Lab 2 Report: Heat Exchanger

By Chris Mechling, Elizaveta Bell, Frances Yih, Adrian Mercado

AB-1

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Introduction

Like many systems involving water distribution, residential showers require heating before being released by the shower head. In order to keep costs low, our partners have requested an investigation of a concentric pipe system to determine a flow and length configuration that recovers the highest heat loss during water heating.

Concentric heat exchangers (HX), known alternatively as double-pipe HX, are specialized systems designed to transfer heat between two substances. We see these HX as A/C or HVAC-type devices. Double-pipe HX transfers heat by having fluid flow adjacent to one another using the pipe walls as a conductive material. Due to the fluid flow being adjacent to one another, HX can maximize or minimize heat transfer by inducing a countercurrent or co-current flow. The preference between the two flows is up to what is deemed practical for the situation.

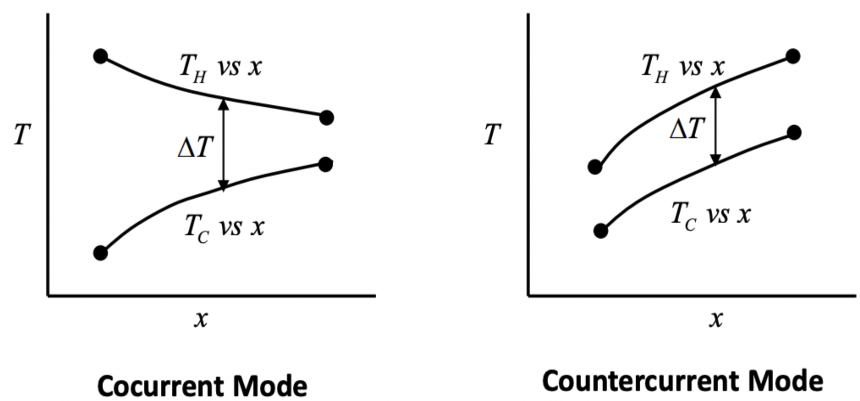

Fig. 1. Co-current vs. Countercurrent Fluid Flow

The graph above visualizes the general principle of countercurrent and co-current flow. Generally, the countercurrent flow has a constant delta T throughout x length. As for co-current flow, at shorter distances, delta T is relatively high. And when x increases, delta T trends toward zero.

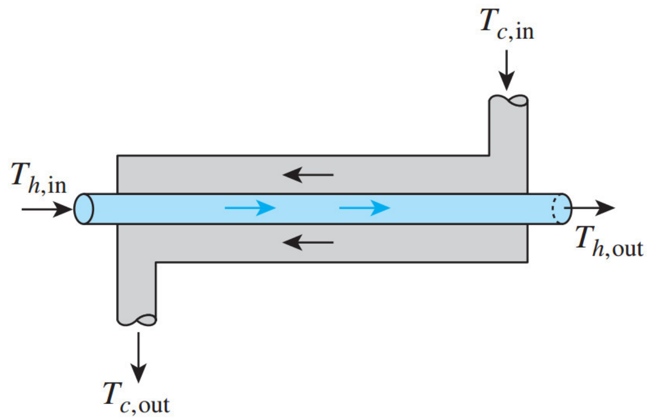

Fig. 2. Concentric Pipe Heat Exchanger

Another principle to address is the variable Temperature log-mean (ΔT_{lm}). Contrastly to delta T, the temperature log-mean accounts for the varying temperature between the inlets and outlets. This is done by finding the average temperature difference at the inlets and outlets. The average temperature difference is essentially the temperature of the liquid at these endpoints since heat transfer occurred at the full-length *x*.

In the experiment, a concentric heat exchanger system was provided to determine the most cost-effective heat exchanger. Cost-effectiveness is related to the overall heat transfer coefficient, U_0. As per request, we observed how U_0 varies under the following conditions: counterflow, co-current flow, and pipe length.

To obtain the necessary data, we varied the flow of hot and cold water at various flow rates. In order to complete the assigned task, there was a collection of equations used.

Assuming that the system was at steady state, we utilized the heat transfer equation for adiabatic heat exchanger systems:

$$Q = U_0 A T_{LM}$$

where:

U_0 $[\frac{W}{m^2K}]$ is the overall heat transfer coefficient

$$A = area\ of\ tube = \pi DL\ [m^2]$$

$$T_{LM} = \frac{\Delta T_{inlet} - \Delta T_{outlet}}{ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}$$

ΔT_{inlet} is the temperature difference across the tube inlets

ΔT_{outlet} is the temperature difference across the tube outlets

(Earle, 1983)

It can be seen that the means for calculating area and T_{LM} are given, but those for U_0 are not. In order to calculate the heat transfer coefficient, we considered the resistances present in the system.

$$U_0 = (R_H + R_C + R_p)^{-1}$$

R_H is the resistance in the hot stream due to convective heat transfer.

R_C is the resistance in the cold stream due to convective heat transfer.

R_p is the resistance of the pipe due to conductive heat transfer.

These resistances can be found using their respective equations:

$$R_H = \frac{1}{\pi h_i D_{inner} L}$$

$$R_C = \frac{1}{\pi h_a D_{outer} L}$$

$$R_p = \frac{ln(\frac{D_{outer}}{D_{inner}})}{2\pi k_{stainless\ steel} L}$$

In order to find these resistances, one must look to the flow regime. In fact, it can be found that the turbulence of the flow determines the resistance. In fact, we can use a combination of the Reynolds number (Re), Prandtl Number (Pr), and the Nusselt number (Nu) to determine the type of flow the system displays.

$$Re = \frac{\rho u L}{\mu}$$

Where ρ is the density of fluid, u is the velocity of flow, L is the length of the pipe, and μ is the viscosity of fluid.

$$Pr = \frac{C_p \mu}{k}$$

Where C_p is the heat capacity, μ is the dynamic viscosity, and k is the thermal conductivity for water = 0.6 $[\frac{W}{mK}]$.

For flow within a circular pipe, the Nusselt Number can be defined in various ways, but we will only use one method:

If the flow is laminar, then the Nusselt number is equal to:

$$Nu = \frac{hD}{k} = 4.36$$

(Bergsman, 2023)

For turbulent flow where 0.7 ≤ Pr ≤ 16 ≤ 16,700, $Re_D \geq 2000$:

$$\overline{Nu}_D = 0.027(Re_D)^{\frac{4}{5}}(Pr)^{\frac{1}{3}}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$where\ \mu_s\ is\ the\ viscosity\ at\ room\ temperature\ (25C) = 10^{-3}\ Pa{\cdot}s$$

Then, the resistances can be calculated using the heat transfer coefficient, h, given by:

$$h = \frac{Nu*k}{D}$$

There will be two solutions, depending on the location of flow. At the center of flow, D is the diameter of the inner tube's inside diameter and h will be known as h_i. If the flow is being examined at the ends of the pipes (annulus), D = $D_{inner} - D_{outer}$ and h is denoted h_a.

Using this method, we will determine the heat transfer coefficient and the total heat each combination of flows produces.

