Lab Report 3 - Transport

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AB-1

Introduction

Tasks:

1. Corporate has asked our team to evaluate the thermal conductivity and thermal diffusivity of a brass rod at steady and unsteady states.
2. Following this, corporate has asked our team to develop an experimental setup for determining the viscosity of various liquids present in the lab.

Per our instructions, this report will be split into two parts. The first will be focused on evaluating thermal properties in steady and unsteady state, and the second will be focused on determination of viscosities. This section is meant to serve as an overview of the core concepts of the experiment, further explained in later sections.

Part One: Thermal Conductivity and Thermal Diffusivity at Steady and Unsteady States

Starting with the first prompt, some key concepts are described below:

Steady State – From a chemical engineering perspective, steady state refers to a system in which all state variables are constant in spite of ongoing processes. Considering the context of thermodynamics, the state variables here would include internal energy, enthalpy, temperature, pressure, volume, and entropy. This approach allows for isolation of Heat and Work variables, which are not state functions. Steady state processes are very common across all sectors of chemical engineering as they allow for close examination of the variables of interest without the added complexity of state properties and their interactions.

$$k = \frac{A_s}{\dot{m}C_p * \Delta x}$$

$$\dot{m}\ [kg/s] = mass\ flow\ rate = \dot{V}\rho$$
$$\Delta x\ [m] = distance\ difference = \Sigma(node\ distances)$$
$$A_s\ [m^2] = Surface\ Area\ along\ pipe = \pi * D * W$$
$$k\ [\frac{kW}{m*K}] = thermal\ conductivity$$
$$C_p\ [kJ/kg] = specific\ heat$$

Unsteady State – Also referred to as transient state. Unlike with steady state operation, unsteady state operations include mass and energy fluctuations over time. This complicates the calculations for heat and work, but can still be determined through a transient energy balance.

We will be using an infinite cylinder model and correlations to solve for diffusivity:

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = C_! exp(\zeta_1^{\ 2} F_o);\ F_o = \frac{\alpha t}{L^2}$$

$$\alpha\,[\frac{m^2}{s}] = thermal\ diffusivity$$

$$t\,[s] = time\ variable$$

$$L^2\ [m^2] = length\ variable$$

$$C\ = C\ coefficient$$

$$\zeta_1^{\ 2}\ = Zeta\ correlation$$

$$F_o\ = Fouriers\ number$$

$$T\ = Temperature$$

Conductive heat transfer: Conduction as heat transfer occurs if there is a temperature gradient, depending on the available directions of transport. The amount of the heat energy transported (Q) being carried from hot to cold is dependent on the cross sectional area (A) and the thermal transfer coefficient of the material (k). Thermal conductivity is a vital component to many industrial processes, and many materials are chosen based solely on their thermal properties. An example of this would be the design of a heatsink used to keep one of our chemical reactors within our optimal temperature range, where we would be using a highly conductive material to draw heat away from the reactor.

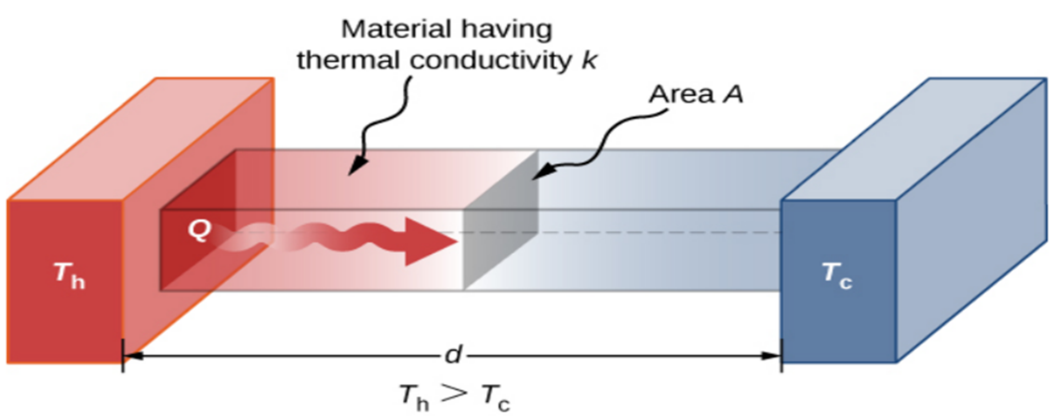


Figure 1. Overview of 1D Heat Flow

Considering our experimental setup is a long brass rod, we can simplify this model into a 1-Dimensional heat flow problem, as we're only concerned with heat flowing in one direction, shown above. With a known input of heat, we can track the temperature changes along the rod with several temperature probes and use that data to calculate the thermal conductivity of brass. After determining the thermal conductivity constant of the material (k), we can now divide this value by density (ρ) and specific heat capacity (Cp) to find the thermal diffusivity (α), which is a measure of the rate of transfer of heat in a material from hot to cold.

Taking this value and applying it to the 1-Dimensional heat equation yields the following:

$$\frac{\partial u}{\partial t} = -\frac{1}{c\rho}\frac{\partial q}{\partial x} = -\frac{1}{c\rho}\frac{\partial}{\partial x}\left(-k\,\frac{\partial u}{\partial x}\right) = \frac{k}{c\rho}\frac{\partial^2 u}{\partial x^2}$$

Figure 2. 1D Heat Equation

This equation allows us to determine the evolution of the temperature profile over time.

Part Two: Experimental Determination of Viscosity

For the second part of the experiment, our goal is the creation of an experimental setup to evaluate the viscosity of different fluids. Viscosity is the quantity that describes a fluid's resistance to flow, or how "thick" the fluid is, and generally varies inversely with temperature. Viscosity is an important consideration in many chemical engineering applications, as it can affect a wide range of other variables such as fluid flow behavior, process efficiency, and internal stresses in the equipment.

