On the New Stopping Criteria of Iterative Turbo Decoding by Using Decoding Threshold

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Abstract—**Although many stopping methods of iterative decoding have been discussed in the literature extensively, many of them only focus on the *solvable decoding* (information is enough for successful decoding). In this paper, we discuss the limitation of the decoding ability based on the extrinsic information transform (EXIT) chart. Then, we propose a new information measurement by using cross correlation to predict the decoding threshold. Moreover, we propose two early termination (ET) schemes (ET-I and ET-II) based on the predicted decoding threshold. The iterative decoding can stop in either high-signal-to-noise ratio (SNR) situations where the decoded bits are highly reliable (solvable decoding), or low-SNR situations where the decoder already has no capability to decode (unsolvable decoding). The simulation results show that the reduced iterations due to the ET-I scheme almost will not affect the SNR performance, and the ones due to the ET-II scheme can still satisfy the requirement of the specification. Based on our analysis and simulation results, we can further modify the conventional GENIE chart by considering the decoding threshold. By using our new ET concepts, the previous stopping techniques can also be modified to stop in low-SNR situations. The ET property for the iterative decoding can help reduce the unnecessary iterations, so as to save computational complexity and power consumptions in digital signal processors (DSPs) or application-specific integrated circuits (ASICs) in mobile handsets.**

Index Terms—**Decoding threshold, early termination (ET), extrinsic information transform (EXIT) chart, iterative decoding, turbo codes, turbo principle.**

I. Introduction

IN 1993, a new class of forward-error-correction (FEC) code, *turbo code*, was introduced by Berrou *et al.* [1], [2]. The turbo code is well known for its near-Shannon-limit superior decoding performance through iterative processes. Consequently, the third generation (3G) mobile wireless communication system standards [5] have adopted the turbo code as the FEC coding scheme. Furthermore, the method that the decoder performs with a "turbo" feedback has been generalized to "turbo-principle" [6]. It has been successfully applied to many detection/decoding problems [7]–[13].

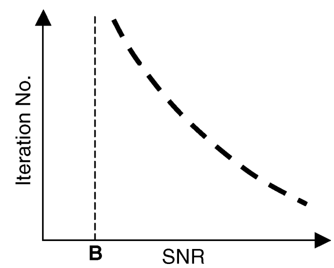

Fig. 1. Iteration number versus SNR.

For the turbo decoding, the smallest iteration number that the decoder can recovery information bits is increased as the channel SNR decreases, such as the bold-dashed line in Fig. 1. Thus, there exists a decoding threshold (B position in Fig. 1), where the decoder must spend infinite iterations to overcome the seriously noisy channel. Considering of the cost of the hardware implementations and power consumptions, several following early termination (ET) schemes are usually employed in practical very large scale integration (VLSI) designs.

- **Fixed iteration chart** [Fig. 2(a)]—The decoder will perform decoding with the maximum iteration under any signal-to-noise ratio (SNR), such as the bold-solid line in Fig. 2(a). The designer usually adopts the fixed decoding iteration number [A position in Fig. 2(a)] according to the requirement of system specifications.
- **Conventional stopping chart** [Fig. 2(b)]—The decoder will stop decoding (early terminate) in high-SNR situations when the decoded bits are highly reliable. In the noiseless (or high-SNR) environments, the decoder can easily complete the decoding. Instead of fixed-iteration decoding, the decoder can stop operating at early stage to save VLSI consumed power or digital signal processor (DSP) computational complexity.

In recent literatures [14]–[23], many stopping criteria for turbo decoders have been presented, and they can be categorized into two classes. One class is based on *soft-bit decisions*, such as cross entropy (CE) [14], absolute log-likelihood ratio (LLR) measurement [15], mean stimation (ME) [16], updated threshold [17], and *a priori* LLR measurement [18]. The other class is based on *hard-bit decisions*, such as sign–change ratio (SCR) [19], hard-decision-aided (HDA) [19], sign–difference ratio (SDR) [20], and improved HDA (IHDA) [21]. In addition, some methods are based on *extra checking policies*, such as cyclic redundancy check (CRC) [22], and valid codeword check (VCW) [23].

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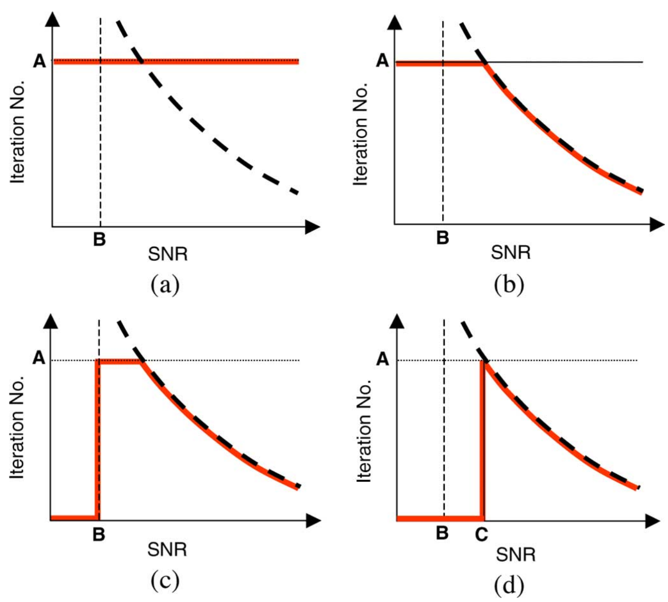


Fig. 2. Iteration number versus SNR. (a) Fixed-iteration chart. (b) Conventional stopping chart. (c) Proposed ET-I scheme. (d) Proposed ET-II scheme.

Most of the previous ET works may assume that the decoder works in a system-specified or a solvable environment. In many severe environments, the decoder can obtain the information streams correctly with the maximum iterations. For most iteratively computing designs, there usually exists an operating limit where any further iteration results in very little improvement. In this paper, we assume the decoder may encounter the unsolvable situation (there is too little information for the decoder to work correctly). Thus, unlimited decoding is still impossible to recover the information. In this case, it is important to derive an effective criterion to stop the iterative processes, so as to prevent from unnecessary computations and decoding delay. However, it is difficult to judge how many iterations of decoding are enough and the limitation of the decoder. In this paper, we focus mainly on how to find a suitable decoding threshold based on the limit of VLSI implementations (e.g., fixed-iteration decoding) and system specifications. Thus, we applied the extrinsic information transform (EXIT) chart [25]–[27] to obtain the decoding threshold. In addition to the original information measurements (SNR [25], [26] and mutual information [26]) in the EXIT chart, we first propose a new measurement by using *cross correlation* to predict the decoding threshold.

In the following, we propose two new ET schemes (ET-I and ET-II) for low-SNR situations where *the decoder no longer has the capability to decode the incoming data*:

- **ET-I scheme** [Fig. 2(c)]—The ET-I scheme is based on the (infinite-iteration) decoding threshold, obtained by the EXIT chart. That is, for any SNR lower than point B, the decoder can stop decoding immediately since it is already below the theoretical bound for correctly decoding.
- **ET-II scheme** [Fig. 2(d)]—The ET-II scheme is based on the fixed-iteration decoding threshold [C position in Fig. 2(d)]. Since the iterative operations are limited by considering the hardware implementation, the ET-II scheme adopts the fixed-iteration decoding threshold (C position) instead of infinite-iteration one (B position).