And finally, in order to find the total cost of the shower, we will use the following equation:

$$Total\ Cost = X\ kW * 0.13\ \frac{\$}{hour}$$

We expected high heat transfer yields with in a system consisting of high flow rates and countercurrent flow, based off of two equations; $Q = UA\Delta T_{lm}$ and $Q = \dot{m}A\Delta T$. The heat transfer rate can be maximized by inducing a high mass flow rate. And this would work for both hot and cold liquids. Using this equation, we can observe the respective liquids separately, setting them up as a convective heat transfer and validating the variable ΔT.

$$Q_{Hot} = \dot{m}A\Delta T$$

ΔT = Can be treated as a constant negative value, due to the fluid getting colder

A = Can be treated as a constant value

After assigning arbitrary values and simplifying we can see the direction of heat transfer and how it relates to mass flow rate:

$$Q_{Hot} = \dot{m}(1)(-1) \quad --> \quad -Q_{Hot} = \dot{m}$$

$$Q_{Cold} = \dot{m}A\Delta T$$

ΔT = Can be treated as a constant positive value, due to the fluid getting hotter

A = Can be treated as a constant value

After assigning arbitrary values and simplifying we can see the direction of heat transfer and how it relates to mass flow rate:

$$Q_{Cold} = \dot{m}(1)(1) \quad --> \quad Q_{cold} = \dot{m}$$

Using these assumptions, Q for the "hot" water goes negative, and the "cold" Q goes positive. Verifying that heat transfer is going from hot to cold. Based on this, we can scale heat transfer with mass flow rate, as claimed in the previous paragraph.

For the equation, $Q = \dot{m}A\Delta T$. The ΔT goes to zero for the co-current flow but stays constant for the countercurrent, the longer the length. Results are dependent on the length. A longer length favors countercurrent, and a short length benefits co-current. Overall, we expected that the countercurrent flow would yield higher heat transfer for the length of pipes provided, and that it would allow for the most cost effective production. As for pipe length, we still favor a 2-pipe system due to area being inversely related to the overall heat transfer coefficient, $\frac{Q}{U^{*}\Delta T_{lm}} = A$. If we were to go smaller (Ex: 1-pipe system), then this may favor co-current flow rather than countercurrent.

Method

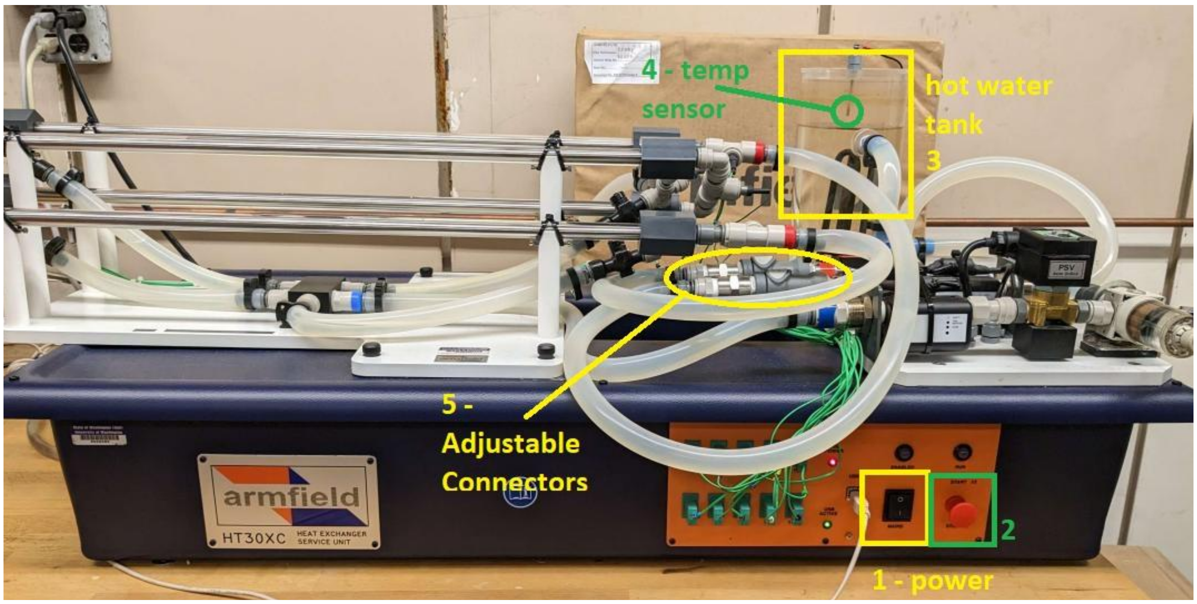

Fig. 3. HT36 Apparatus Diagram

1) Turn on heat exchanger by flipping switch to the “on” position (1, Figure 3), and then pulling the red switch labeled “stop” on the unit (2, Figure 3)
2) Add DI water to the hot water tank (3, Figure 3) until DI water is above the temperature sensor (4, Figure 3)
3) Check the adjustable connectors (5, Figure 3) to see if the flow is set up in a co-current or countercurrent direction
    a) The arrows on the connectors indicate the direction of flow, and this can be used in visual comparison to the rest of the apparatus to determine whether it is a countercurrent or co-current set up
    b) When running this experiment, the heat exchanger was in a countercurrent set up.

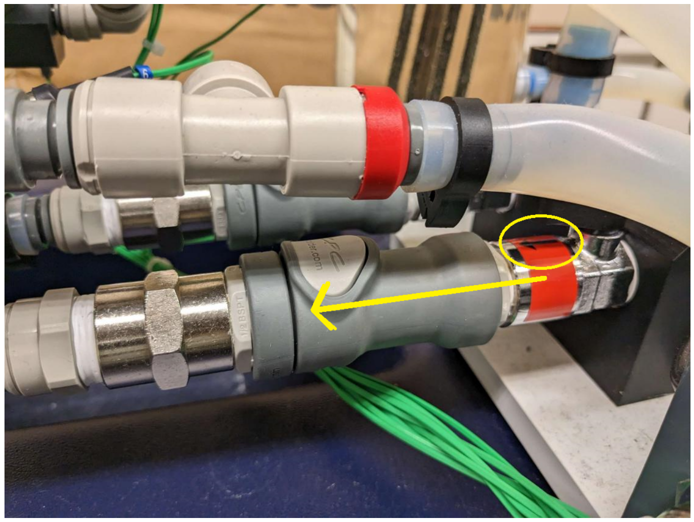

Fig. 4. Enlarged Image of Connectors

4) Boot up software on computer - HX36 Armfield Software
    a) Check that the flow option corresponds to the set up of the heat exchanger
    b) For the first experiment, we will be setting the heat exchanger to 4 tube-lengths
5) Turn on heater and use automatic settings to adjust water temperature to 38 degrees celsius
6) Check that the cold water valve (on the wall behind the heat exchanger) is open to allow cold water through

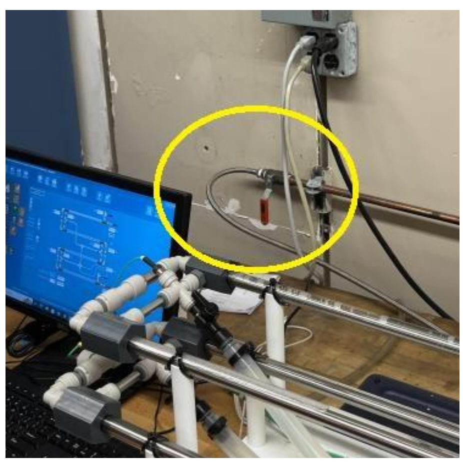

Fig. 5. Cold Water Valve Location (position off)