For our experimental setup, it was suggested that we utilize a falling sphere method. For this method, we first determine the physical characteristics of our sphere (such as mass, volume, and density), fill a beaker with a known amount of experimental fluid, then drop the sphere through the beaker, recording the time it takes for the sphere to reach the bottom. The basic experimental setup is shown below:

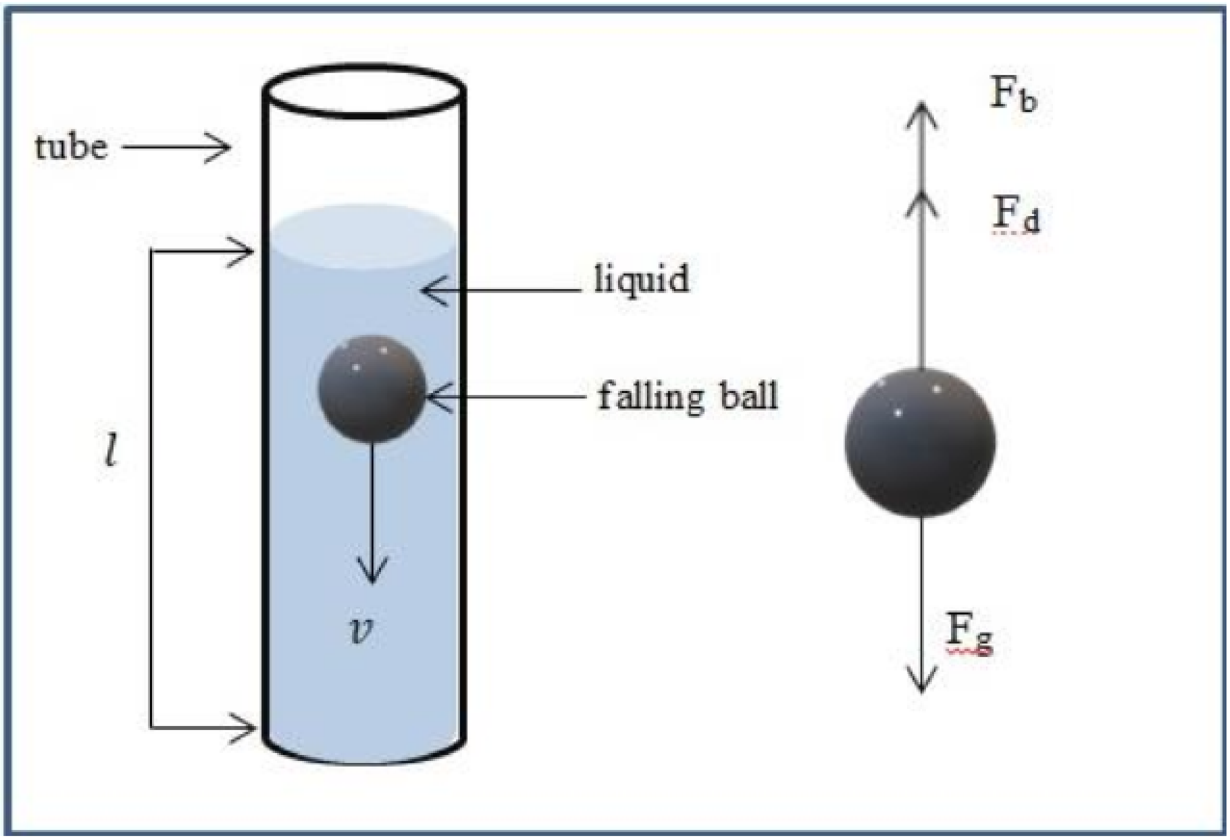


Figure 3. Falling Sphere Experimental setup

Viscosity:
As the free-body diagram illustrates, there are three forces of concern as the ball is falling through the column:

$$\textbf{Gravity: } F_G = -\frac{\pi}{6} p_p d_p^{\ 3} g$$

$$\textbf{Buoyancy: } F_D = (+)\ \frac{\pi}{8} p_f d^3_{\ p}\ g$$

$$\textbf{Drag: } F_D = \frac{\pi}{8} p_f V_p^{\ 2} d_p^{\ 2} C_D$$

Where p_p is the ball density, p_f is the fluid density, d_p is the ball diameter, *g* is gravitational acceleration, V_p is velocity of the sphere, and C_D is the drag coefficient.

$$p \ [\frac{kg}{m^3}] = density$$

$$V\,[\frac{m}{s}] = velocity$$

$$d\,[m] = diameter$$

$$g\,[\frac{m}{s^2}] = gravity\ constant$$

$$F\ [N] = Force$$

$$C_D = drag\ coefficient$$

Note: Subscript designations stated above

Densities were found experimentally through determining mass and volume.

As the sphere approaches terminal velocity, these forces approach equilibrium:

$$\Sigma F_{net} => F_{gravity} = F_{drag} + F_{buoyancy} = 0$$

Note: Due to free body force diagram, positive and negative values can be x-y plane sensitive based on initial assumptions

The drag coefficient can be expressed as a function of the Reynold's number (RE), being the ratio between internal forces to viscous forces. For the scenario of a sphere falling through viscous fluid, the following calculation holds:

$$Re = \frac{p\,V_p d_p}{\mu}$$

Since our flow yields a Reynold's number less than one, we can assume that drag coefficient is constant:

$$C_D = (\frac{24}{RE}).$$

Combining these equations and imposing equilibrium yields the following:

$$V_p = \frac{g d_p^{\ 2}(p_p - p_f)}{18\mu}$$

Finally, we can rearrange this equation and solve for viscosity of the fluid:

$$\mu\,[\frac{N*s}{m^2}] = \frac{g d_p^{\,2}(p_p - p_f)}{18(V_p)}$$

Methods

This section will describe the methods used to achieve our results.

Part 1: Heat Rod and Thermocouple Experiments

Figure 5. Data Acquisition Unit, which is located in between the computer and the test apparatus

1) Boot up computer program “Heated Rod v2” - this is the program that reads data from the data acquisition unit. The first part of the experiment is in unsteady state.
    a) Plug in a flash drive into the PC, and set the data save location to the flash drive

2) Push the green button on the data acquisition unit and make sure it is cycling through all 9 channels. Clicking sounds are normal.