Based on the ET-I and ET-II concepts, we propose a new early termination technique called measurement of reliability (MOR). By using the new ET concepts, the previous techniques can also be modified to stop in low-SNR situations, such as modified-SCR (M-SCR), modified-SDR (M-SDR), and modified-ME (M-ME). Moreover, for the hardware implementation, the MOR technique just uses part of overall frame in stage testing. We also adopt the stage-testing concept in the previous techniques, including of modified-HDA (M-HDA) and modified-IHDA (M-IHDA).

In the wireless or mobile communication systems, because the link performance may be limited by the potentially severe channels, the error correcting codes and retransmission techniques are usually employed. For example, the 3G wireless [28] systems combine automatic repeat request (ARQ) and FEC, called hybrid-ARQ technique, in its physical layer specification. Moreover, the WiMAX/802.16 [29], [30] systems also incorporate the hybrid-ARQ technique into their physical layers. For our proposed schemes, if the decoder cannot work, the receiver can send an ARQ message to the transmitter. Meanwhile, the decoder can be terminated earlier to save the energy of the handset for the extension of the standby time. Thus, our proposed ET schemes can reduce the computational complexity of the DSP processors and/or the power consumptions of the decoder in implementing the iterative decoding in mobile handsets.

The rest of this paper is organized as follows. In Section II, the information measurements are introduced. In Section III, we propose several stopping approaches. In Section IV, we perform extensive computer simulations to validate our arguments. In Section V, we draw the conclusions.

II. Extrinsic Information Transfer Chart

The conventional turbo decoder is composed of two component decoders. Based on iteratively decoding, the input *a priori* LLR of one component decoder comes from the output extrinsic LLR of the other component decoder [1], [2]. Literature [25]–[27] has shown that the EXIT chart of single component decoder can characterize the convergence of the turbo decoder. Fig. 3 shows the EXIT flow, which uses the Gaussian-approximated LLR to measure the relationship between the input and the output of the component decoder.

A. Gaussian-Approximated LLR

For soft-in–soft-out decoding with Bahl–Cocke–Jelinik–Raviv (BCJR) algorithm, the *a priori* information L_a can be modeled by applying an independent Gaussian random variable n_a. The variance and mean of n_a are σ_a^2 and zero. Thus, the variance of L_a is σ_a^2 and the mean of L_a is corresponding to the known transmitted systematic bits x [25]

$$L_a \equiv \mu_a x + n_a \tag{1}$$

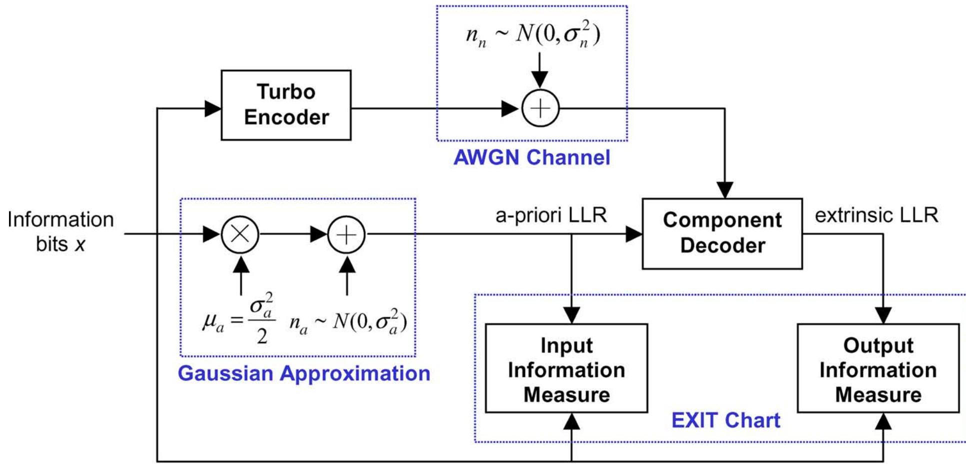


Fig. 3. EXIT diagrams [26].

where

$$\mu_a = \frac{\sigma_a^2}{2}. \tag{2}$$

Then, the probability density function (pdf) can be written as

$$p(L_a|x=\mu) = \frac{1}{\sqrt{2\pi}\sigma_a} e^{-\frac{\left(L_a - \frac{\sigma_a^2}{2}\mu\right)^2}{2\sigma_a^2}}. \tag{3}$$

B. SNR Measurement

To measure the information, let the information bits be the zero sequence. Then, the SNR ratio [25] for the random variable L_a can be computed by

$$S(L_a) \equiv \frac{E[L_a]^2}{Var[L_a]}. \tag{4}$$

Note that the SNR of L_a is different to the channel SNR.

C. Mutual Information Measurement

The mutual information measurement $I(x;L_a)$ [26] is another way to observe the relationship between transmitted systematic bits x and L_a

$$\begin{aligned} I(x,L_a) \equiv & E\left[\log_2 \frac{p(L_a|x)}{P(L_a)}\right] \\ = & \frac{1}{2}\int p(L_a|x=+1)\log_2 \frac{p(L_a|x=+1)}{P(L_a)} dL_a \\ & + \frac{1}{2}\int p(L_a|x=-1)\log_2 \frac{p(L_a|=-1)}{P(L_a)} dL_a. \end{aligned} \tag{5}$$

Then, the mutual information can be obtained by the histogram measurement of $p(L_a|x)$. Furthermore, from (3), we have

$$\begin{aligned} \frac{p(L_a|x=+1)}{P(L_a)} &= \frac{2}{1+e^{-L_a}} \\ \frac{p(L_a|x=-1)}{P(L_a)} &= \frac{2}{1+e^{L_a}}. \end{aligned} \tag{6}$$

Then, (5) can be written as

$$I(x;L_a) = E\left[\log_2 \frac{2}{1+e^{-xL_a}}\right]. \tag{7}$$

Thus, the mutual information can also be obtained by the time-averaging computing.

D. Proposed Cross-Correlation Measurement

For the iterative decoding, the *a priori* probability $p(x)$ is updated by the extrinsic information

$$\begin{aligned} p(x=+1|L_a) &= \frac{e^{L_a}}{1+e^{L_a}} \\ p(x=-1|L_a) &= \frac{e^{L_a}}{1+e^{L_a}}. \end{aligned} \tag{8}$$

Thus, we propose a new information measurement based on the estimation of systematic bits $\hat{x}$. That is

$$\hat{x} = (+1)p(x=+1|L_a) + (-1)p(x=-1|L_a) = \frac{e^{L_a}-1}{1+e^{L_a}}. \tag{9}$$

The new information measurement can be computed by the cross correlation between the estimated systematic bits $\hat{x}$ and the known x

$$C(x;\hat{x}) \equiv E[x\hat{x}] = E\left[x\frac{e^{L_a}-1}{1+e^{L_a}}\right]. \tag{10}$$

