

# Quantum Circuit Optimilization Using Monte Carlo Tree Search and Reinforcement Learning

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# Abstract

...partial- couple of sentences about motivation / task

# Contents

Abstract	i
1 Introduction	1
1.1 Motivation . . . . .	1
1.2 Problem . . . . .	1
1.3 Organisation . . . . .	2
1.4 State-of-the-art . . . . .	2
1.5 Research . . . . .	2
2 Quantum Circuit Routing	4
2.1 The Qubit . . . . .	4
2.2 Quantum Circuits . . . . .	5
2.3 Topology . . . . .	6
2.4 Decoherence . . . . .	7
3 Previously Researched Work	9
3.1 Cambridge’s t ket ’s Routing Algorithm . . . . .	9
3.2 SWAP-based BidiREctional heuristic search algorithm . . . . .	9
3.3 Reinforcement Learning Approaches . . . . .	10
3.4 Integer Linear Program . . . . .	10
3.5 Graph Neural Network with Monte Carlo Tree Search . . . . .	10
4 Initial Placement of the Qubits	11
4.1 Qubit Allocation . . . . .	11
4.2 Weighted Directed Graph . . . . .	12
5 Monte Carlo Tree Search with Reinforcement Learning	14
5.1 Implementation of the MCTS . . . . .	14
5.2 Implementation of the Reinforcement Learning Model . . . . .	16
5.3 Training of the Model . . . . .	16
6 Results and Discussion	17
6.1 Results and Analyses . . . . .	17
6.2 Discussion . . . . .	17
7 Conclusions and Future Work	18

# List of Figures

2.1	Regular bit and quantum bit illustration . . . . .	4
2.2	Example of a logic gate in a regular computer (left), example of a Quantum gate (right) . . . . .	5
2.3	A quantum circuit extracted from a programmed script . . . . .	5
2.4	Quantum topologies of big tech companies' quantum computers. Source: [1] . . . . .	6
2.5	Example of a linear topology. . . . .	6
2.6	Different combinations of qubit mapping. . . . .	7
2.7	A SWAP-gate and its decomposition. . . . .	7
4.1	Toy topology for the initial qubit placement . . . . .	11
4.2	Toy circuit model of CNOT gates for the initial qubit placement .	12
4.3	Coupling Directed Graph of the toy circuit model . . . . .	12
4.4	talk about how we start with the highest out-degree and match them on the highest out-degree of the actual topology, we do this recursively. starting from (a) and going from there. . . . .	13
5.1	Simple illustration of simulation of Tic-Tac-Toe. Rewards being add when crosses wins, subtracted when noughts wins and no reward will be given when it is a tie. . . . .	14
5.2	Monte Carlo Tree Search roadmap for choosing the best action. .	15
5.3	Selection of nodes with Upper Bound Confidence (UCB) and a heuristic value H given by the neural network. . . . .	15



# Chapter 1

## Introduction

### 1.1 Motivation

It is fascinating to think about the fact that the computing power of a military computer from 50 years ago, that was the size of an entire room, are in today's regular computers. However, even with the phenomenal growth we made in technology, there remain problems that regular computers just can't solve.

Quantum computers are expected to make a breakthrough in chemical and biological engineering through the discovery and manipulation of molecules; encryption for cyber security; the processing of very large quantities to aid in artificial intelligence; and the pricing of complex assets in finance [2]. The quantum computers from nowadays may be able to perform tasks which surpass the capabilities of today's regular computers, but the world is not there yet.

### 1.2 Problem

One of the barriers of programming a quantum computer is the loss of information in a qubit. Think of a spinning coin with information: either heads or tails. Eventually the coin will stop spinning and land on one of the sides. This is the same for a quantum system. The algorithms that are written for a quantum computer contain noise, which can ensure that the information provided is unreliable. The way algorithms can be operated is by writing them into quantum gates. They are the same as the regular logic gates, but with quantum phenomena, which will be explained later in the paper. So basically, mathematical algorithms are transformed into circuits of operations. These circuits are written in Quantum Assembly, a language that the quantum computer understands. Each circuit has a defined number of logical quantum bits (qubits). These qubits are what the gates are interacting with. In the moment of processing, the logical qubits in the circuit will then be mapped into the topology of the quantum computer, which is a connectivity architecture of the qubits inside the quantum computer, also refereed as physical qubits. The problem here lies in the connectivity matching. When programming a circuit, one must satisfy the topology. For instance, say two qubits need to interact with each other through a gate. If these qubits are not connected to each other, this operation will be inoperable, which means the whole circuit

cannot be executed. So there needs to be a way to make circuits operable while satisfying the connectivity constraints of the quantum computer. A way to do this is by adding a so called SWAP-gate in the circuit, that causes the qubits to flip. This flip ensures that the qubits come closer to each other or even become connected. The problem with the swap gate is that it is not a known logic gate for the quantum computer. The swap gate is performed by placing 3 CNOT gates one behind the other. This ensures that the circuit becomes larger in depth and therefore also more sensitive to decoherence. So there needs to be an optimization in placing these SWAP gates to ensure that the information is reliable.

