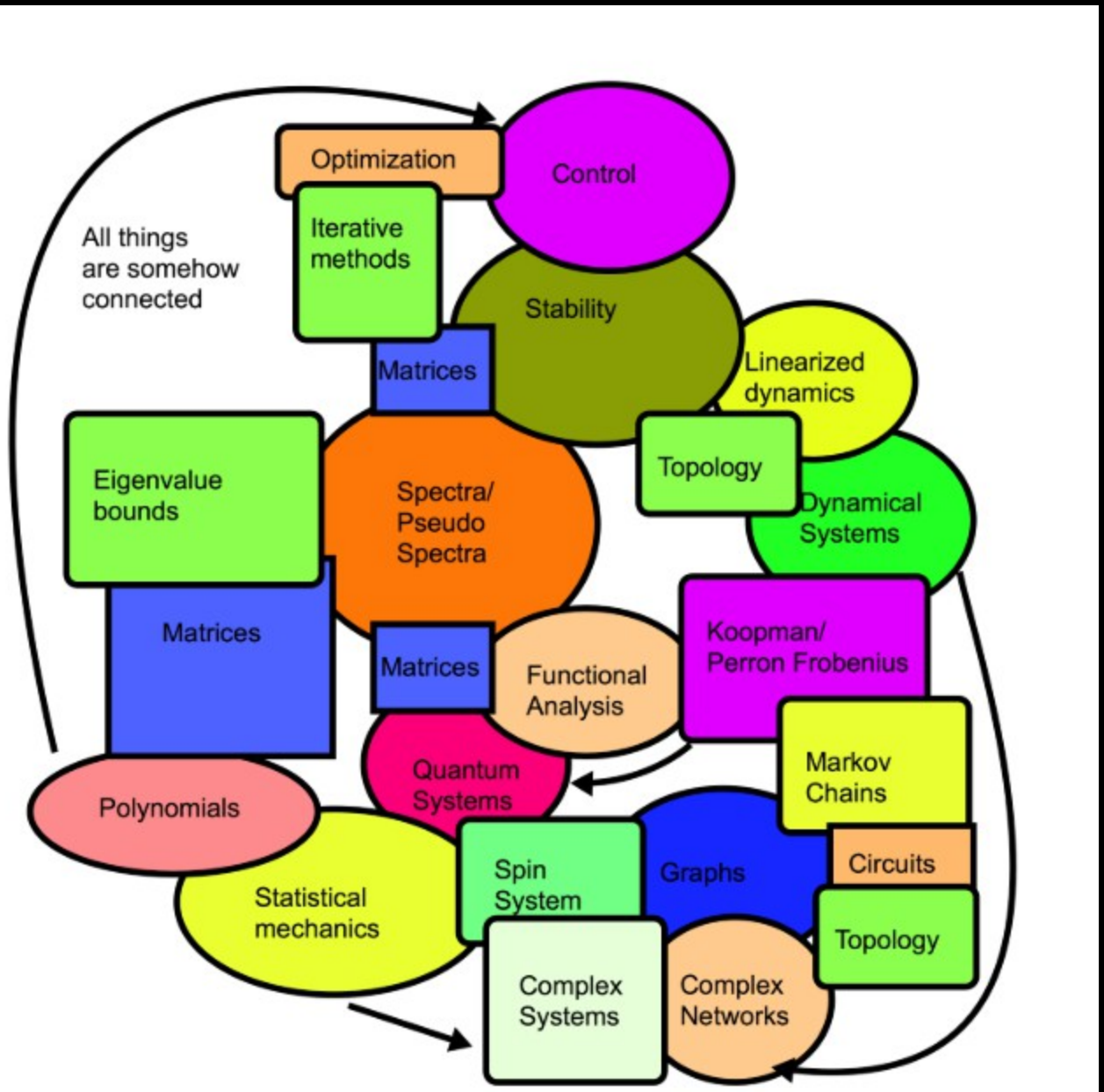
control theory

combinatorics (maths)

spectral methods (maths, physics)

optimisation theory

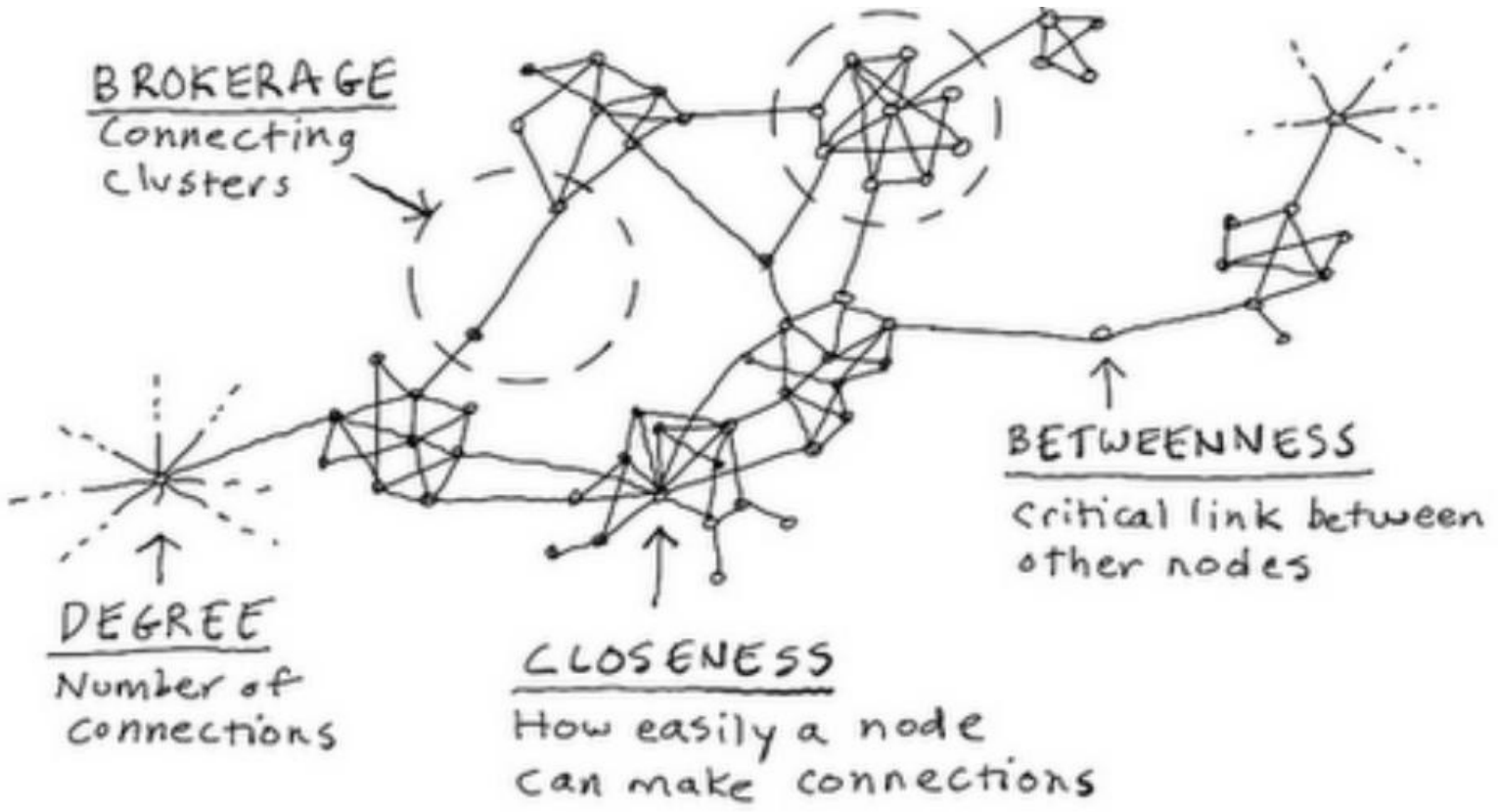
Caravelli, Tupikina et al. “Spectal methods for complex systems”, (2019)



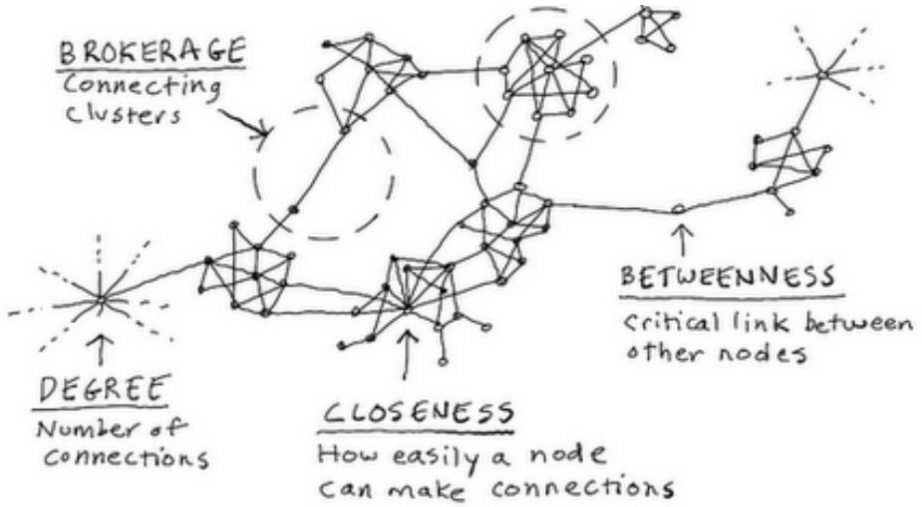
How to describe a network?

