Average-entropy variation in iterative decoding of turbo codes and its application

J. Y. Chen, L. Zhang and J. Qin

A new stopping criterion for turbo codes is proposed. Based on the entropy concept, a metric called average-entropy to measure the average uncertainty of the estimated bits of each iteration is derived. This metric has a close relation to the bit error rate (BER). The average-entropy decreases as BER reduces and vice versa. The proposed stopping criterion stops the iterative algorithm when there is not a, or little, reduction of the average-entropy. Compared with other well-known criteria, this new criterion reduces the average number of iterations while maintaining error correction performance.

Introduction: Turbo codes [1] achieve outstanding performance close to the Shannon limit because of two important features, i.e. the random coding structure and the iterative decoding method. However, the iterative decoding algorithm results in high computational complexity. To reduce this complexity, we explore two characteristics of the turbo decoding process. First, owing to the randomness of the channel, the number of iterations needed to achieve similar performance varies from frame to frame. Secondly, after some number of iterations, any further iteration has little performance improvement. The so-called early stopping criteria are proposed to reduce the average number of iterations by stopping the decoding early. On the one hand, we hope to save overhead and reduce decoding delay; on the other hand, we hope to achieve similar error correction performance. Hence, it is key to devise a metric that characterises the performance of each iteration for an early stopping criterion to work efficiently.

Many early stopping schemes have been put forward. Hagenauer *et al.* [2] developed a stopping criterion base on cross-entropy (CE), which is a measure of 'closeness' between two distributions. Based on the CE concept, Shao *et al.* [3] presented the sign change ratio criterion, which has a simpler implementation than the CE criterion. The sum-reliability criterion [4] were proposed by considering hardware implementation issues. Reference [5] proposed a criterion by performing a consistency check between the input and output bits of the decoder. Recently, nonlinear dynamics was applied to analyse the iterative decoding process [6, 7]. By viewing the turbo decoding as a nonlinear dynamic process, [7] studied various properties of this process and presented an early stopping scheme based upon computing the *a posteriori* average entropy.

In this Letter, we propose an early stopping scheme from a new point of view. We devise a metric called average-entropy that has a well-defined physical meaning from the information theoretical point of view, i.e. it represents the uncertainty of the estimated bits of the current iteration. Also, by simulation, the variation of average-entropy is closely related to that of bit error rate (BER). Thus, the average-entropy well characterises the performance nature of the iterative decoding process. A novel efficient early stopping criterion based on this metric is proposed. It requires less average number of iterations than other well-known criteria, while suffering very little performance degradation.

Turbo decoding: Assume the information bit vector is $\boldsymbol{C} = (c_1, c_2, \ldots, c_N)$ and the received vector is $\boldsymbol{r} = (r_1, r_2, \ldots, r_N)$, where N represents the frame size. We consider systematic parallel concatenated turbo codes and BPSK transmission over an AWGN channel. Using the MAP algorithm [8, 9], the log likelihood ratio (LLR) for the estimated information bit $\hat{c}_k$ $(k = 1,2,\ldots,N)$ is (we use the notation similar to [8, 9]):

$$\Lambda(\hat{c}_k) = \log \frac{\sum_{(S_{k-1},S_k)\in B_k^1} \alpha_{k-1}(S_{k-1})\gamma_k^1(S_{k-1},S_k)\beta_k(S_k)}{\sum_{(S_{k-1},S_k)\in B_k^0} \alpha_{k-1}(S_{k-1})\gamma_k^0(S_{k-1},S_k)\beta_k(S_k)} \tag{1}$$

Average-entropy: The proposed stopping rule is based on the concept of entropy which is a measure of uncertainty about a random variable. The average-entropy metric we derive in this Section is used to describe the decoder's uncertainty on the estimated bits, i.e. the smaller this metric is, the higher the probability that the estimated bits are correct.