Moreover, (9) can be expressed as

$$\hat{x} = \frac{p(L_a|x=+1) - p(L_a|x=-1)}{2p(L_a)}. \tag{11}$$

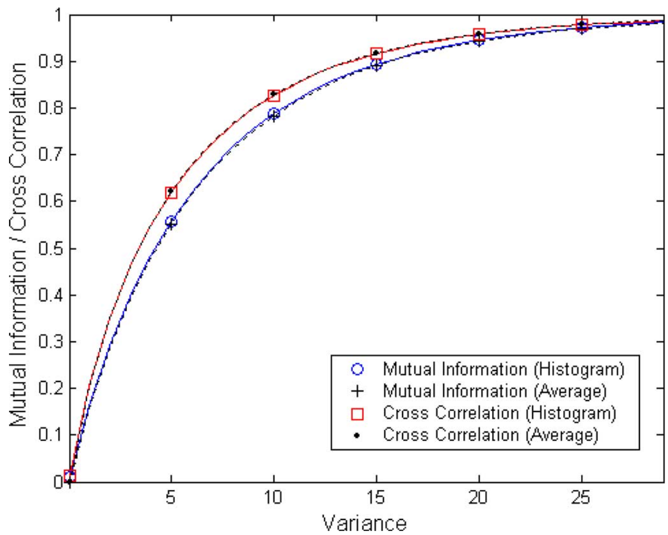


Fig. 4. Simulations for information measurement versus variance σ_a^2.

Thus, $C(x;\hat{x})$ can be formulated in another form as

$$\begin{aligned}
C(x;\hat{x}) &= \iint x \frac{p(L_a|x=+1) - p(L_a|x=-1)}{2p(L_a)} \\
&\quad \times p(x, L_a) dx dl_a \\
&= \frac{1}{2}\int p(L_a|x=+1) \\
&\qquad \times \frac{p(L_a|x=+1) - p(L_a|x=-1)}{2p(L_a)} dL_a \\
&\quad + \frac{1}{2}\int p(L_a|x=-1) \\
&\qquad \times \frac{p(L_a|x=-1) - p(L_a| = +1)}{2p(L_a)} dL_a \\
&= \frac{1}{2}\int p(L_a|x=+1)\left(\frac{p(L_a|x=+1)}{p(L_a)} - 1\right) dL_a \\
&\quad + \frac{1}{2}\int p(L_a|x=-1)\left(\frac{p(L_a|x=-1)}{p(L_a)} - 1\right) dL_a \\
&= E\left[\frac{p(L_a|x)}{p(L_a)} - 1\right]. \qquad (12)
\end{aligned}$$

Compared with the mutual information measurement, the cross correlation also can be computed by the histogram measurement of (12) and time average of (10). The simulations for the variance σ_a^2 are shown in Fig. 4. It shows the result of the histogram measurement is the same as the one of the time average. Also, the variation of the cross correlation is similar to the one of the mutual information. Both measurements are bounded between zero and one.

E. EXIT Chart

To observe the convergence of iterative decoding, the information-measurement transfer function of the component decoder is plotted twice. The axes of one decoder are swapped. Thus, the output of decoder 1 (DEC1) is the input of decoder 2 (DEC2), and the output of DEC2 is the input of DEC1. The exchange of extrinsic information can be visualized as a decoding trajectory. Next, the cross-correlation measurement is plotted for $E_b/N_0 = 1$ and 0.8 dB in Fig. 5. For the noisier channel (0.8 dB), the decoder must spend more iterations to pass the narrow tunnel. When the connection of both curves occurs, the decoding trajectory cannot pass through the tunnel. Thus, the decoding threshold can be obtained.

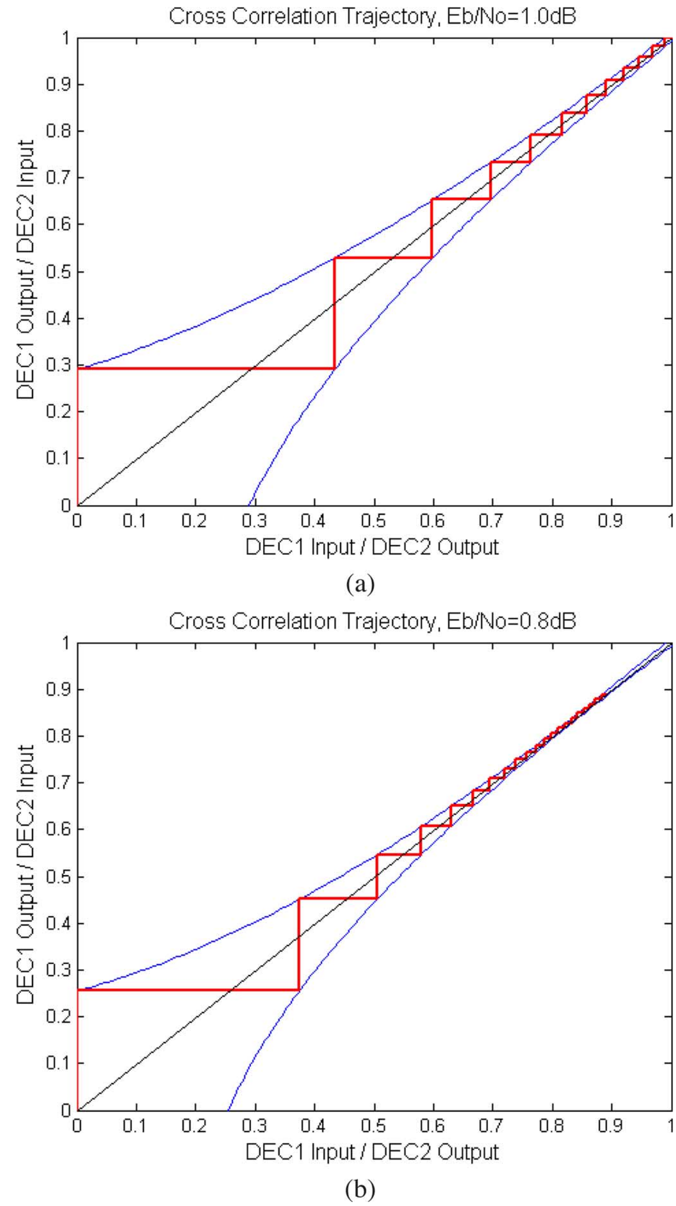


Fig. 5. Simulations for EXIT charts: (a) $E_b/N_0 = 1$ dB and (b) $E_b/N_0 = 0.8$ dB.