### 1.3 Organisation

The research of this thesis is done for the organisation SURF. A cooperative association of Dutch educational and research institutions. Universities, universities of applied sciences, MBO institutions, UMCs and research institutes work together within SURF to purchase or develop the best possible digital services. The company owns supercomputers which are available to complete high performance processes. Within the company there are multiple departments including quantum innovation, machine learning, and high performance computing. Which will be useful for this research.

### 1.4 State-of-the-art

There are already a number of studies done in this area, for example a state-of-the-art research was the one of Pozzi, et al. [3], where he used Reinforcement Learning and something called Quantum Annealing to optimize the number of SWAP-gates in a circuit. His research was based on a random logical qubit allocation on the physical qubits. When creating an initial qubit placement, the number of SWAP-gates can be reduced. Another recently published study is done by Sinha, et al [4], where they managed to minimize the number of SWAP gates for a random qubit allocation using Monte Carlo Tree Search in combination with Reinforcement Learning.

### 1.5 Research

Both papers discussed the need to look into initial qubit placement, which will also ensure that the SWAP gates are minimized in addition to using Reinforcement Learning. With this in mind, research will be conducted on quantum circuit optimization using the Monte Carlo Tree Search in combination with Reinforcement Learning. An initial qubit placement will be used for this.

This paper answers the main question: 'How can quantum circuit routing be optimized using Reinforcement Learning applied on Monte Carlo Tree Search?' This question will be answered by means of the following sub-questions:

- What is quantum circuit routing?



- What has already been done to successfully accomplish quantum circuit routing?
- How can the initial qubit placement procedure be performed?
- How does the Monte Carlo Tree Search work on optimizing the number of SWAP-gates?

This research paper is organised as follows. Section ?? offers a description of basic knowledge about quantum computing phenomenon and techniques needed for understanding the main question. Section ?? demonstrates what research has already been done by others. Section ?? describes the sub-problem initial qubit placement and how to solve it. Section ?? provides an overview of the experimental setup, which is how the Monte Carlo Tree Search is put together with the Reinforcement Learning. Section ?? shows the results of what the reinforcement learning model delivered. Finally, in Section ?? and ?? states the discussion about the research that is performed and the final conclusion.

## Chapter 2

# Quantum Circuit Routing

It is recognized that quantum computers can provide revolutionary developments. This chapter explains what a quantum computer is and how it can be programmed. It also explains in detail what the problem is that will be optimized.

### 2.1 The Qubit

A quantum computer can be seen as a regular computer that stores information and performs operations using quantum mechanics. A regular computer stores all its information such as numbers, text, and images, in series of 0's and 1's. These units are called bits. The way a quantum computer stores information is through the use of quantum bits (qubits). The difference between a bit and a qubit is that a qubit can carry not only the information 0 or 1, but also 0 and 1 on top of each other. This phenomenon is called superposition. A well-known example of superposition is the Schrödinger's cat example, where a hypothetical cat is illustrated in a closed box with a bottle of poison. It is not certain whether the cat is alive or dead unless it is checked. Before checking, the cat finds itself in a situation where it has a 50% chance of being dead or alive. This is the same with a qubit, there is no certainty whether it will hold 0 or 1, only a 50% probability that it will be one of the two. An illustration of a bit and a qubit can be found in figure 2.1.

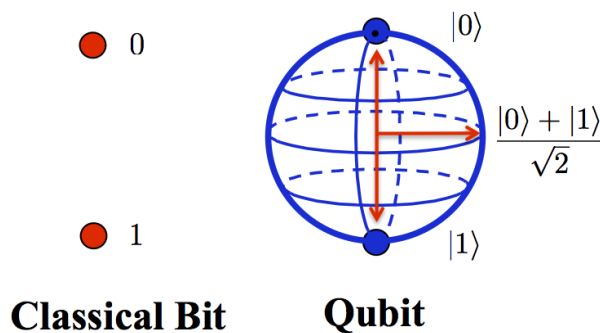


Figure 2.1: Regular bit and quantum bit illustration

## 2.2 Quantum Circuits

A regular computer operates by means of directions (algorithm) that are provided in the form of a script. This script then tells the computer processor what calculations have to be done to arrive at a certain output. These scripts can be written in different programming languages and the computer translates this into binary language, to control the logic gates inside the processor. The logic gates provide the calculations necessary for the operations. This principle, of programming gates that do calculations to perform a certain operation, works the same in a quantum computer. A script is written that determines which quantum gates should operate. An example of a gate that will be discussed a lot in this thesis is the Controlled NOT gate (CNOT-gate), see figure 2.2.