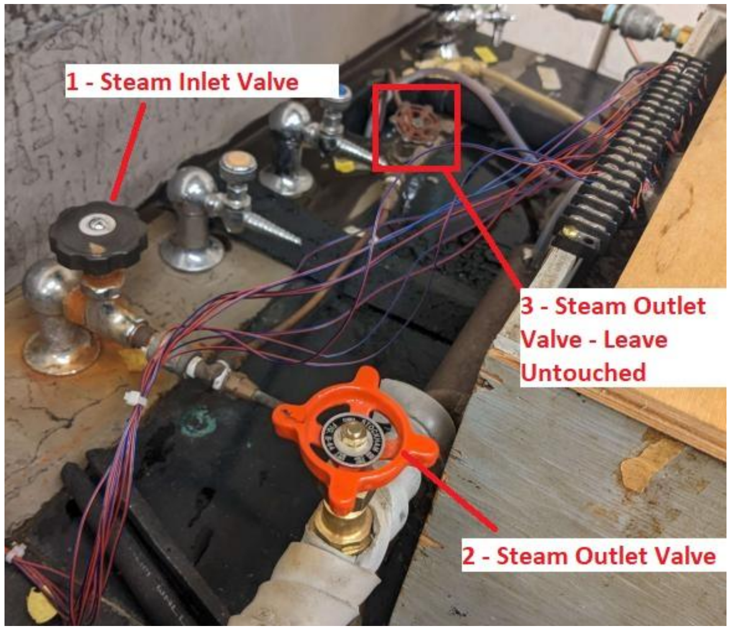


Figure 6. Steam Valves, located behind apparatus

3) Put on safety gloves (Figure 3) and turn on the steam outlet valve (2, Figure 2) by twisting it counter clockwise, then open the steam inlet valve (1, Figure 2). At the same time the inlet valve is opened, start a timer.
    a) Hot water should be ejected from the apparatus into the drain
4) When there is a temperature change detected in the 7th thermocouple (T7 on the computer software), note the time on the stopwatch and the computer.
    a) The data acquisition unit takes measurements at even time intervals, though the times listed are not accurate. To find temperature at different times, we can scale the timeframe of the computer's data to our measured time frame, and "take data" from different time points.

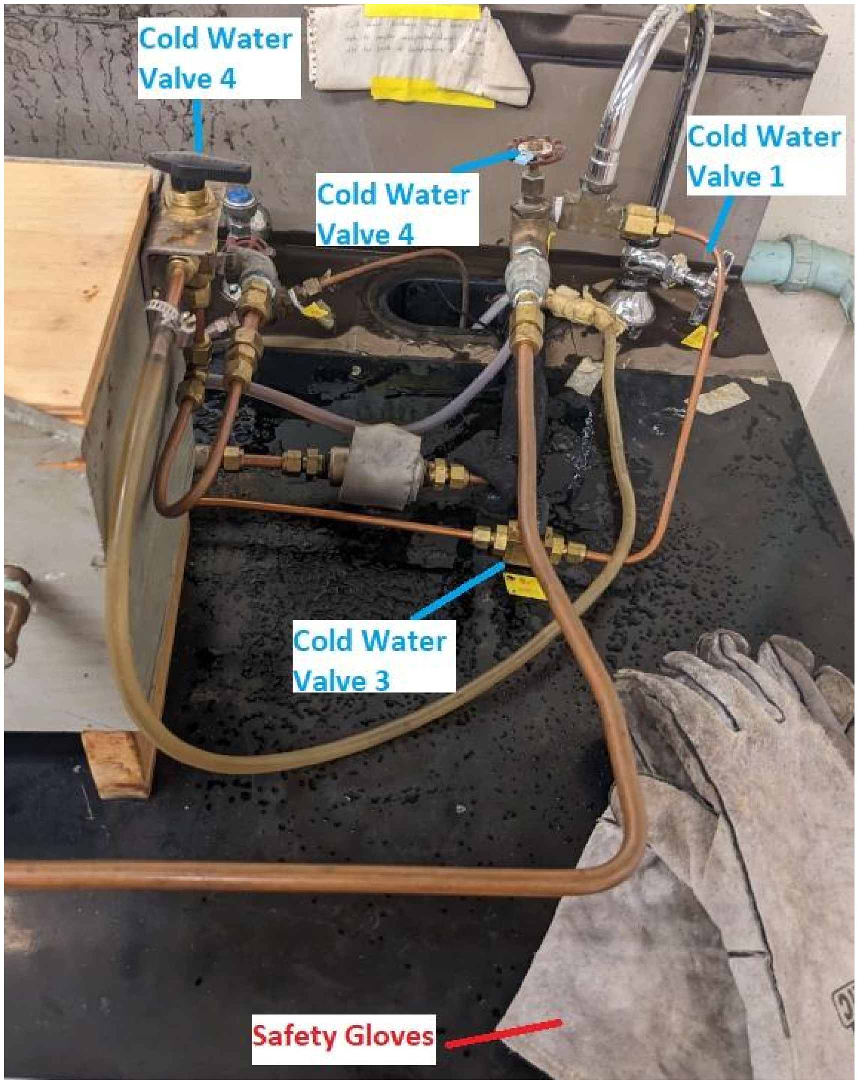


Figure 7. Cold Water Valves and Safety Gloves for Steam Valves

5) To start the steady-state portion of the experiment we will turn on the cold water in order from valve 1 to 4 (Figure 3). Water valve 3 should be turned 45 degrees instead of 90.
6) When the temperature of the water flowing into the apparatus is equal to the water flowing out of the apparatus the system has reached steady state. Note the time on the computer and start the stopwatch. Let the system continue running for 3 minutes.
7) Stop data collection by clicking the "stop," and note the time on the stopwatch.
8) Turn off the steam valves, starting with the inlet valve, then the outlet valve.
9) Measure the flow rate of the water by placing a beaker under the cold water outlet, and measuring the time it takes to fill to a certain volume. This volume will be decided during the experiment and kept constant. Repeat this measurement two more times.
10) Turn off cold water valves, starting with valve 4 and progressing to valve 1.
11) Check that data is saved before closing the program and progressing to part 2 of the experiment.

