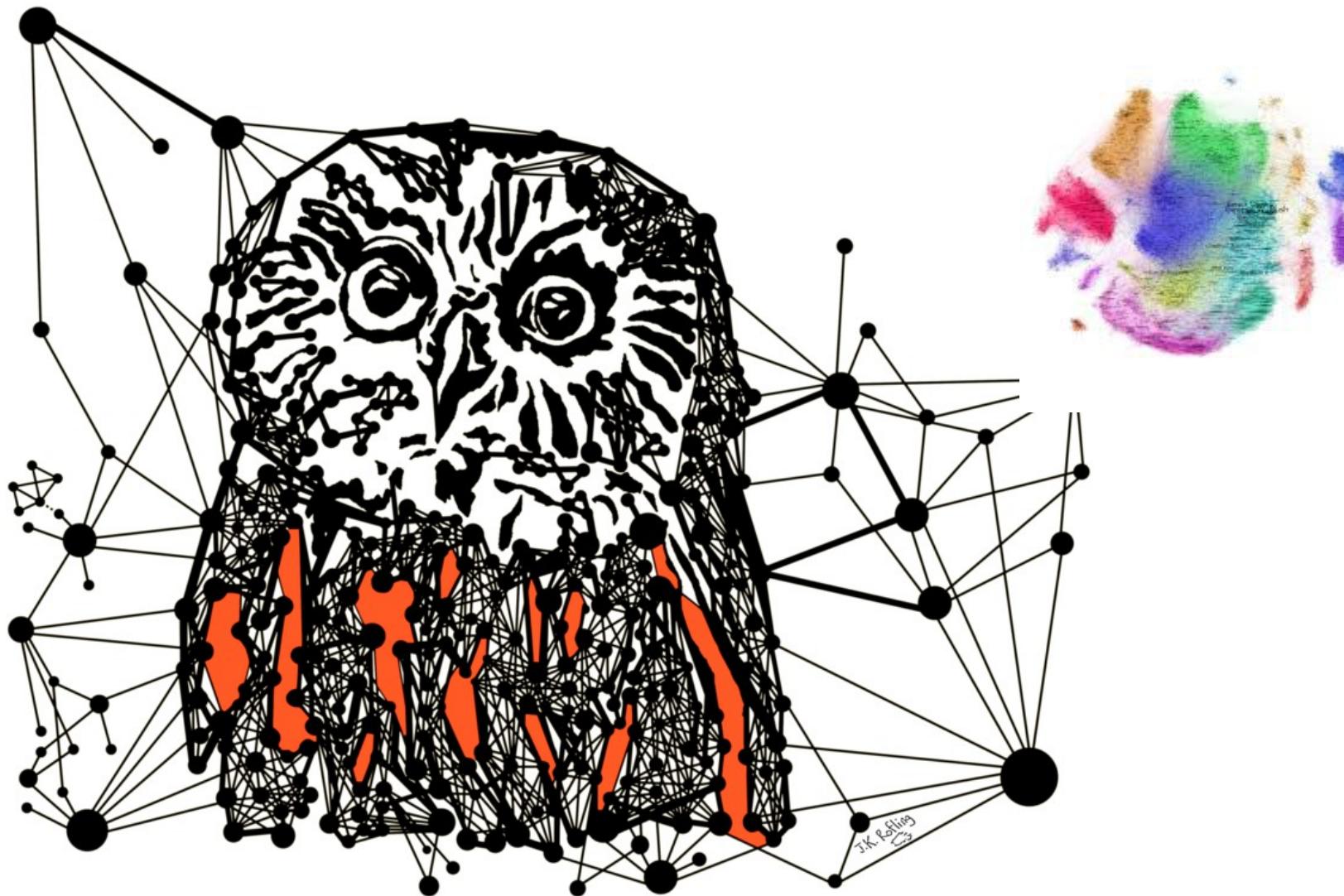




# **Networks theory: 2 methods for network analysis**

Liubov Tupikina (CRI)

## Network quiz



Brendan network artist  
Barabasi book on network science

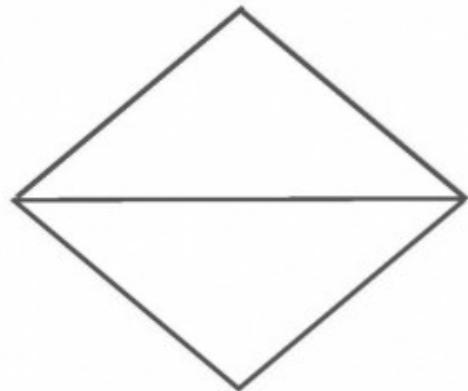
## Network quiz



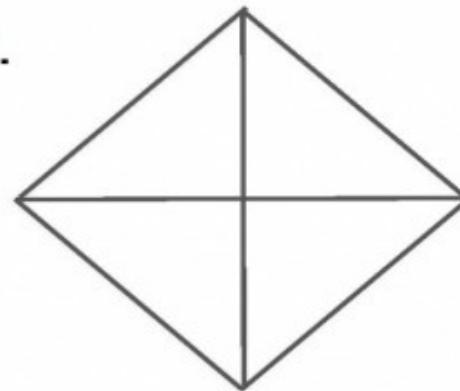
Brendan network artist  
Barabasi book on network science

## Network quiz

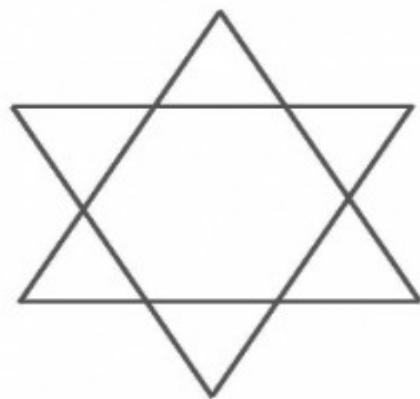
a.



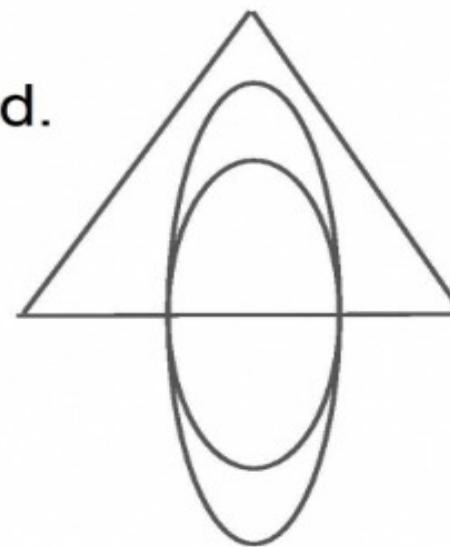
b.



c.



d.



Brendan network artist  
Barabasi book on network science

# Lecture 2

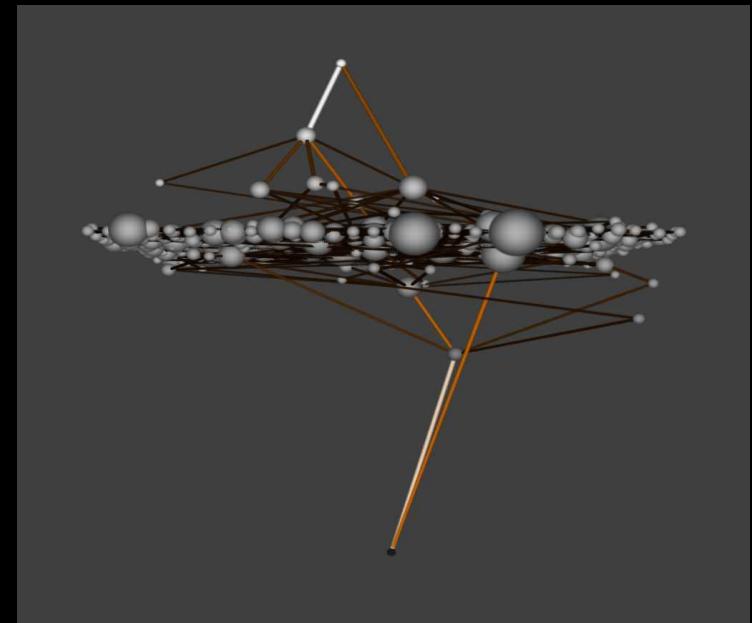
**Network games**

**Network theory and graph theory (random graph theory)**

**Network measures**

**Networks and statistical mechanics**

**Study networks in groups**



# Network theory and other fields

There are methods, which people brought to network science from other fields

probability theory

control theory

combinatorics (maths)

spectral methods (maths, physics)

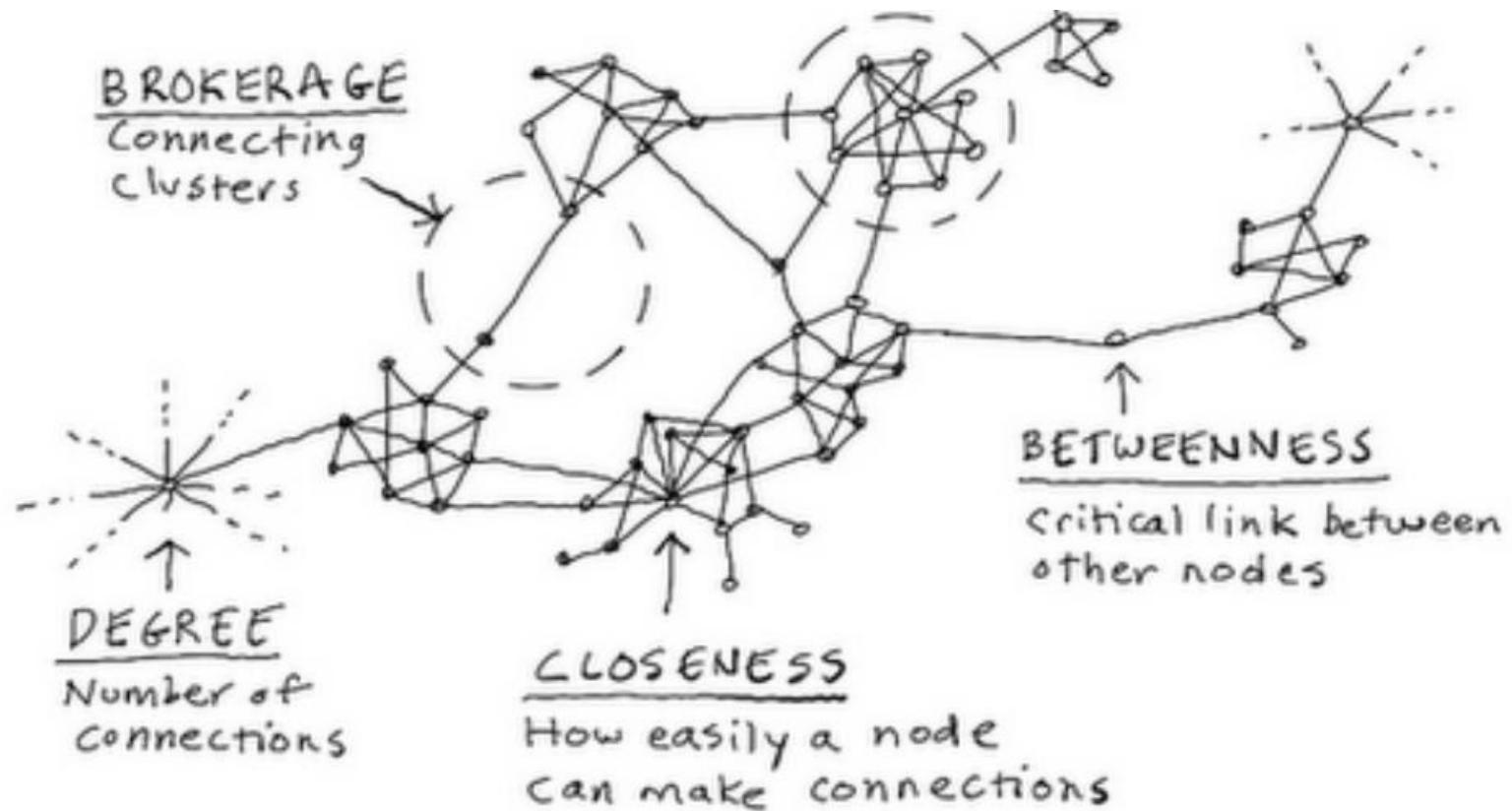
optimisation theory

Caravelli, Tupikina et al. "Spectral methods for complex systems", (2019)

