Binary Star System
Derivations

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Consider a binary star system:

Let m_1, r_1, v_1 be respectively the mass, radius of revolution around center of mass and orbital velocity of first binary star and m_2, r_2, v_2 be the mass, radius of revolution around center of mass and orbital velocity of second binary star, respectively.

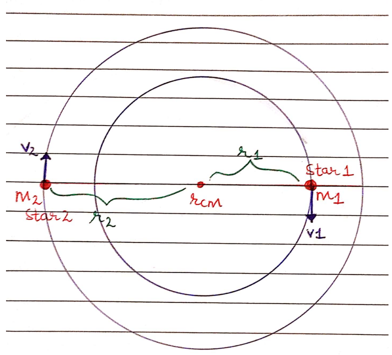

Figure 1: Binary star system

Using the center of mass formula for the two stars, we get,

$$m_1 r_1 = m_2 r_2$$

Since it is a binary star system, the time period of revolution, T for both the stars will be equal. Now using **Kepler's Third Law**, which is stated as follows:

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

Here, for the binary stars we have, $r = r_1 + r_2$, and $m = m_1 + m_2$. Hence, for binary stars the above formula translates as:

$$T^2 = \frac{4\pi^2 (r_1 + r_2)^3}{G(m_1 + m_2)}$$

Put formula for m_2 above equation to calculate mass of star 1,

$$T^2 = \frac{4\pi^2(r_1+r_2)^3}{G(m_1 + \frac{m_1 r_1}{r_2})}$$

Take m_1 as common and simplify for radius terms,

$$T^2 = \frac{4\pi^2 r_2 (r_1+r_2)^2}{G m_1}$$

Now, take m_1 on LHS:

$$m_1 = \frac{4\pi^2 r_2 (r_1+r_2)^2}{G T^2}$$

Here's the formula for m_1 is expressed in terms of r_1 and r_2, lets rather express it in terms of orbital velocities, using:

$$r_1 = \frac{Tv_1}{2\pi} \qquad r_2 = \frac{Tv_2}{2\pi}$$

Hence putting these in original equation,

$$m_1 = \frac{4\pi^2 \frac{Tv_2}{2\pi}(\frac{Tv_1}{2\pi} + \frac{Tv_2}{2\pi})^2}{GT^2}$$

Factoring out common terms, we get m_1 as:

$$m_1 = \frac{Tv_2(v_1+v_2)^2}{2\pi G}$$

And similarly, we get m_2 as:

$$m_2 = \frac{Tv_1(v_1+v_2)^2}{2\pi G}$$

Lets also calculate the velocity of center of mass using this formulas,

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Lets put the masses calculated in above formula,

$$v_{cm} = \frac{\frac{Tv_2(v_1+v_2)^2}{2\pi G} v_1 + \frac{Tv_1(v_1+v_2)^2}{2\pi G} v_2}{\frac{Tv_2(v_1+v_2)^2}{2\pi G} + \frac{Tv_1(v_1+v_2)^2}{2\pi G}}$$

Cancelling out common terms, we get:

$$v_{cm} = \frac{2v_1 v_2}{v_1 + v_2}$$