Part 2: Viscosity Data

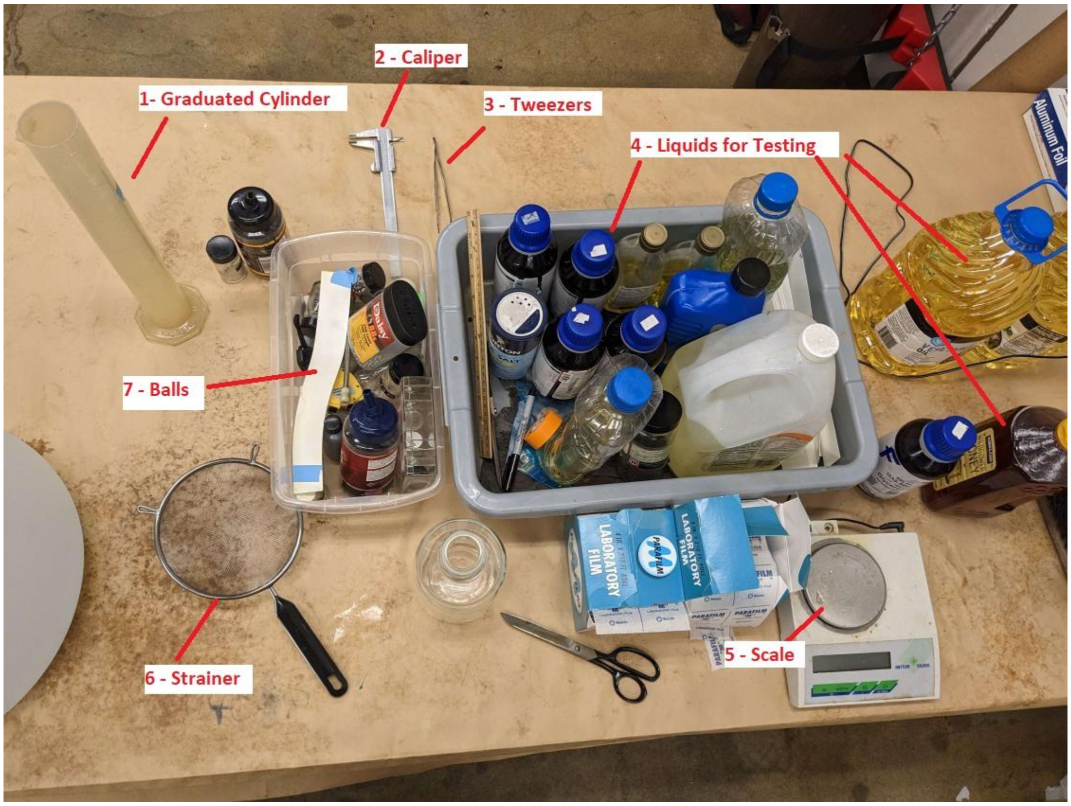


Figure 8. Viscosity Experiment Equipment

1) Pick a type of ball to use for the experiment, then using three different balls, measure the diameter with the caliper.
2) Use the scale to determine the mass of the three balls three separate times, noting each value.
3) Pick three or four different liquids to test viscosity. For this experiment, we used vegetable oil, glycerol, motor oil, and soybean oil.
4) Put the graduated cylinder on the scale and note its mass.
5) Fill the graduated cylinder with liquid until the bottom of the meniscus reaches the chosen volume. Measure the distance from the top of the liquid to the bottom of the liquid with a ruler.
    a) This height will be determined during the experiment, noted, and kept constant throughout trials
6) Note the mass of the liquid and graduated cylinder.
7) Starting from the top of the graduated cylinder, drop the ball and measure the time it takes for the ball to travel from the top of the liquid to the bottom of the graduated cylinder. This will require a designated timer.
8) Repeat step 7 four more times for 5 total trials. We are assuming that the displacement of fluid by each ball is negligible and that any difference in distance traveled is also negligible.

9) To reset, remove the balls from the liquid, either by pouring the liquid through the strainer and back into its original container. Then, wash the graduated cylinder with soap and water, and use a paper towel to dry out the inside.
10) Repeat steps 5 through 7 for the other chosen liquids.
    a) Return testing liquids to their original containers, and rinse out the graduated cylinder and other equipment before changing liquids.

Results

This section displays the data collected throughout the experiment, as well as detailed error analysis and relevant figures.

Part 1: Thermocouple Data

Table 1-A: Unsteady State Thermocouple Data

Actual Time (m)	Data Time (appx)	T1 (C)	T2 (C)	T3 (C)	T4 (C)	T5 (C)	T6 (C)	T7 (C)	Water out	Water in
0:00:00	1:30:37	21.8	21.6	21.6	21.7	21.7	21.6	21.6	21.8	21.5
0:51	1:31:06	40.7	24.7	22.0	21.7	21.6	21.6	21.6	21.8	21.5
1:42	1:31:35	55.8	33.4	25.4	22.3	21.8	21.6	21.6	21.9	21.5
2:33	1:32:04	63.2	40.4	29.8	24.1	22.3	21.8	21.7	21.9	21.5
3:24:20	1:32:32 (when T7 changed)	68.1	46.0	34.2	26.5	23.4	22.1	21.8	21.9	21.6
Version 2										
0:00:00	1:30:37	21.8	21.6	21.6	21.7	21.7	21.6	21.6	21.8	21.5
0:51	1:31:28	52.8	31.0	24.2	22.0	21.7	21.6	21.6	21.8	21.5
1:42	1:32:19	66.4	43.3	32.0	25.2	22.8	21.9	21.7	21.8	21.6
2:33	1:33:10	73.8	51.8	39.5	30.2	25.5	23.1	22.2	21.9	21.5
3:24:20	1:34:01 (3:24 after)	78.5	57.6	45.3	34.8	28.7	24.9	23.2	22.0	21.6

Table 1-B: Steady State Thermocouple Data

Actual Time (min:s)	Data Time	T1 (C)	T2 (C)	T3 (C)	T4 (C)	T5 (C)	T6 (C)	T7 (C)	T8 (C)	T9 (C)
0	1:38:15	89.7	73.6	62.8	51.6	43.1	36.6	32.5	23.0	23.0
0:51	1:40:49	93.1	78.6	68.9	58.3	49.9	43.3	38.8	25.1	26.1
1:42	1:43:23	85.3	77.1	70.7	62.5	55.1	49.0	44.5	28.3	30.0
2:33	1:45:57	79.4	73.7	69.4	63.4	57.6	52.7	48.9	32.1	34.3
3:26:13	1:48:30	77.2	71.9	68.4	63.5	58.9	55.0	51.8	35.6	39.0
Version 2										
0	1:45:04 (3 min 26 seconds before stop)	80.9	74.8	63.9	69.9	56.9	51.7	47.6	30.8	32.9
0:51	1:45:55	79.4	73.7	69.4	63.4	57.6	52.7	48.9	32.1	34.2
1:42	1:46:47	78.5	72.9	68.9	63.4	58.1	53.5	50.0	33.3	35.6
2:33	1:47:38	77.8	72.4	68.6	63.5	58.5	54.3	50.9	34.4	39.0
3:26:13	1:48:30	77.2	71.9	68.4	63.5	58.9	55.0	51.8	35.6	39.0