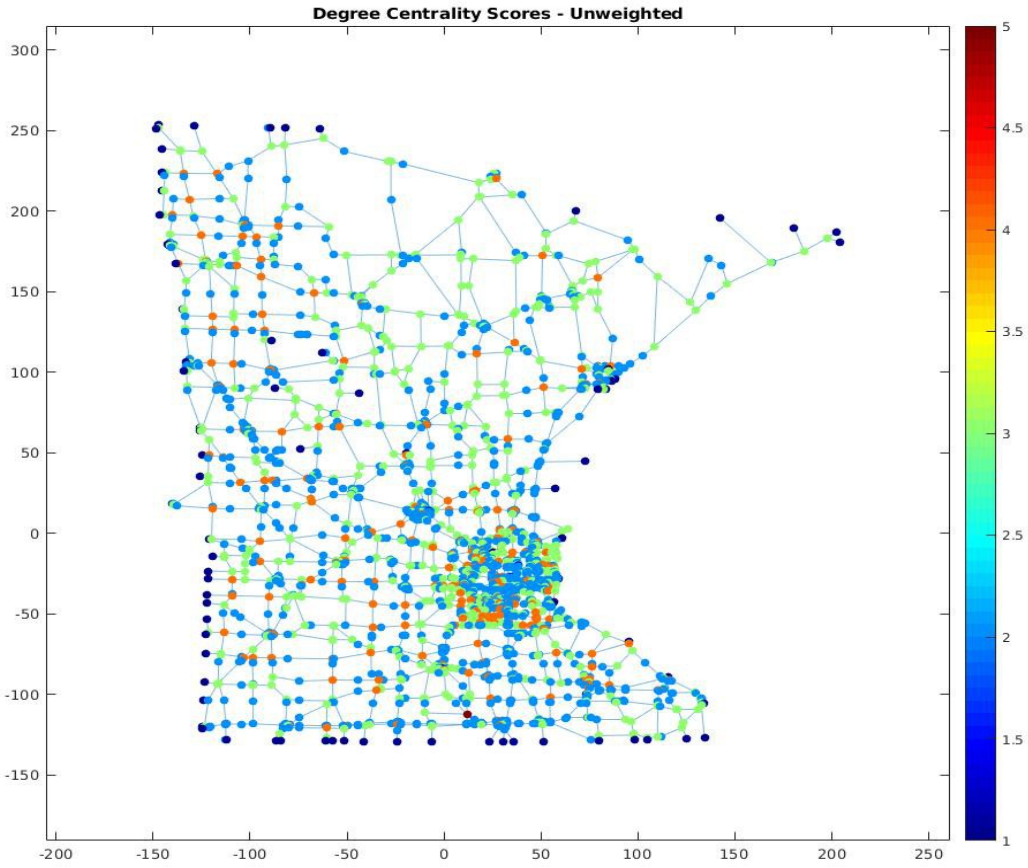
To use network measures

Network measures

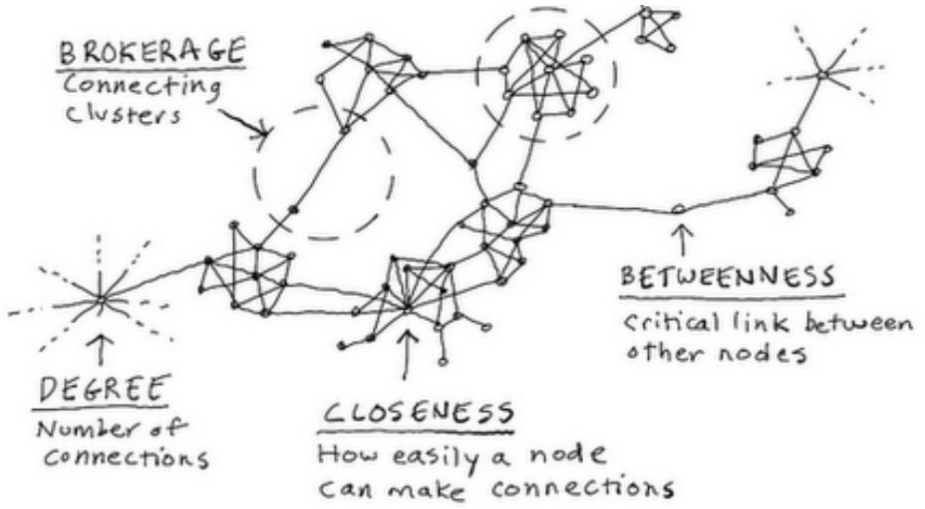


Network measures: degree



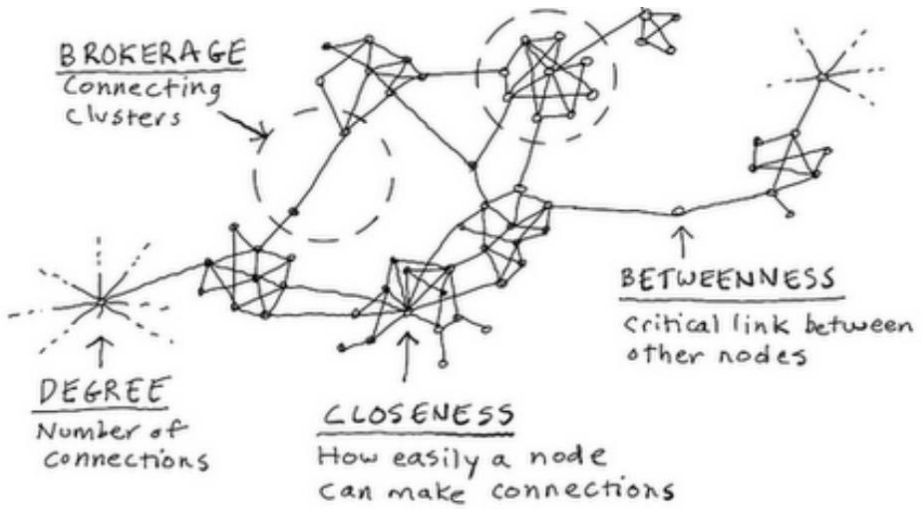


Network measures: betweenness centrality

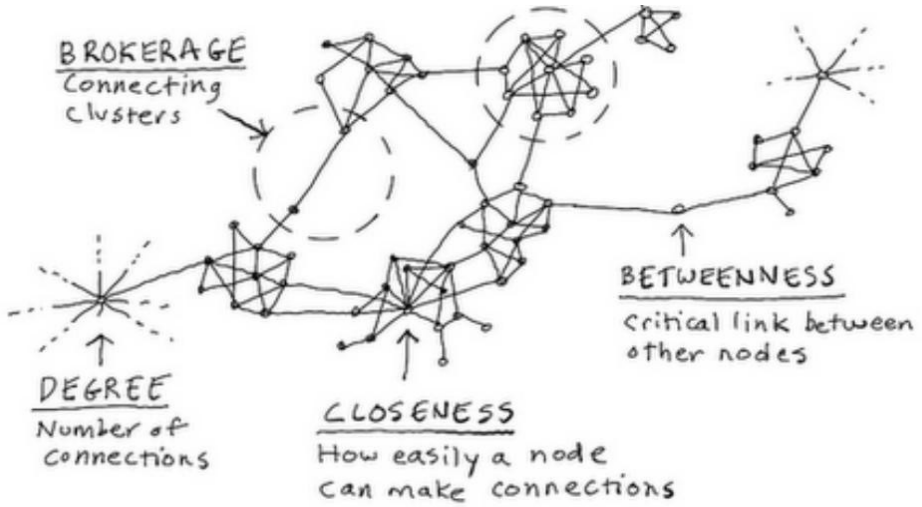


$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

Network measures: betweenness centrality



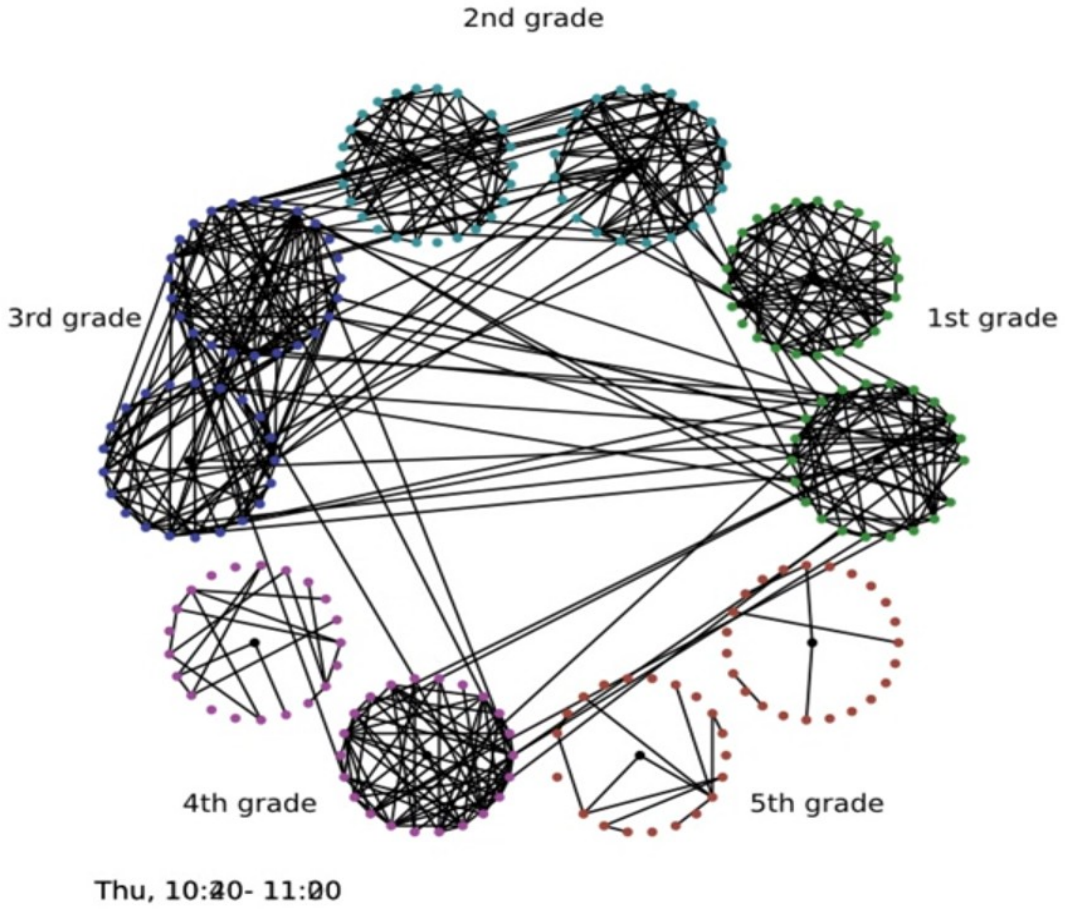
Network measures: clustering



$$C_i = \frac{|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

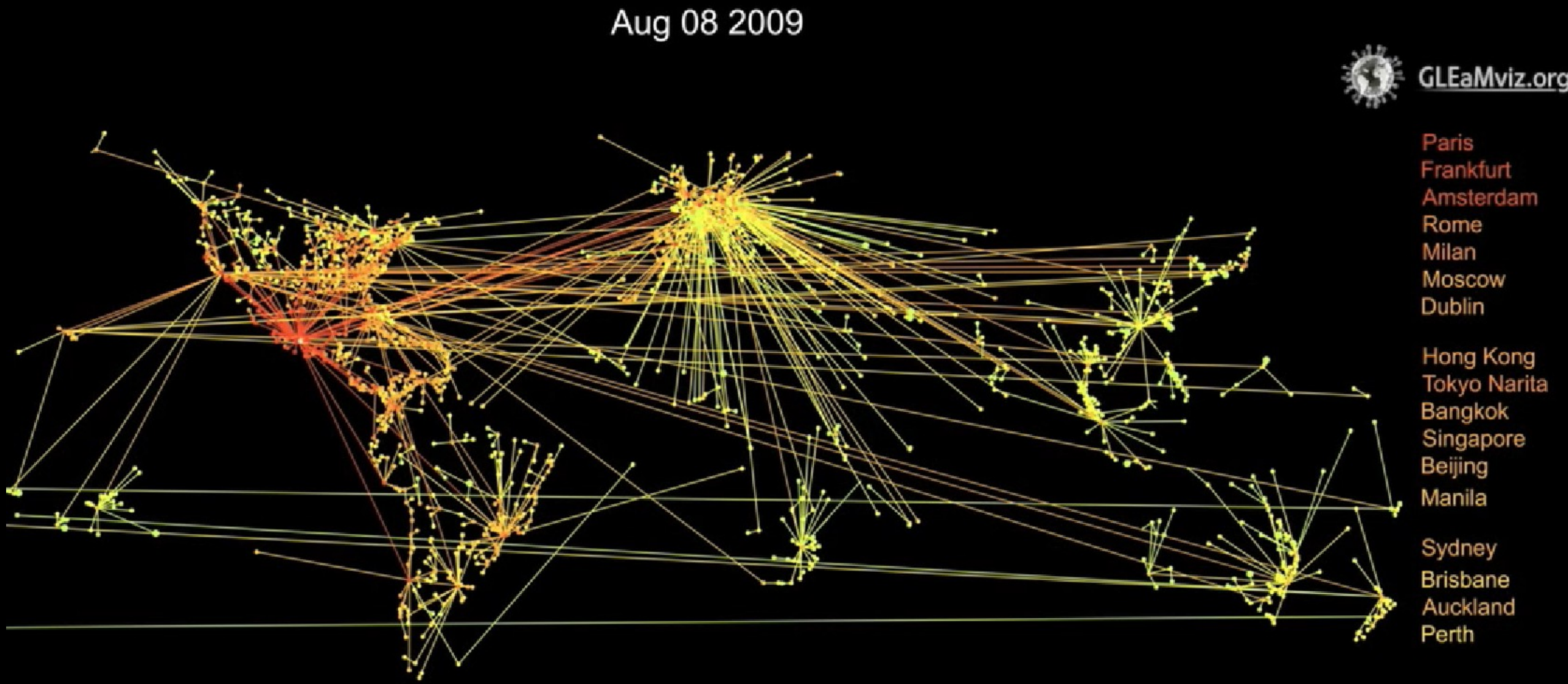
Open questions:
How to analyze temporal networks?