III. Proposed Early Termination Schemes

A. Proposed ET-I and ET-II Schemes

If the decoder has decoded successfully in high-SNR environments, it can stop operating at early stage without further iterations. On the other hand, the decoder should also stop unnecessary iterations in unsolvable environments. From the EXIT discussion in Section II, it was shown that the decoder indeed has the decoding limitation. Therefore, we propose two new ET schemes. In the ET-I scheme, the decoder can stop operating behind the (infinite-iteration) decoding threshold. Moreover, for

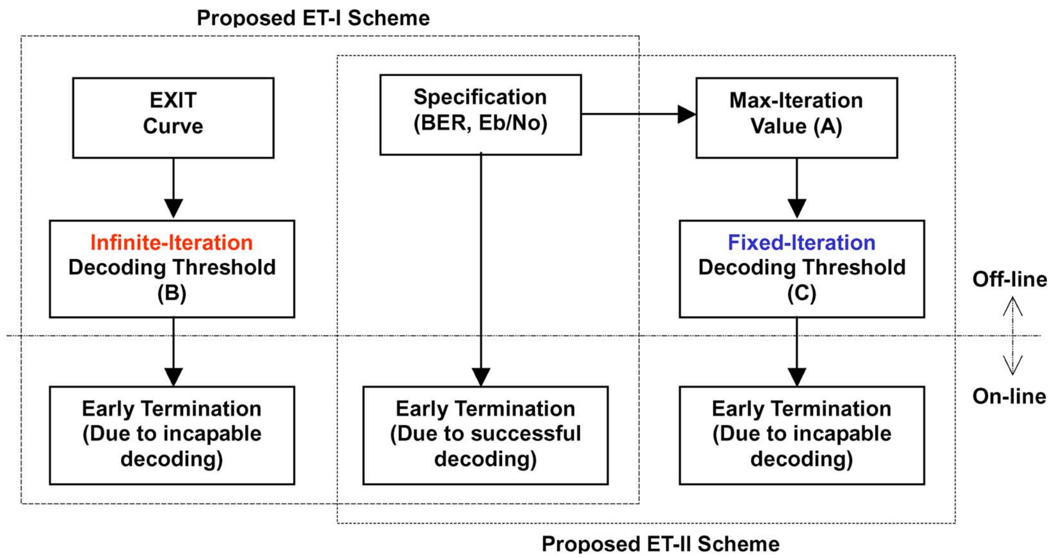


Fig. 6. Proposed early termination schemes (ET-I and ET-II).

the conventional hardware design flow, the decoder just operates with fixed iteration that can satisfy the system specification. In the ET-II scheme, the decoder can stop operating behind the fixed-iteration decoding threshold. Fig. 6 shows the design flow of our approaches, which are partitioned into offline and online computing blocks. The offline blocks are to obtain the ET thresholds according to the EXIT charts or specifications. The online blocks are to operate real time with the decoder.

B. Characteristics of ET Indicators

Consider a rate-$1/n$ turbo decoder that consists of two soft-output component decoders. Let $y_k = (y_k, 1, y_k, 2, \ldots, y_{k,n})$ be the output of additive white Gaussian noise (AWGN) channel and $\hat{u}_k$ be the estimate of the information bit u_k at time k. The mean and variance of the Gaussian noise is zero and σ^2. The frame length is N. At the ith iteration, $L_m^{(i)}(\hat{u}_k)$ and $\mathrm{Le}_m^{(i)}(\hat{u}_k)$ are the LLR and the extrinsic values delivered by the mth component decoder, respectively, for $m = 1, 2$. It has been shown in [1] and [2] that

$$L_1^{(i)}(\hat{u}_k) = \mathrm{Le}_2^{i-1}(\hat{u}_k) + \frac{2}{\sigma^2} y_{k,1} + \mathrm{Le}_1^{(i)}(\hat{u}_k) \tag{13}$$

$$L_2^{i}(\hat{u}_k) = \mathrm{Le}_1^{(i)}(\hat{u}_k) + \frac{2}{\sigma^2} y_{k,1} + \mathrm{Le}_2^{i}(\hat{u}_k). \tag{14}$$

Most of the ET schemes [14]–[23] use the $L_m^{(i)}(\hat{u}_k)$ and $\mathrm{Le}_m^{(i)}(\hat{u}_k)$ or their translations (the maximum, minimum, mean, hard-decision, and so on) to be the ET indicators. Fig. 7 shows the ET indicator variation with the channel SNR. In the high-SNR situation, all indicators can work well since they vary monotonically. However, in the low-SNR situation, the indicators of SCR, HDA, and IHDA are near quadratic variation, which interferes the judgment of the ET function. As a result, the SCR, HDA, and IHDA approaches are not suitable in the low-SNR situation.

C. Measurement of Reliability

In the turbo decoding algorithm [1], [2], the magnitude of LLR itself is an index that indicates the reliable degree of decoded information bit. Thus, we define the magnitude of LLR $|L_2^{(i)}(\hat{u}_k)|$ as the reliability

$$R_k \equiv \left| L_2^{(i)}(\hat{u}_k) \right|. \tag{15}$$

Clearly, the decoding is complete and can be terminated if the reliability of all decoded bits is greater than a predefined threshold value Th_1 [15]

$$R_k > \mathrm{Th}_1, \qquad k \in \{1, 2, \ldots, N\}. \tag{16}$$

On the other hand, if the noise is too large, the decoder cannot work successfully. That is, the reliability of all decoded bits is smaller than a given threshold value Th_2

$$R_k > \mathrm{Th}_2, \qquad k \in \{1, 2, \ldots, N\}. \tag{17}$$

When (17) occurs, the decoder can stop decoding and send the ARQ message to the transmitter for data retransmitting.

D. Frame Testing Versus Stage Testing

Both (16) and (17) show that the testing condition is based on the overall frame (frame testing). Thus, the approach needs $2N$ comparison operations per iteration, illustrated as the pseudocode in column (a) of Table I. Note that, in real hardware implementation, two comparators can be reused.

The decoder stops according to the stopping condition that the reliabilities of all decoded bits are greater (smaller) than the threshold. The stopping condition can be changed by the logical relationship. Thus, the decoder can continue decoding when the reliability of any decoded bit is smaller (greater) than the threshold (stage testing). If the reliability of any decoded bit matches the condition, the others need not be tested again and the comparator can be turned off, illustrated as the pseudocode in column (b) of Table I. As a result, the comparison operations can be reduced while the hardware complexity is the same.