The entropy for the estimated bit vector $\hat{\boldsymbol{C}}$ of the ith iteration is:

$$\begin{aligned} H^{(i)}(\hat{\boldsymbol{C}}|\mathbf{r}) &= -\sum_{\hat{C}} p(\hat{\boldsymbol{C}}|\boldsymbol{r})\log p(\hat{\boldsymbol{C}}|\boldsymbol{r}) \\ &= -\sum_{k=1}^{N}[p^{(i)}(\hat{c}_k=0|\boldsymbol{r})\log p^{(i)}(\hat{c}_k=0|\boldsymbol{r}) \\ &\quad + p^{(i)}(\hat{c}_k=1|\boldsymbol{r})\log p^{(i)}(\hat{c}_k=1|\boldsymbol{r})] \end{aligned} \tag{2}$$

where the second equation is obtained by assuming independence between estimated bits. $p^{(i)}(\hat{c}_k|\boldsymbol{r})$ is the belief about the estimated bit $\hat{c}_k$ and can be computed from the LLRs as:

$$p^{(i)}(\hat{c}_k = d|\boldsymbol{r}) = \begin{cases} \dfrac{e^{\Lambda(\hat{c}_k)}}{1+e^{\Lambda(\hat{c}_k)}}, & for\ \hat{c}_k = 1 \\ \dfrac{e^{-\Lambda(\hat{c}_k)}}{1+e^{-\Lambda(\hat{c}_k)}}, & for\ \hat{c}_k = 0 \end{cases} \tag{3}$$

The average-entropy per bit is then obtained as:

$$\begin{aligned} H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) &\stackrel{def}{=} -\frac{1}{N}\sum_{k=1}^{N}[p^{(i)}(\hat{c}_k=0|\boldsymbol{r})\log p^{(i)}(\hat{c}_k=0|\boldsymbol{r}) \\ &\quad + p^{(i)}(\hat{c}_k=1|\boldsymbol{r})\log p^{(i)}(\hat{c}_k=1|\boldsymbol{r})] \end{aligned} \tag{4}$$

In (3), the conditional probabilities $p^{(i)}(\hat{c}_k = 0|\boldsymbol{r})$ and $p^{(i)}(\hat{c}_k = 1|\boldsymbol{r})$ are calculated from the LLRs $\Lambda(\hat{c}_k)$, while in the *a posteriori* average entropy they are calculated from the extrinsic information $\Lambda_e(\hat{c}_k)$ [6, 7]. That is the difference between average entropy and the *a posteriori* average entropy. The average-entropy is calculated in each iteration to denote the decoder's uncertainty about the decisions. If this value approaches zero, the decoder makes the right decisions in high probability, and thus BER reduces. If it approaches one, the decisions are wrong in high probability and BER increases. In the following Section, we illustrate such a statement and explain how the average-entropy varies in the iterative decoding process.

Average-entropy against number of iterations: In the following simulations, we use the rate-1/3 parallel concatenated turbo code. The generator is $G = [7\ 5]$. The frame length is $N = 400$ and the random interleaver is adopted.

In Fig. 1, we have the following observations:

1. For a fixed SNR, the average-entropy decreases as the number of iteration increases and after reaching some iterations (called turning point), it becomes nearly unchanged.
2. The higher the SNR, the smaller the average-entropy becomes, which coincides with our interpretation of the average-entropy, i.e. the higher the SNR, the less uncertainty on the decoder's decisions.
3. The variations of the average-entropy accord with those of BER. Therefore, we can track variations of BER by monitoring the average-entropy which is feasible to be calculated in the receiver.

For other turbo codes of different frame length and code structures, similar observations can be obtained and thus the statements are general.

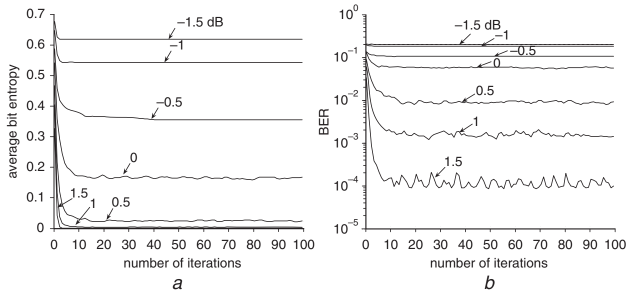

Fig. 1 *Average-entropy and BER variation in turbo decoding*

a Average-entropy against number of iterations
b BER against number of iterations

New stopping criterion: With the above observations, we propose a new early stopping criterion for turbo decoders based on the average-entropy $H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r})$. A threshold value $H_{th}(SNR)$ can be fixed for every SNR interval. When $H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) \leq H_{th}(SNR)$, little extra information can be obtained for improving the performance of the decoder, and then the decoding can be stopped.