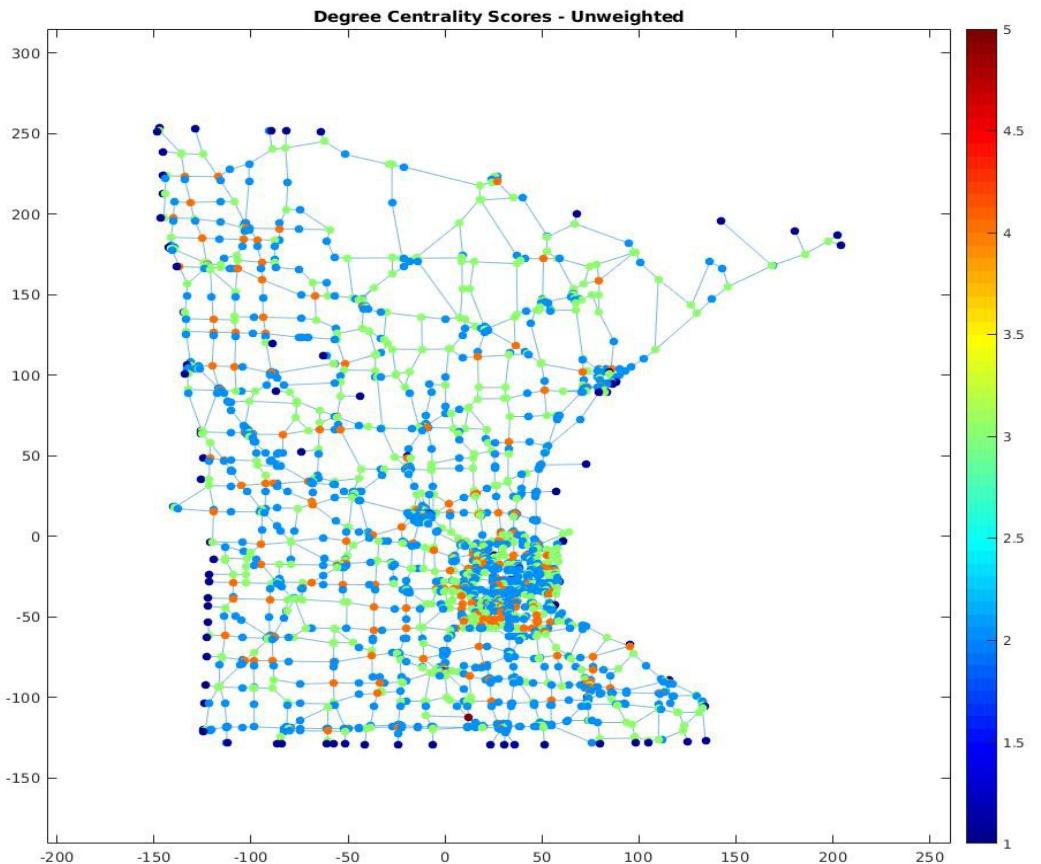
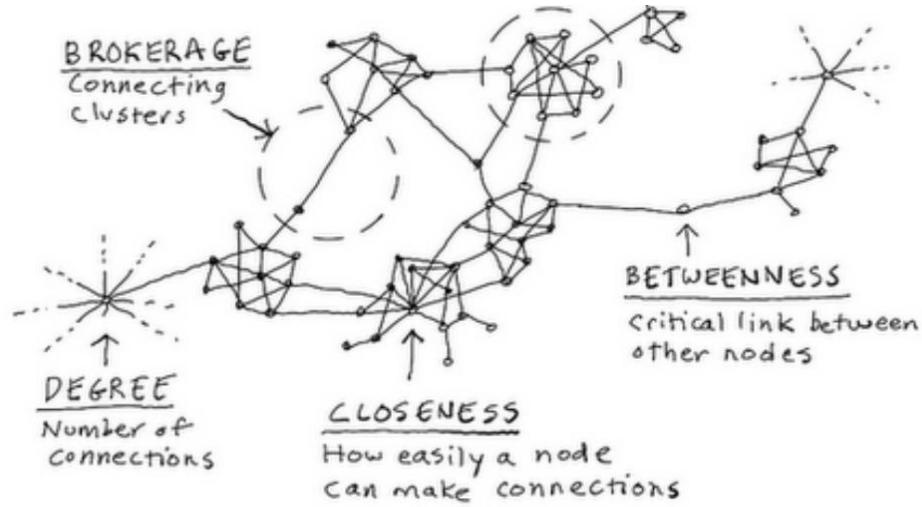