Table 1-C: Cold Water Flow Rate

	Volume (ml)	Time (s)	Flow Rate (ml/s)
	1000	18.23	54.85
	1000	17.18	58.21
	1000	17.05	56.65
	1000	17.58	56.88
	1000	17.45	57.31
AVG	1000	17.50	57.15
STD	0	0.412	1.103

DEV			
STD ERR	0	0.180	0.493

Part 2: Viscosity

Tables 2-0: Ball Information

# of balls	Ball Mass (g)	Mass/Ball (g)
1	0.34	0.340
5	1.72	0.344
10	3.46	0.346
AVG	_	0.343
STD DEV	_	0.002
STD ERR	_	0.001

Table 2-1: Ball Diameter:

Trial	Diameter
1	0.18
2	0.18
3	0.18
AVG	0.18
STD DEV	0
STD ERR	0

*Note that the information on the supplier container reads the diameter of each ball as 4.5mm, which is about 1.77 inches.

Graduated Cylinder Mass: 53.6 g
Distance Traveled in all trials: 21.9 cm

Table 2-2: Vegetable Oil Trials

Trial	Time for Ball Drop (s)	Mass

		(Liquid+Graduated Cylinder) (g)
1	0.88	143.62
2	0.69	143.96
3	0.84	144.30
4	0.80	144.64
5	0.70	144.98
AVG	0.78	144.30
STD DEV	0.06	0.4808
STD ERR	0.03	0.2150

Table 2-3: Glycerol Trials

Trial	Time for Ball Drop (s)	Mass (Liquid+Graduated Cylinder) (g)
1	4.55	175.33
2	4.79	175.67
3	4.53	176.01
4	4.45	176.35
5	4.72	176.89
AVG	4.608	176.00
STD DEV	0.127	0.5404
STD ERR	0.057	0.2417

Table 2-4: Motor Oil Trials

Trial	Time for Ball Drop (s)	Mass (Liquid+Graduated Cylinder) (g)
1	1.01	137.25
2	0.91	137.59

3	1.04	137.93
4	0.89	138.27
5	1.08	138.61
AVG	0.986	137.93
STD DEV	0.074	0.4808
STD ERR	0.033	0.2150

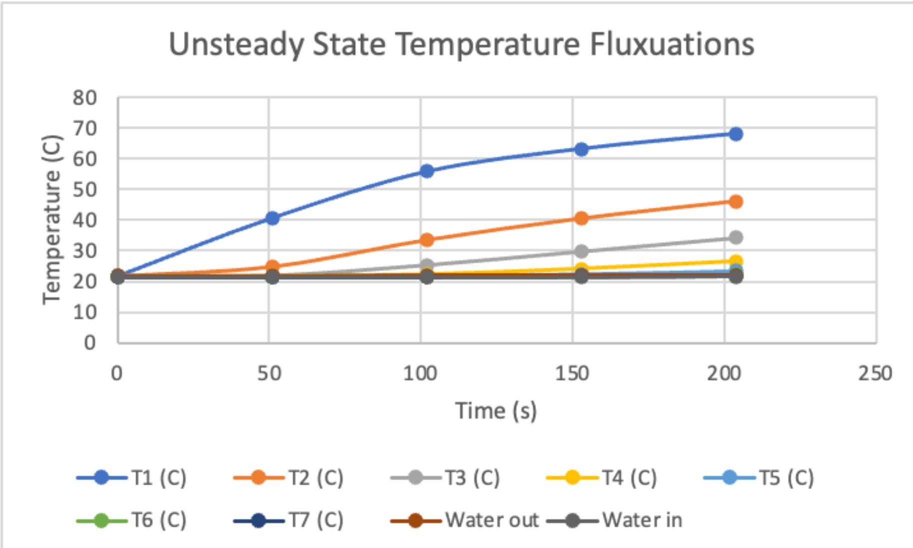
Table 2-5: Soybean Oil Trials

Trial	Time for Ball Drop (s)	Mass (Liquid+Graduated Cylinder) (g)
1	1.27	145.25
2	1.22	145.79
3	0.95	146.13
4	0.91	146.47
5	0.96	146.81
6	1.22	147.15
7	1.10	147.49
AVG	1.09	146.44
STD DEV	0.14	0.7250
STD ERR	0.05	0.2740

Figures

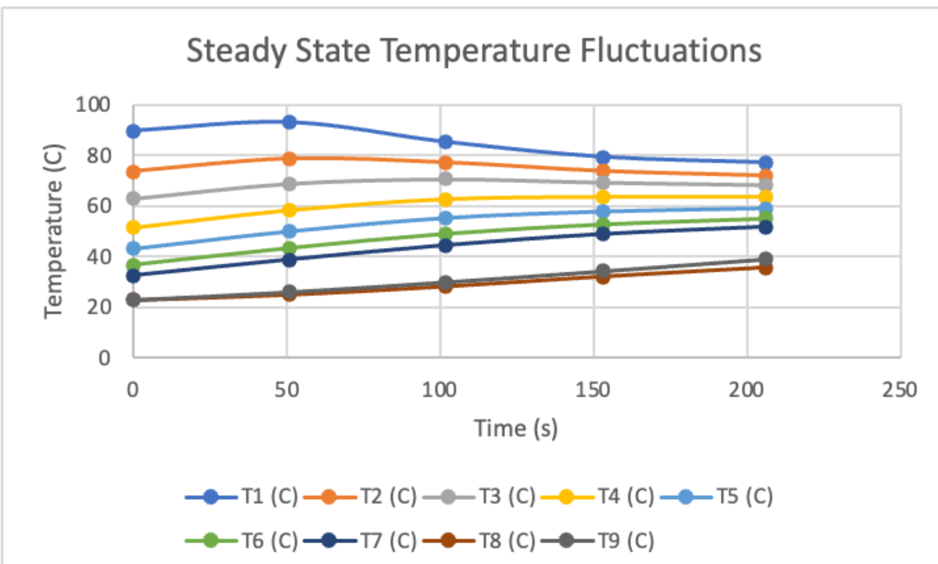
This section will display all figures relevant to collected and calculated data.