Open questions:
How to analyze temporal networks?



http://www.sociopatterns.org/gallery/

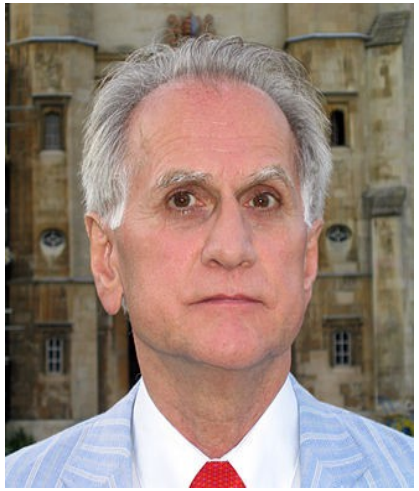
Real-world network examples



Real-world network examples

Network theory and probability theory

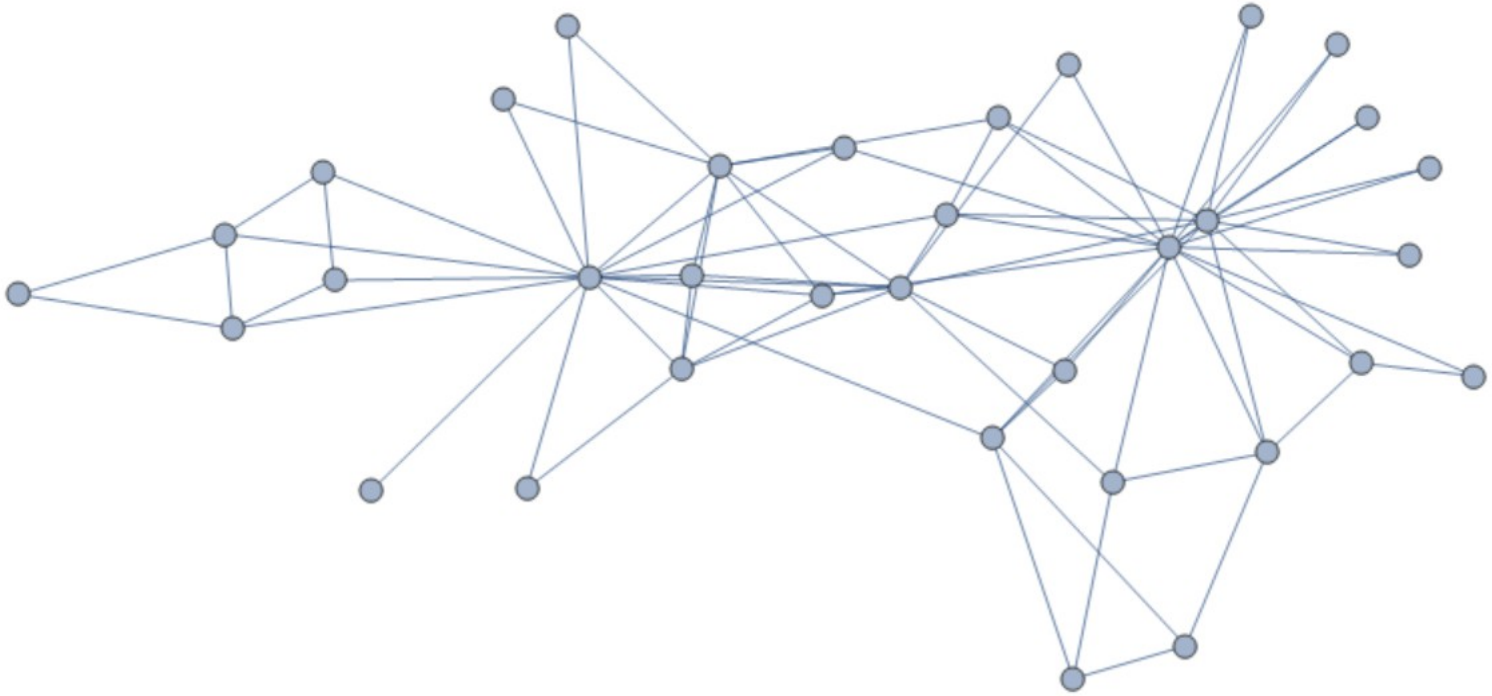
Network theory history



L.Euler (1735)
P.Erdos, A.Renyi (1959)
B.Bollobas (1981)
Watts, Strogatz (1989)
M.Newman (2000)

Network theory and probability theory

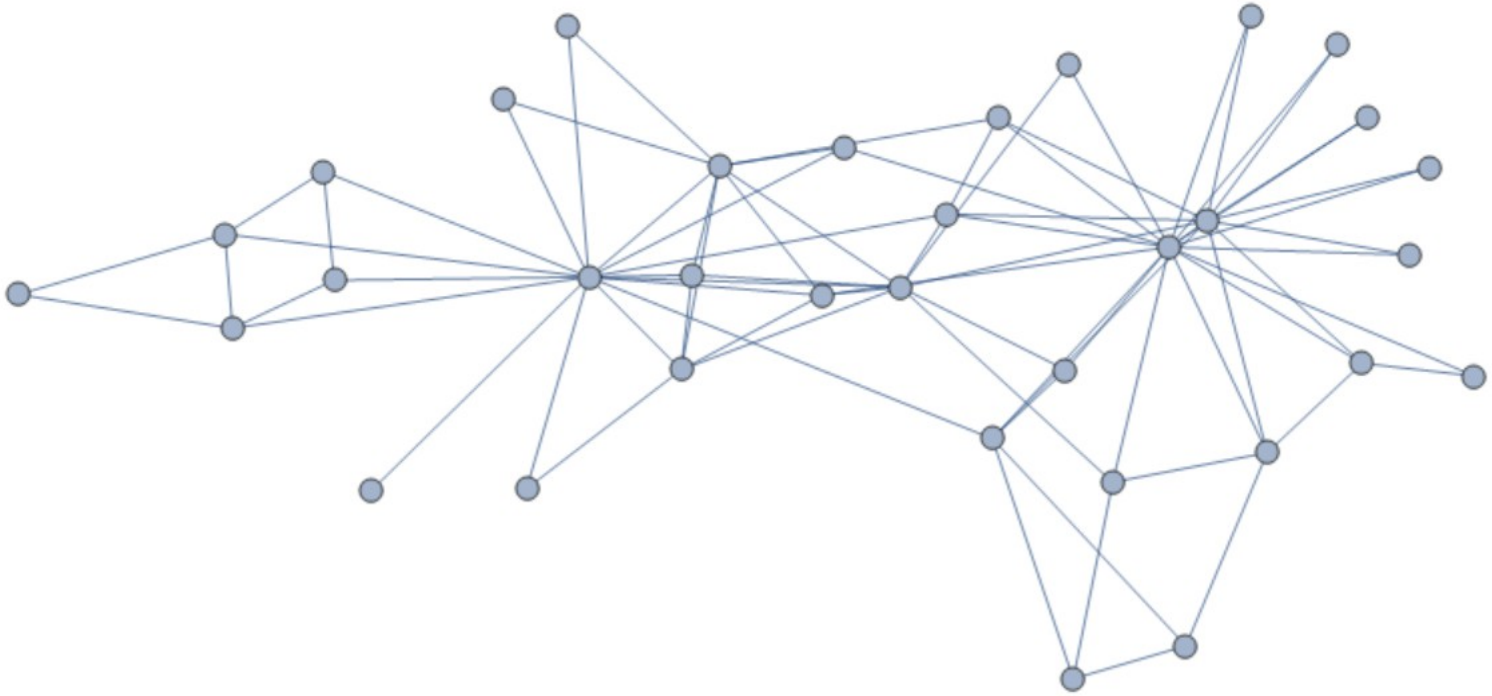
The question: how does random graph looks like?



Network theory and probability theory

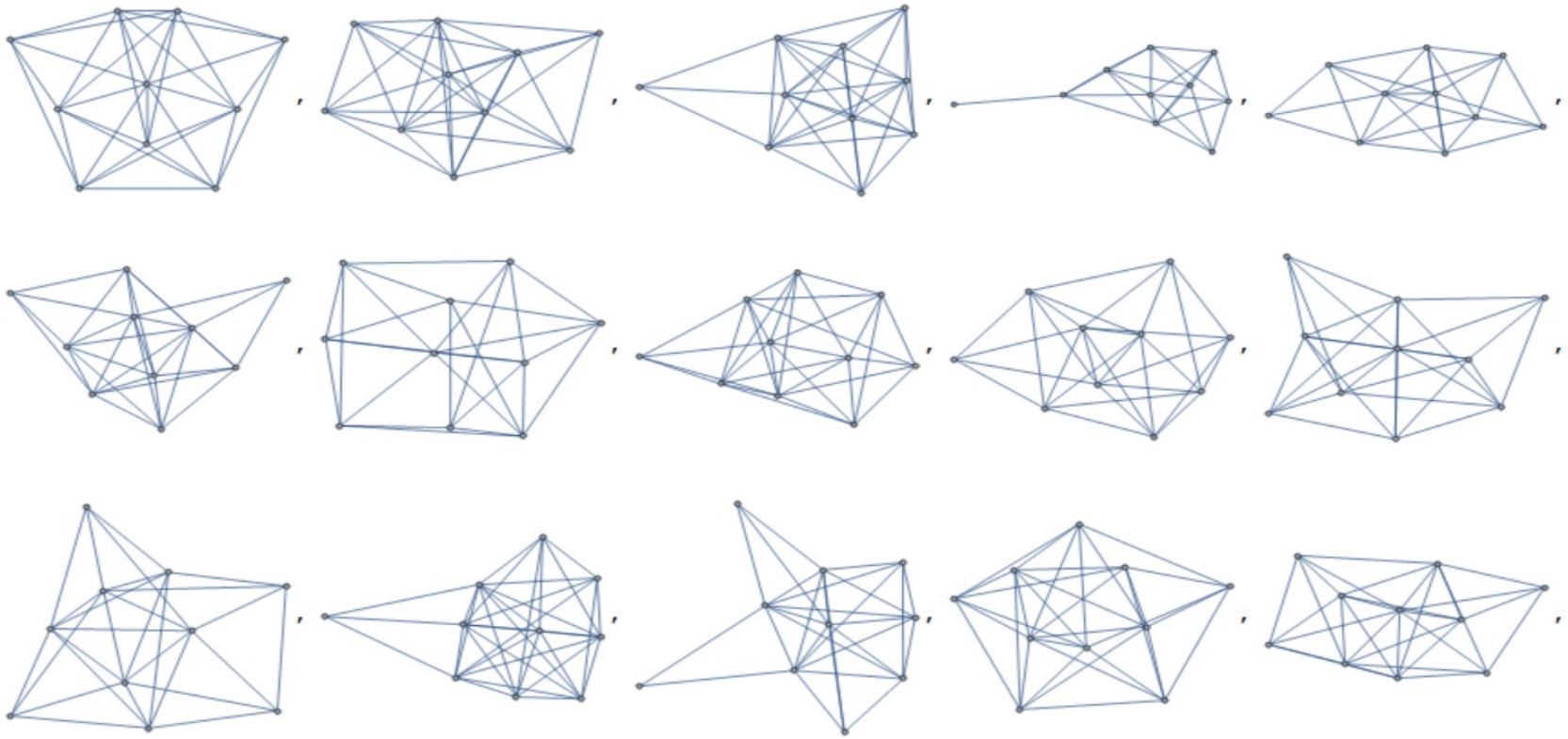
The question: how does random graph looks like?

Three models:
Erdos-Renyi model
Watts-Strogatz model
Barabasi-Albert model

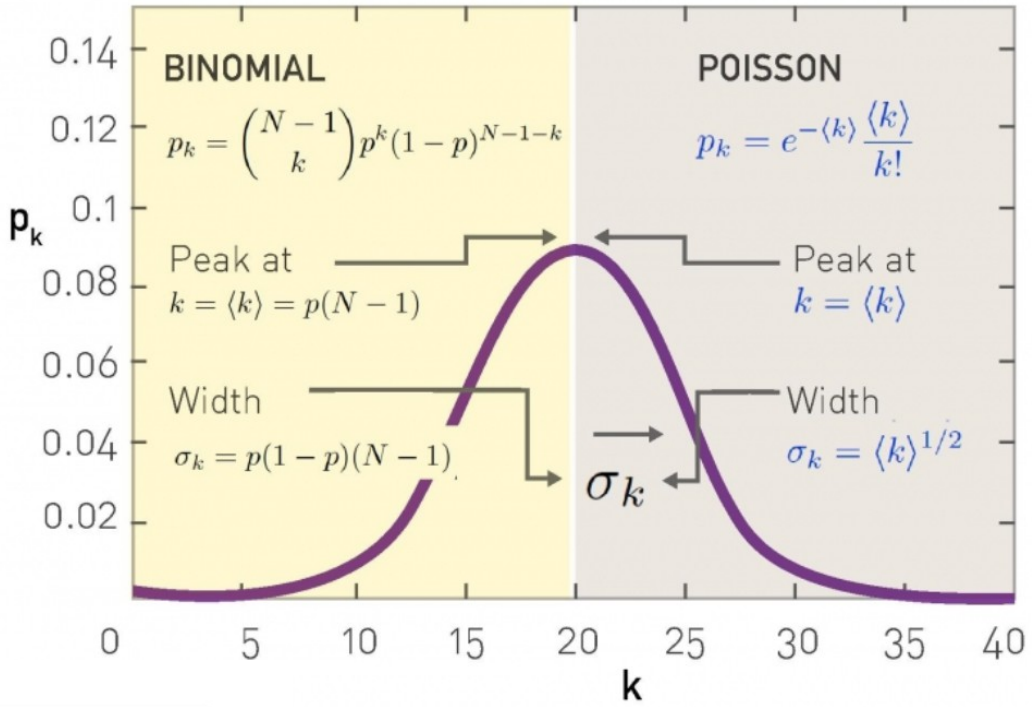


Network theory and probability theory

The question: how does random graph looks like?
Which properties does this graph have?