**How to describe a network?**  
**To use network measures**

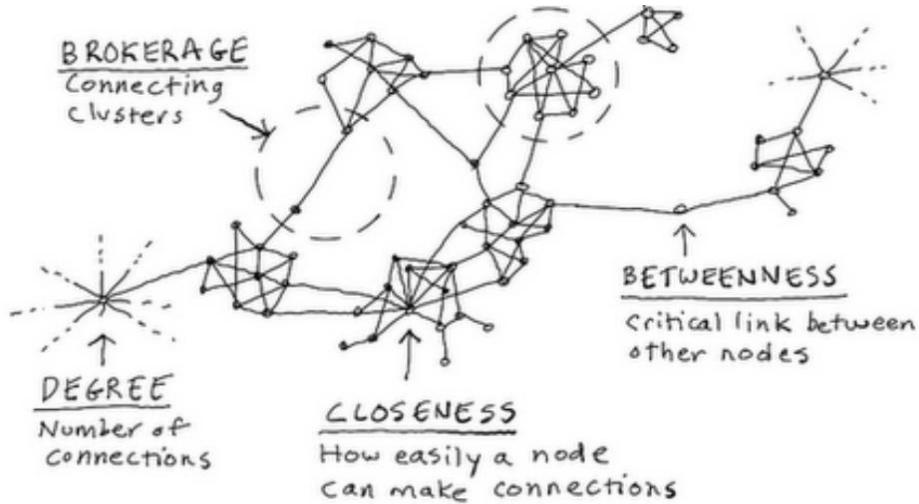
# Network measures



# Network measures: degree

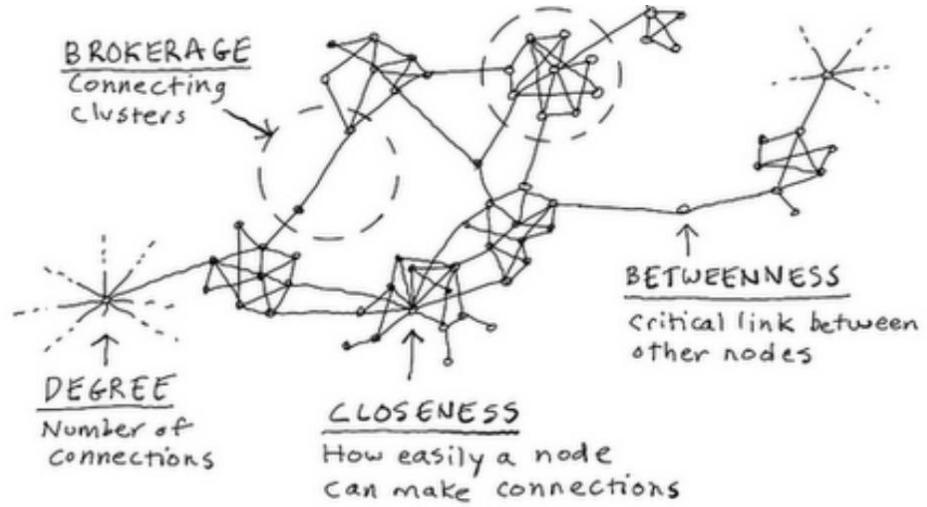


# Network measures: betweenness centrality

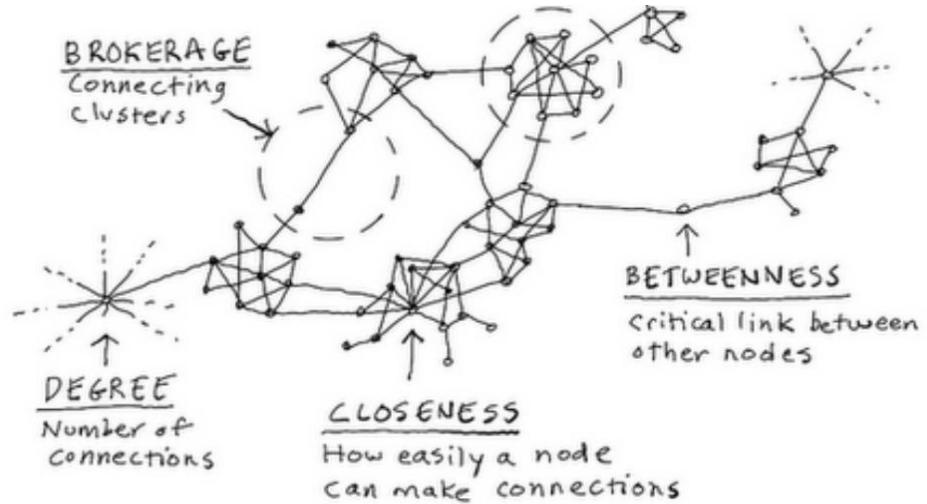


$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

# Network measures: betweenness centrality



# Network measures: clustering

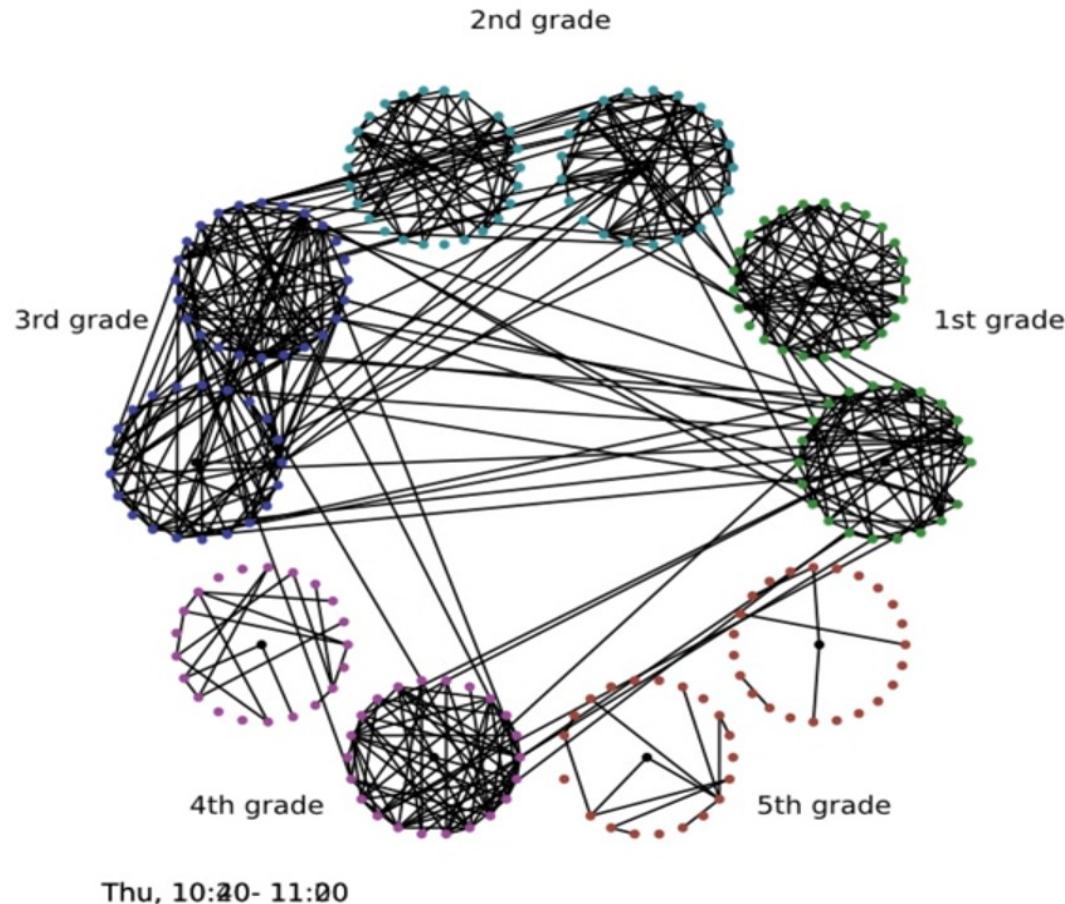


$$C_i = \frac{|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}$$

# **Open questions:**

## **How to analyze temporal networks?**

# Open questions: How to analyze temporal networks?



<http://www.sociopatterns.org/gallery/>

## Real-world network examples

Aug 08 2009



Paris  
Frankfurt  
Amsterdam  
Rome  
Milan  
Moscow  
Dublin

Hong Kong  
Tokyo Narita  
Bangkok  
Singapore  
Beijing  
Manila

Sydney  
Brisbane  
Auckland  
Perth

## Real-world network examples



# **Network theory and probability theory**

# Network theory history



**L.Euler (1735)**

**P.Erdos, A.Renyi (1959)**

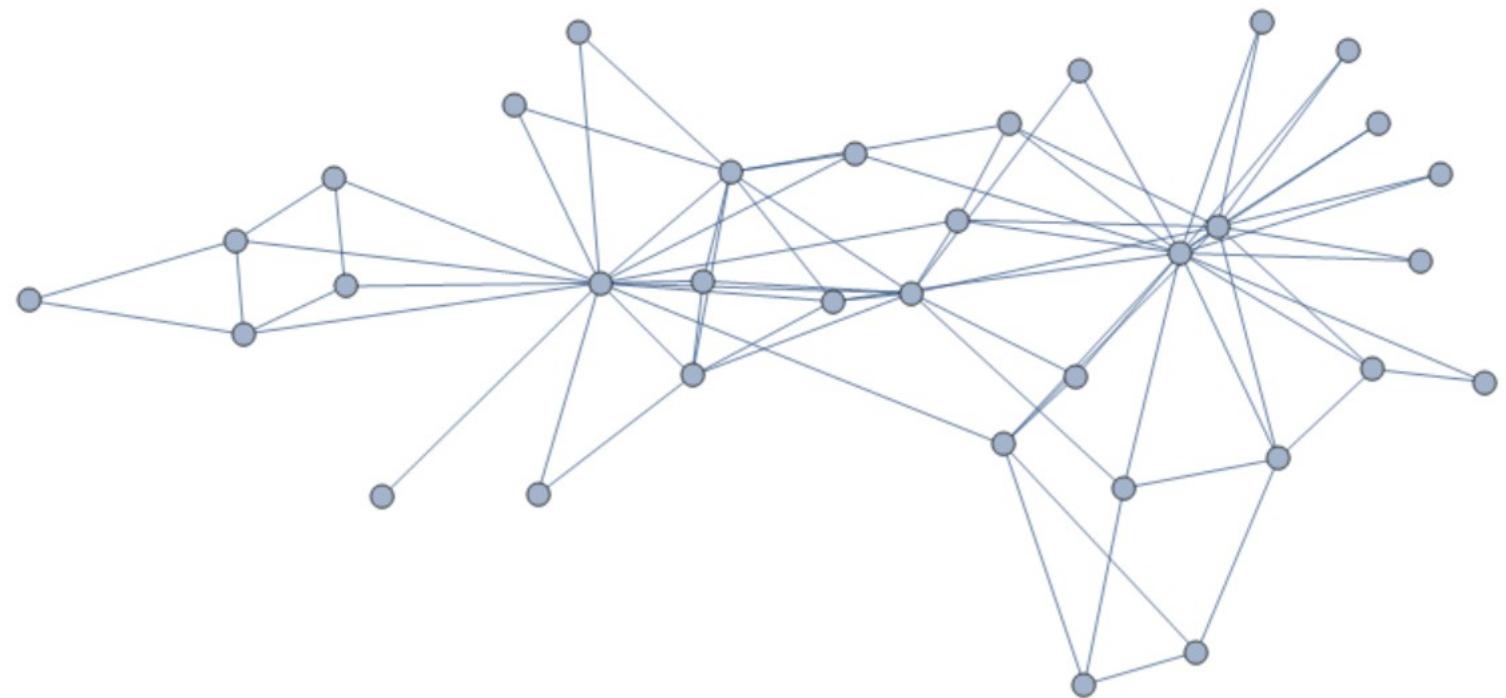
**B.Bollobas (1981)**

**Watts, Strogatz (1989)**

**M.Newman (2000)**

# Network theory and probability theory

The question: how does random graph looks like?



# Network theory and probability theory

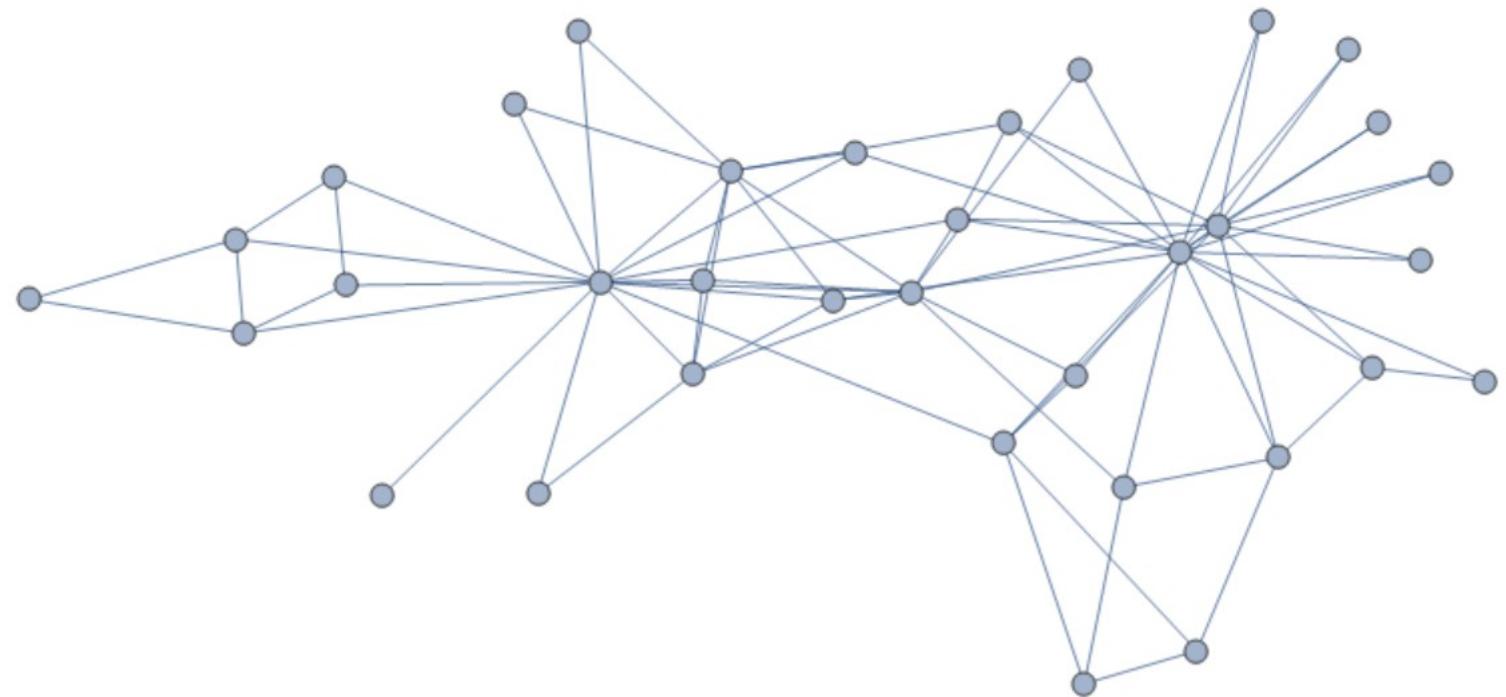
The question: how does random graph looks like?

Three models:

Erdos-Renyi model

Watts-Strogatz model

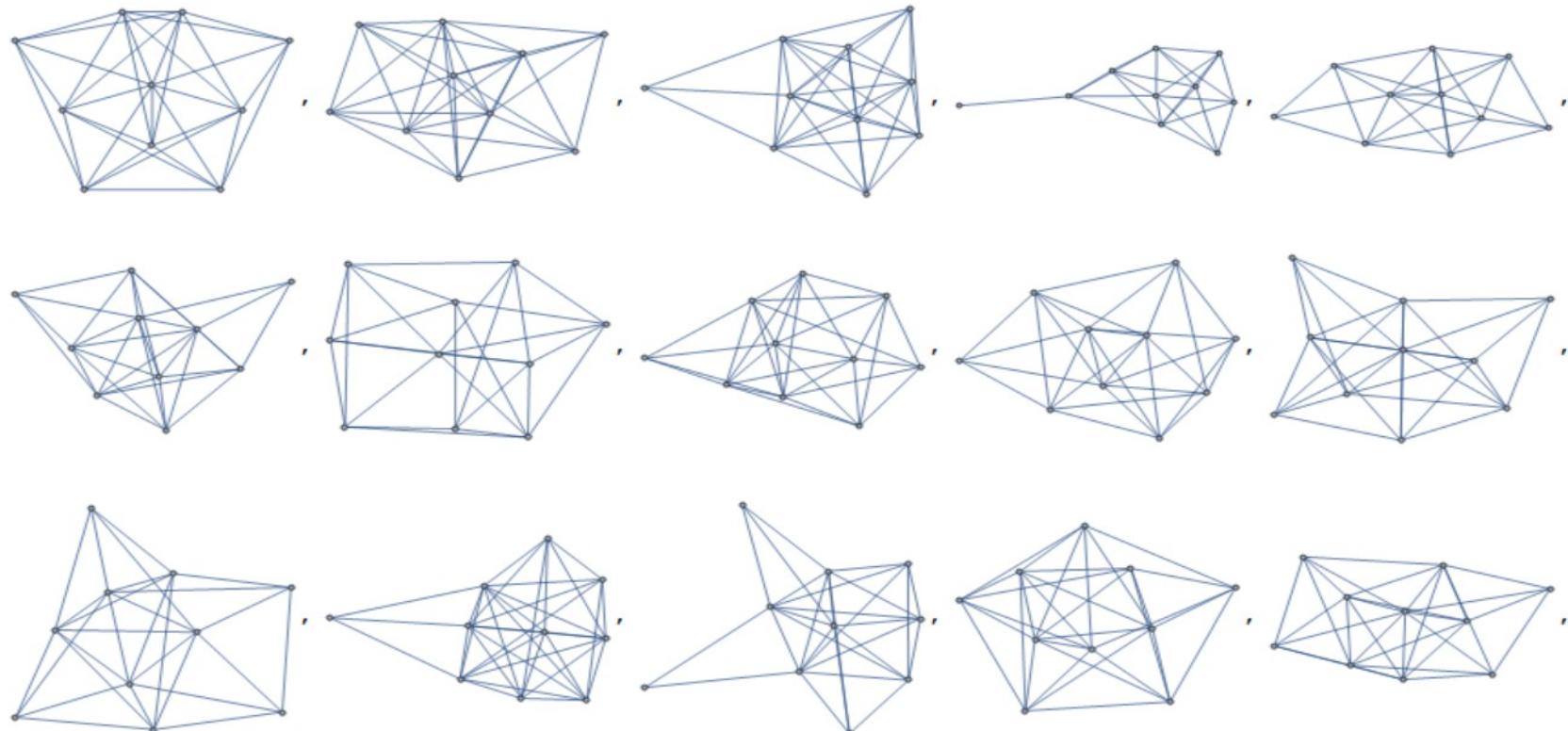
Barabasi-Albert model



# Network theory and probability theory

The question: how does random graph looks like?

Which properties does this graph have?