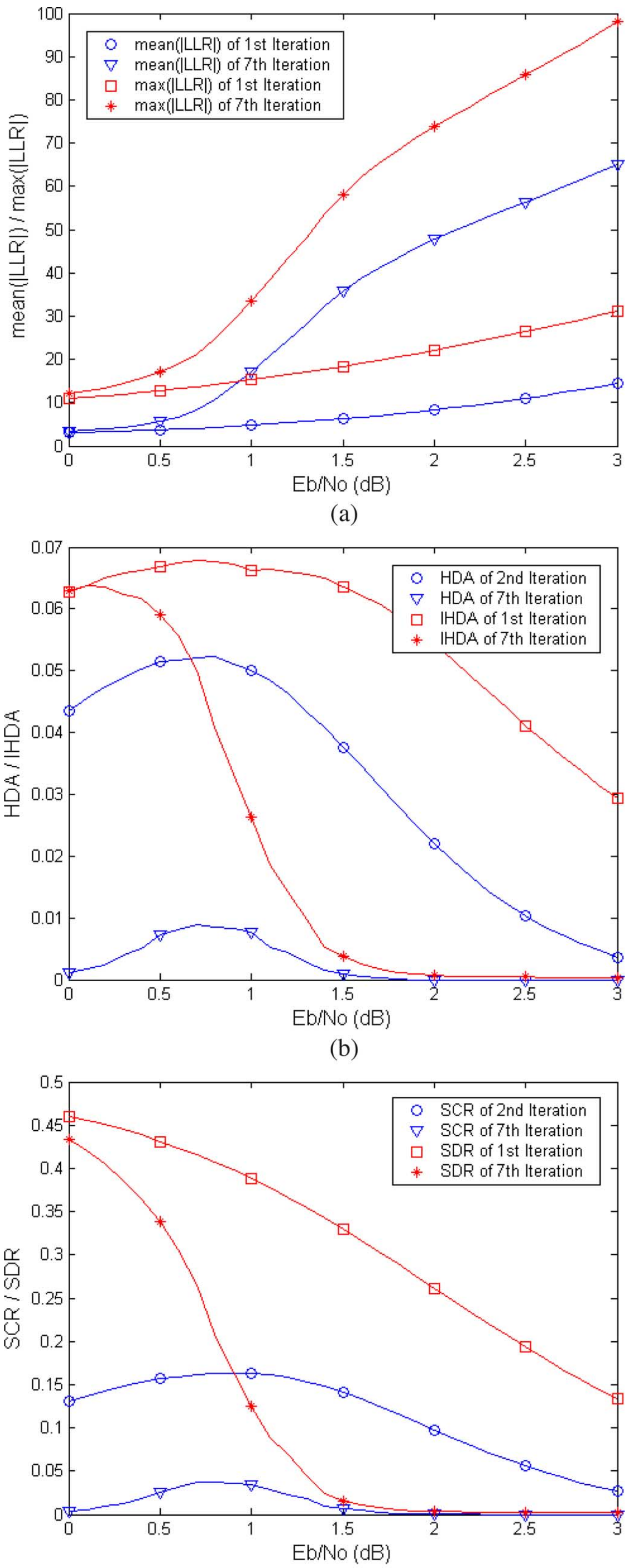


Fig. 7. ET indicators versus E_b/N_0. (a) Mean and maximal values of absolute LLR. (b) HDA and IHDA (normalized by N). (c) SCR and SDR (normalized by N).

E. Modification on Existing ET Methods

For the several ET approaches [14]–[23], most of them focus on the stopping criteria in high-SNR situations. If the noise is serious, the decoder usually performs with maximum iterations, even out of the decoding ability. According to our stopping concepts, some existing ET approaches can be modified to stop in low-SNR situations, such as SCR, SDR, and ME methods. Besides, most of them operate based on one overall frame, and some of them also can be modified to operate in stage testing, such as HDA and IHDA methods.

TABLE I
PSEUDOCODE: (a) FRAME TESTING AND (b) STAGE TESTING

(a)	(b)
$R := [R_1, R_2, \ldots, R_N]$	$R := [R_1, R_2, \ldots, R_N]$
if $R > Th_1$ or $R < Th_2$	*continue_tag*1 := 0
Stop := 1	*continue_tag*2 := 0
else	for $k := 1$ to N
Stop := 0	if *continue_tag*1 = 0
end	if $R(k) < Th_1$
	*continue_tag*1 := 1
	end
	end
	if *continue_tag*2 = 0
	if $R(k) > Th_2$
	*continue_tag*2 := 1
	end
	end
	end
	if *continue_tag*1 = 1 and *continue_tag*2 = 1
	Stop := 0
	else
	Stop := 1
	end

1) Modified ME: This ME [16] approach is based on monitoring the mean of the absolute LLR values of the component decoder output over a frame. That is

$$M_{|L|} \equiv E\left(\left|L_2^{(i)}(\hat{u}_k)\right|\right). \tag{18}$$

From simulations, $M|L|$ increases as the number of errors decreases. Therefore, the ME stopping rule is to compare $M|L|$ with a single specified threshold Th. If

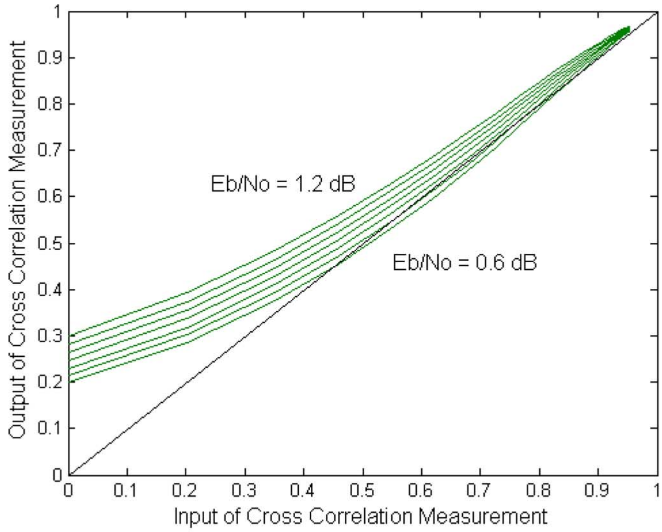
$$M_{|L|} > \mathrm{Th} \tag{19}$$

the decoder can stop the iterative processes and consider the frame to be error free. Otherwise, the decoder will continue iterating.

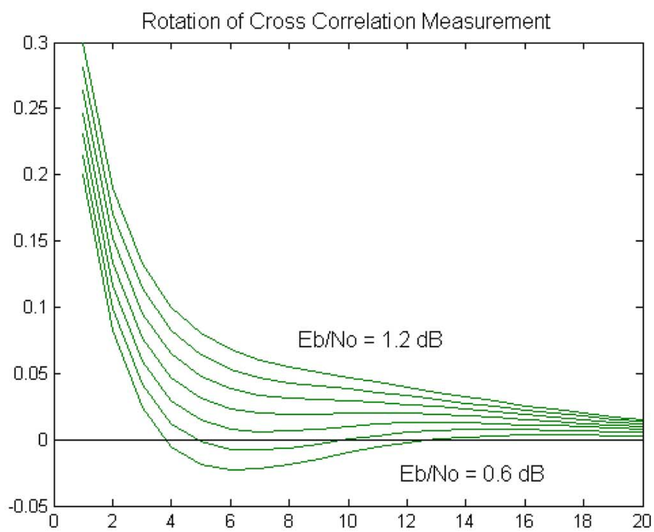
In severe noisy channel, the decoder should stop when the mean of the absolute LLR is smaller than a given threshold value Th'. Thus, in addition to the condition in (19), we add extra stopping condition for low SNR situations

$$M_{|L|} < \mathrm{Th}'. \tag{20}$$

2) Modified SCR: The SCR approach was presented in [19] based on the CE concept [14]. Let $C(i)$ be the sign changes of $\mathrm{Le}_2(\hat{u})$ from iteration $(i-1)$ to iteration i. The SCR stopping



(a)



(b)

Fig. 8. Simulations for the decoding trajectory.