Figure 1: Unsteady State Thermocouple Variations



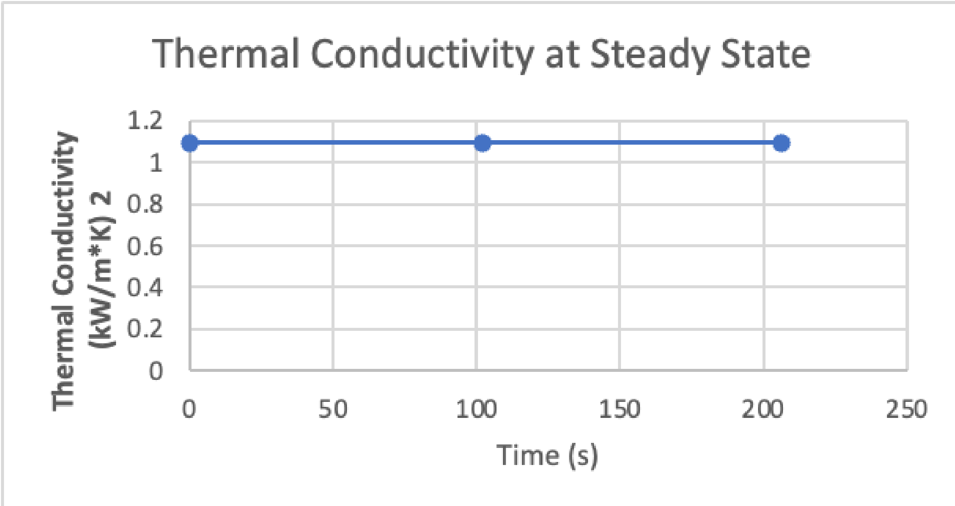
This figure displays the temperature changes throughout the pipe over a period of about 3 minutes at unsteady state. Each thermocouple point has a designated color.

Figure 2: Steady State Thermocouple Variations



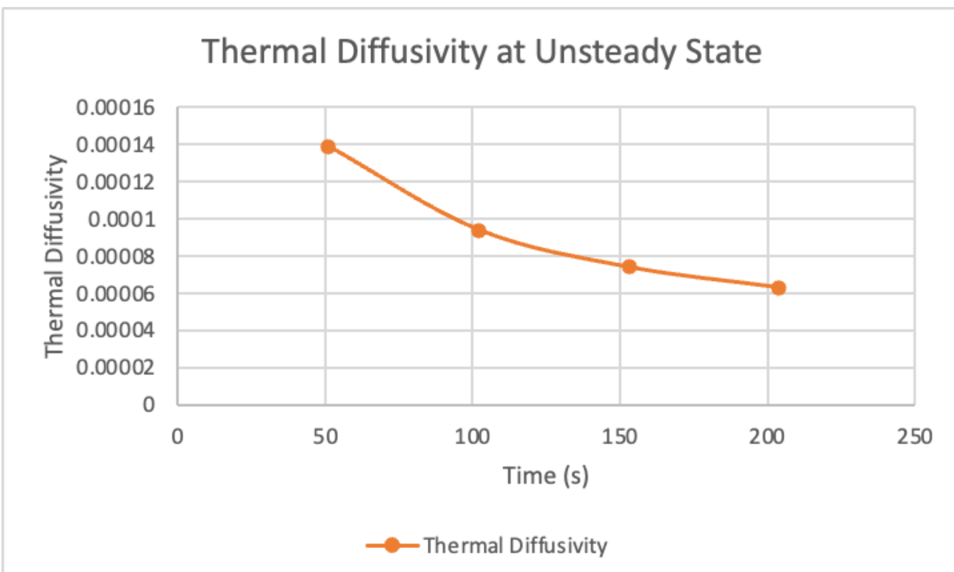
This figure displays the temperature changes throughout the pipe over a period of about 3 minutes at steady state. Each thermocouple point has a designated color.

Figure 3: Thermal Conductivity at Steady State



This figure shows the thermal conductivity of the brass rod over a period of time at steady state. The line is flat and does not stray from 1.092 at any point.

Figure 4: Thermal Diffusivity at Unsteady State



This figure shows the downward trend of thermal diffusivity over time in seconds of the brass rod at an unsteady state.

Calculations

Assume steam is at a constant temperature on one end

Part 1: Heat Conductor

Steady State:

The apparatus has a few constant variables, pipe dimensions, and water flow rate. In relation to heat transfer, we can isolate thermal conductivity and plug-and-chug.

Things to note are the conversions that must be done prior to solving for thermal conductivity. For example, the dimensions of the apparatus have been recorded in imperial bust be converted to metric to have the desired units [W/m^2*K] for thermal conductivity.

There is one last variable needed to calculate thermal conductivity. The heat capacity value is extrapolated from tabulated reference data in correlation to the water temperature.