7) Adjust flow rates to an "intermediate flow rate"
    a) Experimentation with the unit showed flow rates to be within a general range of 2 liters per minute to 4 liters per minute
    b) Both hot water and cold water flow rates were set to 2 liters per minute

8) Run 3 trials by leaving the heat exchanger on and taking measurements a few seconds apart
    a) We will be taking heat exchanger temperature data, as well as water temperatures and water flow rates from HX software
9) Turn off the heater and heat exchanger from the HX software
10) Turn off HX36 heat exchanger by pushing red switch and turning power switch to “off” position
11) Swap the direction of flow by detaching direction-labeled tubing and switching their positions
12) Repeat step 7 for the alternate direction.
13) Reset to countercurrent flow set up.
14) At this point we will move on to the next experiment - keep cold water flow constant, we will try three different flow rates of hot water, taking measurements for each
    a) We will be taking 5 trials, ten seconds apart, automatically, through the armfield software unit
15) Adjust the cold water flow rate to the next flow rate, and reset the hot water to the first hot flow rate “H1”
16) Repeat steps 14 and 15.
    a) This should give us a total of 9 combinations of flow rates with 5 trials each
17) At this point we move on to the next experiment - leaving the heat exchanger in the same configuration for sake of time, shorten the apparatus to two tube lengths in the HT software
    a) Take 5 trials for comparison data, again, ten seconds apart
18) At this point in the experiment, to create more consistent data for co-current and countercurrent flow calibration, we repeated 5 trials with the software on a co-current set up
    a) We then used the C2+H2 data for comparison instead of the original 3-trial data for our counter calibration, since the C2+H2 trial also used 3 L/min for each flow.

Results

This section displays the data collected throughout the experiment, as well as detailed error analysis and relevant figures.

Table 1: Counter Current Flow Calibration

	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	Cold Flow Rate [L/min]	Hot flow rate [L/min]
4 Tubes						
1	20.4	28.1	37.8	30.4	2.92	3.1

2	20.1	27.9	37.7	30.2	2.99	3.2
3	20.1	27.9	37.8	30.2	2.86	3.2
AVG	20.2	28.0	37.8	30.3	2.92	3.2
STD Dev	0.173	0.115	0.0577	0.115	0.0651	0.058
STD ERROR	0.100	0.0664	0.0333	0.0664	0.0376	0.033

This table represents the collected data for countercurrent flow with all four tubes in use. It described the temperature differences between the inlets and outlets of the cold and hot water flows, respectively in degrees celsius. In addition, this table contains flow rates for the two flows. All data was taken in 10 second intervals. Note that this data was not used in favor of the later "C2+H2" data, which was set up in the same manner but was run in a consistent manner to the rest of the experiment.

Table 2: Co-current Flow Calibration

Trials	$T_{C,\,in}$ (°C)	$T_{C,\,out}$ (°C)	$T_{H,\,in}$ (°C)	$T_{H,\,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
4 Tubes						
1	16.1	22.8	33.8	27.5	3.08	2.9
2	16.1	23.0	33.8	27.8	3.08	3.0
3	16.0	22.9	33.7	27.7	3.03	3.0
4	16.0	23.0	33.9	27.8	3.11	3.0
5	16.0	22.9	34.0	27.7	2.86	3.0
AVG	16.0	22.9	33.8	27.7	3.03	3.0
STD Dev	0.0548	0.0837	0.114	0.122	0.100	0.041
STD ERROR	0.0245	0.0374	0.0510	0.0548	0.0449	0.018

This table represents the collected data for co-current flow with all four tubes in use. It described the temperature differences between the inlets and outlets of the cold and hot water flows, respectively in degrees celsius. In addition, this table contains flow rates for the two flows. All data was taken in 10 second intervals.

Table 3: Flow rates used the following data tables, with their respective labels and abbreviations.

Hot Flow Rates (L/min)	Cold Flow Rates (L/min)
2 (H1)	2 (C1)
3 (H2)	3 (C2)
4 (H3)	3.5 (C3)

This is a “key” table of the hot and cold flow rates in units of [L/min] that were aimed for during this experiment for reference.

Table 4: Counterflow based temperature data collection for each flow rate combination.

C1 + H1	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	39.1	29.2	17.1	26.5	2.04	2.0
2	39.0	29.3	17.1	26.5	2.16	2.0
3	38.9	29.1	17.1	26.4	2.04	2.0
4	38.8	29.0	17.1	26.4	2.14	1.9
5	38.7	29.0	17.1	26.3	2.08	1.9
AVG	38.9	29.1	17.1	26.4	2.09	2.0
STD Dev	0.14	0.117	0	0.0753	0.0500	.052
STD ERROR	0.062	0.0523	0	0.0336	0.0223	0.023
C1+ H2	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	37.4	29.9	16.9	27.0	2.14	3.0
2	37.3	29.9	16.9	27.0	2.08	3.0
3	37.5	30.0	16.9	27.0	2.17	3.0
4	37.4	30.0	16.9	27.0	2.11	2.9
5	37.4	30.0	16.9	27.0	2.20	3.0
AVG	37.4	30.0	16.9	27.0	2.14	3.0

STD Dev	0.0707	0.0516	0	0	0.0387	0.050
STD Error	0.0316	0.0230	0	0	0.0173	0.022
C1 + H3	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	37.5	31.1	16.9	27.9	2.17	3.9
2	37.5	30.9	16.9	27.7	2.16	4.0
3	37.3	31.0	16.9	27.7	2.14	4.0
4	37.1	30.8	16.9	27.6	2.15	3.9
5	37.0	30.8	16.9	27.6	2.23	3.9
AVG	37.3	30.9	16.9	27.7	2.17	3.9
STD Dev	0.228	0.117	0	0.122	0.0354	0.055
STD Error	0.101	0.0523	0	0.0545	0.0158	0.0245
C2 + H1	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	37.1	26.9	16.8	24.2	2.92	2.1
2	37.2	27.0	16.8	24.2	3.08	2.1
3	37.1	26.8	16.8	24.1	2.95	2.1
4	37.4	26.8	16.7	24.0	3.14	2.0
5	37.6	26.9	16.7	24.1	2.86	2.0
AVG	37.3	26.9	16.8	24.1	2.99	2.1
STD Dev	0.217	0.0837	0.0548	0.0837	0.116	0.055
STD Error	0.0970	0.0374	0.0245	0.0374	0.0518	0.0245
C2 + H2	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]