rule is to compute $C(i)$ at each iteration i and stop further iterations when

$$C(i) \leq (0.005 \sim 0.03)N. \tag{21}$$

For low SNR channel, it is shown that the sign of LLR changes rapidly. Thus, the decoder should stop when the frequency of sign change $C(i)$ is greater than a predefined threshold Th. Therefore, in addition to (21), we can add extra stopping condition for low SNR situations

$$C(i) > \mathrm{Th}. \tag{22}$$

3) Modified SDR: The SCR method needs storage to store sign values from the previous iteration. Thus, an improved SCR approach called SDR was proposed in [20]. Let $D(i)$ be the number of sign difference between $\mathrm{Le}_1^{(i)}(\hat{u})$ and $\mathrm{Le}_2^{(i)}(\hat{u})$ at the same iteration i. The SDR stopping rule is to compute $D(i)$ at each iteration i and stop further iterations when

$$D(i) \leq (0.001 \sim 0.01)N. \tag{23}$$

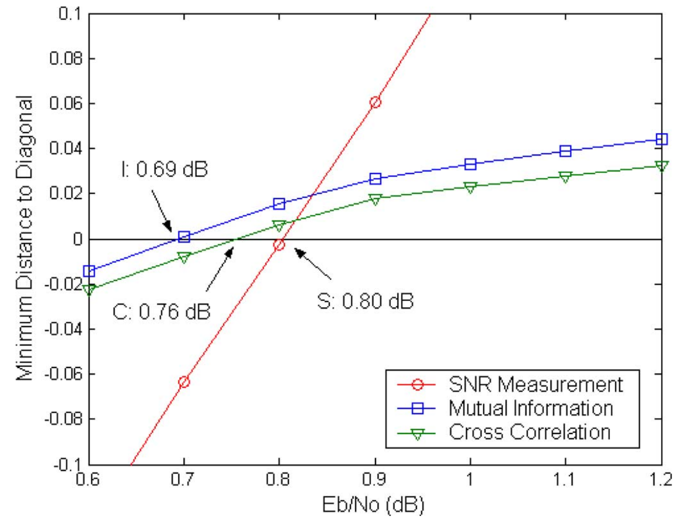


Fig. 9. Simulations for the decoding threshold.

Similar to M-SCR, in addition to (23), we add anther stopping condition for low SNR situations

$$D(i) > \mathrm{Th} \tag{24}$$

where Th is a predefined threshold.

4) Modified HDA : Related to the CE concept [14], another effective method called HDA approach was presented in [19]. The HDA stopping rule is to check the hard decisions of the information bits between two consecutive iterations and stop further iterations when

$$\mathrm{sign}\left(L_2^{(i-1)}(\hat{u}_k)\right) = \mathrm{sign}\left(L_2^{(i)}(\hat{u}_k)\right), \qquad \text{for } k = 1, 2, \ldots, N. \tag{25}$$

The decoder stops according to the condition that the signs of all decoded bits in one frame are the same as those of previous iteration. By using the stage-testing concept, the decoder can continue operating when the sign of any decoded bit is different from the one of previous iteration. If any decoded bit matches the condition, the others need not be tested again and the comparator can be turned off. As a result, the comparison operations can be reduced.

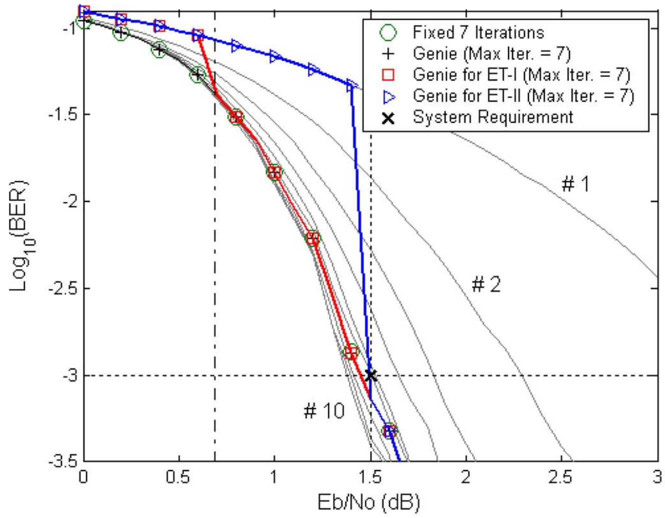
5) Modified IHDA: The IHDA approach extends the existing HDA method. Unlike the HDA, the IHDA approach [21] requires no storage by checking the information from the same iteration. The IHDA stopping rule is that the decoder can stop further iterations when

$$\mathrm{sign}\left(\mathrm{Le}_1^{(i)}(\hat{u}_k) + \frac{2}{\sigma^2} y_{k,1}\right) = \mathrm{sign}\left(L_2^{(i)}(\hat{u}_k)\right), \qquad \text{for } k = 1, 2, \ldots, N. \tag{26}$$

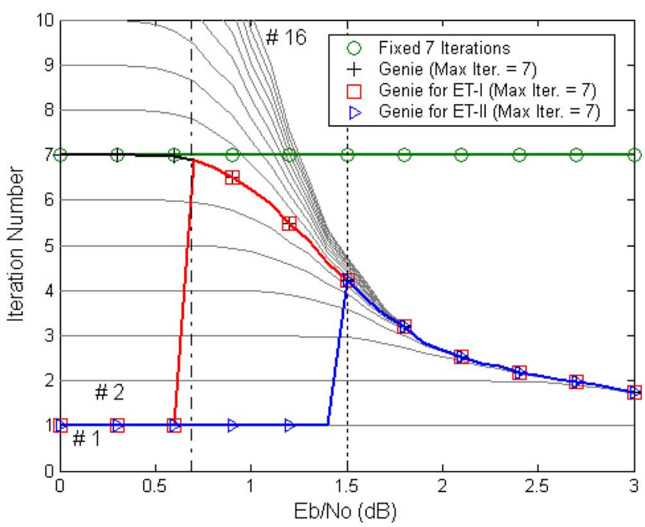
Similar to the M-HDA, the IHDA method can also adopt the stage-testing concept, so as to reduce the comparison operations.

IV. Simulations and Comparisons

We perform simulations for $K = 3$ turbo codes with different stopping criteria. The generator polynomials are $(111)_2$ and $(101)_2$. The hundreds of frames, whose length N is 1000,

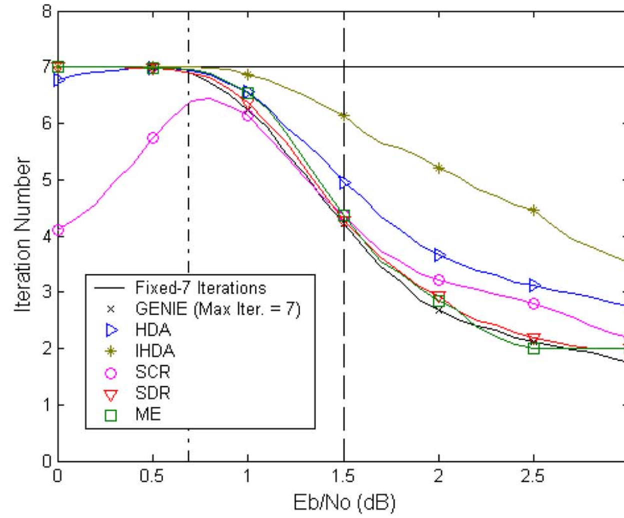


(a)

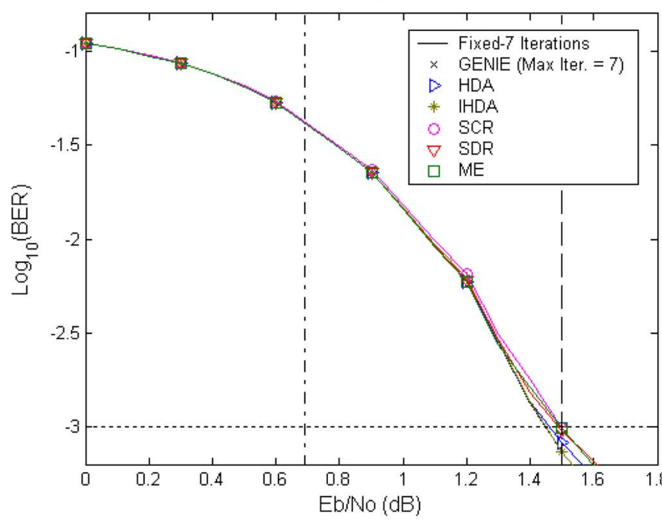


(b)

Fig. 10. (a) Simulations for BER versus E_b/N_0. (b) Simulations for Iteration number versus E_b/N_0.