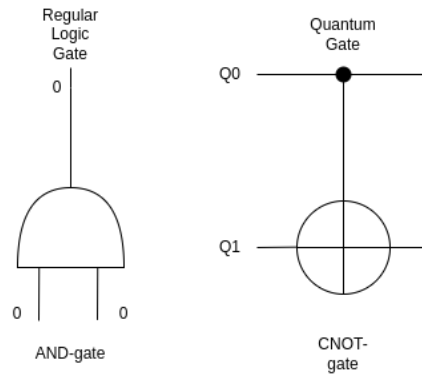


Figure 2.2: Example of a logic gate in a regular computer (left), example of a Quantum gate (right)

This script then creates a circuit consisting of the sequence of the quantum gates that were defined in the script. The algorithm that has been programmed is thus represented as a circuit with quantum gates. An example of a circuit can be seen in figure 2.3. The circuit illustrates between which qubits operations have to be performed. The circuit of figure 2.3 illustrates a four qubit circuit consisting of CNOT-gates with each interacting with two qubits. Usually, qubits are represented in Dirac notation [5], but in this thesis that is not relevant and the qubits will be represented with a capital Q.

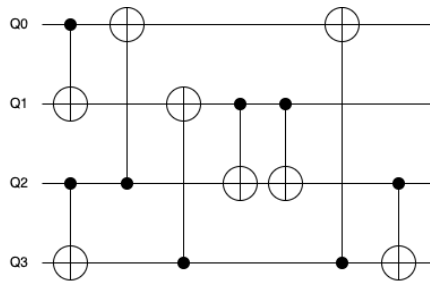


Figure 2.3: A quantum circuit extracted from a programmed script

Each horizontal line in the circuit indicates the 'path' of a qubit, each path can have a quantum gate through which the qubit has to pass. Ultimately, the paths can be read to obtain a final result. As mentioned earlier in the example of

Schrödinger's cat, there is a 50% chance that the cat is alive or dead unless the box is put up for checking. Reading or measuring a circuit forces the qubits to choose a state, either 0 or 1. The moment it chooses one of the two, the qubit collapses to a regular bit with the one value. This means that the output of the circuit will represent an ordinary series of bits, just like in a regular computer.

## 2.3 Topology

The step after feeding a quantum circuit to the quantum computer is the process in which the defined qubits in the circuit (logical qubits) are allocated to the qubits in the hardware (physical qubits). The logical qubits are, as it were, assigned a location on the hardware. The form in which these qubits can be allocated depends on the connectivity architecture of the physical qubits. This means that the qubits in the hardware are attached to each other in a certain form. This architecture is also known as the topology of the quantum computer.

There are different types of topologies. For example, figure 2.4b shows the topology of Rigetti's Aspen-4 quantum computer with a linear structure, or figure 2.4e shows the topology of Google's Sycamore, which consists of more connected qubits in a grid structure.

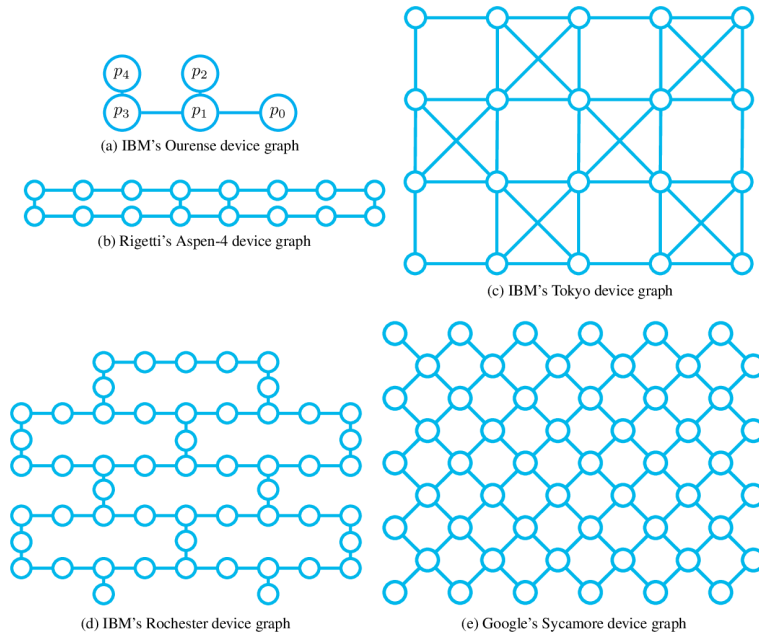


Figure 2.4: Quantum topologies of big tech companies' quantum computers. Source: [1]