Probability theory and statistics



BRIEF STATISTICS REVIEW

Four key quantities characterize a sample of N values $x_1, \ldots, x_N$:

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The n^{th} moment:

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \ldots + x_N^n}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i^n$$

Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(x_i - \langle x \rangle \right)^2}$$

Distribution of x:

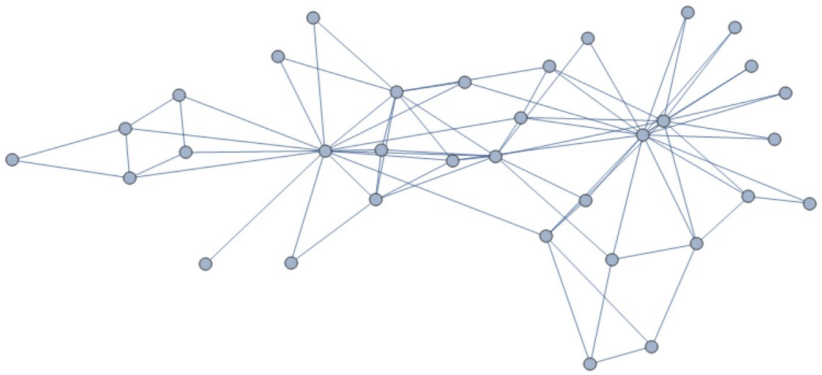
$$p_x = \frac{1}{N} \sum_i \delta_{x, x_i}$$

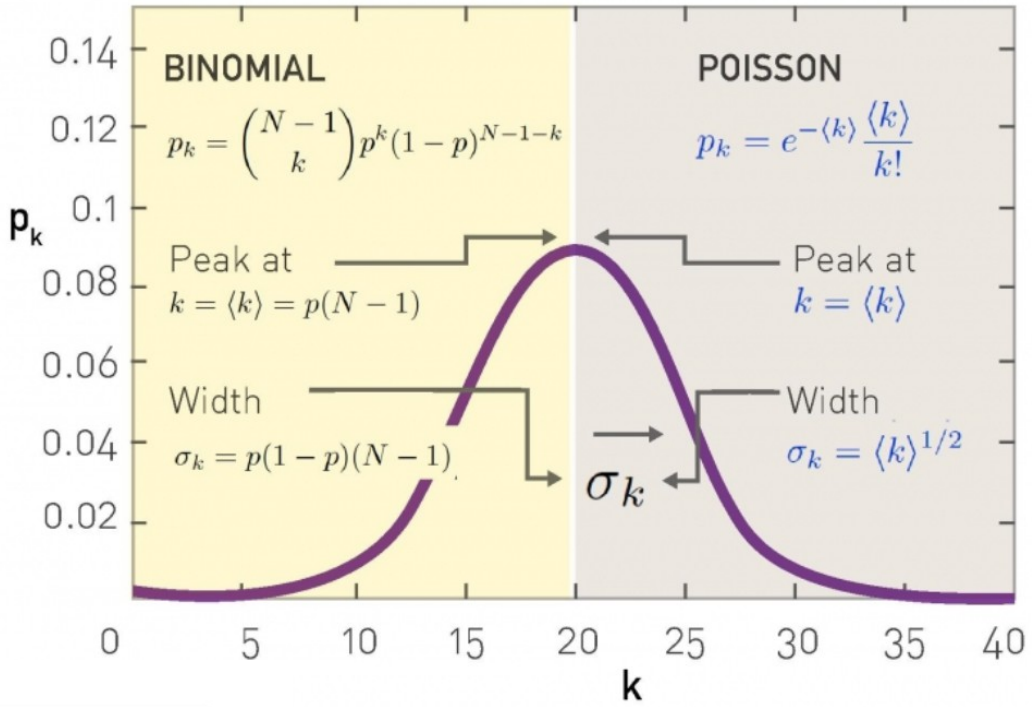
where p_x follows

$$\sum_i p_x = 1 \quad \left(\int p_x \, dx = 1 \right)$$

Barabasi book on network science

Probability theory and statistics





BRIEF STATISTICS REVIEW

Four key quantities characterize a sample of N values $x_1, \ldots, x_N$:

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The n^{th} moment:

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \ldots + x_N^n}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i^n$$

Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(x_i - \langle x \rangle \right)^2}$$

Distribution of x:

$$p_x = \frac{1}{N} \sum_i \delta_{x, x_i}$$

where p_x follows

$$\sum_i p_x = 1 \quad \left(\int p_x \, dx = 1 \right)$$

Barabasi book on network science

1st model: Erdos-Renyi random network model

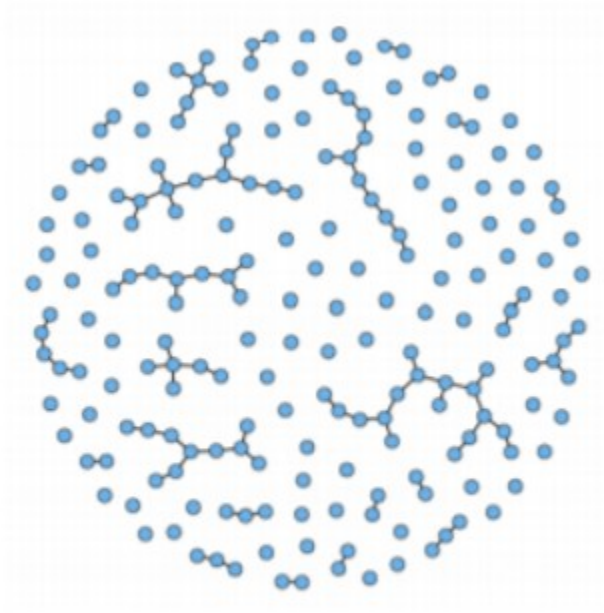
Erdos-Renyi networks

G(N,p), N number of nodes
p is probability to have link between nodes

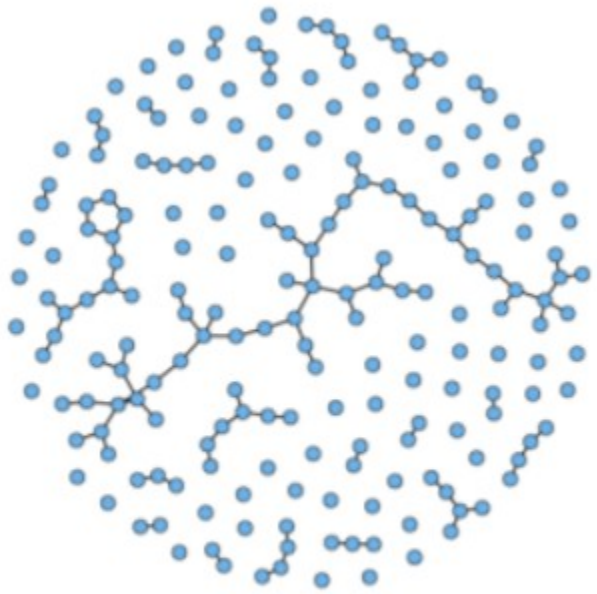
1st model: Erdos-Renyi network model

Erdos-Renyi networks

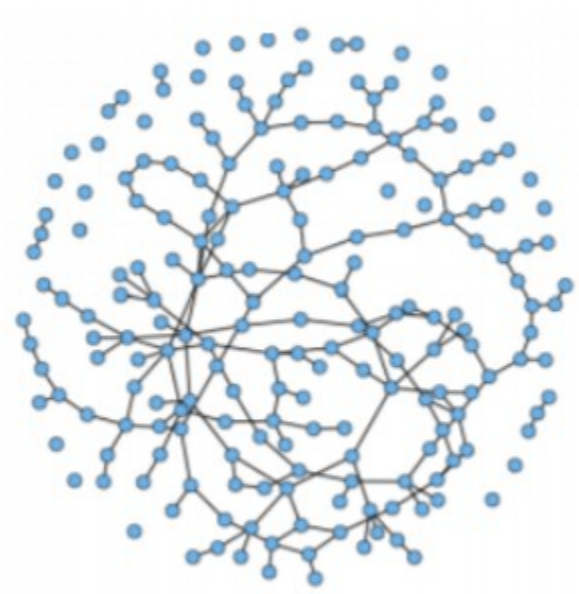
G(N,p), N number of nodes
p is probability to have link between nodes



$p < p_c$



$p = p_c$



$p > p_c$

2nd model: Watts-Strogarz small-world model

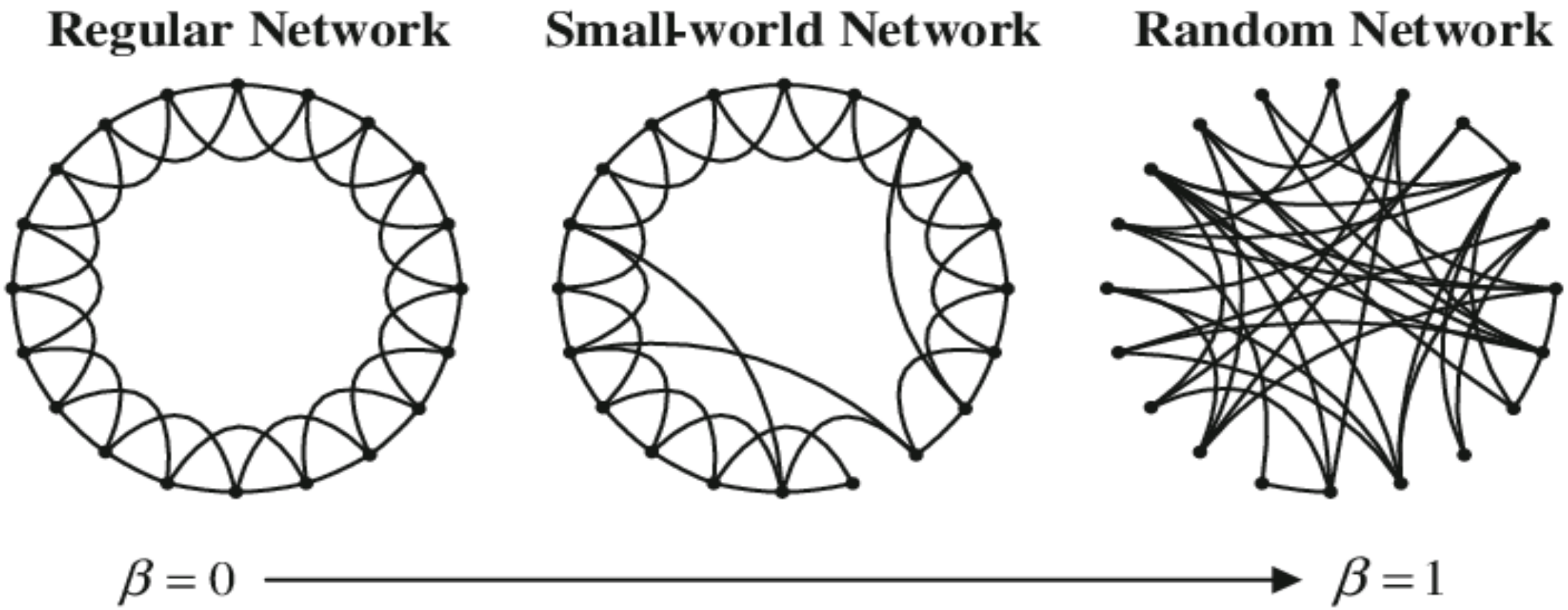
Watts-Strogatz networks

G(N,k,q), N number of nodes, k-average degree, q is probability of rewiring

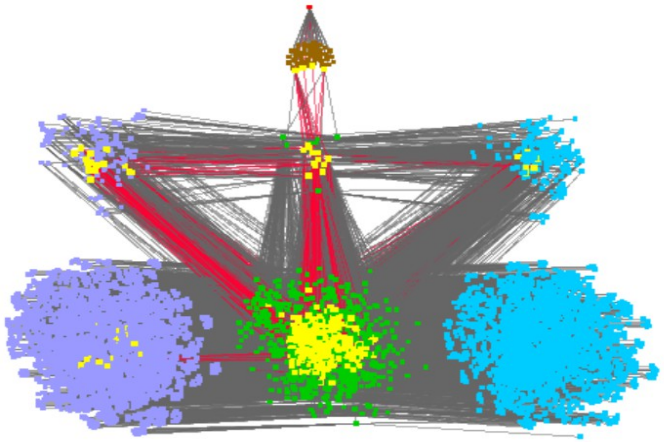
2nd model: Watts-Strogarz small-world model