# Probability theory and statistics

## BRIEF STATISTICS REVIEW

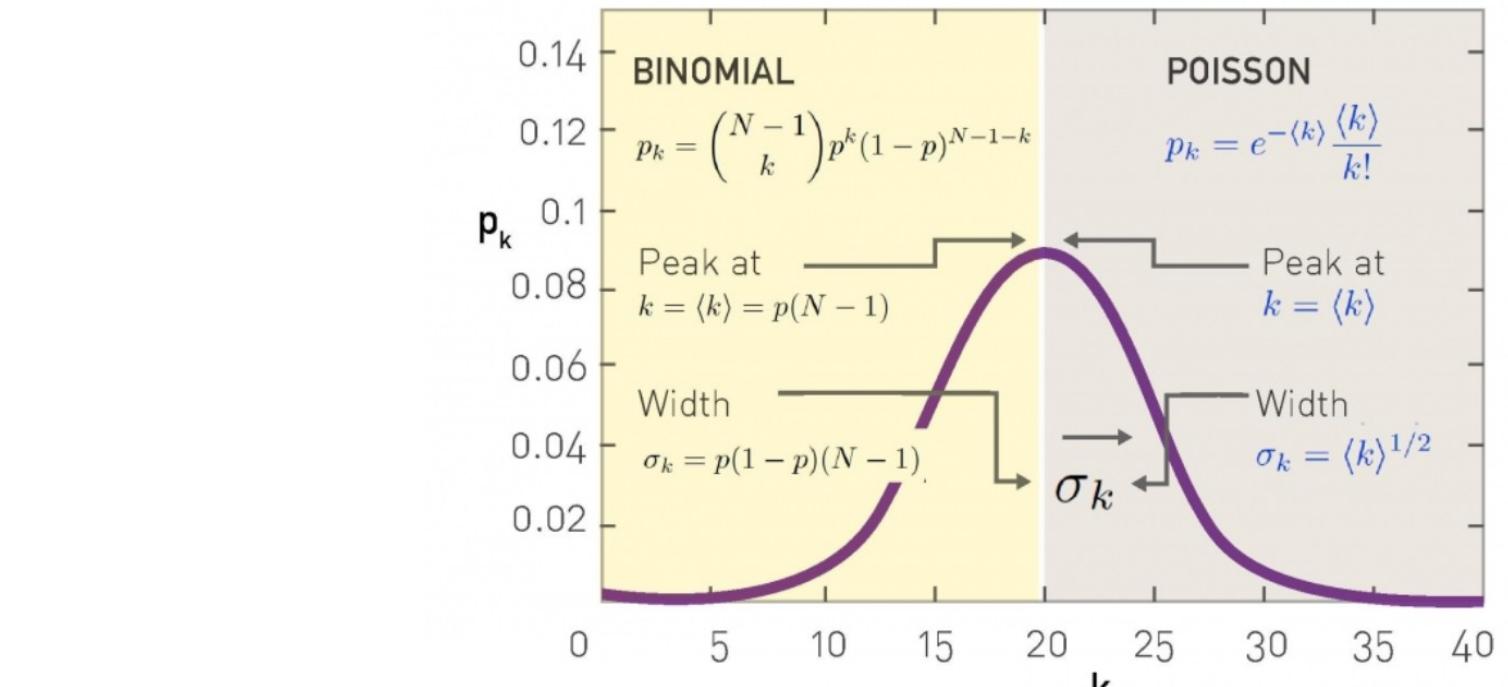
Four key quantities characterize a sample of  $N$  values  $x_1, \dots, x_N$ :

*Average (mean):*

$$\langle x \rangle = \frac{x_1 + x_2 + \dots + x_N}{N} = \frac{1}{N} \sum_{i=1}^N x_i$$

*The  $n^{\text{th}}$  moment:*

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \dots + x_N^n}{N} = \frac{1}{N} \sum_{i=1}^N x_i^n$$



*Standard deviation:*

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \langle x \rangle)^2}$$

*Distribution of  $x$ :*

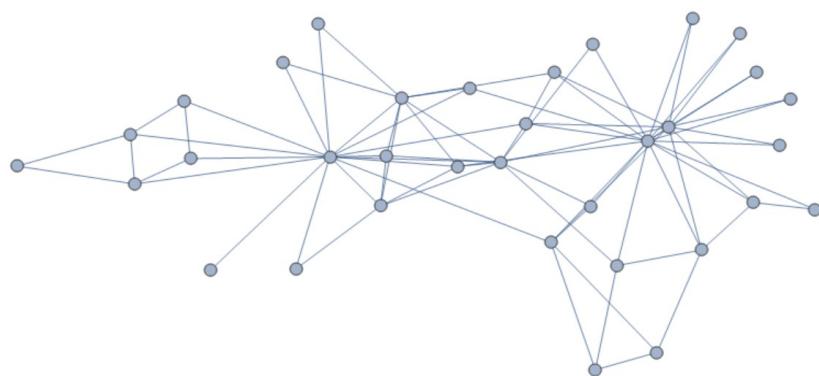
$$p_x = \frac{1}{N} \sum_i \delta_{x, x_i}$$

where  $p_x$  follows

$$\sum_i p_x = 1 \left( \int p_x dx = 1 \right)$$

Barabasi book on network science

# Probability theory and statistics



## BRIEF STATISTICS REVIEW

Four key quantities characterize a sample of  $N$  values  $x_1, \dots, x_N$ :

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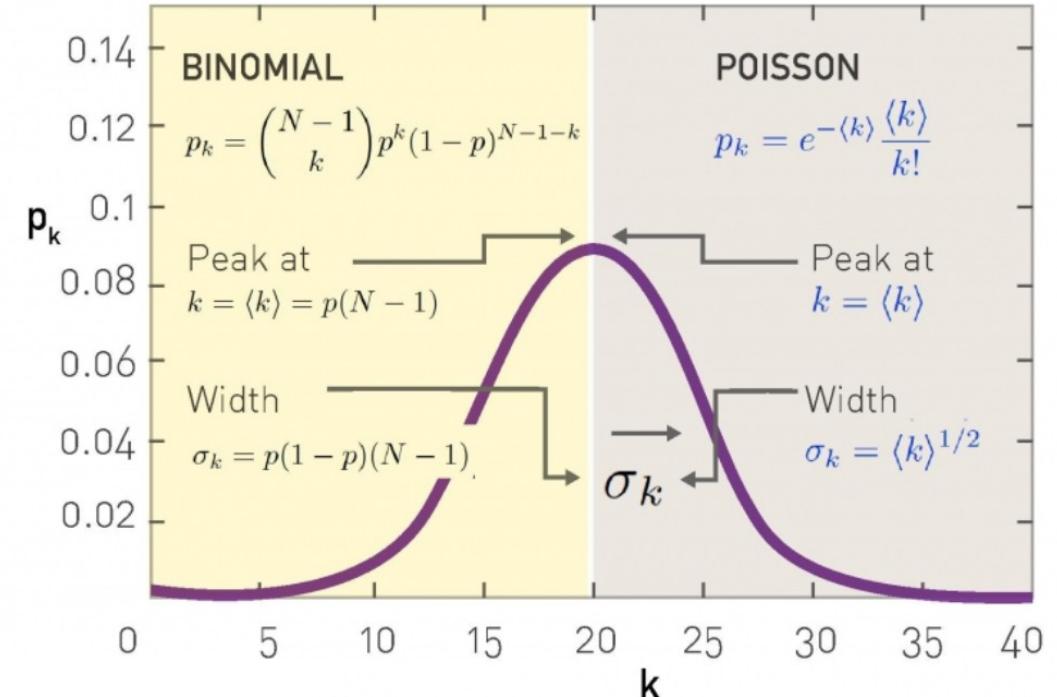
$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \langle x \rangle)^2}$$

*Distribution of  $x$ :*

$$p_x = \frac{1}{N} \sum_i \delta_{x, x_i}$$

where  $p_x$  follows

$$\sum_i p_x = 1 \quad \left( \int p_x dx = 1 \right)$$



Barabasi book on network science

# **1<sup>st</sup> model: Erdos-Renyi random network model**

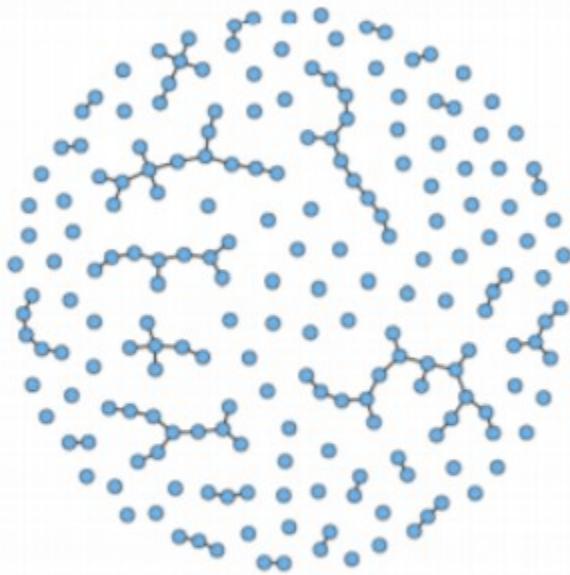
**Erdos-Renyi networks**

**$G(N,p)$ , N number of nodes  
p is probability to have link between nodes**

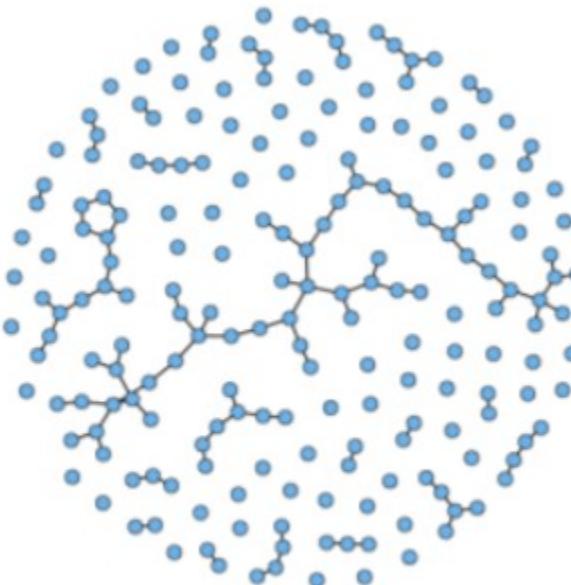
# 1<sup>st</sup> model: Erdos-Renyi network model

Erdos-Renyi networks

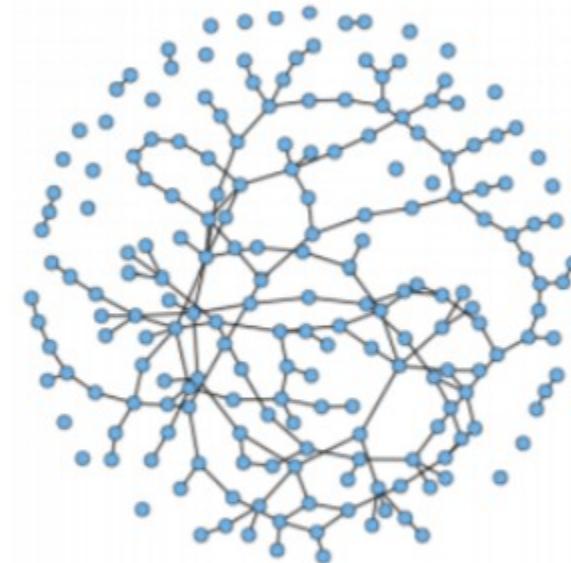
$G(N,p)$ , N number of nodes  
p is probability to have link between nodes



$$p < p_c$$



$$p = p_c$$



$$p > p_c$$

## **2<sup>nd</sup> model: Watts-Strogatz small-world model**

**Watts-Strogatz networks**

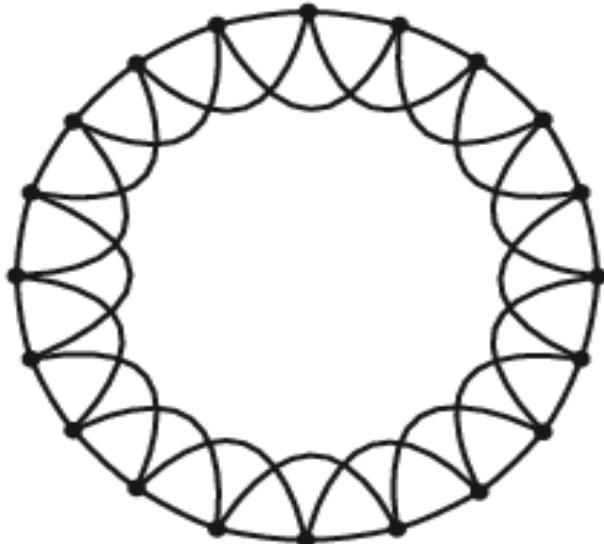
**$G(N,k,q)$ , N number of nodes, k-average degree, q is probability of rewiring**