These connectivities determine which qubits are allowed to interact with each other. Imagine the topology of figure 2.5 and the circuit of figure 2.3.

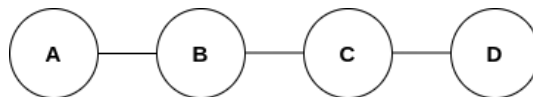


Figure 2.5: Example of a linear topology.

There are several ways to map the circuit's logical qubits into the computer's physical qubits, see figure 2.6. It can be done in sequence, randomly, but also with an initial placement, where the allocation is based on the qubits that need to interact in the circuit.

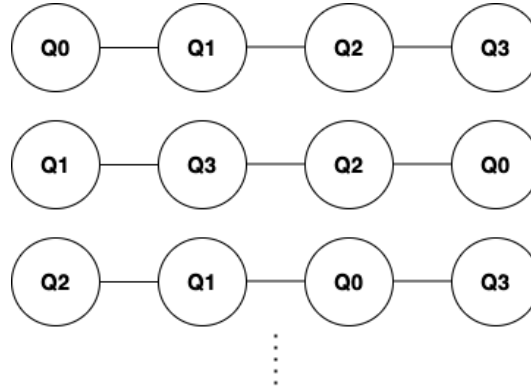


Figure 2.6: Different combinations of qubit mapping.

After mapping the logical qubits, the operations in the circuit will be performed. The moment interactions have to be performed in the circuit between qubits that have no connection in the topology, the operation cannot be performed, after which the entire circuit is rejected by the quantum computer. This appears to be a common problem when writing quantum algorithms. One way to make the circuit perform on a quantum computer is to add SWAP-gates to the circuit 2.7.

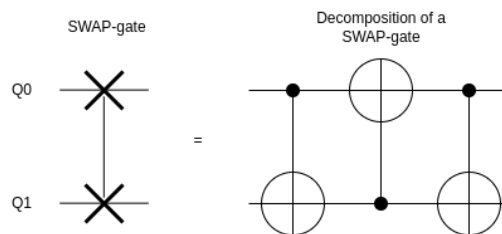


Figure 2.7: A SWAP-gate and its decomposition.

The SWAP gate will ensure that the qubits, that interact with the SWAP-gate, will switch positions in the quantum computer. This means that the qubits that have to operate with each other have a connection in the topology. This can be done over the entire circuit, until all operations are executable hardware-based.

A SWAP-gate is not a standard gate known to the quantum computer. A SWAP-gate can be simulated by placing three CNOT gates in a row, as shown in figure 2.7. Because there are actually three extra gates added per SWAP gate, this will ensure that the circuit will have a greater depth.

## 2.4 Decoherence

Qubits can be compared to a spinning coin. Eventually the coin will stop spinning and land on heads or tails. A qubit is also spinning between 0 and 1, but will also

eventually stop spinning, after which it will end up at 0 or 1. This phenomenon of breaking out of the superposition, after which it has to choose a state is called decoherence. The lifespan of a qubit differs per hardware composition of the quantum computer. Based on the temperature and network in the quantum chip, it can be ensured that the lifespan of a qubit is longer. The longest achieved so far by a qubit is 50 seconds, but the most common is somewhere between 15-120 $\mu$ seconds [6].

Because qubits decohere, it causes parts of information to be lost. As a result, the output information will be unreliable. For this reason, it is important to set up a circuit that is small in depth, so that it can be quickly executed by the quantum computer while the qubits last. This is a tricky problem when a large algorithm has to be executed, where SWAP-gates are essential.

# Chapter 3

## Previously Researched Work

This chapter will provide a picture of what has been done before to optimize circuit routing and what has been recommended by these researchers for further research.

### 3.1 Cambridge's t|ket 's Routing Algorithm

Cambridge students describe a solution for the qubit routing problem implemented with t|ket , a platform independent compiler developed by Cambridge Quantum Computing Ltd. They believe that this heuristic method in t|ket matches or beats the results of other circuit mapping systems in terms of depth and total gate count of the compiled circuit, and has much reduced run time allowing larger circuits to be routed. They represent a quantum computer as a graph where nodes are physical qubits and edges are the allowed 2-qubit interactions.