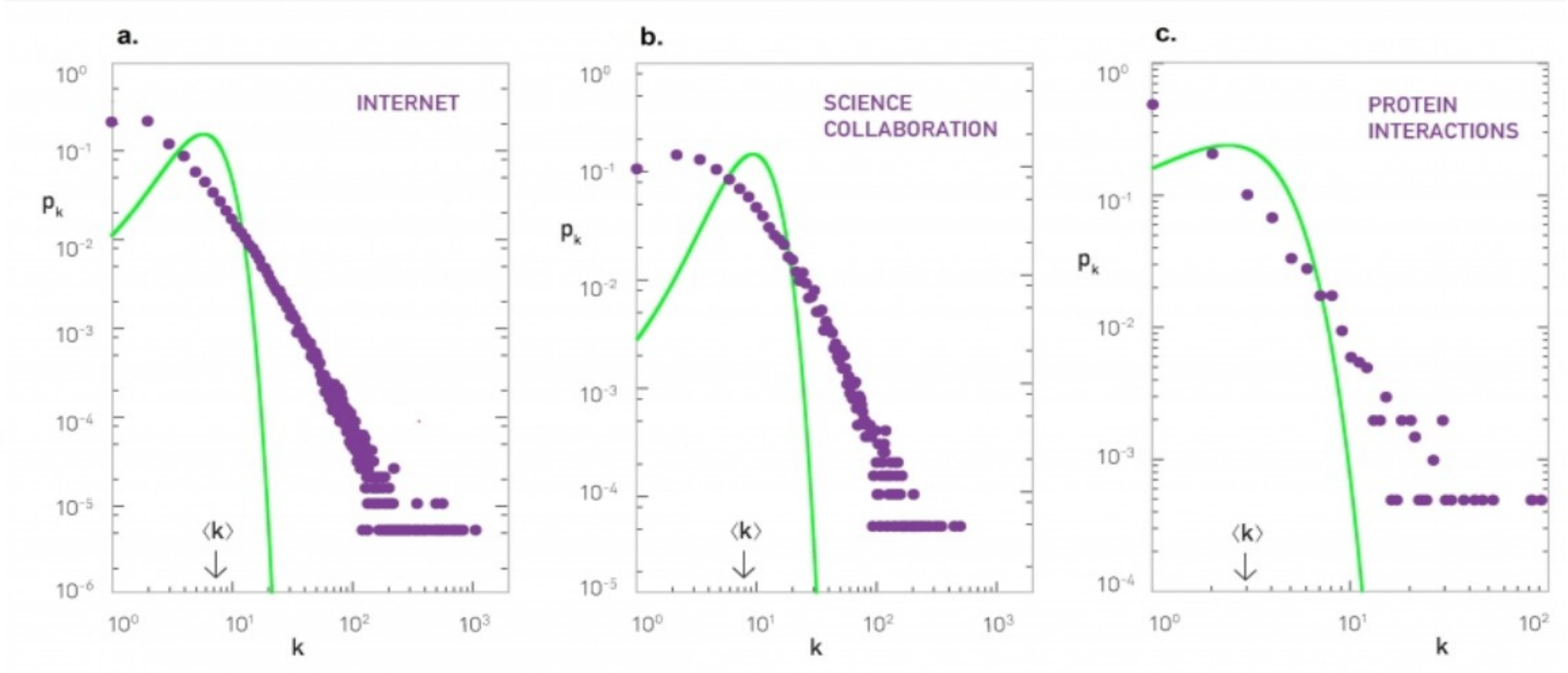
Erdos-Renyi networks

G(N,p), N number of nodes
is probability to have link between nodes



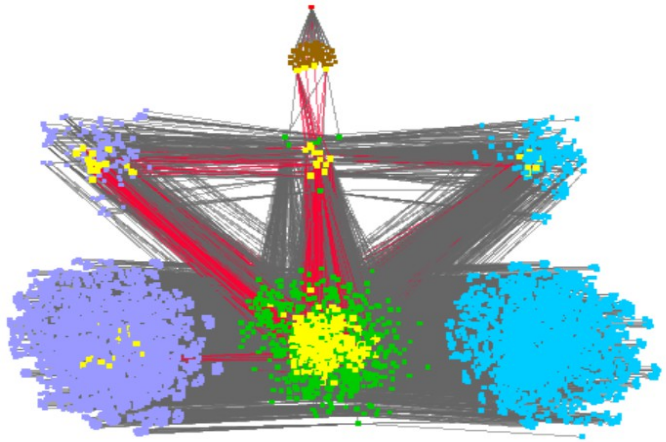
3rd model: Barabasi-Albert network model

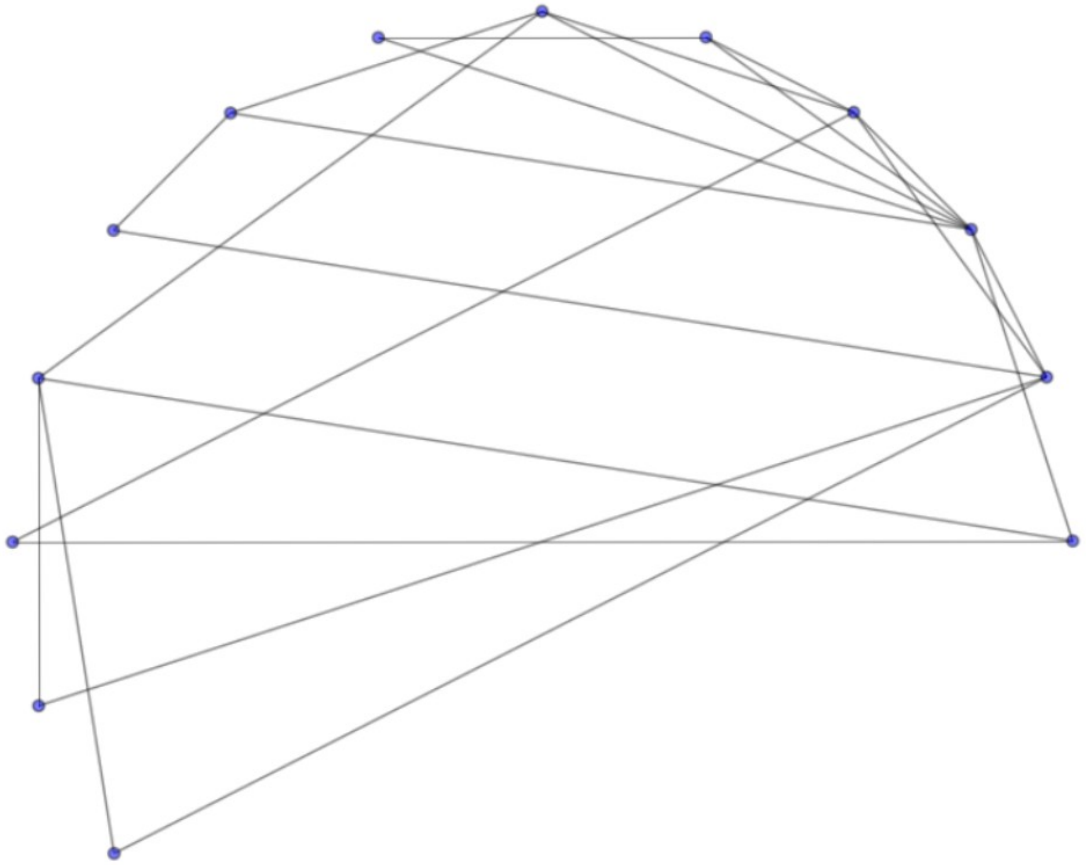




Barabasi book on network science
Cytoscape software

Scale-free networks: how to generate them?

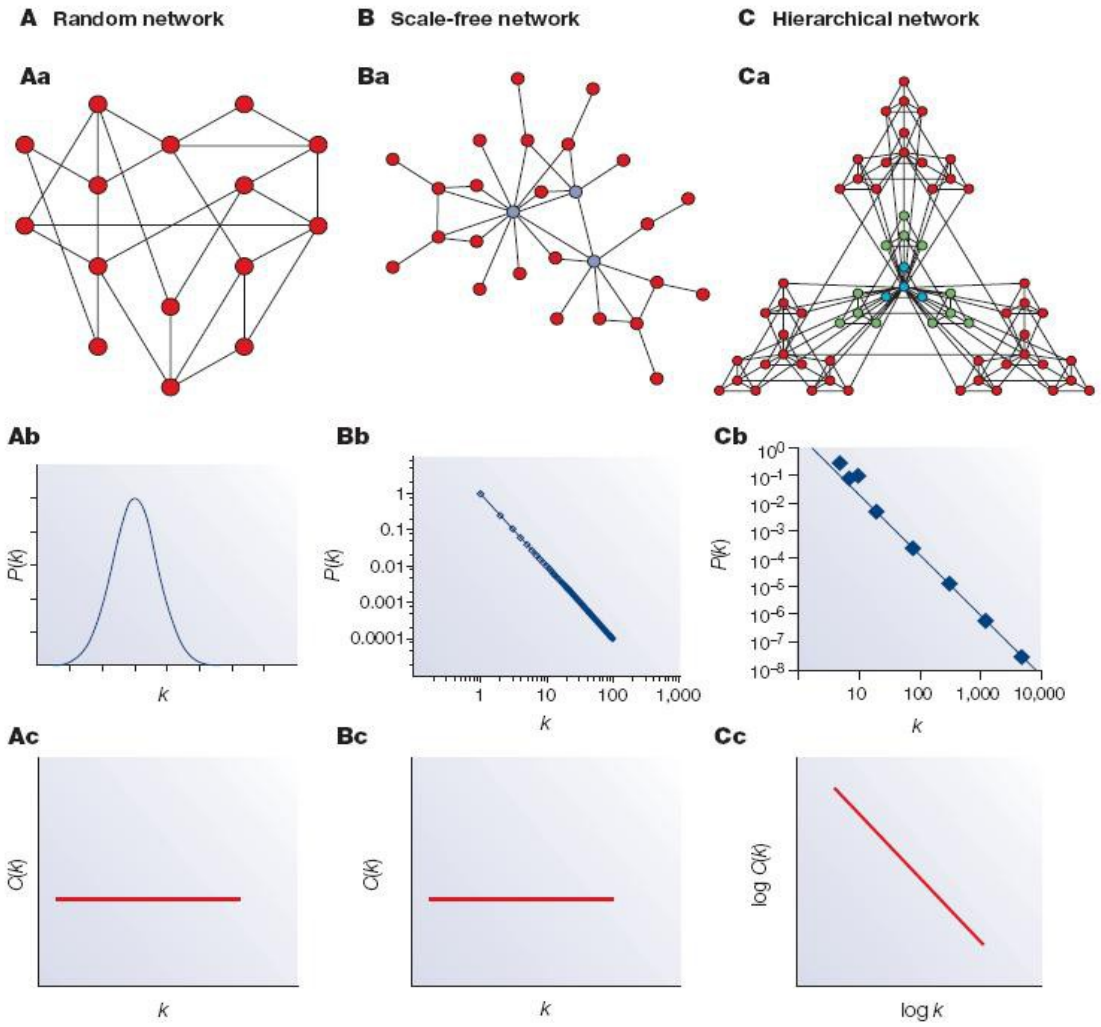




Barabasi book on network science
Cytoscape software

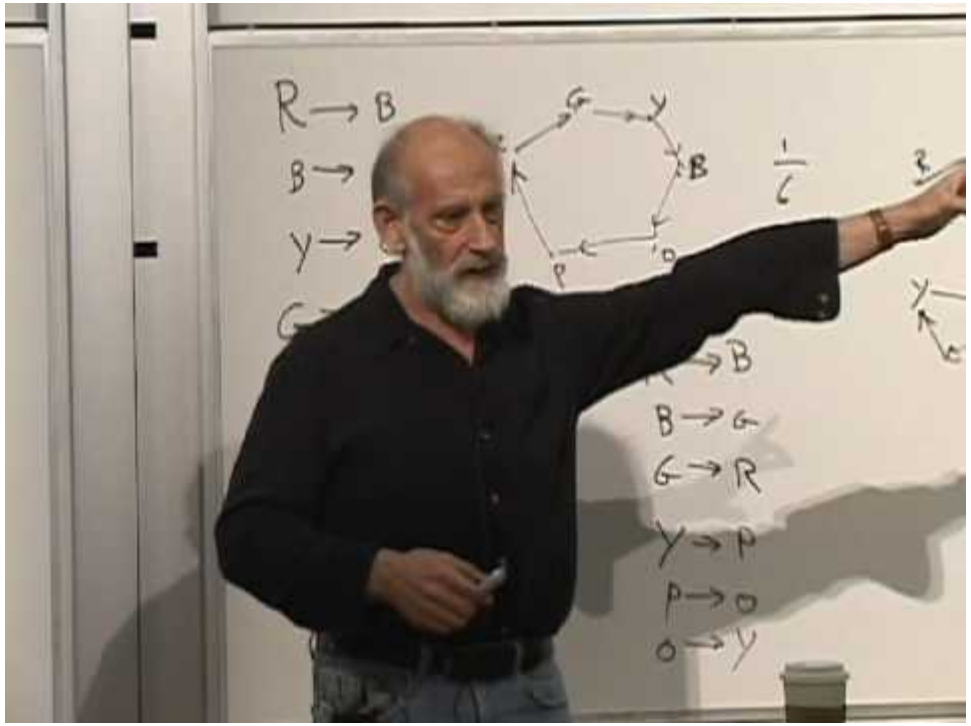
Practical part:
are properties of random networks random?