## 2<sup>nd</sup> model: Watts-Strogarz small-world model

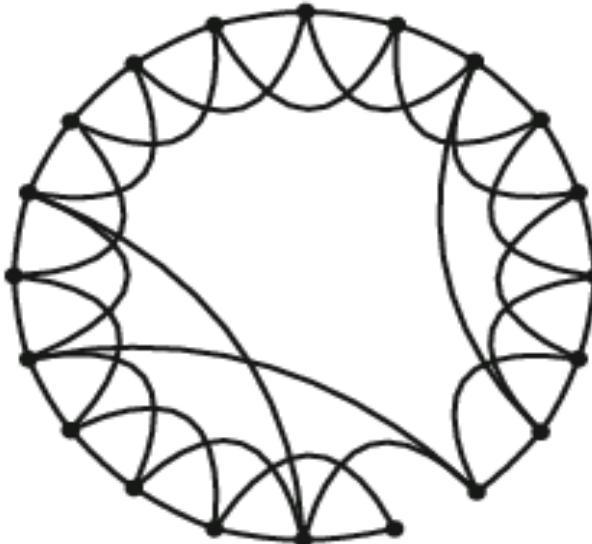
Erdos-Renyi networks

$G(N,p)$ , N number of nodes  
is probability to have link between nodes

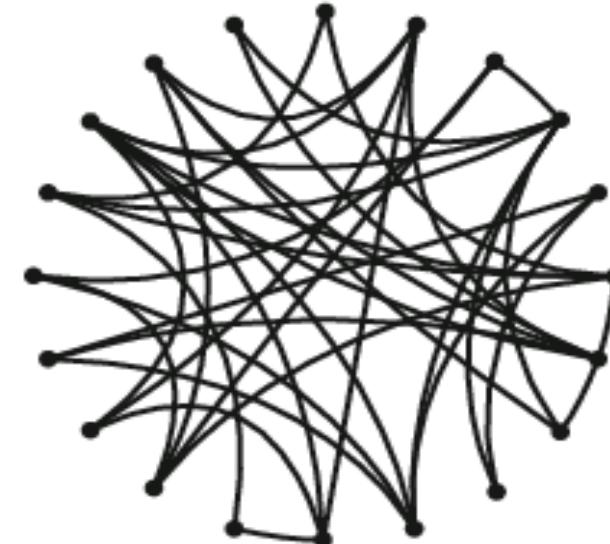
**Regular Network**



**Small-world Network**



**Random Network**

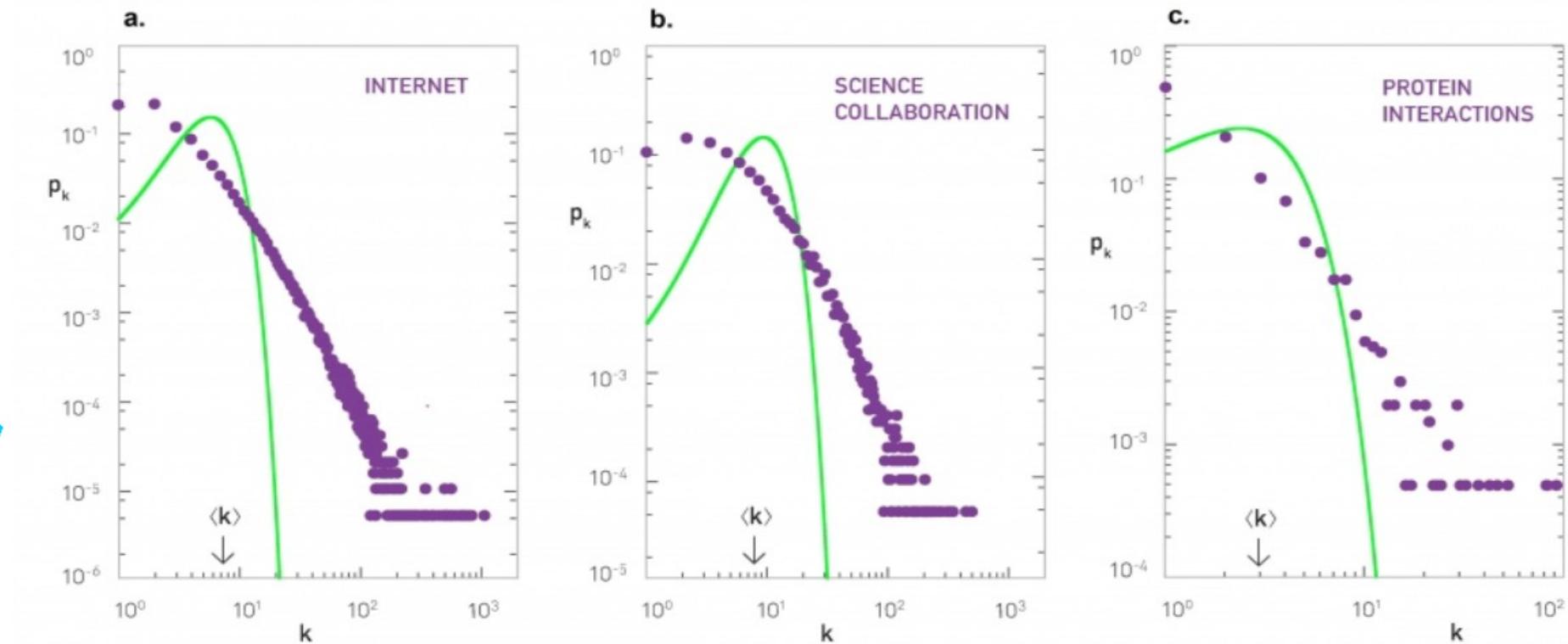
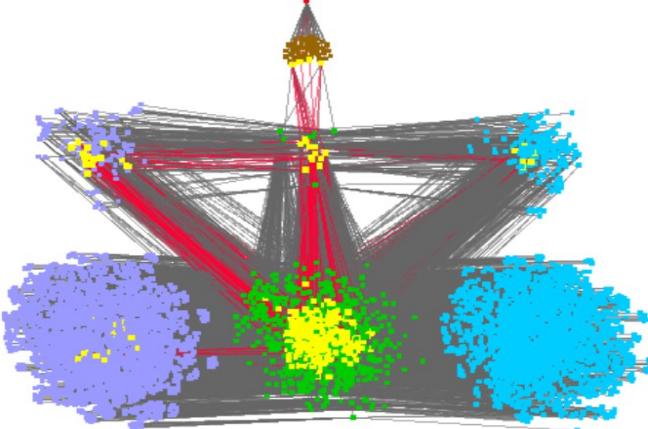


$\beta = 0$



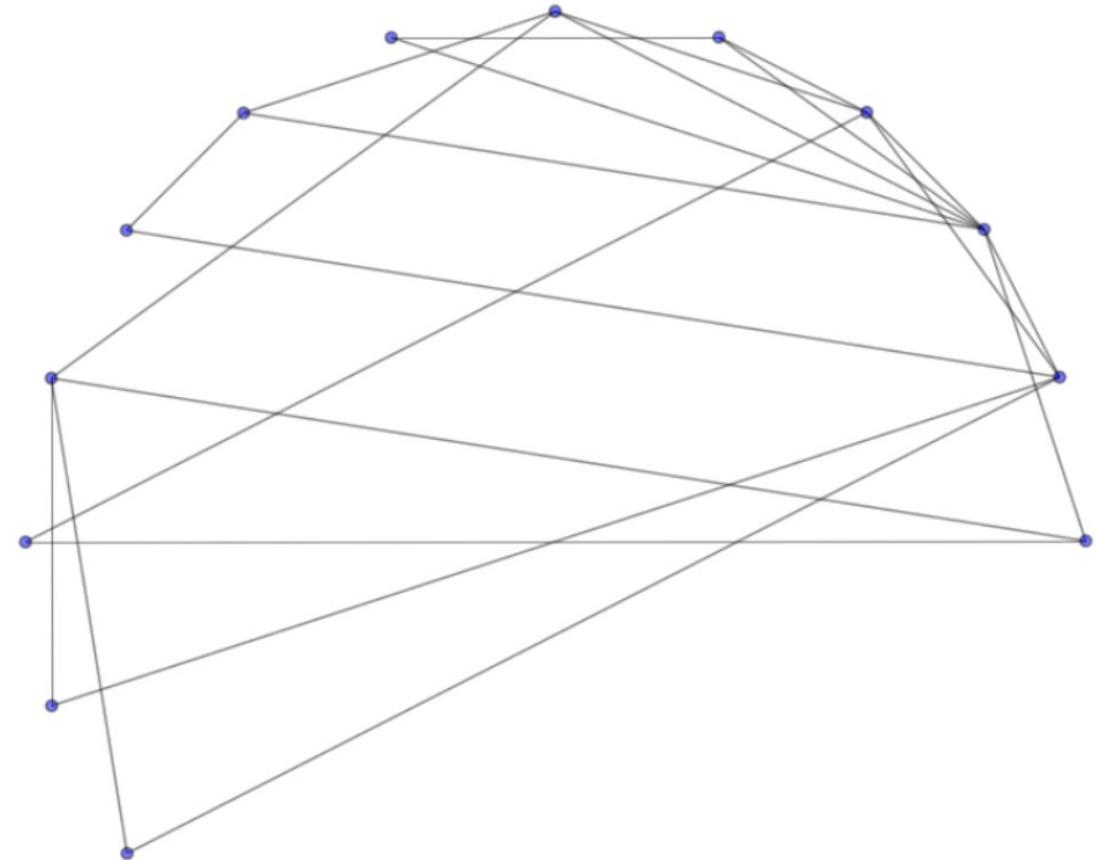
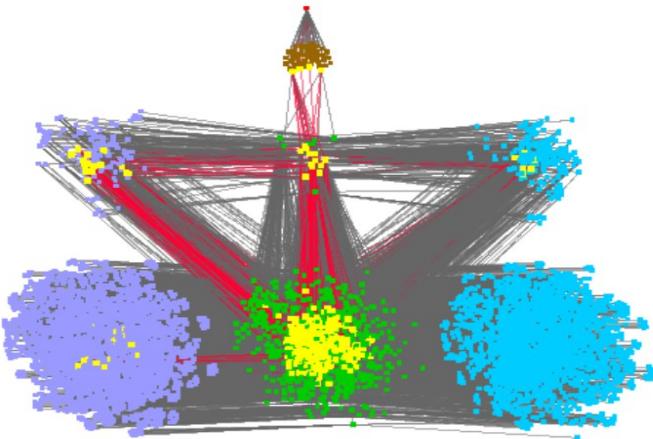
$\beta = 1$