To determine the threshold values for the average-entropy algorithm, we first investigate the average-entropy variations. As observed above, there exists some turning point in Fig. 1 after which there is a smaller, or not, reduction of average-entropy as well as BER. The threshold values are set to be the values that are near the turning points, and for convenience we can group the SNRs which have the similar average-entropy in practical applications. As an example, we suggest the following scheme for the simulation settings presented above. In addition, the early stopping scheme could also be developed for other turbo codes of different frame length and code structures in a similar way.

Average-entropy (AE) based criterion:

For $SNR < 0.8$ dB, $H_{av}^{(i-1)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) - H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) \leq 10^{-3}$;
For $0.8 \text{ dB} \leq \text{SNR} < 1.2$ dB, $H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) \leq 1.5 \times 10^{-2}$;
For $SNR \geq 1.2$ dB, $H_{av}^{(i)}(\hat{\boldsymbol{C}}|\boldsymbol{r}) \leq 2.5 \times 10^{-3}$

We now compare the performance of the AE-based (AEB) criterion with other existing criteria, e.g. the cross-entropy criterion (CE) [2], sign-change ratio (SCR) [3], sum-reliability (SUM) [4] and zero-entropy detection (ZED) [7]. Fig. 2 gives simulation results of the stopping criterions performance. It can be observed that the AE criterion is superior to other criterions in that it requires less average number of iterations while achieving almost the same BER performance. AEB, ZED and CE are nearly the same in computational complexity, while SUM and SCR require less computation.

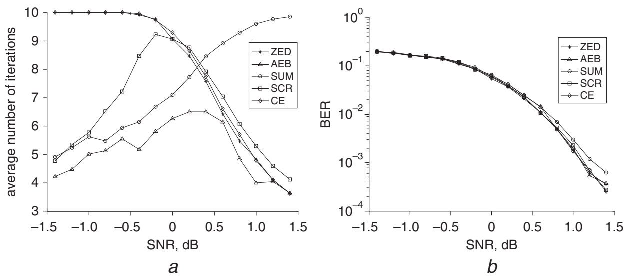

Fig. 2 *Performance comparison of different stopping criteria for turbo decoding (G = [7, 5], N = 400, rate = 1/3)*

a Average number of iterations against SNR
b BER against SNR

To verify the robustness of the proposed early stopping scheme, we tested the criterion under different frame length and code structures. Fig. 3 shows the results of using a rate 1/2 turbo code of generator $G = [15\ 17]$ and frame length $N = 2048$. Fig. 3 shows that for the low SNR region (SNR < 1 dB), the AEB criterion requires fewer iterations than the others, while BER degradation is small.

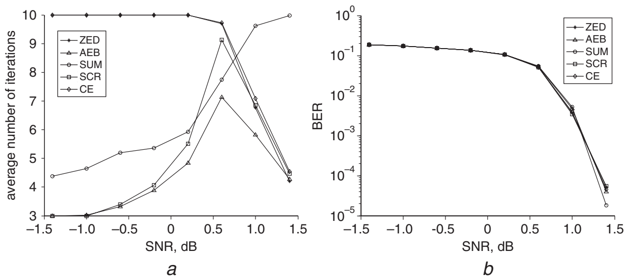

Fig. 3 *Performance comparison of different stopping criteria for turbo decoding (G = [15 17], N = 2048, rate = 1/2)*

a Average number of iterations against SNR
b BER against SNR

Conclusions: We have presented a new metric, average-entropy, which represents the amount of uncertainty of the turbo decoder's decisions. An efficient stopping criterion is thus proposed based on the average-entropy. The effectiveness and efficiency of the new scheme is verified by simulations. The general observation is that the average number of iterations reduces, while almost the same bit error rate performance is maintained.

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