Practical part:
are properties of random networks random?



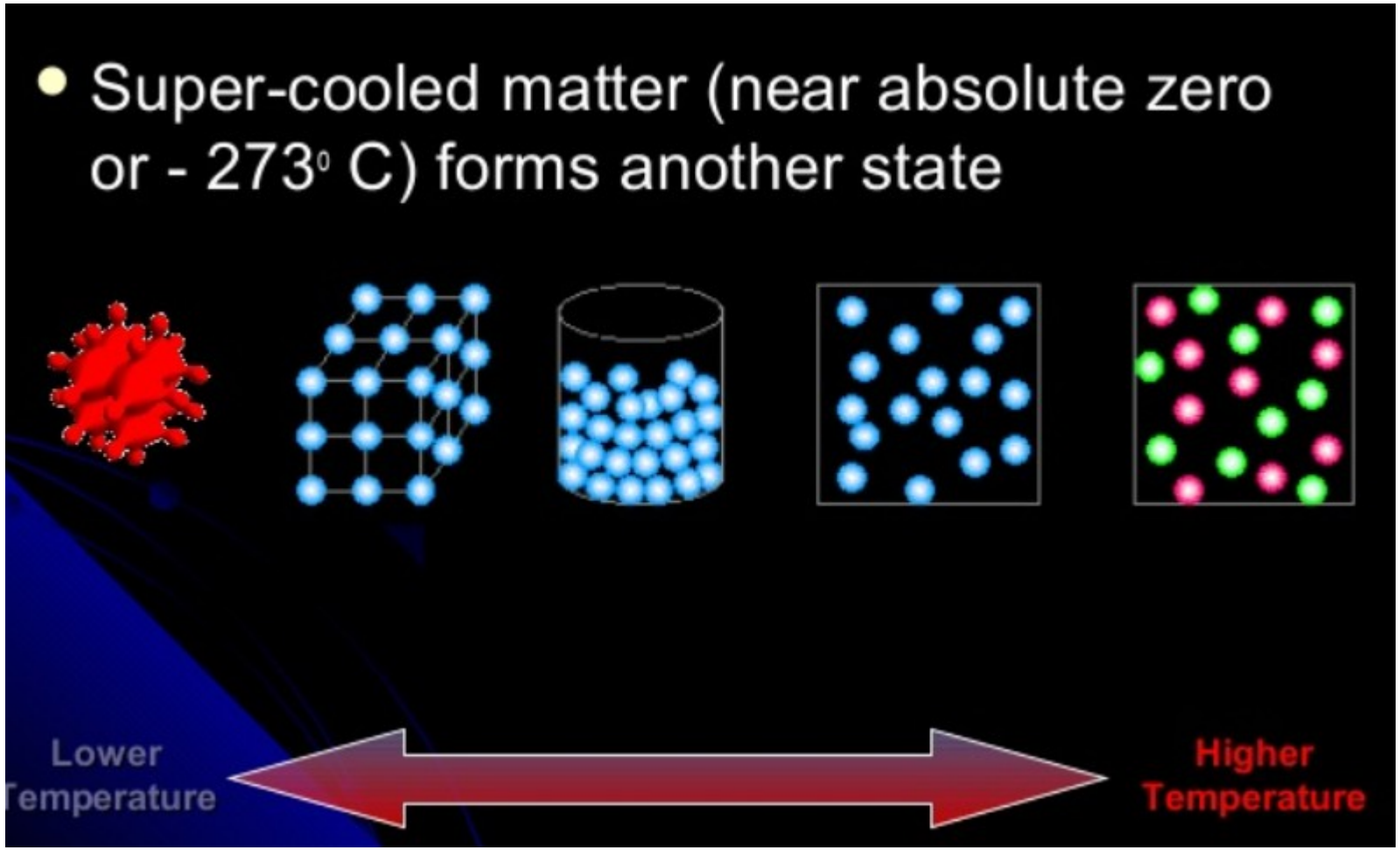
Network theory and statistical mechanics

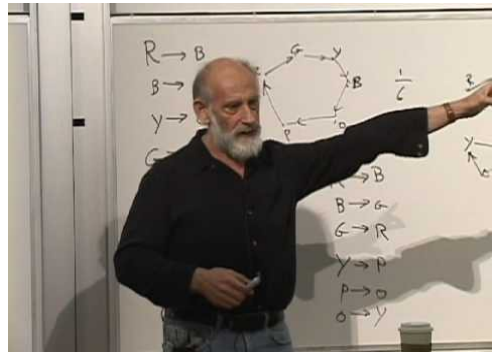
Statistical mechanics models



Stanford course on statistical mechanics

Statistical mechanics models





Stanford course on statistical mechanics

Network theory and matrix theory

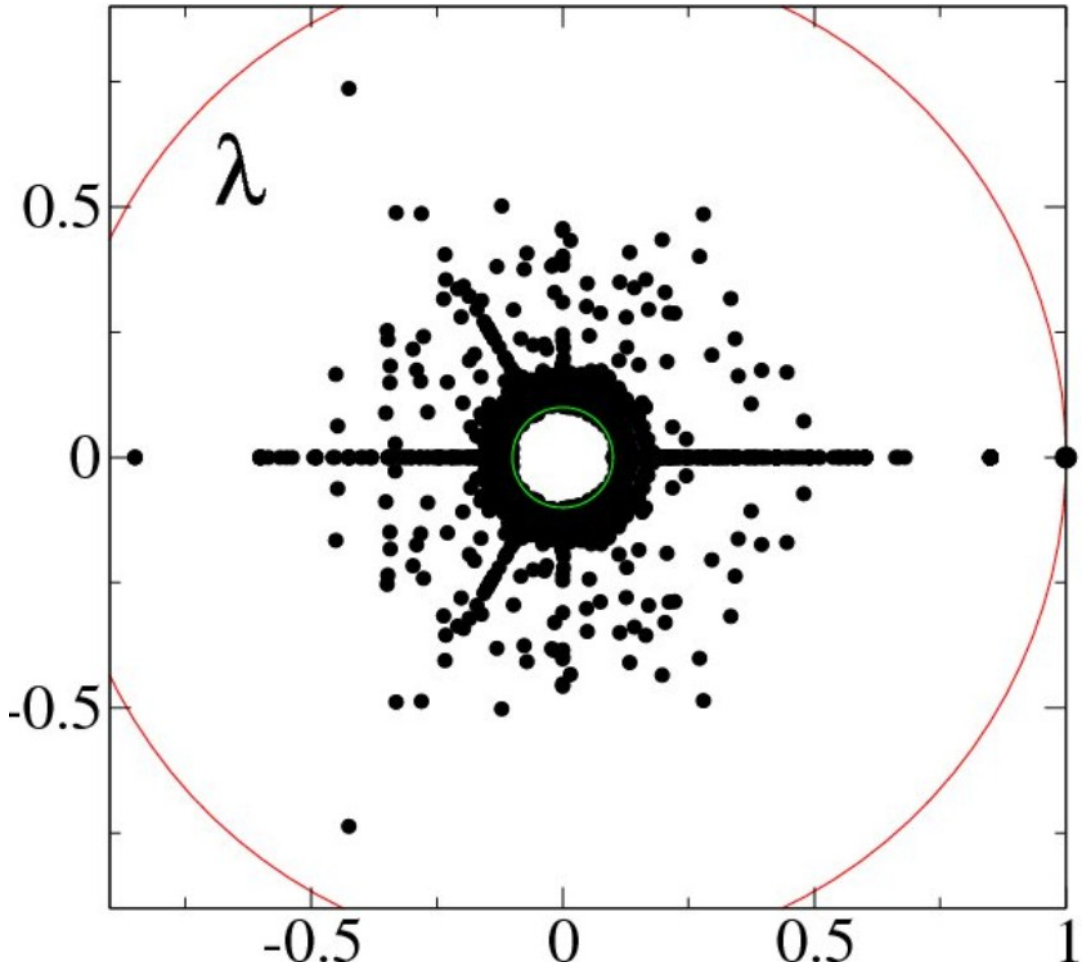
Matrix theory

matrix representation of unweighted undirected graph

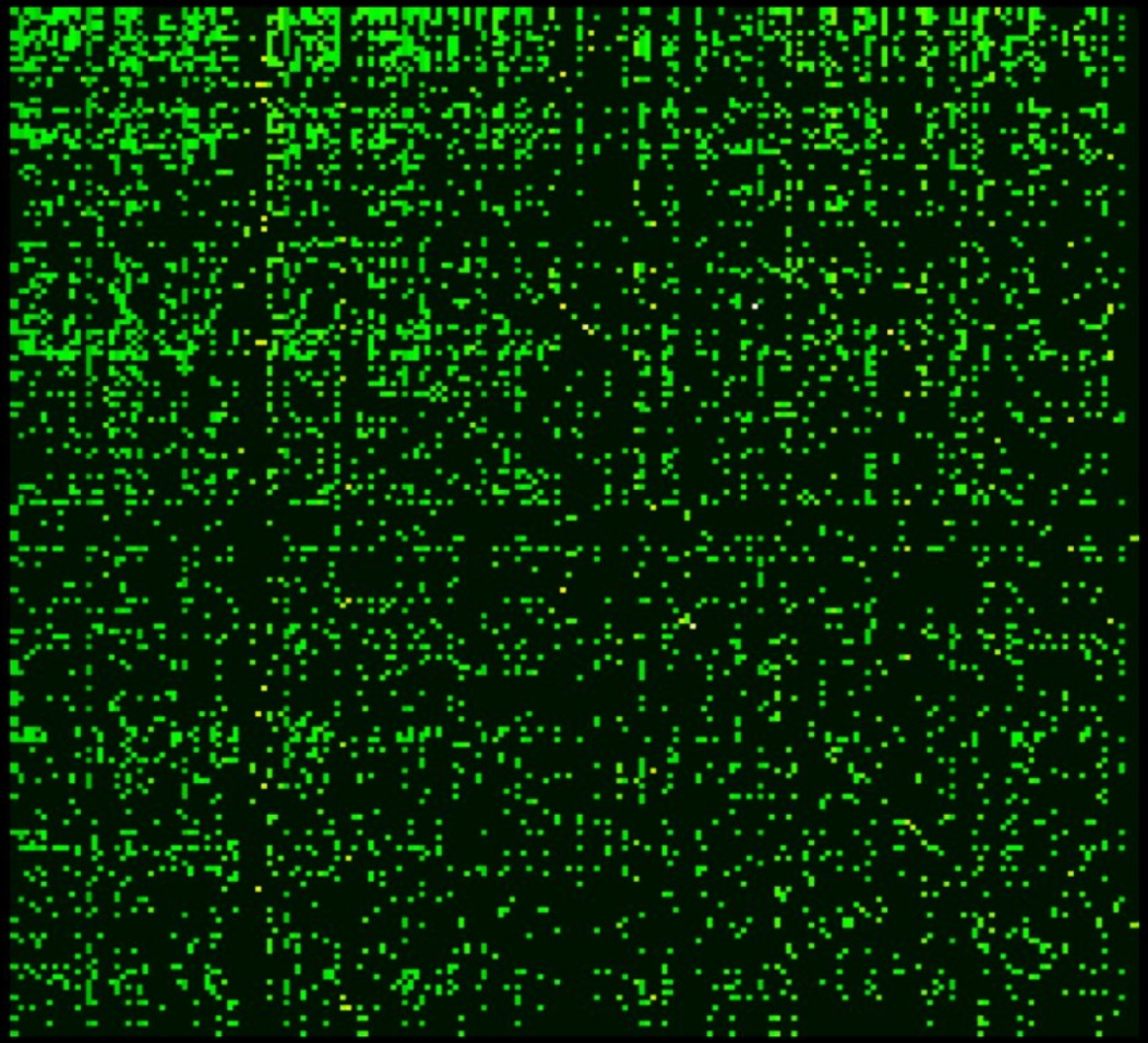
Spectral methods for complex systems, Caravelli et al.
Barabasi et al.