# 3<sup>rd</sup> model: Barabasi-Albert network model



Barabasi book on network science  
Cytoscape software

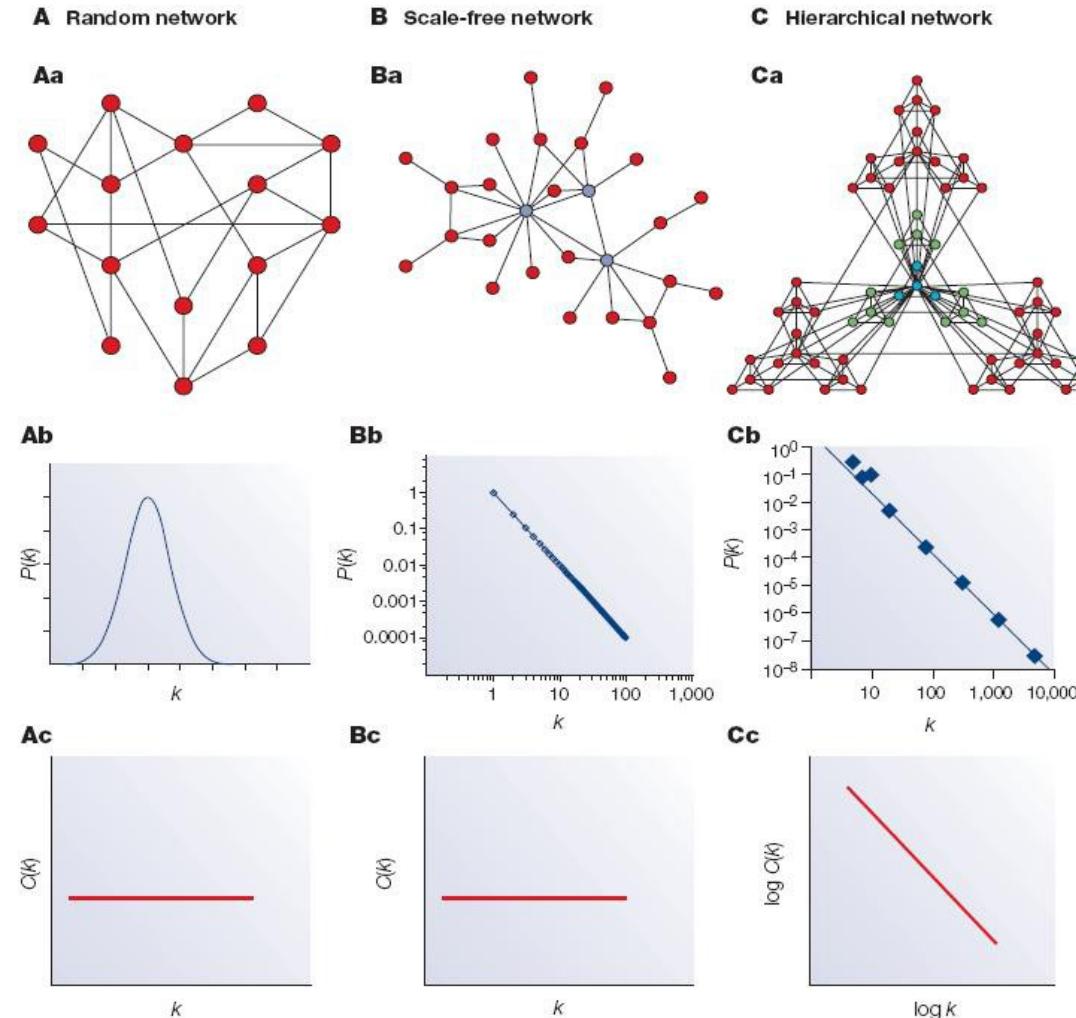
# Scale-free networks: how to generate them?



Barabasi book on network science  
Cytoscape software

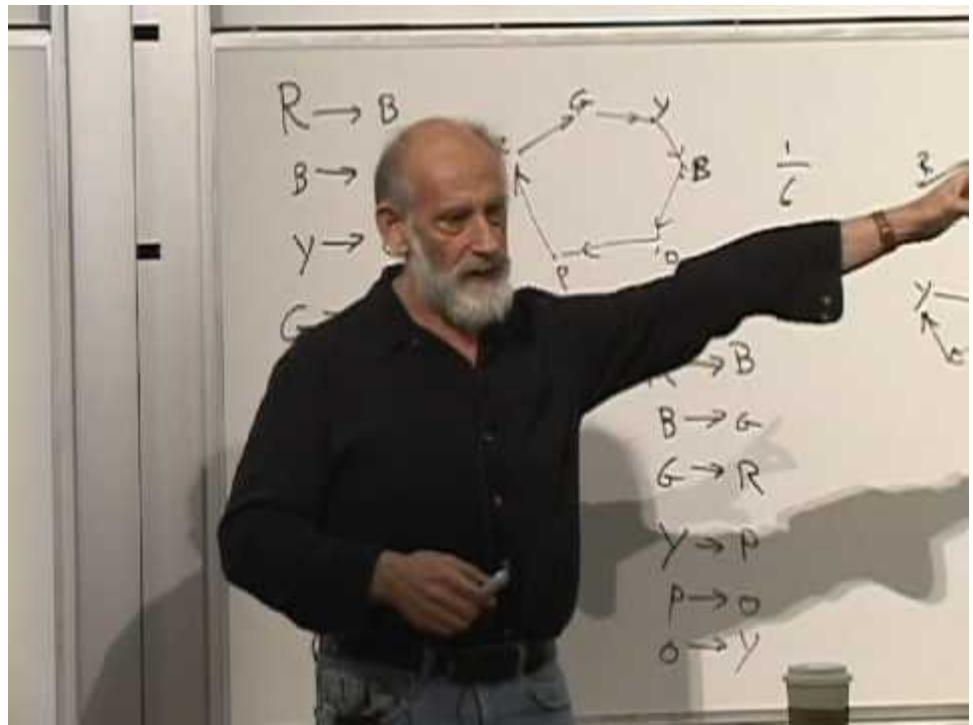
**Practical part:**  
**are properties of random networks random?**

# Practical part: are properties of random networks random?



# **Network theory and statistical mechanics**

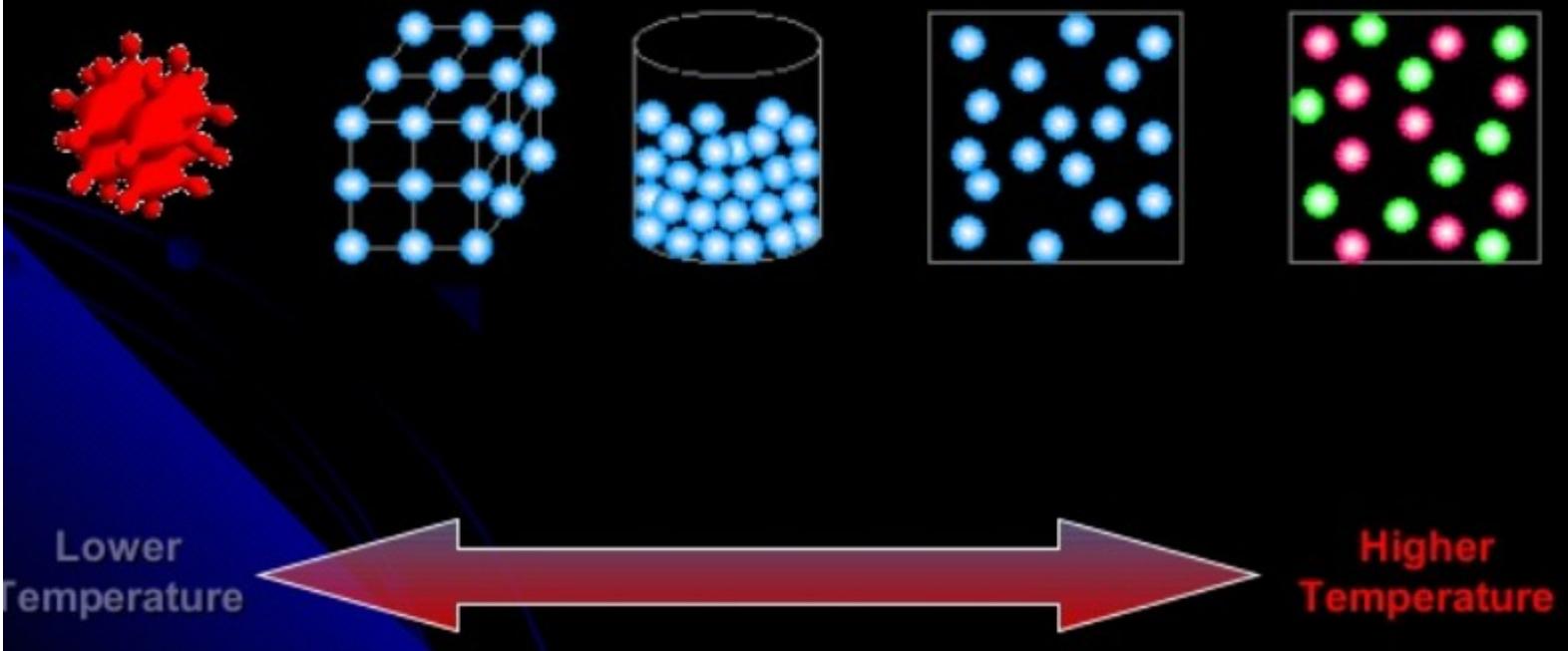
# Statistical mechanics models



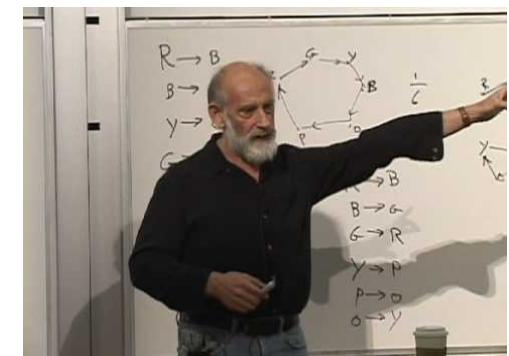
Stanford course on statistical  
mechanics

# Statistical mechanics models

- Super-cooled matter (near absolute zero or  $-273^{\circ}\text{C}$ ) forms another state



