Skoltech

Skolkovo Institute of Science and Technology, Skoltech
http://www.skoltech.ru/

Course "Experimental data processing"
Term 1B, October 2 – October 27, 2023

Final project
Estimating the time and height of a meeting of two rockets using barometer and accelerometer measurements. Linear Kalman filter.

The objectives of this assignment are to encourage you to think creatively and critically to extract a useful signal from noisy experimental data, find best estimation method of a dynamical process and make forecast of its future development.

This assignment is to be done in groups of 3 students maximum, and only one document is submitted for the group. You may also freely talk with students in other groups, but the final documents that you submit must be done only by your group.

Formulation

1. Determine the time and height at which two rockets meet using barometer and accelerometer measurements. Second rocket is faster, but started its flight 2 seconds later.

2. There are following measurements for your availability
   files/folder/Final projects/Project 12/data/
   - **'z1.txt', 'z2.txt'**
     Measurements of position s by barometer and acceleration a by the accelerometer for two rockets. File '**t1.txt**' and '**t2.txt**' indicates the corresponding time of measurements in seconds.

     Format
     First column - measurements of position s by barometer.
     Second column - measurements of acceleration a by the accelerometer.

3. How to solve this problem?
   - Construct two Linear Kalman filters that uses data sets **'z1.txt' and 'z2.txt'.**
   - Develop optimal smoothing based on obtained filtered results for two filters.
   - Determine the time and height at which two rockets meet using filtered and smoothed results. Are there any difference in estimations? Which estimations are more accurate? You could us estimation error covariance matrices from filtration and smoothing for that. Why?

Useful hints:

(a) State vector $X_i = \begin{vmatrix} s_i \\ V_i \\ a_i \end{vmatrix}$

Here s is the position of the rocket, V_i – velocity, a_i – acceleration.

The state of a rocket at step i is represented by the following state equation

$$X_{i+1} = \Phi_{i+1,i} X_i$$

Here transition matrix

$$\Phi_{i+1,i} = \begin{vmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{vmatrix}$$

Δt – time step between measurements.

(b) Measurement vector consists of position s by barometer and acceleration a by the accelerometer

$$z_i = \begin{vmatrix} s_i^m \\ a_i^m \end{vmatrix}$$

From a prior information we know that the variance of measurement noise for position s is $\sigma_s^2 = 12^2, \sigma_a^2 = 0.09^2$. Covariance matrix of measurement noise R thus consists of diagonal elements of these variances. Corresponding observation matrix H.

$$H = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Construct a linear forward filter and backward smoothing assimilating these measurements Make plots of measurements, filtration and smoothing. Determine time and height to apogee based on the estimation results. Calculate estimation errors over the whole estimation interval as well at the time when apogee is reached according to estimations.

(c) Develop a filter in these conditions.

(d) Determine the time and height at which two rockets meet using filtered and smoothed results. Are there any difference in estimations? Which estimations are more accurate? You could us estimation error covariance matrices from filtration and smoothing for that. Why?

Compare your results with the results of team doing Project 14. They have the same task, but in their case they have gaps in measurements. Compare accuracy of estimation with your and their approach. You can also use Kalman filter algorithm for the accuracy estimation. Which results are more accurate? Why?

And please answer the main question of this project!

Friday, October 27

1. Present your results. You will have 15 minutes (12 minutes for presentation, 3 minutes for Q&A). The presentation should include the problem formulation, why it is important, model and measurement, methods and tools used, nice figures, grounds why the chosen method is the

best method (visual analysis, quantitative criteria, simplicity of implementation, and any other arguments). Which regularities are found. Discuss what are the risks of obtained estimations and conclusions about the process. Make general conclusions about the efficiency of method.

Try **to share** with the audience a practical and useful idea behind the project together with useful tips applied in the project, **to make the overall exchange of practical and efficient tools and approaches and for what they can be applied.** Please share the responsibility in presenting between your team mates, each team member should present.

2. Submit the final version of your project to canvas before end of term.