Matrix theory

	a	b	c	d	e	f	g	h
a	0	1	0	0	1	0	1	0
b	1	0	1	0	0	0	0	1
c	0	1	0	1	0	1	1	0
d	0	0	1	0	1	0	0	0
e	1	0	0	1	0	0	0	0
f	0	0	1	0	0	0	1	0
g	1	0	1	0	0	0	0	0
h	0	1	0	0	0	0	0	0

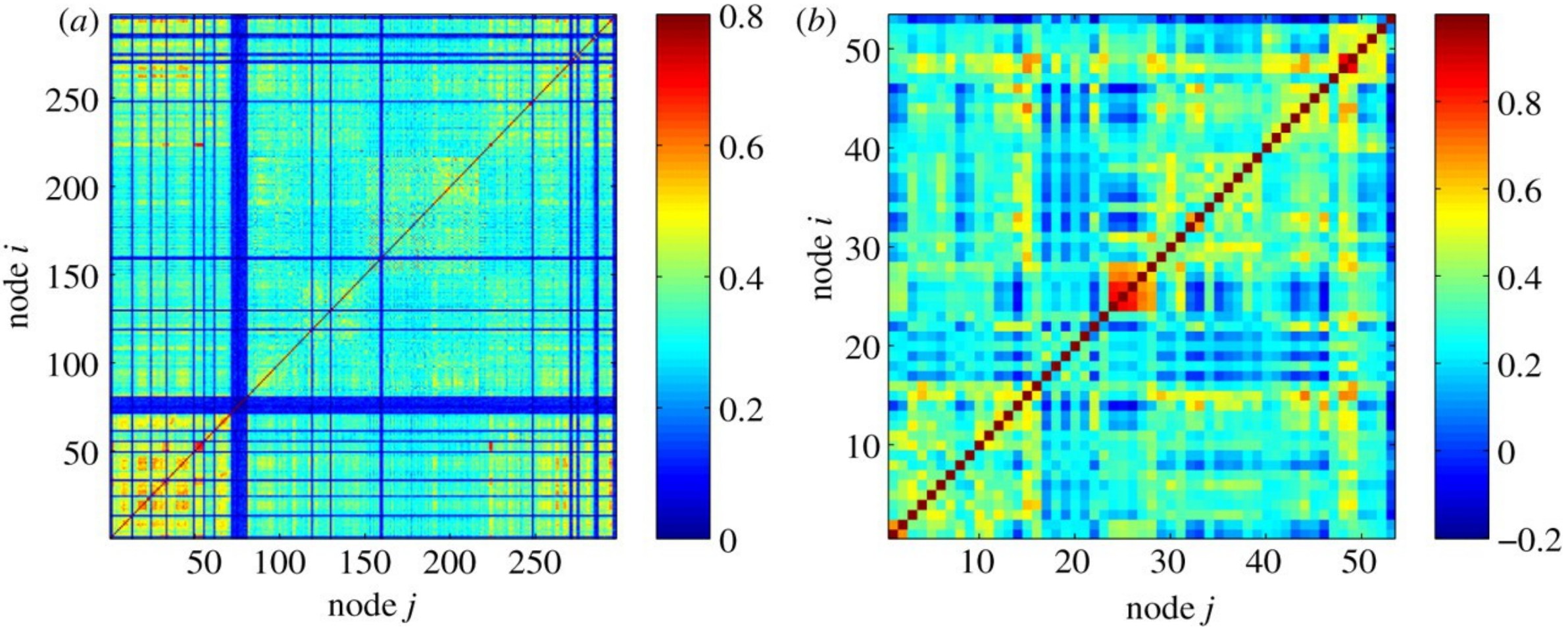


Spectral methods for complex systems, Caravelli et al.
Barabasi et al.



Google matrix of Wikipedia articles network, written in the bases of PageRank index; fragment of top 200 X 200 matrix elements is shown, total size N=3282257 (from [Wikipedia])

Matrix theory



Spectral methods for complex systems, Caravelli et al.

Literature overview

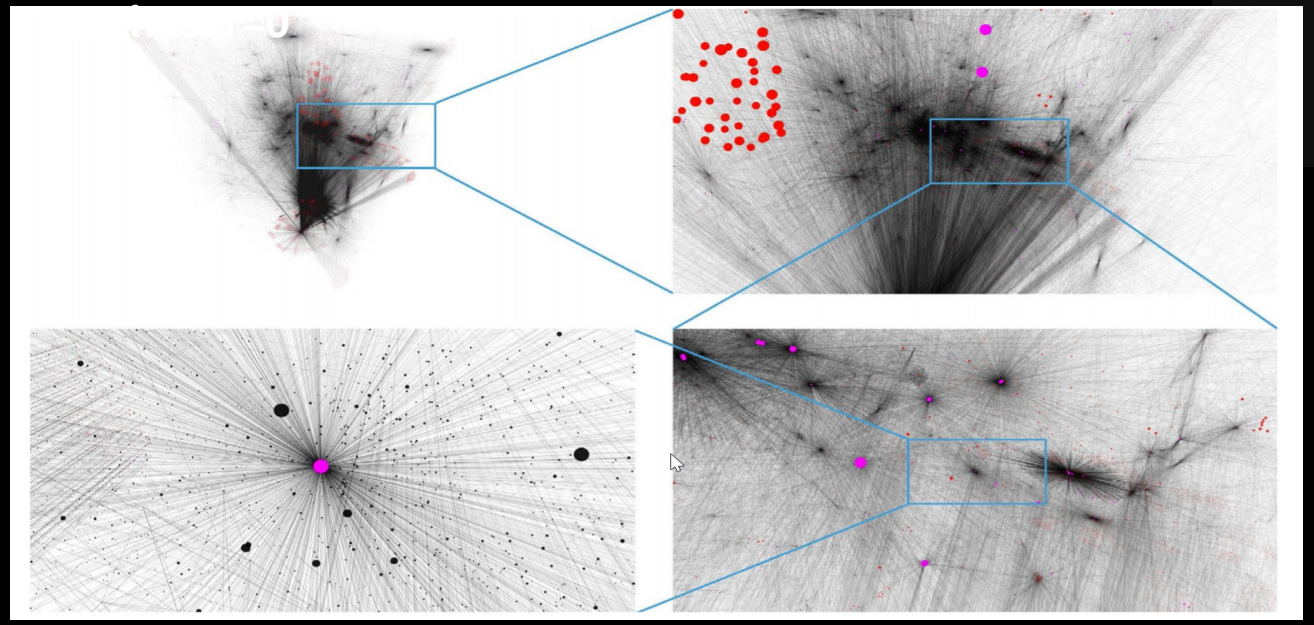
http://networksciencebook.com/

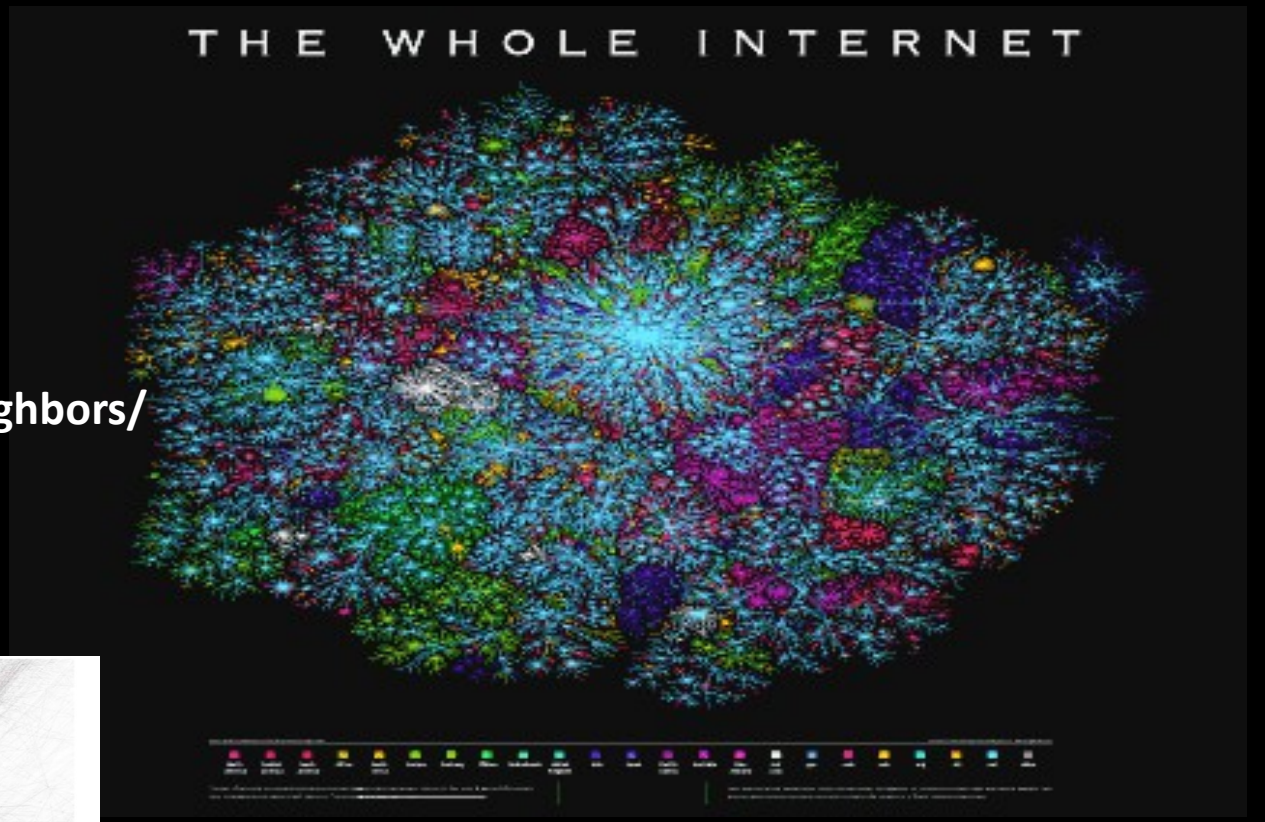
https://www.barabasilab.com

http://networkrepository.com/graph-vis.php

http://www.complexity-explorables.org/explorables/neighbors/

Spectral methods for complex systems online lectures
https://sites.google.com/view/cssm/online-courses?