(a)



(b)

Fig. 11. Conventional ET strategies. (a) Simulations for iteration versus E_b/N_0. (b) Simulations for BER versus E_b/N_0.

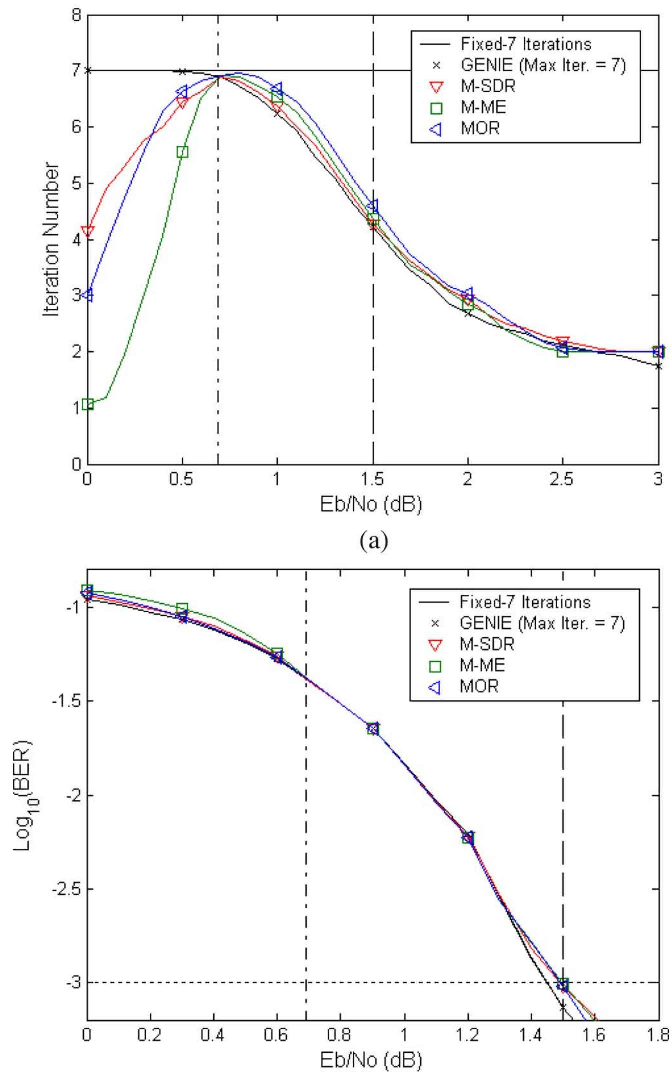
are obtained. The overall rate R is 1/2. The component decoder adopts the maximum *a posteriori* probability (MAP) [3], [4] algorithm. Moreover, the requirement of the system specification is assumed that the decoder must satisfy 10^{-3} bit error rate (BER) above 1.5 dB AWGN channel.

A. Simulations for the Decoding Threshold

To find the decoding threshold, three information measurements are adopted in the AWGN channel from 0.6 to 1.2 dB with step size of 0.1 dB. Fig. 8(a) showing that the cross-correlation transfer curves are close to diagonal line as the channel becomes noisier. Then, Fig. 8(a) is rotated 45° clockwise and plotted in Fig. 8(b). SNR and mutual information measurements are performed with the same steps as Fig. 8. Then, Fig. 9 shows the curves composed of the lowest points of the transfer curves in Fig. 8(b). As a result, the decoding thresholds, obtained from the cross with the zero horizontal line, are 0.69 dB by the mutual information measurement, 0.76 dB by the cross-correlation measurement, and 0.80 dB by the SNR measurement. For the following simulations, we adopt the optimistic decoding threshold 0.69 dB (B position in Fig. 2) to avoid missing the decodable patterns.

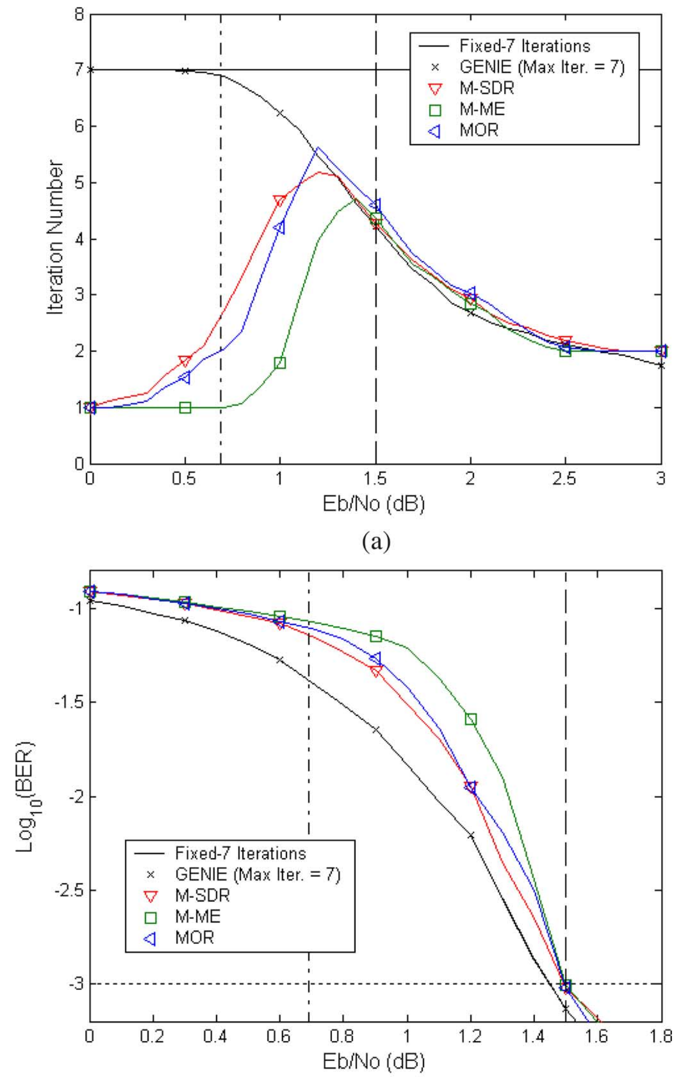
B. Simulations for the GENIE-Aided Benchmark

For the specification of 10^{-3} BER at 1.5 dB, the required iteration number of the decoder is seven (A position in Fig. 2), given by the simulation result in Fig. 10(a). Before simulating the aforementioned stopping approaches, it needs a GENIE-aided benchmark to be the performance boundary. In the GENIE stopping rule [20], the decoder is given the correct information bits, and the decoder stops immediately after the frame is correctly decoded. It can be found that the performances of last iterations are so close that further iterations can only achieve slight improvement. Besides, with increasing iterations, the BER cannot be improved in low SNR. Fig. 10(b) illustrates the several GENIE curves with different maximal iteration from 1 to 16. The GENIE curve (maximal iteration = 7) is used in previous stopping approaches, and its BER per-



(a) (b)

Fig. 12. Proposed ET-I scheme. (a) Simulations for iteration versus E_b/N_0. (b) Simulations for BER versus E_b/N_0.