1	37.4	28.3	16.6	25.3	3.04	3.0
2	37.1	28.3	16.6	25.3	3.05	3.0
3	36.9	28.2	16.5	25.1	3.05	3.0
4	36.7	28.0	16.5	25.0	3.08	2.9
5	36.5	28.0	16.5	25.0	2.89	3.1
AVG	36.9	28.2	16.5	25.1	3.02	3.0
STD Dev	0.349	0.152	0.0548	0.152	0.0753	0.071
STD Error	0.156	0.0679	0.0245	0.0679	0.0336	0.0317
C2 + H3	$T_{H,in}$ (℃)	$T_{H,out}$ (℃)	$T_{C,in}$ (℃)	$T_{C,out}$ (℃)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	34.6	28.0	16.4	24.8	3.05	3.9
2	34.5	28.0	16.3	24.8	3.05	4.0
3	34.4	27.9	16.4	24.6	2.93	3.9
4	34.1	27.7	16.3	24.5	2.99	4.1
5	34.0	27.6	16.3	24.5	2.98	4.1
AVG	34.3	27.8	16.3	24.6	3.00	4.0
STD Dev	0.259	0.182	0.0548	0.152	0.0510	0.10
STD Error	0.116	0.0813	0.0245	0.0679	0.0228	0.0447
C3 + H1	$T_{H,in}$ (℃)	$T_{H,out}$ (℃)	$T_{C,in}$ (℃)	$T_{C,out}$ (℃)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	34.7	24.6	16.2	21.8	3.52	1.9
2	34.9	24.7	16.2	21.9	3.88	2.0
3	35.2	24.8	16.2	22.0	3.49	1.9
4	35.2	24.9	16.2	22.0	3.65	2.0
5	35.4	24.9	16.2	22.0	3.65	2.0

AVG	35.1	24.8	16.2	21.9	3.64	2.0
STD Dev	0.278	0.130	0	0.0894	0.153	.055
STD Error	0.124	0.0581	0	0.0399	0.0684	0.0245
C3 + H2	$T_{H, in}$ (°C)	$T_{H, out}$ (°C)	$T_{C, in}$ (°C)	$T_{C, out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	35.3	26.4	16.1	23.2	3.47	3.0
2	35.0	26.3	16.1	23.1	3.45	3.0
3	34.6	26.1	16.1	22.9	3.71	3.1
4	34.5	26.0	16.1	22.8	3.71	3.1
5	34.4	25.9	16.1	22.8	3.59	2.9
AVG	34.8	26.1	16.1	23.0	3.59	3.0
STD Dev	0.378	0.207	0	0.182	0.125	0.084
STD Error	0.169	0.0375	0	0.0375	0.0559	0.0375
C3 + H3	$T_{H, in}$ (°C)	$T_{H, out}$ (°C)	$T_{C, in}$ (°C)	$T_{C, out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	33.5	26.4	16.1	23.1	3.55	4.0
2	33.2	26.3	16.1	23.0	3.71	4.0
3	33.1	26.1	16.1	22.9	3.55	4.0
4	32.8	26.2	16.1	22.9	3.65	4.0
5	32.6	25.9	16.1	22.7	3.71	4.0
AVG	33.0	26.2	16.1	22.9	3.63	4.0
STD Dev	0.351	0.192	0	0.148	0.0805	0
STD Error	0.156	0.0858	0	0.0661	0.0360	0
STD Dev (All)	1.83	1.89	0.365	1.90	.622	.83

This data table shows all collected data for every combination of the flow rates above (9 combinations) with their respective 5 trials. All data was taken in intervals of 10 seconds. All four tubes were used.

Table 5: Comparative co-current temperature data for the C2 + H3 flow rates.

C2 + H3	$T_{H,in}$ (°C)	$T_{H,out}$ (°C)	$T_{C,in}$ (°C)	$T_{C,out}$ (°C)	Cold flow rate [L/min]	Hot flow rate [L/min]
1	31.9	25.9	16.0	23.0	3.27	4.0
2	32.0	26.0	16.0	23.0	3.17	3.8
3	32.1	26.1	16.1	23.1	3.17	3.9
4	32.0	25.9	16.1	23.0	3.17	4.0
5	32.2	26.1	16.0	23.1	3.30	3.9
AVG	32.0	26.0	16.0	23.0	3.22	3.9
STD Dev	0.114	0.100	0.0548	0.0548	0.0639	0.084
STD Error	0.0509	0.0447	0.0245	0.0245	0.0285	0.0375

This table displays the collected data for C2 + H3 in 2 tubes in co-current flow, chosen to serve as a comparison for the calculations below.

Table 6: Software calculated Reynold’s Number, dimensionless.

	Cold Water Re	**Hot Water Re**
4 Tubes		
C1 + H1	2091	6728
C1 + H2	2209	10263
C1 + H3	2273	13338
C2 + H1	2892	6732
C2 + H2	3053	9831
C2 + H3	3047	12437
C3 + H1	3358	5998
C3 + H2	3371	9619

C3 + H3	3456	12480
Co-current Calibration	3027	9192
2 Tubes (countercurrent):		
C2 + H3	3313	12541
Assumed Error		

This table displays the software collected Reynold's number for each flow rate combination tested, including the nine 4 tube countercurrent combinations, the comparative 4 tube co-current calibration, and the 2 tube countercurrent trial. The data is dimensionless and represents all trials for each data set.

Table 7: Software collected heat capacity data.

4 Tubes	**Cold Water Cph** $\frac{kJ}{kg*K}$	**Hot Water Cph** $\frac{kJ}{kg*K}$
C1 + H1	4.181	4.178
C1 + H2	4.180	4.178
C1 + H3	4.180	4.178
C2 + H1	4.181	4.178
C2 + H2	4.181	4.178
C2 + H3	4.181	4.178
C3 + H1	4.182	4.178
C3 + H2	4.182	4.178
C3 + H3	4.182	4.178
Co-current Calibration	4.181	4.178
2 Tubes		

C2 + H3	4.181	4.178

This table contains the software collected heat capacity data for the nine 4-tube trials, as well as the co-current calibration, and the 2 tube countercurrent trial.

Table 8: Software collected dynamic viscosity data.

	Cold Water $10^{-3}Pa * s$	**Hot Water** $10^{-3}Pa * s$
4 Tubes		
C1 + H1	0.961	0.729
C1 + H2	0.953	0.735
C1 + H3	0.942	0.725
C2 + H1	0.994	0.766
C2 + H2	0.981	0.749
C2 + H3	0.987	0.773
C3 + H1	1.032	0.804
C3 + H2	1.014	0.782
C3 + H3	1.013	0.795
Co-current Calibration	1.002	0.792
2 Pipes		
C2 + H3	0.973	0.786

This table represents the software collected data of the dynamic viscosity of the fluid. It shows collections for the nine 4-tube countercurrent trials, as well as the 4-tube co-current calibration, and the 2-tube countercurrent comparison.

Calculations

In order to complete the calculations required accurately, we made some assumptions. To begin, bends will not be considered as part of the length of each pipe. We will assume that the flow is at steady state and the dependence on temperature is linear and one dimensional. In order to simplify numerical calculation, we assume that there are no frictional losses within the tubes and that resistances will add in series.

The sample calculations will be done using the C1 + H1 data.

To begin, the type of flow within the tube needs to be determined. Using the software calculated values, we held the threshold for turbulent flow at a Reynolds number of 2000 (Rogers, 2017). Anything over this value was considered turbulent, and any flow under laminar.

With this reasoning, all data points collected in this lab were presumed to be turbulent per the data table above.

For turbulent flow, the Prandtl (Pr) and Nusselt (Nu) numbers must be found in order to assess the flow and reach the end goal of determining the heat transfer coefficient, U.