Converting Volumetric flow rate to Mass flow rate:

$$\dot{m} = \dot{V}\rho \implies 57.15\,[\frac{mL}{s}] * 1\,[\frac{g}{mL}] * 10^{-3}\,[\frac{kg}{g}]$$
$$\dot{m} = 0.0572\,[\frac{kg}{s}]$$

Calculate Surfacel Area:

$$A_s = \pi\,(2\,[in])(11.1\,[in]) = 69.708\,[in^2] * \frac{1}{1550}\,[\frac{m^2}{in^2}]$$
$$A_s = 0.0449\,[m^2]$$

To find delta x:

$$\Delta x = (0.76 + 1.60 + 1.38 + 1.25 + 1.50 + 1.50)\,[in] * (0.0254)\,[\frac{m}{in}]$$
$$\Delta x = 0.205\,[m]$$

Note: Sum of the length designations starting from the first node (T1) to the seventh (T7)

Find tabulated specific heat value for water(vapor):

$$C_p = 1.187\ [\frac{kJ}{kg}]$$

Plug-in:

$$k = \frac{(0.0572)(1.187)\ (0.205)}{(0.0449)} = 0.309\ [\frac{kW}{m^*K}]$$

Table 3: Steady State Calculations

Time (sec)	Mass Flow Rate (g/m^3)	Heat Capacity (J/K)	Area (m^2)	Length (m)	Thermal Conductivity $[\frac{kW}{m^*K}]$
0	0.0572	1.186	0.0449	0.205	0.309
102	0.0572	1.186	0.0449	0.205	0.309
206.13	0.0572	1.187	0.0449	0.205	0.309

Percent Error in Comparison to literature value (0.146 [kW/m*K]) (Muldiani, et.al)

$$\frac{0.309-0.109}{0.109} * 100 = 183.5\%$$

Unsteady-State Calculations

For our unsteady-state calculation, we can model the brass rod as an infinite cylinder with a sudden temperature change, and use correlations to solve for the fourier number, which we can use to solve for the thermal diffusivity of the rod. Assuming 1D heat transfer, we can use the solution for the center of the rod, eg. radius = outer radius.

Referencing a correlation table (Bergsman), we can determine our constants, since in cases of sudden temperature change h is infinite, and thus so is the Biot number. We are also assuming a uniform initial temperature of 22 celsius based on room temperature. This gives us the following:

$$\theta = \frac{T(x)-100}{22-100} = 1.602exp((-\ 2.4050)^2(\frac{\alpha t}{L^2}))$$

Here, we are plugging in a data point and its associated time value to solve for diffusivity. L was earlier determined. In this case we are using T(x=1, t=51s)

$$\theta = \frac{52.8-100}{22-100} = 1.602exp((-\ 2.4050)^{2}(\frac{\alpha*51}{0.205^{2}})) \rightarrow \alpha = 0.000138699$$

Table 4: Unsteady State Calculations

Time (sec)	T(x=0)	Thermal Diffusivity $[\frac{m^2}{s}]$.
51	52.8	0.000139
102	66.4	0.000094
153	73.8	0.000074
204	78.5	0.000063
AVG	_	0.000093
STD DEV	_	2.605E-05
STD ERR	_	1.299E-05

3.41 × 10-5

Percent Error in Comparison to literature value (0.146 [kW/m*K]) (Talaghat, et.al)

$$\frac{0.000093-0.0000341}{0.0000341} * 100 = 172\%$$

Viscosity Calculations

The following is a demonstration of the equations shown in the introduction using experimental data from Glycerol. First, some constants used in the calculations:

Mass of ball: 0.34g

Drop length: 21.9 cm

Cylinder Internal radius: 32.3 mm

Cross-sectional area: $A_c = \pi(0.0323m)^2 = 0.00328\, m^2$

The next step is to determine the density of the ball and of the experimental fluid:

Density (p) = $\frac{mass}{volume}$

$$p_b = \frac{3.4\, g}{0.382\, cm^3} = 8.9\, \frac{g}{cm^3} = 890\, \frac{kg}{m^3}$$

The volume of fluid displaced by the falling sphere is given by:

$$V_f = A_c * L = \pi(.0323m)^2 * (0.219m) = 7.18 * 10^{-4}\,m^3$$

$$p_f = \frac{1.224\,kg}{(7.18*10^{-4}m^3)} = 1705\,\frac{kg}{m^3}$$

Now we can plug our experimental and calculated values into our combined forces equation and solve for viscosity:

$$\mu = \frac{(-9.81\,m/s)(2*0.0045\,m)^2(890.74 - 1705\,kg/m^3)}{18(0.04753\,m/s)} = 0.757\,\frac{N*s}{m^2}$$

Table 5: Calculated viscosity data

Fluid	Mass of fluid(kg)	Density fluid (kg/m^3)	Fall duration (s)	Velocity (m/s)	Viscosity $[\frac{N*s}{m^2}]$
Vegetable Oil	0.907	1260	0.782	0.280	0.0588
Glycerol	1.224	1705	4.608	0.04753	0.757
Motor Oil	0.8433	1175	0.986	0.2221	0.0565
Soybean Oil	0.9284	1293	1.09	0.2009	0.0800

Error Analysis

Sources of Error

When conducting the experiment, our main source of error in the heat conductor rod was a mismatch of data timing - we had expected that the time printed in the data file would match the time on display in the software, but this was not the case. This led to some guesswork on which times matched which points in the experiment, and the data points may not have matched the procedure of the experiment. There was also some variation in flow rate of cold water.

Additionally, in our calculations, our results deviated strongly from the expected results or reasonable values, but we were unable to determine the calculation problems. It’s possible that a calculation error propagated through the rest of the calculations, or that data was poorly identified.

In the viscosity portion of our experiment, error was induced especially by delays in reaction time, both between the start of the timer and the ball being dropped, but also between the ball hitting the bottom of the graduated cylinder and the timer. Additional error could have come from differing drop heights, or deviations from a straight path when the ball was falling through the liquid. Also, we assumed that the height displacement by the balls in the liquid from previous trials was negligible, which may have introduced further error. Also, residual water, or other liquids in the graduated cylinders between trials may have influenced the density of fluid in the trials.