(a) (b)

Fig. 13. Proposed ET-II scheme. (a) Simulations for iteration versus E_b/N_0. (b) Simulations for BER versus E_b/N_0.

formance is almost the same as the one of the fixed-7-iteration decoding [the seventh iteration curve in Fig. 10(a)].

For the proposed ET-I scheme, there is another GENIE curve in Fig. 10(b), where the decoder, below the infinite-iteration decoding threshold 0.69 dB, just runs an iteration to obtain the LLR for stopping-criterion testing. Note that the BER performance of the GENIE curve (maximal iteration = i) is almost the same as the one of the fixed-i-iteration decoding. Therefore, the BER performance curve of the GENIE for ET-I scheme in Fig. 10(a) can be composed of the fixed-1-iteration and fixed-7-iteration decoding at 0.69 dB.

For the proposed ET-II scheme, there is also another GENIE curve, where the fixed-7-iteration decoding threshold is 1.5 dB in Fig. 10(b). Note that although the BER performance simulation in Fig. 10(a) gives the real fixed-7 decoding threshold, 1.45 dB at 10^{-3} BER, we still adopt 1.5 dB (C position in Fig. 2) as the fixed-7 decoding threshold. The reason is that the most important of the design principle is to satisfy the system specification. Thus, the BER performance curve of the GENIE for ET-II scheme is composed of the fixed-1-iteration and fixed-7-iteration decoding at 1.5 dB in Fig. 10(a).

C. Simulations for Different ET Schemes

With the conventional GENIE curve, the simulations for several approaches discussed previously are performed. Fig. 11(a) illustrates the simulations of iterations versus E_b/N_0 for SCR, SDR, HDA, IHDA, and ME approaches. It can be found that, in high SNR, the order of reduced iterations of all ET schemes is as follows:

$$\text{ME, SDR} > \text{SCR} > \text{HAD} > \text{IHDA}. \tag{27}$$

However, most of them perform with maximal iterations in low-SNR situations. Because of the nonmonotonic variation of the SCR indicator in Fig. 7(c), the SCR based on threshold comparing is ineffective in the low-SNR situation in Fig. 10(a). Last, the BER performance simulations are performed in Fig. 11(b). Besides, we also add the simulations of conventional GENIE

TABLE II
COMPARISON SUMMARY

	Measurement Operation	Threshold Operation	Comparing Operation	Storage	Minimum Iteration	Stopping Demand		Test Unit
						Hi-SNR	Lo-SNR	
CE [14]	(5N-1)-Real	Pre-sim	1-Real	(N+2)-Real	1	○	×	Frame
SCR [19]	(3N-1)-Integer	Pre-sim	1-Integer	(N+2)-Integer	2	○	×	Frame
SDR [20]	(3N-1)-Integer	Pre-sim	1-Integer	1	1	○	×	Frame
HDA [19]	N-Binary	-	N-Logic	N-Integer	2	○	×	Frame
IHDA [21]	N-Binary	-	N-Logic	-	1	○	×	Frame
Absolute LLR Measurement [15]	-	Pre-sim	N-Real	-	1	○	×	Frame
ME [16]	N-Real	Pre-sim	2-Real	2	1	○	×	Frame
Updated Threshold [17]	N-Real	Pre-sim, 2-Real	2-Real	3	2	○	×	Frame
A-priori LLR Measurement [18]	2N-Real	Pre-sim	1-Real	-	1	○	×	Frame
CRC [22]	1-CRC Decoder	-	1-Logic	-	1	○	×	Frame
VCW [23]	1-VCW Check	-	1-Logic	-	1	○	×	Frame
***M-HDA**	N-Binary	-	<N-Logic	N-Integer	2	○	×	Stage
***M-IHDA**	N-Binary	-	<N-Logic	-	1	○	×	Stage
***M-SCR**	(3N-1)-Integer	Pre-sim	2-Integer	(N+2)-Integer	2	○	○	Frame
***M-SDR**	(3N-1)-Integer	Pre-sim	2-Integer	2	1	○	○	Frame
***M-ME**	N-Real	Pre-sim	2-Real	2	1	○	○	Frame
***MOR**	-	Pre-sim	<2N-Real	2	1	○	○	Stage

Notation * indicates the works that are introduced in this literature.

(maximal iteration = 7) and fixed-7–iteration decoding as the benchmark. All of them can satisfy the specification.

Next, Fig. 12(a) illustrates the simulations for MOR, M-SCR, M-SDR, and M-ME approaches by using the proposed ET-I scheme. Because of the nonmonotonic variation of the SCR indicator in Fig. 7(c), the low- and high-SNR stopping criteria of M-SCR will interfere with each other and fail. Note that the presimulated ET thresholds must satisfy the constraint that the iterations at 0.69 dB must exceed the ones of the GENIE (maximal iteration = 7) in Fig. 12(a). Thus, the decoder can normally work when the channel SNR is above the infinite-iteration decoding threshold. It can be found that in low-SNR situations, the order of the reduced iterations of the ET schemes is as follows:

$$\text{M-ME} > \text{MOR} > \text{M-SDR}. \tag{28}$$

Moreover, the performance of them is close either in low- or high-SNR situations in Fig. 12(b) and also satisfies the specification.

For the proposed E-II scheme, although the BER performance degrades below the fixed-iteration decoding threshold, the presimulated ET thresholds can keep the BER to be smaller than 10^{-3} at 1.5 dB in Fig. 13(b). Thus, the decoder can still satisfy the specification. Moreover, Fig. 13(a) shows that the reduced iterations of the proposed ET-II scheme are more than the ones of the proposed ET-I scheme in low-SNR situations.

D. Summary of All ET Approaches

Last, all stopping methods are summarized in Table II. The main operations include measurement, threshold, and comparing operations. Some approaches need at least two iterations to compare the information, and they need storage, such as SCR, HDA, M-HDA, and M-SCR. Besides, within the approaches operated with ET thresholds, their thresholds are usually presimulated according to a specific system. Even the ET threshold of the updated threshold method is obtained by presimulating and updating with iterations. Last, by the comparison, our proposed MOR approach and the modified approaches M-ME and M-SDR can perform efficiently either in high- or low-SNR situations. Additionally, the MOR approach and the modified approaches M-HDA and M-IHDA may perform by part of an overall frame.

V. CONCLUSION

In this paper, we discuss the decoding threshold of the turbo decoder and propose the new information measurement by using the cross correlation. Then, two ET schemes are proposed by using the decoding threshold. Thus, the iterative decoding can stop in either high-SNR situations where the decoded bits are highly reliable, or in low-SNR situations where the decoder already has no capability to decode. Moreover, several previous ET works can be modified based on the new ET concepts. By the simulation results, our proposed ET schemes can reduce the unnecessary iterations and still satisfy the system specification.

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