To begin, we calculated the Prandtl number, followed by the Nusselt number using the Reynold's numbers obtained by the system.

For hot water, we found that:

$$Pr = \frac{(4.180 \frac{kJ}{kg*K})(0.729 \frac{kg}{m*s})}{(0.006 \frac{kJ}{s*m*K})} = 507.6$$

And for cold water:

$$Pr = \frac{(4.178 \frac{kJ}{kg*K})(0.961 \frac{kg}{m*s})}{(0.006 \frac{kJ}{s*m*K})} = 669.7$$

Due to the turbulence of the flow, the Nusselt number was calculated using the Pr value above:

And for the turbulent flow hot water:

$$Nu_D = 0.027(6728)^{\frac{4}{5}}(507.6)^{\frac{1}{3}}(\frac{0.961*10^{-3}}{8.9*10^{-4}})^{0.14} = 241.7$$

Table 9: Reynold's, Prandtl, and Nusselt Numbers for corresponding flows

Countercurrent 4-tubes	Cold Water Re	Pr	Nu	Hot Water Re	Pr	Nu
C1 + H1	2091	669.7	108.2	6728	507.6	241.7
C1 + H2	2209	663.9	112.6	10263	511.8	340.2

C1 + H3	2273	656.3	114.6	13338	504.8	416.8
C2 + H1	2892	692.7	142.5	6732	533.4	247.6
C2 + H2	3053	683.6	147.9	9831	521.6	331.6
C2 + H3	3047	687.8	148.1	12437	538.3	406.3
C3 + H1	3358	719.3	163.5	5998	559.9	231.0
C3 + H2	3371	706.8	162.6	9619	544.5	332.6
C3 + H3	3456	706.1	165.8	12480	553.6	412.9
Co-current Calibration	3027	698.3	147.9	9192	551.5	322.7

(This table displays the calculated Pr and Nu values for each flow combination of 4 tubes, and the co-current calibration data).

Now, we can use these calculations to find the resistances of each pipe in order to calculate the overall heat transfer coefficient.

Due to the set up of the heat exchanger, we knew that the cold flow was within the annulus of the tubes (between the inner and outer tubes), and the hot flow was in the inner tube. This allowed us to calculate the heat transfer coefficients and their respective resistances using the following methods.

First, it was required to find the heat transfer coefficient h:

Using C1 + H1 as an example, we find that the coefficient for the cold water's inner tube is:

$$h_i = \frac{108.2 * 0.00865 \frac{kJ}{s*m*K}}{0.0083\ m} = 174.7\ \frac{kJ}{m^2 sK} = 174.7 \frac{W}{m^2 K}$$

And again for annulus flow:

$$h_a = \frac{241.7 * 0.00865 \frac{kJ}{s*m*K}}{0.0012\ m} = 541.0\ \frac{kJ}{m^2 sK} = 541.0 \frac{W}{m^2 K}$$

Table 10: Heat transfer coefficient calculations

Countercurrent 4-tubes	$h_i\ [\frac{W}{m^2 K}]$	$h_a\ [\frac{W}{m^2 K}]$

C1 + H1	174.7	541.0
C1 + H2	245.9	563.0
C1 + H3	301.3	572.8
C2 + H1	179.0	712.5
C2 + H2	239.7	739.4
C2 + H3	293.7	740.4
C3 + H1	167.0	817.4
C3 + H2	240.4	813.1
C3 + H3	298.5	829.1
Co-current Calibration	233.3	741.8

(This table contains the calculated values of the inner and annulus heat transfer coefficient for all combinations of flow, along with the co-current flow values).

We can now calculate the three thermal resistances, R_H, R_C, and R_p for C1 + H1:

$$R_H = \frac{1}{\pi(222.9\ \frac{W}{m^2K})(0.0083\ m)(0.66\ m)(4\ tubes)} = 0.0430$$

$$R_C = \frac{1}{\pi(690.2\ \frac{W}{m^2K})(0.0095\ m)(0.66m)(4\ tubes)} = 0.0121$$

$$R_p = \frac{ln(\frac{0.0095\ m}{0.0083\ m})}{2\pi(14.4\ \frac{W}{m*K})(0.66\ m)(4\ tubes)} = 0.00814\frac{K}{W}$$

k value for stainless steel retrieved from (Engineering ToolBox, 2005)

Thus, the heat transfer rate is:

$$U_0 = (0.0430\ +\ 0.0121\ +\ 0.00814)^{-1} = 15.8\ \frac{W}{m^2*K}$$

Finally, we used the steady state heat transfer equation to find the average heat loss Q from each flow combination:

$$Q = U_0 A T_{LM}$$

where

$$T_{LM} = \frac{\Delta T_{inlet} - \Delta T_{outlet}}{ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})} \text{ and } A = \pi D L$$

(Earle, 1983)

Continuing with the sample calculations, we obtained that:

$$Q = (15.80 \frac{W}{m^2 * K})(\pi * 0.0095m * 4(0.66m))(\frac{(38.9-17.1)-(29.1-26.4)}{ln(\frac{38.9-17.1}{29.1-26.4})}) = 11.39\, W$$

Table 11: Resistances, Overall heat transfer coefficients, and Q rates for all

4-Tubes	R_H [$\frac{W}{m^2 * K}$]	R_C [$\frac{W}{m^2 * K}$]	R_p [$\frac{W}{m^2 * K}$]	U [$\frac{W}{m^2 * K}$]	Q [W]	Q [kW]
C1 + H1	0.05487	0.01548	0.008140	12.74	9.179	0.00918
C1 + H2	0.03899	0.01488	0.008141	16.13	11.571	0.01157
C1 + H3	0.03182	0.01462	0.008141	18.32	13.403	0.01340
C2 + H1	0.05357	0.01176	0.008141	13.61	9.535	0.00953
C2 + H2	0.03999	0.01133	0.008141	16.82	12.167	0.01217
C2 + H3	0.03264	0.01131	0.008141	19.19	12.959	0.01296
C3 + H1	0.05742	0.01023	0.008141	13.19	8.871	0.00887
C3 + H2	0.03988	0.01030	0.008141	17.15	11.728	0.01173
C3 + H3	0.03212	0.01010	0.008141	19.85	13.025	0.01302
Co-current Calibration	0.04110	0.01129	0.008141	16.52	9.179	0.00918
2-Tubes						
C2 + H3	0.02446	0.00810	0.00814	24.57	9.535	0.00953

(This table displays the calculated heat transfer coefficient, heat Q, and resistance values for all nine 4-pipe flows, as well as the co-current calibration for 4-pipes, and the 2-pipe flow rate comparison)

And finally, in order to find the total cost of the shower, we assumed a basis that each shower taken in the building lasts for one hour to find:

$$\text{Total Cost C1 + H1} = 0.01139\text{ kW} * \$0.13\frac{\$}{kWhe} = \$0.00148$$

Assuming that a person showers 20 times in one month, the total yearly cost at this rate is equal to $0.3552 for C1 + H1 flow.