Error Propagation

For the heat conductor experiment:

For calculating mass flow rate, the density is constant and the volumetric flow rate is our variable:

$$\dot{m} = \dot{V}\rho$$

$$\delta\dot{m} = \rho * \delta\dot{V} \rightarrow \frac{\delta\dot{m}}{\dot{m}} = \frac{\delta\dot{V}}{\dot{V}}$$

Surface Area calculations were based on constants exclusively, so error propagation was not conducted. The same goes for finding the length of the rod, because those were based on constant, given values. For calculating the k constant, our variables are mass flow rate - other values were based on constants.

$$k = \frac{-A_c}{\dot{m}C_p * \Delta x} \rightarrow \delta k = \frac{-\delta\dot{m}}{\dot{m}^2} * \frac{-A_c}{C_p * \Delta x} \rightarrow \frac{\delta k}{k} = \frac{-\delta\dot{m}}{\dot{m}} = \frac{+\delta\dot{m}}{\dot{m}} = \frac{\delta\dot{V}}{\dot{V}}$$

For the viscosity experiment:

Density equation variables were both mass and volume:

$$\rho = \frac{m}{V} \rightarrow \frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{\delta V}{V}$$

The viscosity equation variables were density and time (related to velocity):

$$\mu = \frac{-gr^2(\rho_{ball} - \rho_{fluid})}{18v} = \frac{-gr^2 t(\rho_{ball} - \rho_{fluid})}{18*L}$$

$$\rightarrow \delta t * \frac{-gr^2(\rho_{ball} - \rho_{fluid})}{18*L} + \frac{-gr^2 t}{18*L}(\delta\rho_{ball} - \delta\rho_{fluid}) = \frac{\delta\mu}{\mu} = \frac{\delta t}{t} + \frac{(\delta\rho_{ball} - \delta\rho_{fluid})}{\rho_{ball} - \rho_{fluid}}$$

Statistics

Statistical significance for viscosity can be addressed using z-score and chi-square tests on the data. The null hypothesis expects that the viscosity observed does not reflect significance relative to the literature viscosity. The alternative hypothesis expects the opposite, there will be significance among the observations.

The following equations were used to calculate for the z-score and chi-squared values of the viscosity measurements:

$$\text{Z-Score} \dashrightarrow Z = \sum \frac{p - p_o}{\sqrt{\frac{p_o(1 - P_o)}{n_{tot}}}}$$

$$\text{Chi-Squared} \dashrightarrow X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Table 7: Statistical Values of Viscosity

Fluid	z-score	chi-squared	P-value (from chi-squared)
Vegetable Oil	0.569	0.182	0.999
Glycerol	1.01	0.0959	0.999
Motor Oil	-64.929	30.937	0.00000315
Soybean Oil	-0.816	0.294	0.999

Note: Degrees of Freedom (5(rows)-1)(2(columns)-1) = 4
Number of rows and columns are representative of the number of trials and viscosity data respectively

Concerning z-score, vegetable oil, glycerol, and soybean oil have been found within the range of $-1.96 < z < 1.96$.

Concerning chi-squared, vegetable oil, glycerol, and soybean oil also have p values at ~0.99, indicating that their data does not contain significance. Signifying the null hypothesis cannot be rejected.

However, motor oil falls outside the $-1.96 < z < 1.96$ for z-score and has an incredibly small p-value. Signifying the null hypothesis can be rejected.

Discussion

For the first section of this lab, we found that at a steady state, the thermal conductivity is constant over time at 0.309 $[\frac{kW}{m*K}]$. Compared to literature values of 0.109 $[\frac{kW}{m*K}]$, this value has a percent error of 183.5%. Following this calculation, we found that the average thermal diffusivity was 0.000093 $\frac{m^2}{s}$. Here the unsteady state had a 172% error when compared to literature values of similar brass alloys. Note the brass alloy composition is unknown, so the percent error values act as a comparison to other common brass compositions. The values themselves give us an idea of how much our alloy diverges from that value but do not necessarily describe how inaccurate the data is, implying that the alloy tested here is of a different composition that shares different properties with the literature data. With that, these values found signify that the alloy is a good conductor of heat. The higher the conductivity value is, the greater the ability to transfer heat. Likewise, with thermal diffusivity, the brass alloy has a good ability to distribute heat along its length.

The objective of the determination of viscosity was to write and correct a procedure that would produce accurate results. Compared to the following expectation values listed in table 7:

Table 7: Expected Viscosity Values

	Motor Oil	**Vegetable Oil**	**Soybean Oil**	**Glycerol**
Viscosity $\frac{N*s}{m^2}$	6.3	0.04	0.1729	0.648
Source	(Toolbox, 2011)	(Smith, 2023)	(Drug Future, n.d.)	(RheoSense, 2017)

We can see that the falling ball method was successful in only certain cases. For example, the expected viscosity of motor oil was 6.3 $\frac{N*s}{m^2}$, but we obtained 0.0564 $\frac{N*s}{m^2}$. For glycerol we expected a value of 0.648 $\frac{N*s}{m^2}$, and obtained 0.757 $\frac{N*s}{m^2}$. The same can be said for vegetable oil with an expected value of 0.04 and an obtained of 0.0588, whereas soybean oil had an expected value of 0.1729 $\frac{N*s}{m^2}$ and we calculated 0.0565 $\frac{N*s}{m^2}$. The data suggests that highly viscous fluids will suffer great error from the falling ball method, while low viscosity fluids will not. This is likely due to the fact that we did not account for the fluctuation of fall that the ball experiences. Although the other forces are equally important, we did not account for the fact that the fall will not fall in a straight line due to fluid dynamics. In fact, this would explain the difference in values of the expected and calculated viscosity of the motor oil. With higher viscosity comes greater fluctuations in fall, resulting in larger error than the lower viscosity fluids.

Recommendations

Our first objective was to determine whether steady state or unsteady state was the best option for routine analysis for the handmade heat exchanger device. Based on the procedure we described in the methods section, we believe that the steady state approach is a more efficient use of resources and provides an accurate enough result. It requires less time, and the given equipment is easier to operate when going in a steady state.

We also needed to create and test a procedure that determines the viscosity of various liquids. While the procedure described above worked okay for low viscosity liquids, it did not hold the same standard for fluids of high expected viscosity. Due to this reasoning, we do not recommend using this method for future testing. The procedure followed is fine if it is a last resort, but a different method would be preferred.

Equal Contribution

	Group Members			
Task	Adrian	Liza	Chris	Frances
Safety Incident Report		✔		
Introduction			✔	
Presenter		✔		
Powerpoint Slides		✔		
Script		✔		
Calculations	✔		✔	✔
Data				✔
Presentation of Results				✔
Figures		✔		
Error Analysis				✔
Statistics	✔			
Discussion		✔		
Recommendations		✔		

References		✔		
Writing Quality Check	✔	✔	✔	✔
Equal Contribution	✔	✔	✔	✔

Chris did not attend the lab and was thus not able to present (it was his turn to present Lab 3). Therefore, Liza switched with Chris on the presentation.

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[Intro+Viscosity Calculations]

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