Stanford course on statistical mechanics



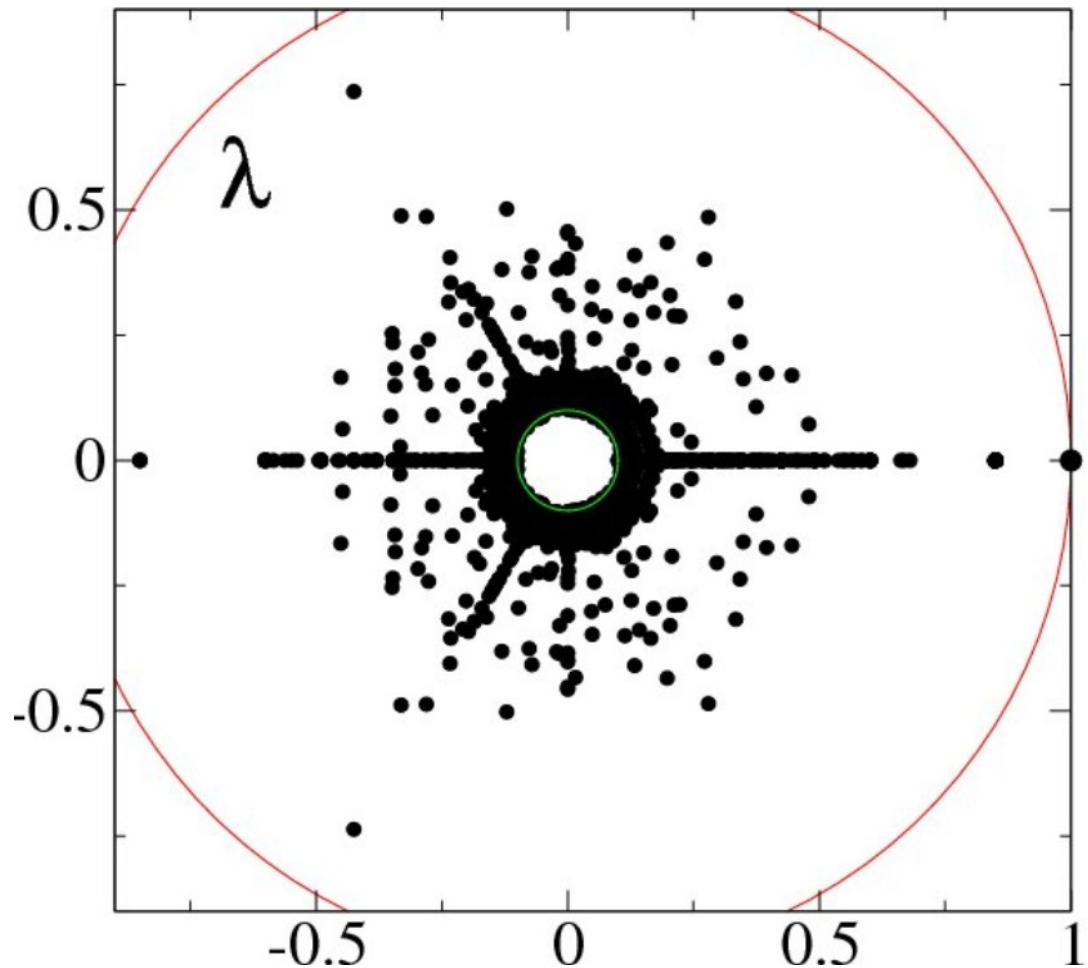
# **Network theory and matrix theory**

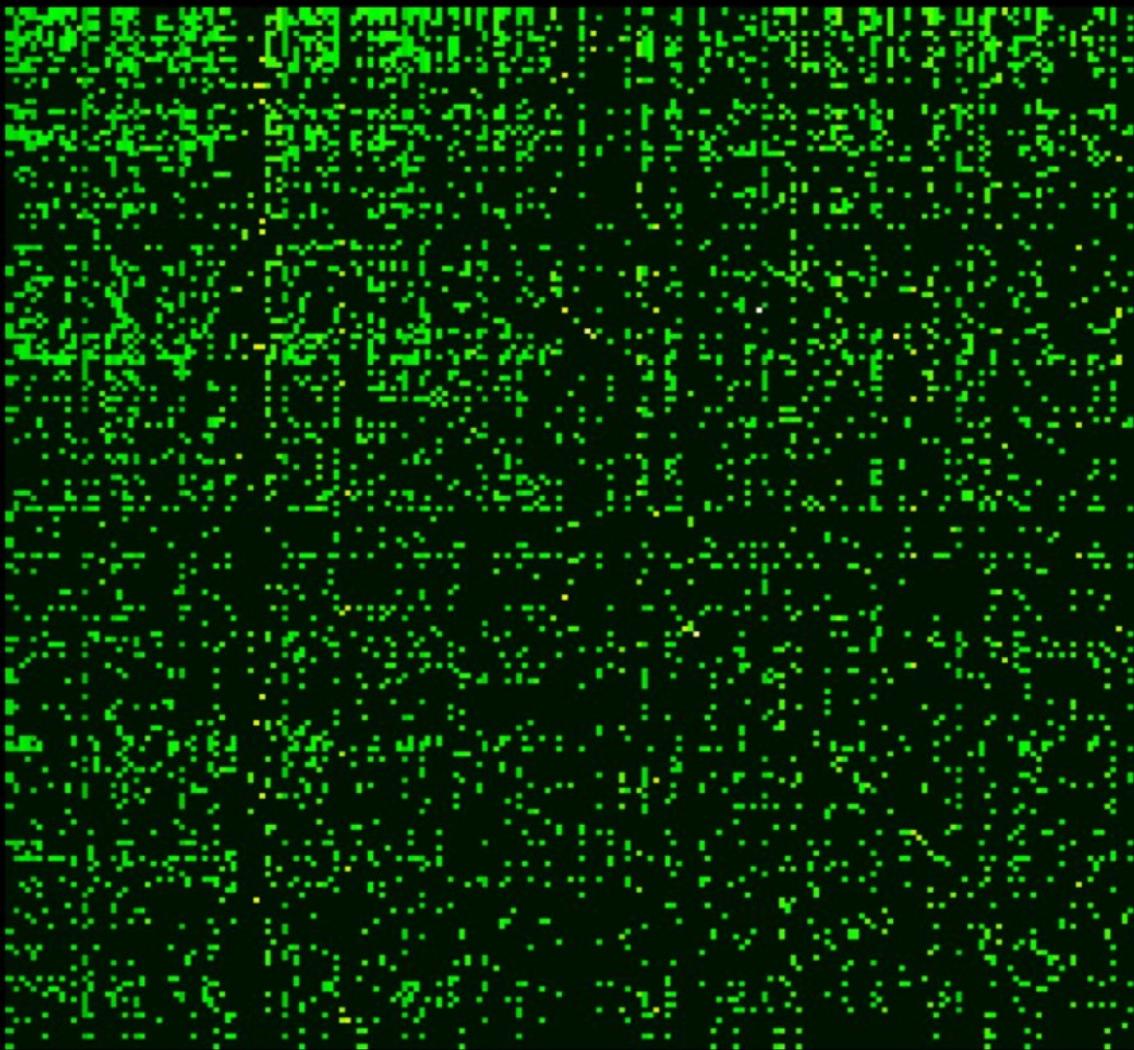
# Matrix theory

matrix representation of unweighted undirected graph

# Matrix theory

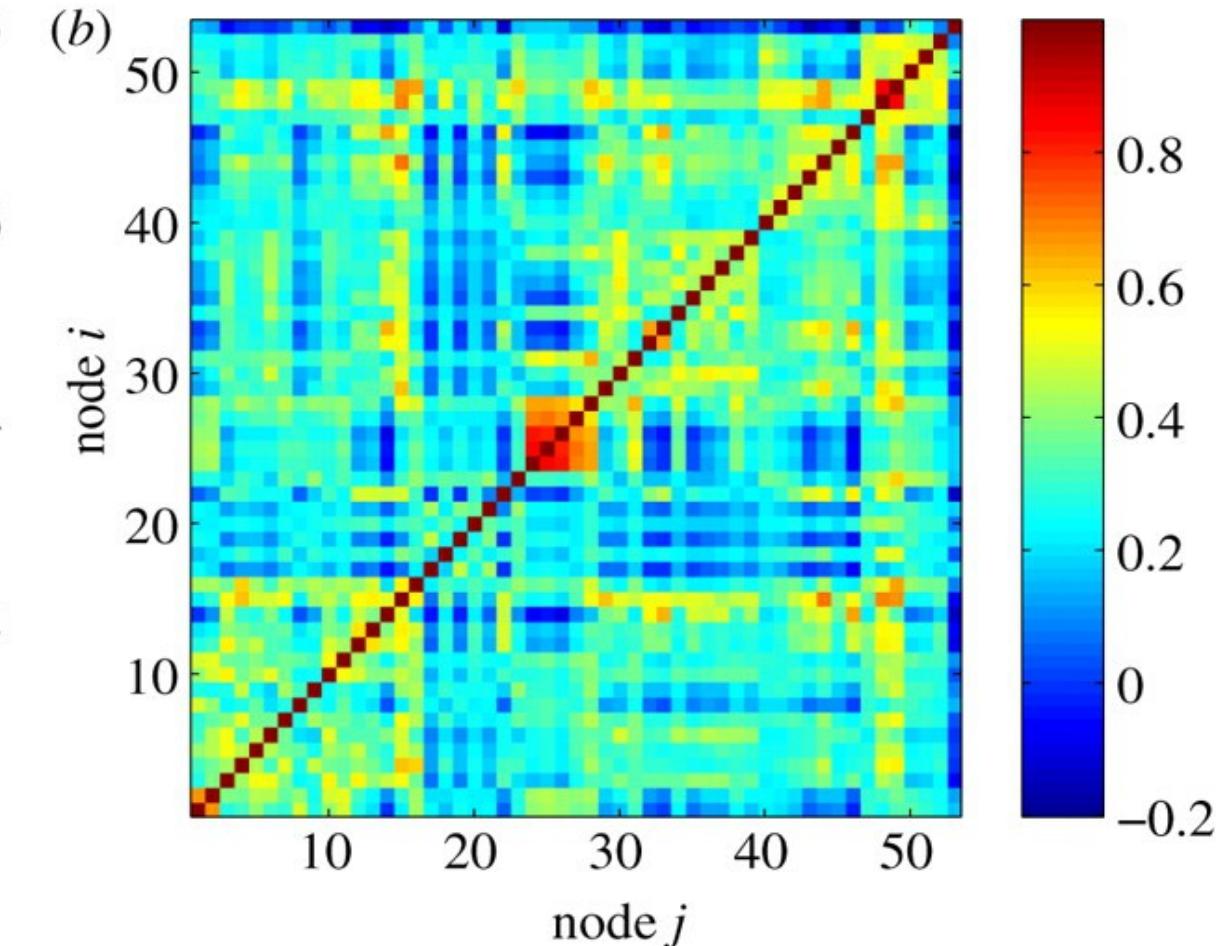
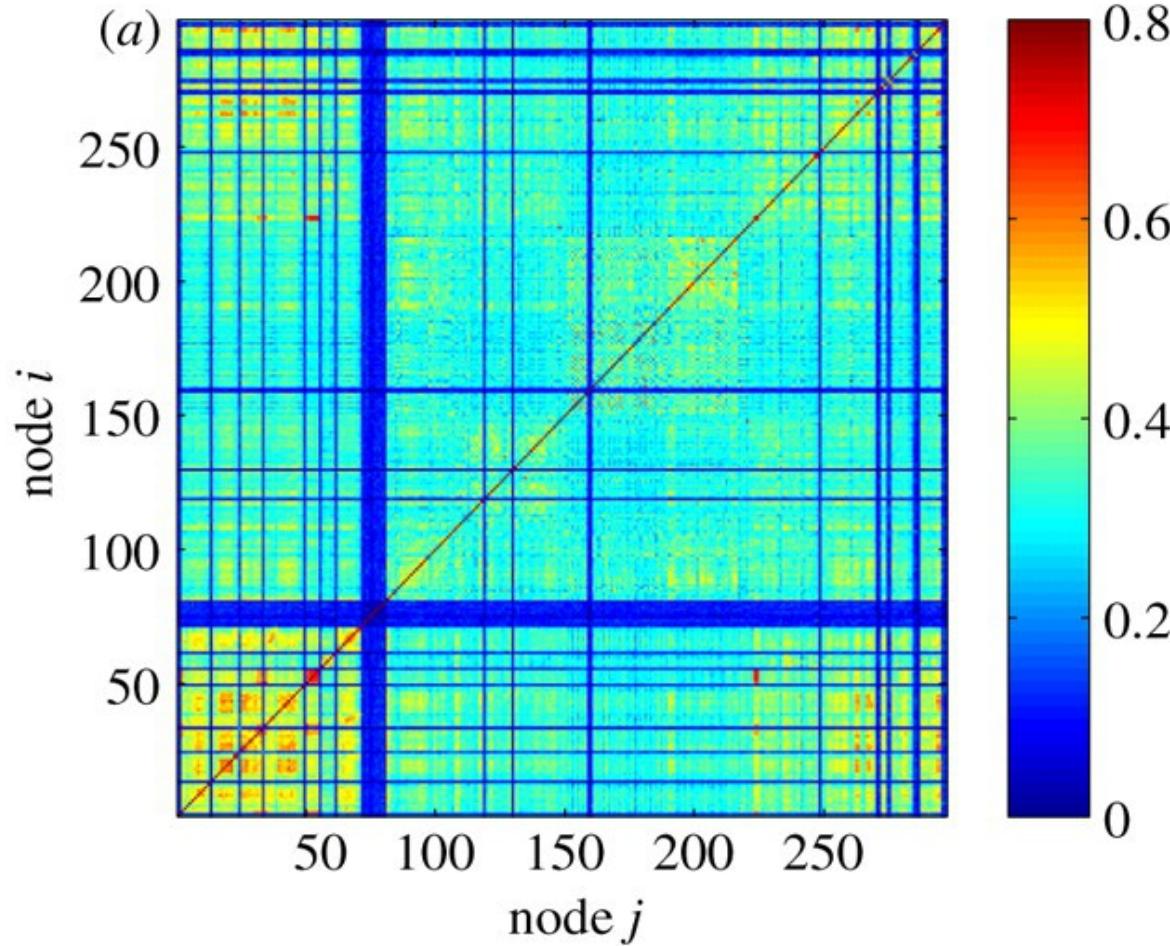
	a	b	c	d	e	f	g	h
a	0	1	0	0	1	0	1	0
b	1	0	1	0	0	0	0	1
c	0	1	0	1	0	1	1	0
d	0	0	1	0	1	0	0	0
e	1	0	0	1	0	0	0	0
f	0	0	1	0	0	0	1	0
g	1	0	1	0	0	0	0	0
h	0	1	0	0	0	0	0	0





**Google matrix of Wikipedia articles network, written in the bases of PageRank index; fragment of top 200 X 200 matrix elements is shown, total size N=3282257 (from [Wikipedia])**

