Supplemental Material for Paper "Fairness-Constrained Throughput Optimization for Coexistence of WiFi and Duty-Cycle 5G NR-U"

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Appendix A: Proof of Theorem 1

Proof. With the throughput fairness constraint as given in (12), the total effective throughput of the network can be written as

$$\hat{\eta}_{out} = 2\hat{\eta}_{out}^{(NR)} = 2\left(\beta_{DC} - \left(1 - \frac{\lfloor\delta\rfloor}{2\delta}\right)(\lfloor\delta\rfloor + 1)\frac{\sigma^{(NR)}}{T_{DC}}\right), \tag{A-1}$$

which is a monotonically increasing function of β_{DC}. Therefore, the optimization problem of (11) is equivalent to the maximization of β_{DC} by adjusting W, which can be obtained as

$$\max_{W}\ \beta_{DC} = 1 - \frac{(\tau_T - 1)\sigma^{(W)}}{2T_{DC}} + \frac{\left(1 - \frac{\lfloor\delta\rfloor}{2\delta}\right)(\lfloor\delta\rfloor + 1)\frac{\sigma^{(NR)}}{T_{DC}} - 1 + \frac{(\tau_T - 1)\sigma^{(W)}}{2T_{DC}}}{1 - \tau_T\alpha_W p_W \ln p_W \cdot \frac{L_P}{\sigma^{(W)} R \tau_T}}. \tag{A-2}$$

by substituting (3) and (9) into (12). It can be clearly seen from (A-2) that only the third term in Eq. (A-2) is dependent on W. As the nominator of the third term in (A-2) is negative than zero, the optimization problem in (A-2) is therefore equivalent to the maximization of its denominator, which is given by

$$\hat{\lambda}_{\max} = \max_{W} \quad \frac{-\tau_T p_W \ln p_W}{1 + \tau_F - \tau_F p_W - (\tau_T - \tau_F)\, p_W \ln p_W}, \tag{A-3}$$

by combining (5). By following a similar derivation in [13], we have

$$\hat{\lambda}_{\max} = \frac{-\mathbb{W}_0\left(-\frac{1}{e(1+1/\tau_F)}\right)}{\tau_F - (\tau_T - \tau_F)\,\mathbb{W}_0\left(-\frac{1}{e(1+1/\tau_F)}\right)}, \tag{A-4}$$

which is achieved when

$$p_A^* = -(1 + 1/\tau_F)\mathbb{W}_0\left(-\frac{1}{e(1 + 1/\tau_F)}\right). \tag{A-5}$$

Eq. (14) can then be obtained by substituting (A-4) into (A-2). Eq. (15) can then be obtained by substituting (A-5) into (4). Eq. (13) can then be obtained by substituting Eq. (14) into (A-1). □

APPENDIX B: PROOF OF THEOREM 2

Proof. By substituting (3) and (10) into (16)-(17), the 3GPP fairness constrained throughput optimization problem can be written as

$$\hat{\eta}_{\max}^{GF} = \max_{W,\beta_{DC}} \frac{n^{(W)}}{n^{(W)}+n^{(NR)}} \hat{\eta}_{\max}^{WiFi} + \beta_{DC} - \left(1 - \frac{\lfloor\delta\rfloor}{2\delta}\right)(\lfloor\delta\rfloor + 1)\frac{\sigma^{(NR)}}{T_{DC}}, \tag{B-1}$$

$$s.t. \quad \beta_{DC} \leq 1 - \frac{(\tau_T - 1)\sigma^{(W)}}{2T_{DC}} - \frac{\hat{\eta}_{\max}^{WiFi} \cdot \dfrac{\sigma^{(W)} R \tau_T}{L_P} \cdot \dfrac{n^{(W)}}{n^{(W)}+n^{(NR)}}}{-\tau_T \alpha_W p_W \ln p_W}. \tag{B-2}$$

It is clear that (B-1) is a monotonically increasing function of β_{DC}. To maximize β_{DC}, we can see that the denominator of the third item of (B-2) should be maximized, which is the same as Eq. (A-3) in the above Appendix A.

By substituting (A-3) and (18) into (B-2), the optimal duty cycle ratio of 5G NR-U is then given by

$$\beta_{DC,m}^{GF} = 1 - \frac{(\tau_T + 1)\,\sigma^{(W)}}{2T_{DC}} - \frac{n^{(W)}}{n^{(W)}+n^{(NR)}}. \tag{B-4}$$

Eq. (19) is derived by substituting $\beta_{DC} = \beta_{DC,m}^{GF}$ into (B-1). Eq. (21) can then be obtained by substituting (A-5) into (4).

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