Table 12: Total Cost per Shower

4 Tubes	Total Spendings per Hour Usage	Estimated yearly
C1 + H1	$0.001193	$0.28632
C1 + H2	$0.001504	$0.36096
C1 + H3	$0.001742	$0.41808
C2 + H1	$0.001240	$0.2976
C2 + H2	$0.001582	$0.37968
C2 + H3	$0.001685	$0.4044
C3 + H1	$0.001153	$0.27672
C3 + H2	$0.001525	$0.366
C3 + H3	$0.001693	$0.40632
2 Tubes		
C2 + H3	$0.000977134	$0.28368

(This table displays the total cost per hour usage and the estimated yearly cost of every person showering).

In order to understand the meaning of this data, we gathered information on the typical cost, flow rate, and temperature of showers.

First, we set a basis for what a typical shower heat exchanger looks like, sourced from the EPA website (U.S. Environmental Protection Agency, 2017):

Average volumetric flow rate = 7.8 $\frac{L}{min}$

Cold water temperature = $20\ (°C)$

Hot water temperature = $37\ (°C)$

C_p of water = $4.18\ [\frac{J}{g*K}]$

Using this information, we were able to determine the average cost per year of a typical shower. We utilized the basic heat transfer equation given by:

$$Q\ =\ mC_p\Delta T$$

Where m is the mass flow rate and ΔT is the change in temperature.

The average mass flow rate can be found through the following process:

$$7.8\ \frac{L}{min}\ *\ 60min\ =\ 468L$$

Thus,

$$Q = mCp(\Delta T) = 35{,}992{,}224\ J$$

$$\frac{Q\ [J]}{3{,}600{,}000[\frac{J}{kWh}]}\ =\ 9.998kWh$$

9.998 kWh * \$0.13$\frac{\$}{kWh}$ = \$1.30 for a generic 1 hour shower

Thus, the estimated yearly cost of 20 showers per month is:

$$\$1.30\ \ *\ \ 20\ showers\ \ *\ \ 12\ months\ \ =\ \ \$312$$

Now, redoing this for the 4 $\frac{L}{min}$ and 2 $\frac{L}{min}$ flow rates used in data collection, we find that the average cost per year is ~\$0.15 per year and ~\$0.074 per year, respectively.

Figures

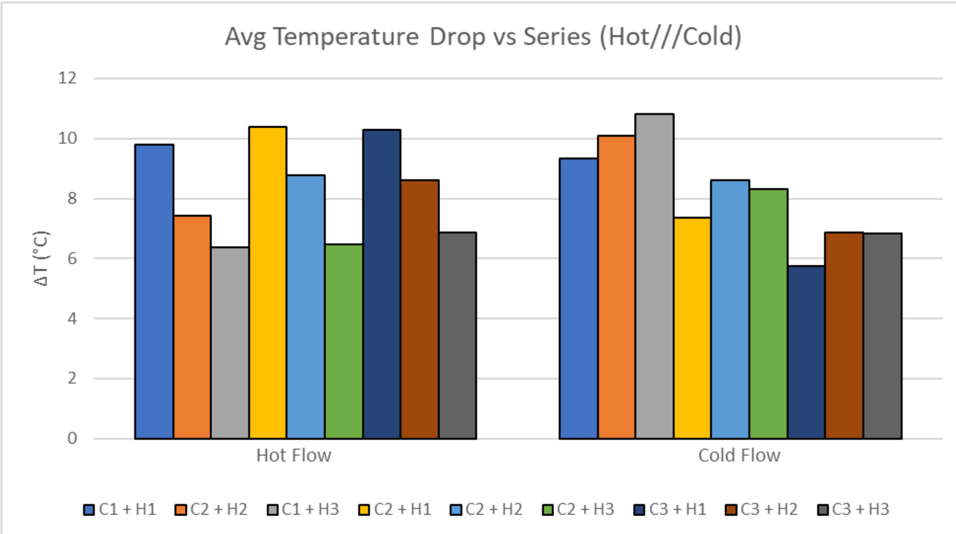

Fig. 6: This figure shows the breakdown of temperature changes throughout the 4 pipe configurations, split between the change in hot flow on the left and cold flow on the right.

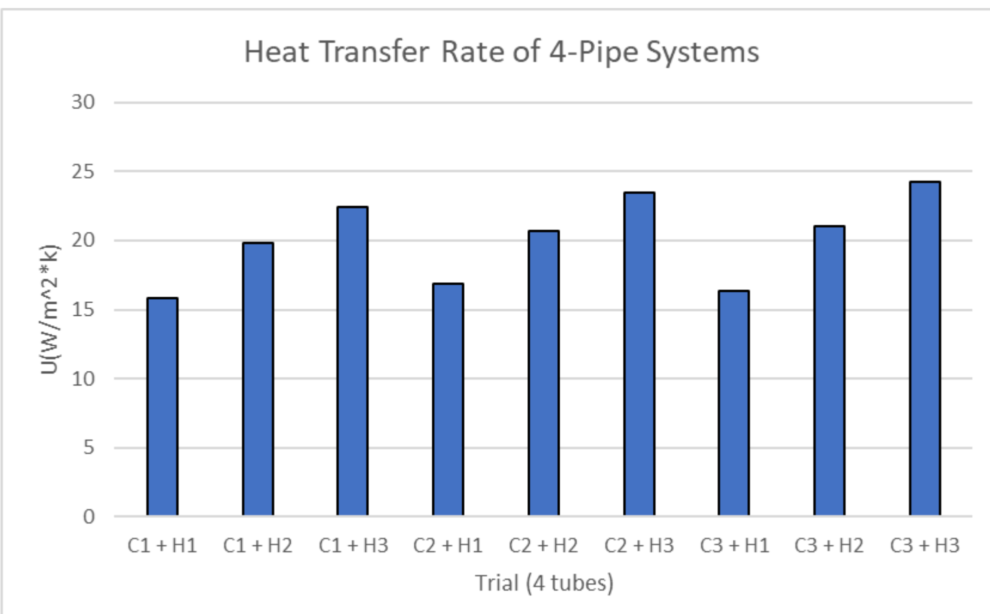

Fig 5. This figure shows the distribution of the heat transfer rate (U) throughout the 4 pipe configurations.