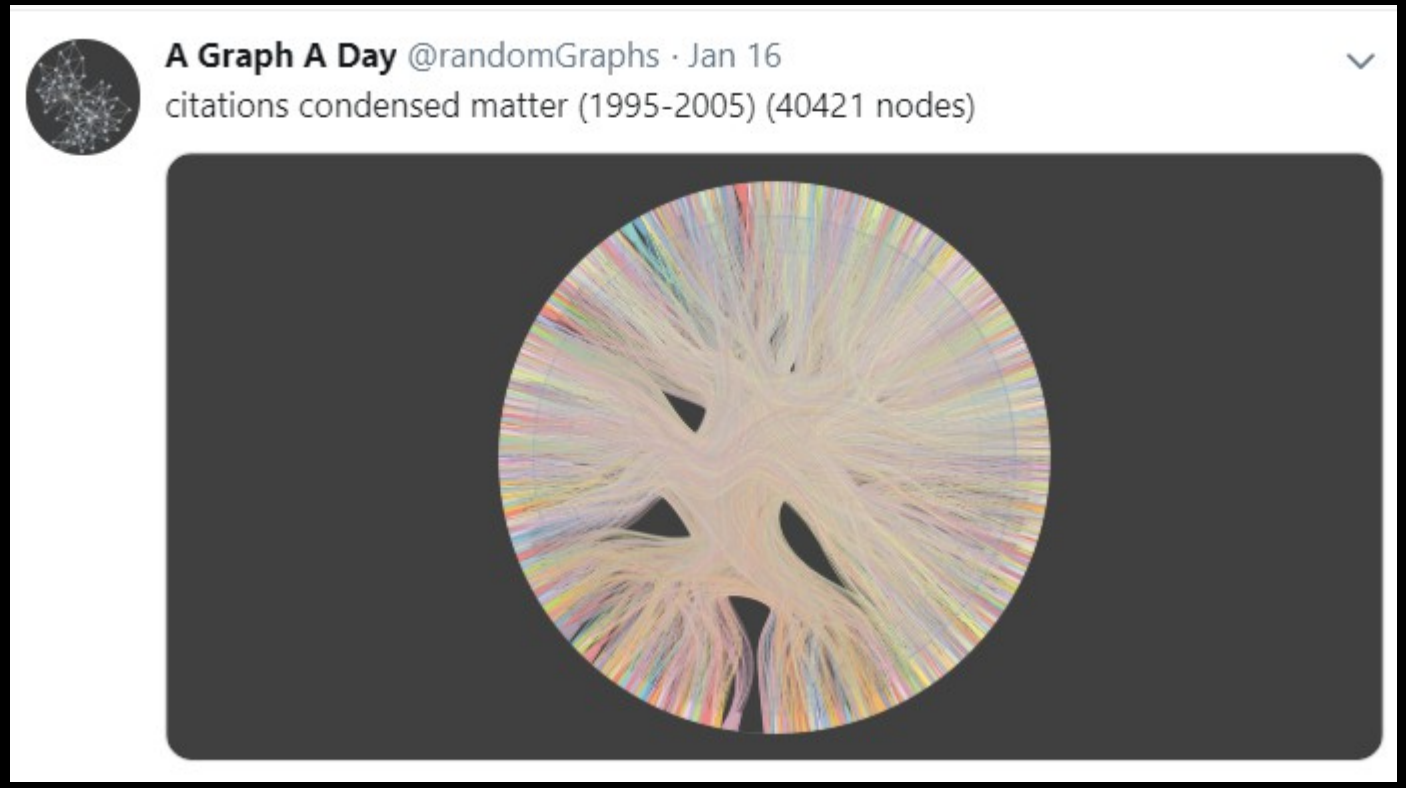




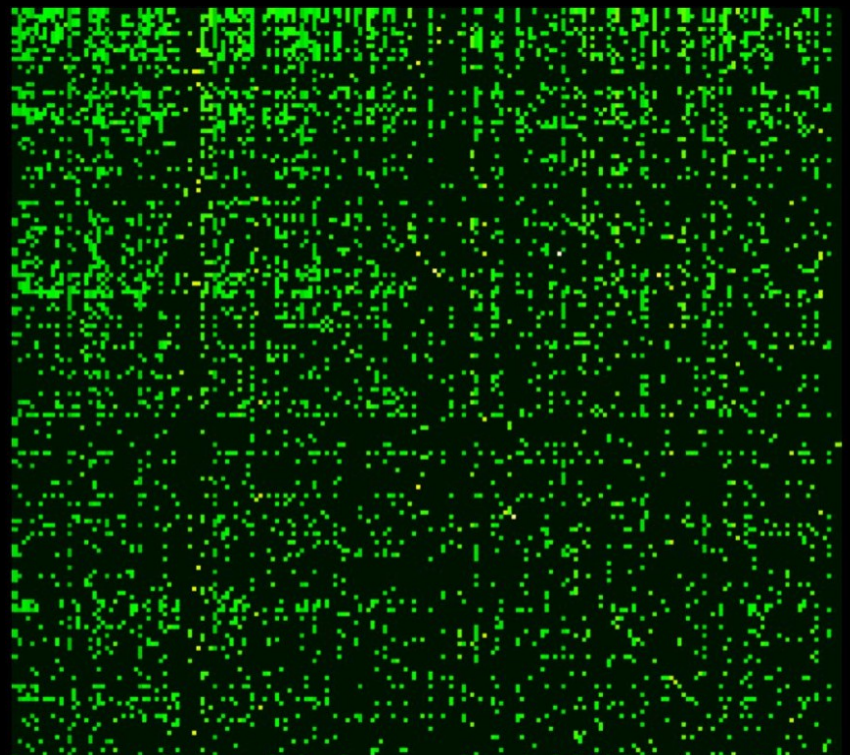
https://snap.stanford.edu/data

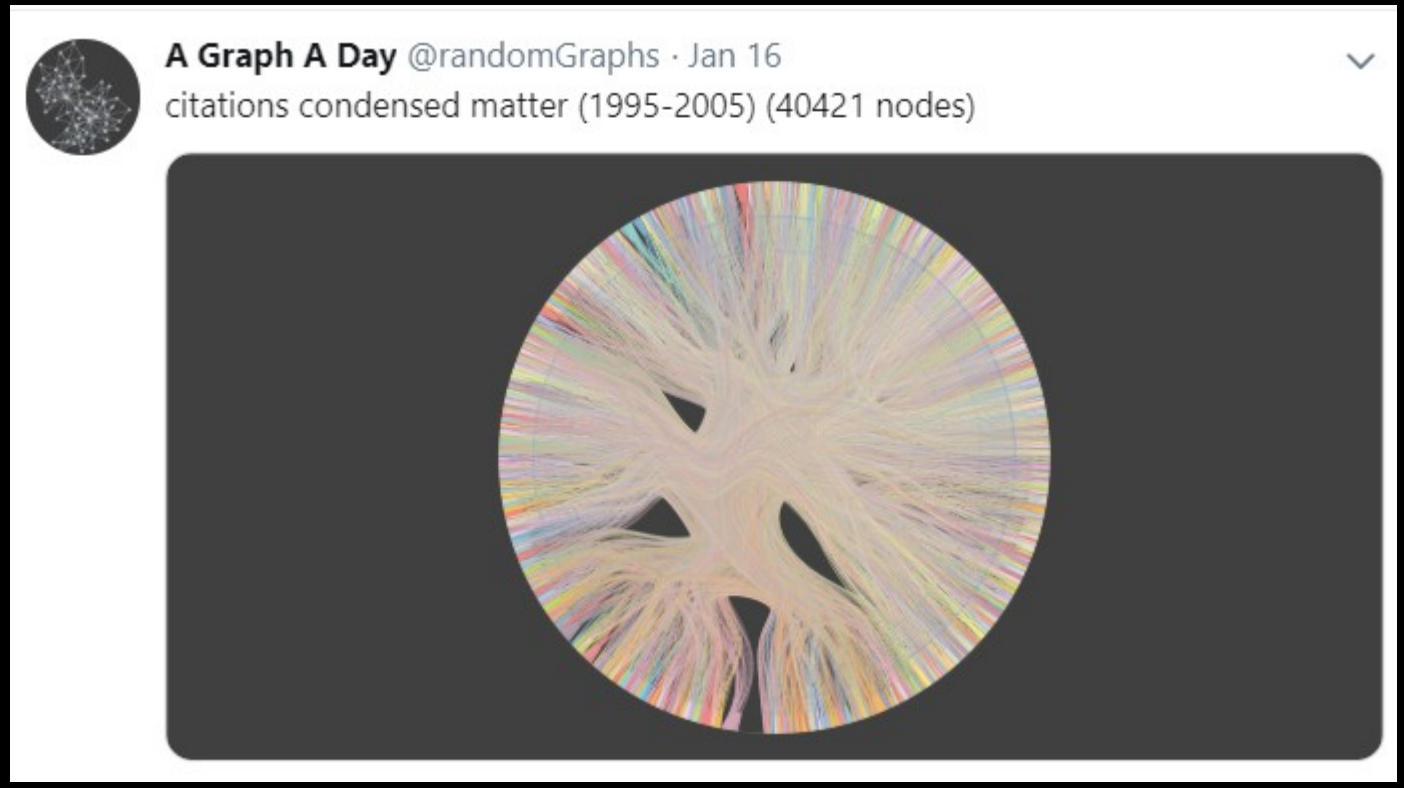
https://sites.google.com/a/binghamton.edu/netscied/teaching-learning/network-concepts

Which methods are used in network theory?



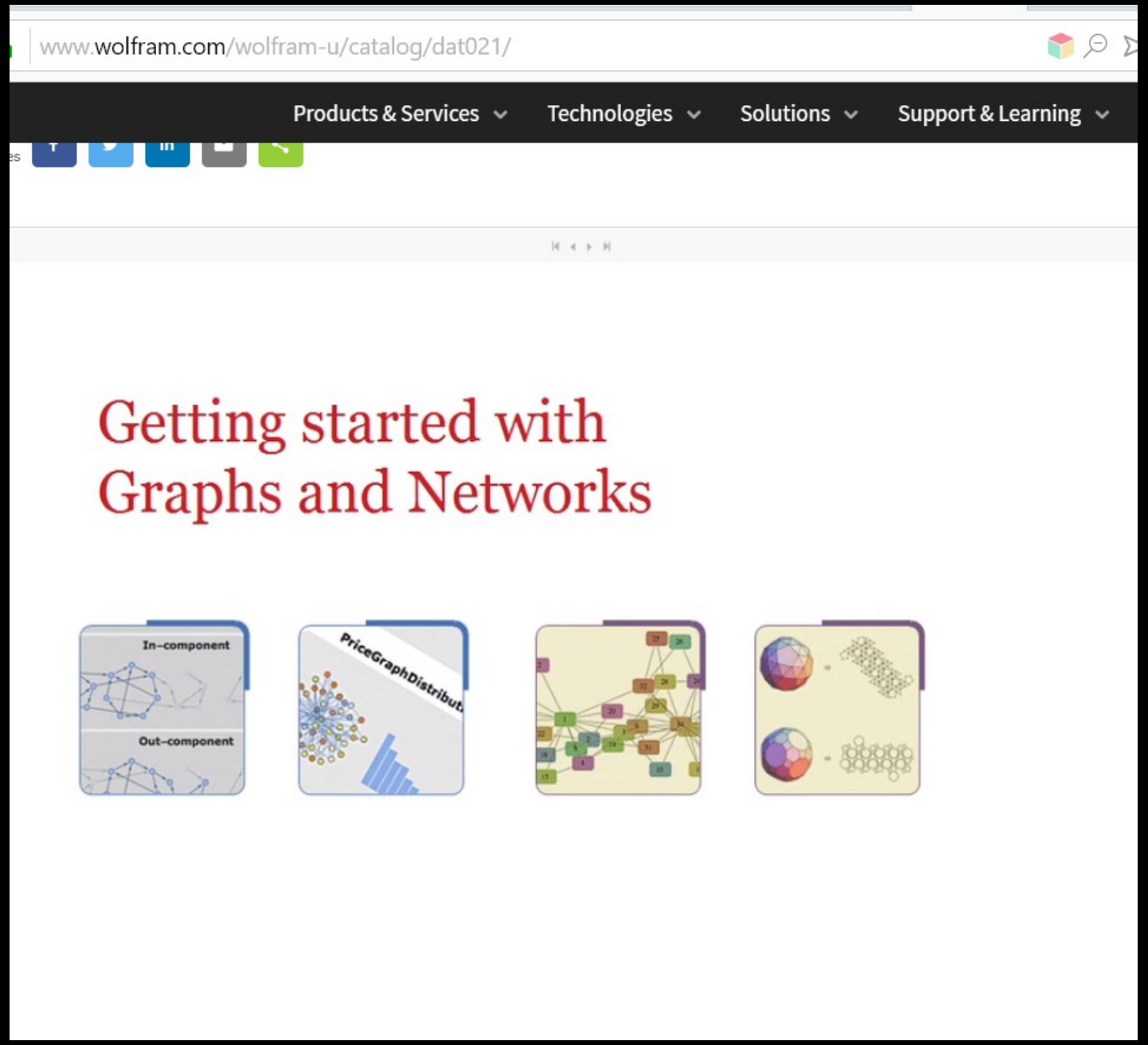
Which methods are used in network theory?





Hands-on part:

go to http://networkrepository.com



Dynamical models on networks

Hands-on part:

go to http://networkrepository.com

Models on networks

Spreading dynamics:

Modelling crowded dynamics:

Data analysis:

2.16

D.Grenbenkov, LT. PRE (2017)

P. Holme, LT, New Jour. Physics (2018)

M.Lindner, N.Molkenthin, LT, R.Donner Springer (2019)