# Matrix theory



# Literature overview

<http://networksciencebook.com/>

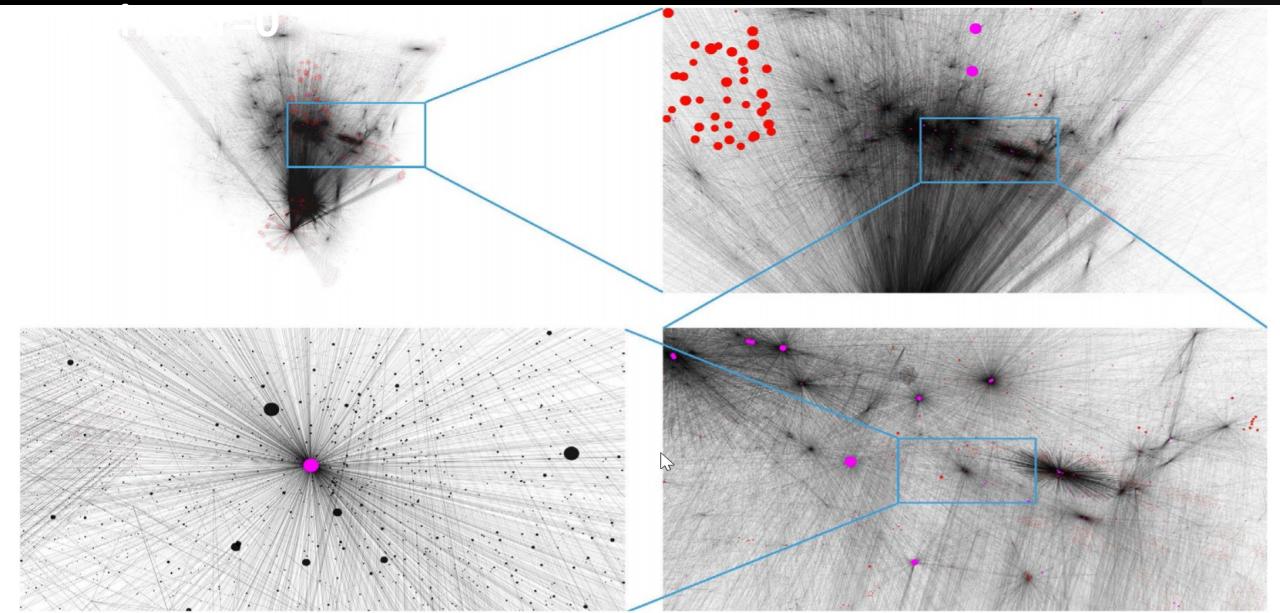
<https://www.barabasilab.com>

<http://networkrepository.com/graph-vis.php>

<http://www.complexity-explorables.org/explorables/neighbors/>

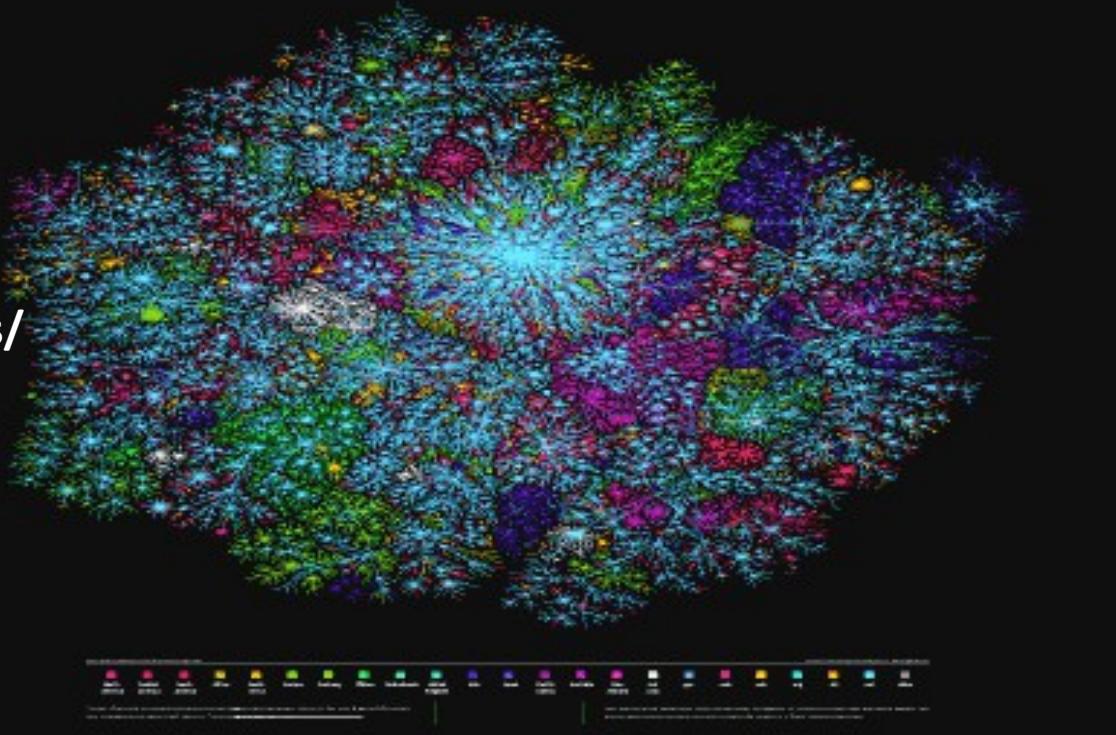
Spectral methods for complex systems online lectures

<https://sites.google.com/view/cssm/online-courses?>



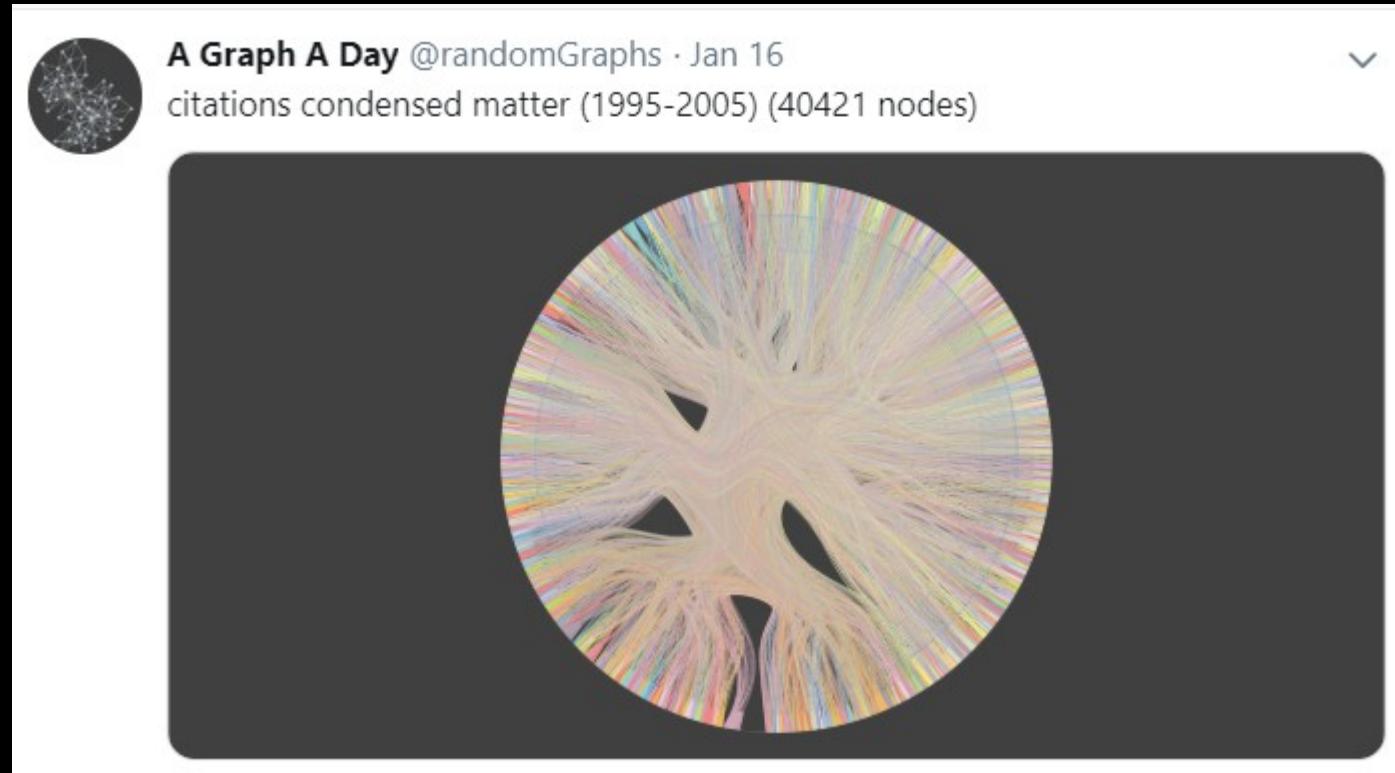
<https://sites.google.com/a/binghamton.edu/netscied/teaching-learning/network-concepts>

THE WHOLE INTERNET

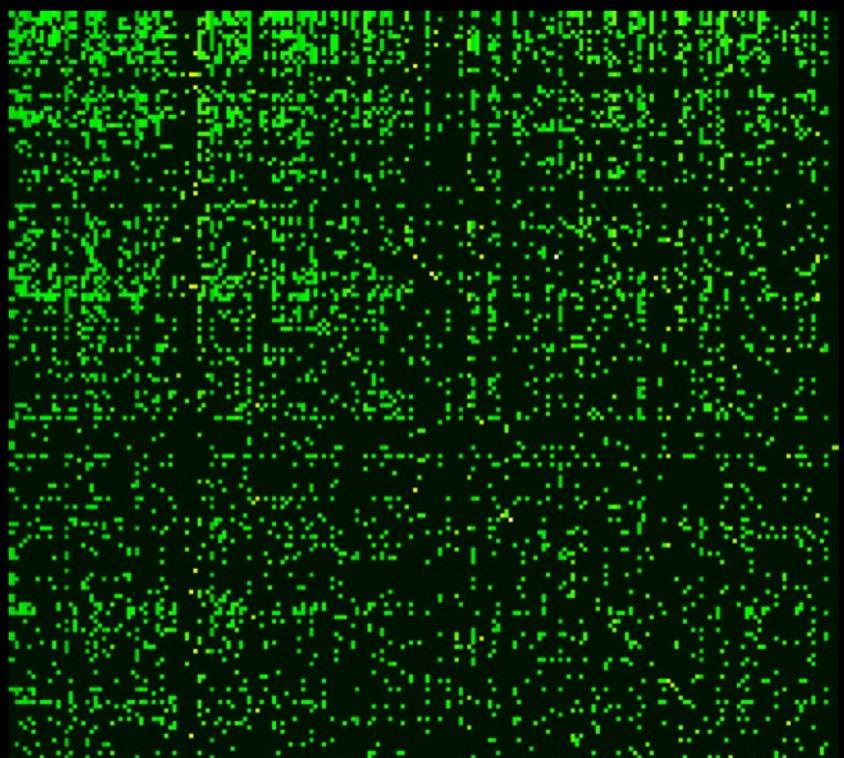


<https://snap.stanford.edu/data>

## Which methods are used in network theory?



**Which methods are used  
in network theory?**



**Hands-on part:**

go to <http://networkrepository.com>

www.wolfram.com/wolfram-u/catalog/dat021/

Products & Services ▾ Technologies ▾ Solutions ▾ Support & Learning ▾

# Getting started with Graphs and Networks

The image shows a screenshot of a Wolfram website page titled "Getting started with Graphs and Networks". The page features four cards illustrating various network concepts:

- In-component vs Out-component:** Shows two network components: one labeled "In-component" and another labeled "Out-component".
- PriceGraphDistribution:** Shows a network structure next to a distribution plot.
- Complex Network Visualization:** Shows a dense network graph with nodes numbered 1 through 30.
- Comparison:** Shows a spherical network structure on the left and a linear chain of nodes on the right.

# Dynamical models on networks

**Hands-on part:**

**go to <http://networkrepository.com>**

# Models on networks

Spreading dynamics:

Modelling crowded dynamics:

Data analysis:



2.16

D.Grenbenkov, LT. PRE (2017)  
P. Holme, LT, New Jour. Physics (2018)  
M.Lindner, N.Molkenthin, LT, R.Donner Springer (2019)