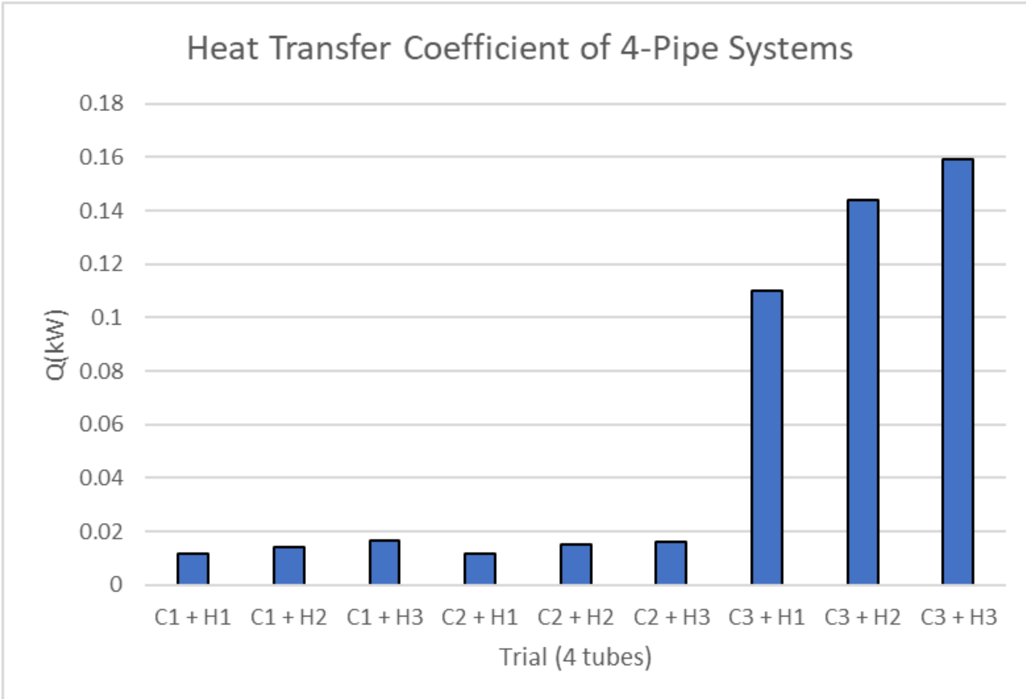

Fig. 6. This figure shows the distribution of Heat Transfer Coefficients(Q) throughout the 4 pipe configurations, scaled for kW.

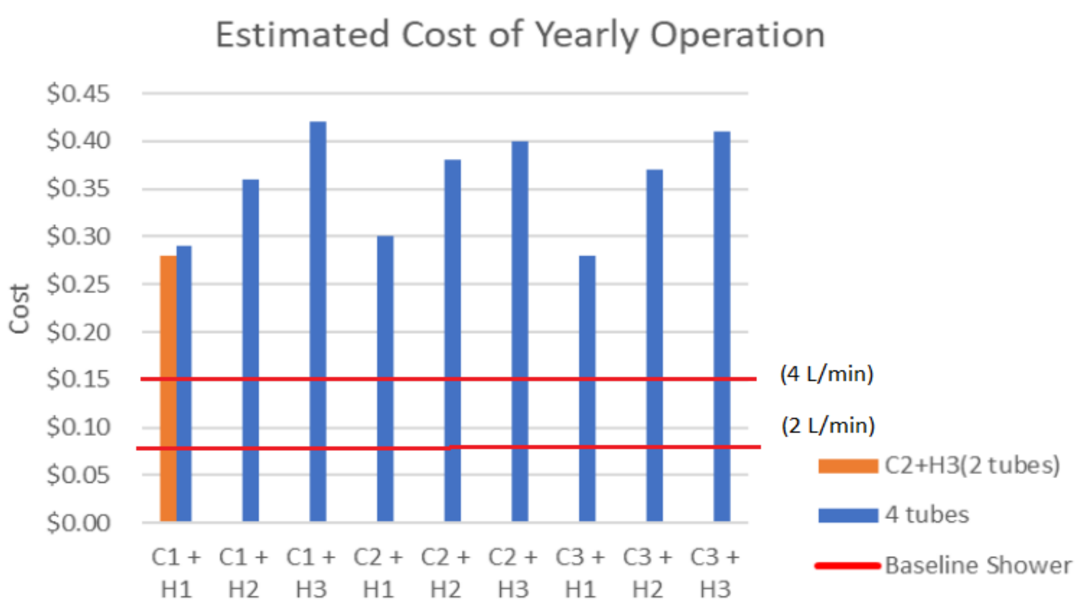

Fig. 8. This figure compares the evaluated costs of yearly operation for the entirety of the 4-pipe configuration compared to a sample 2-pipe configuration, relative to the estimated costs of a generic residential shower with similar operating conditions to our experimental setup.

Error Analysis

Although the standard error of the results obtained in the laboratory are listed in the results tables, there are many instances where these inconsistencies may have occurred.

To begin, the water used in the heat exchanger was not pure. There were visible impurities in the water, with large chunks of a black solid. This would have impacted both the fluidity of the water's movement (viscosity), as well as the heat capacity. This may have caused the values to be higher than expected. The impurities also caused a buildup of air bubbles within the tubes of the heat exchanger, possibly affecting the temperature readings of the fluid. This is due to the possibility that the temperature readings were taken in the air bubbles, rather than in the fluid creating a false reading of the air inside the tubes rather than the desired water.

Another possible source of error is the software used. The Armfield HT36 software was detailing data fluctuations for flow rate very frequently, and would often not be able to settle on a short range of values. It was not able to hold high flow rates (4 L/min) for long periods of time without dragging the flow rates down and causing a decreased rate. Due to this, the group was forced to use non matching flow rates of 3.5 L/min for the cold flow rather than the 4 L/min used for the hot flow, as that was the highest that the software could handle. In fact, this flow rate was not the average flow rate that is used in showers, detailed in the calculations.

A source of error within the calculations is the lack of consideration for friction. Frictional losses occur in any environment in which the fluid is touching a material, stainless steel and plastic in this case. This would have decreased the heat transfer rate, sequentially increasing the heat loss and increasing the cost spent. This accounts for how small our calculated spending rates are in comparison to the expected cost.

As well as these problems, this experiment relies entirely on the accuracy of data reported by the computer. The user manual makes no mention of specific uncertainties implicit in the temperature or flow rate sensors. Pending a response from the Armfield customer service with this data, we proceed under the assumption that reported figures are accurate to their digit of least significance. This forms the basis of our uncertainty analysis and is propagated throughout all calculations.

Following the calculation section above, the error propagation was calculated for each step and then combined.

For the first step, calculating Pr, assuming that Cp and k are constant, the only variable is dynamic viscosity, which has systematic uncertainty according to the equipment.

$$Pr = \frac{C_p\mu}{k} \rightarrow \delta Pr = \frac{Cp}{k}\delta\mu \rightarrow \frac{\delta Pr}{Pr} = \frac{\delta\mu}{\mu}$$

Next, for calculating Nu: here our variables are Pr and the dynamic viscosity.

$$Nu = 0.027(Re_D)^{4/5}(Pr)^{1/3}(\frac{\mu}{\mu_s})^{0.14}$$

$$\delta Nu = 0.027(Re_D)^{4/5}(Pr)^{1/3}(\frac{0.14}{\mu_s^{0.14}})\mu^{-0.86}\delta\mu + \frac{0.027}{3}(Re_D)^{4/5}(Pr)^{1/3}(\frac{\mu}{\mu_s})^{0.14}Pr^{-2/3}\delta Pr$$

$$\frac{\delta Nu}{Nu} = \frac{0.14}{\mu_s^{0.28}\mu^{0.72}}\delta\mu + \frac{\delta Pr}{3Pr} = \frac{0.14}{\mu_s^{0.28}\mu^{0.72}}\delta\mu + \frac{\delta\mu}{3\mu}$$

Next, for calculating the heat transfer coefficient h, where our only variable is Nu:

$$h = \frac{Nu*k}{D} \rightarrow \frac{\delta h}{h} = \frac{\delta Nu}{Nu} = \frac{0.14}{\mu_s^{0.28}\mu^{0.72}}\delta\mu + \frac{\delta\mu}{3\mu}$$

