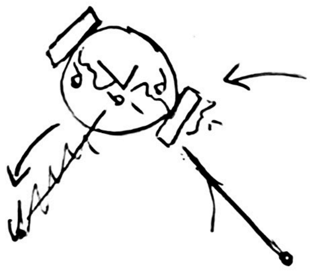
## Inverse Kinematics

Since there is no slipping there can be no velocity in the y direction so my velocity vector is:

$$U = \begin{bmatrix} V_x \\ \emptyset \\ \omega \end{bmatrix} \quad b/c \;\; V_y = \emptyset$$



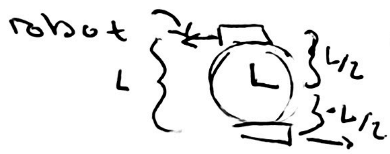
Since this is my diff drive bot I know that my velocity of each wheel is its velocity times the radius of my wheel (in the body frame)

so: $V_l = r\phi_r$ & $V_r = r\phi_l$ ★ ①

Now if there is a case of pure translation then $\phi_r = \phi_l$ & $V_x = V_{left} = V_{right}$ (moving in a straight line)
$(r\phi_l) \quad r(\phi_r)$

$$\therefore \quad \boxed{\phi_r = \phi_l = \frac{V_x}{r}} \text{ ★ ②}$$

For pure rotation I can use the geometry of my robot



& the fact that for the robot to be spinning I know the wheels are going in opposite directions.

This means $V_l = -V_r$ so if $V_l = -\frac{L}{2}\omega = -r\phi_r$

$$\phi_l = -\frac{L}{2}\frac{\omega}{r} \quad \text{then} \quad \phi_r = \frac{L}{2}\frac{\omega}{r} \text{ ★ ③}$$

Since this is a 2D vector space I can add equations 2 & 3 to get this:

$$\phi_l = \frac{V_x}{r} - \frac{L}{2}\frac{\omega}{r} \qquad \phi_r = \frac{V_x}{r} + \frac{L}{2}\frac{\omega}{r} \text{ ★ ④}$$

~~In $Y_R$-$\dot{\theta}_y$ =~~

# Forward Kinematics

Using equations four from calculating inverse kinematics I can use algebra to find $V_x$ & $w$ for forward kinematics:

$$\phi_r = \frac{V_x}{r} + \frac{L}{2}\frac{\omega}{r} \qquad \phi_L = \frac{V_x}{r} - \frac{L}{2}\frac{\omega}{r}$$

$$\left(\phi_r - \frac{V_x}{r}\right) = \frac{L}{2}\frac{\omega}{r}$$

$$\phi_L = \frac{V_x}{r} - \cancel{\frac{L}{2}}\left(\phi_r - \frac{V_x}{r}\right)$$

$$\phi_L = \frac{V_x}{r} - \phi_r + \frac{V_x}{r}$$

$$(\phi_L + \phi_r) = \frac{2V_x}{r} \Rightarrow V_x = \frac{r}{2}(\phi_L + \phi_R)^{*} \quad (5)$$

&

$$\phi_r - \left(\frac{L}{2}\frac{\omega}{r}\right) = \frac{V_x}{r}$$

$$\phi_L = \phi_r - \left(\frac{L}{2}\frac{\omega}{r}\right) - \frac{L}{2}\frac{\omega}{r}$$

$$\phi_L - \phi_r = -\frac{\cancel{2}L\omega}{\cancel{2}r} \Rightarrow \omega = \frac{L}{r}(\phi_r - \phi_L)^{*} \quad (6)$$

To go from 5 & 6 to calculations in the world frame for x, y, θ I use the following

$$V_{world} = \begin{bmatrix} V_x \cos\theta \\ V_x \sin\theta \\ \dot{\theta} \end{bmatrix} \quad {}^{*}(7)$$