For the resistance, where our only variable was h, from the previous step:

$$R = \frac{1}{\pi hDL} \rightarrow \frac{\delta R}{R} = \frac{-\delta h}{h} \rightarrow \frac{\delta h}{h} = \frac{0.14}{\mu_s^{0.28}\mu^{0.72}}\delta\mu + \frac{\delta\mu}{3\mu}$$

For the total/overall heat transfer coefficient U, which uses addition:

$$U = R_i + R_o + R_{pipe} \rightarrow \frac{\delta U}{U} = \frac{\delta R_i + \delta R_o}{R_i + R_o}$$

Note that the uncertainty equation can be rewritten in terms of kinematic viscosity, but was left relative to make simplifying other equations easier. After U was calculated we also went on to calculate the total heat transferred, in which the area was the only constant.

$$Q = UAT_{LM} \rightarrow \frac{\delta Q}{Q} = \frac{\delta U}{U} + \frac{\delta T_{LM}}{LM}$$

$$T_{LM} = \frac{\Delta T_{inlet} - \Delta T_{outlet}}{ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}$$

$$\frac{\delta T_{LM}}{T_{LM}} = [\frac{1}{\Delta T_{in} - \Delta T_{out}} - \frac{1}{\Delta T_{in} ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}]\delta(\Delta T_{in}) + [\frac{1}{\Delta T_{out} - \Delta T_{in}} + \frac{1}{\Delta T_{out} ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}]\delta(\Delta T_{out})$$

$$\frac{\delta Q}{Q} = \frac{\delta R_i + \delta R_o}{R_i + R_o} + [\frac{1}{\Delta T_{in} - \Delta T_{out}} - \frac{1}{\Delta T_{in} ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}]\delta(\Delta T_{in}) + [\frac{1}{\Delta T_{out} - \Delta T_{in}} + \frac{1}{\Delta T_{out} ln(\frac{\Delta T_{inlet}}{\Delta T_{outlet}})}]\delta(\Delta T_{out})$$

Discussion

Our first objective was to determine whether countercurrent or co-current flow setup on the heat exchanger provided the best heat transfer coefficient. Between the co-current set up, with a calculated heat transfer coefficient of 16.52 W/m^2*K, and the countercurrent set up, with a calculated heat transfer coefficient of 16.82 W/m^2*K, the countercurrent set up had a higher coefficient by 0.30 W/m^2*K with the same flow rates. We expected the countercurrent flow to be more efficient at heat transfer, so this result corresponds with our expectations. In our planning report, we decided that the more consistent change in temperature across heat exchangers for countercurrent modes in comparison to co-current modes, where the change in temperature drops as the fluids propagate along the heat exchanger, creating a heat transfer drop (see figure 8 below).

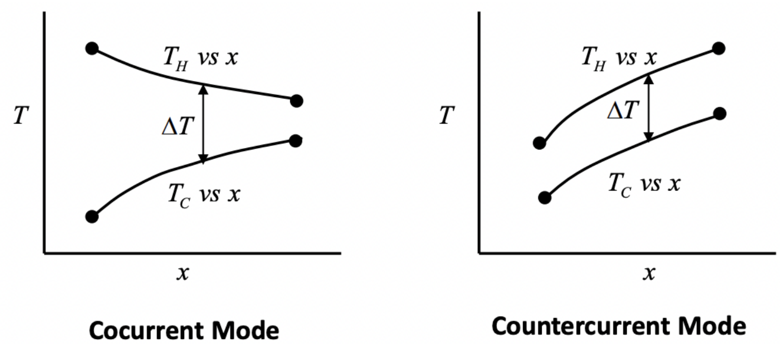

Fig 8. T vs Mode (Subramanian, n.d.)

Because the flow rates were kept constant between counter and co-current modes in this part of the experiment, the difference in heat transfer coefficient seems reasonable for the size of the heat exchanger unit. It could be expected that the difference in performance between the co-current and countercurrent flow may be more stark on an industrial scale.

For calculated heat transfer values and costs, it is worth noting that some of our trials had a hot water temperature of around 34 degrees, and some trials 38 degrees. Because of this variation, it is more reliable to determine the efficiency and practicality of systems with the heat transport coefficient. Overall, the highest heat transfer coefficient was derived from the combination of the highest possible cold water flow rate and the highest possible hot water flow rate, 4 L/min of hot water and 3.5 L/min of cold water, with a coefficient of 19.85 W/m^2*K. Notably, when cold water flow rate was increased, but hot water flow rate held constant, the heat transfer coefficient remained about the same. However, when cold water flow rate was held constant, and hot water flow rate increased, the heat transfer coefficient steadily increased. Generally, the heat transfer coefficient increased by 2-3 W/m^2*K for each L/min the hot water flow rate increased. This result coincides with our expectations, since a faster flow rate “replaces” the hot water faster, and so the change in temperature is better maintained for more efficient heat transfer.

Also as expected, the heat transfer coefficient increased when switching from a 4 pipe system to a 2 pipe system. Between the 4-pipe system with 3 L/min hot and cold water flow rates and the equivalent 2-pipe system, the heat transfer coefficient increased by 5.38 W/m^2*K. This surpassed the 4-pipe system maximum of 19.85 W/m^2*K by 4.72 W/m^2*K. It would be worth testing a 1-pipe system as well for a further heat transfer coefficient increase.

Price calculations indicated that yearly savings amounted to about 0.4 US dollars, so for the device to pay itself back in a year, it would need to be priced at 0.4 US dollars. This was based on averages from the environmental protection agency to determine the typical shower, and using basic energy equations to determine heat transfer. We were also assuming 20 showers a month, for 12 months.

40 cents is clearly not a practical cost, but even if the timeline were to be extended, the pricing of the device would remain impractical in comparison to the cost of the materials used to build the unit. Cost analysis also noted that the price of a shower with the unit was more expensive than without. This could be attributed to the size of the testing unit. However, in a circumstance where showers are more frequent, for example in a larger scale shower room, potential for savings increase.

Recommendations

From our data, we recommend a counter-current, 2-pipe system be used, with cold water and hot water flow rates adjusted to 3.5 L/min and 4 L/min, respectively, or higher if possible, for the highest possible heat transfer coefficient. However, it may be worth considering a shorter 1-pipe system.

For this device to break even in the span of a year, the system would need to be priced at around 40 cents, but it is worth noting that we did not have specific data for the optimized device we are recommending, and this is a rough approximation. However, given the goal of creating a system that can pay itself back within a year of savings, it is highly unlikely that the system can be optimized to a point where it can achieve that goal and be profitable at the same time.

A heat exchanger unit may be more useful in a public showering facility on a larger scale, where more heat can be transferred, and the frequency of showers would be higher.

Equal Contribution

	Group Members			
Task	Adrian	Liza	Chris	Frances
Safety Incident Report			✔	
Introduction	✔	✔		
Presenter	✔			
Powerpoint Slides	✔			
Calculations		✔		
Data		✔		✔
Presentation of Results		✔	✔	
Figures/Tables			✔	
Error Analysis		✔	✔	✔
Discussion				✔
Recommendations				✔
References		✔		
Writing Quality Check	✔	✔	✔	✔
Equal Contribution	✔	✔	✔	✔

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