### 3.2 SWAP-based BidiREctional heuristic search algorithm

In the paper written by Gushu Li et al. [7] they describe, a SWAP-based BidiREctional heuristic search algorithm, named SABRE, is proposed to solve this qubit mapping problem and overcome the drawbacks mentioned above. With the observation that many attempts in exhaustive search can be redundant and effective mapping transition needs to start from the qubits in the two-qubit gates that need to be executed, we design an optimized SWAP-based heuristic search scheme in SABRE with significantly reduced search space. They present a novel reserve traversal search technique in SABRE to naturally generate a high-quality initial mapping through traversing a reverse circuit, in which more consideration is given to those gates at the beginning of the circuit without completely ignoring the rest of the circuit. Moreover, we introduce a decay effect, which will slightly increase our heuristic cost function values when evaluating overlapped SWAPs, to let SABRE tend to select non-overlapped SWAPs. This optimization enables the control of parallelism in the additional SWAPs and can further generate different hardware-compliant circuits with a trade-off between circuit depth and the number of gates.

### 3.3 Reinforcement Learning Approaches

Several approaches have been taken to optimize the circuit routing problem. Most are done through deep reinforcement learning. In Matteo Pozzi’s paper [3], it talks about a qubit routing procedure that uses a modified version of the deep Q-learning model. Where they benchmark with the most advanced quantum compilers currently available (Qiskit and t|ket), on both random and realistic circuits. This research was a follow-up to the original research by Herbert and Sengupta [8]. Introducing the concept of reinforcement learning for the qubit routing problem. They mentioned that in order to achieve this, it is necessary to alter the conventional RL framework to allow combinatorial action space, where they mention numerical results that are included to demonstrate the advantage of the RL-trained qubit routing policy over using a sorting network.

### 3.4 Integer Linear Program

In the paper written by Giacomo Nannicini, et al. [9] the qubit routing problem is modelled as an integer linear program, using a network flow formulation with binary variables that includes the initial allocation of qubits and their routing. We consider several cost functions: an approximation of the fidelity of the circuit, its total depth, and a measure of cross-talk, all of which can be incorporated in the model. Numerical experiments on synthetic data and different hardware topologies indicate that the error rate and depth can be optimized simultaneously without significant loss. We test our algorithm on a large number of quantum volume circuits, optimizing for error rate and depth; our algorithm significantly reduces the number of CNOTs compared to Qiskit’s default transpiler SABRE, and produces circuits that, when executed on hardware, exhibit higher fidelity.

### 3.5 Graph Neural Network with Monte Carlo Tree Search

Another approach is to use the Monte Carlo Tree Search (MCTS) guided by a Graph neural network. The depth of the transformed quantum circuits is minimized by utilizing the MCTS to perform qubit routing by making it both construct each action and search over the space of all actions. It is aided in performing these tasks by a Graph neural network that evaluates the value function and action probabilities for each state. Along with this, we propose a new method of adding mutex-lock like variables in our state representation which helps factor in the parallelization of the scheduled operations, thereby pruning the depth of the output circuit. Overall, our procedure (referred to as QRoute) performs qubit routing in a hardware agnostic manner, and it outperforms other available qubit routing implementations on various circuit benchmarks.



# Chapter 4

## Initial Placement of the Qubits

This section introduces the qubit allocation problem and gives a understanding of one of the possible approaches to solve this problem.

### 4.1 Qubit Allocation

Given a quantum circuit and an architecture, we want to know if it is possible to map the logical qubits in the former to the physical qubits in the latter. This is called initial qubit placement. The moment that an initial qubit placement ensures that all operations in a circuit are possible, the use of SWAP-gates is no longer necessary. However, it is possible that not all gates can be executed. The moment an allocation is set up in such a way that, for example, 80% of the gates are operable, it may be that fewer SWAP-gates are needed to apply, than when a random allocation is performed. In the paper written by Siraichi et al. [10] a mathematical definition is given for the qubit allocation problem. A simplified explanation would be, suppose there is a topology of the coupling between the physical qubits, of which a list  $\phi$  is made with qubits that interact with each other and a list  $\psi$  with qubits involved in a CNOT-gate operations of the circuit. The moment an order of  $\psi$  can be arranged in such a way that most or all CNOT-gate operations become possible. To illustrate this, the following example is taken.

Assume the topology of Figure 4.1.

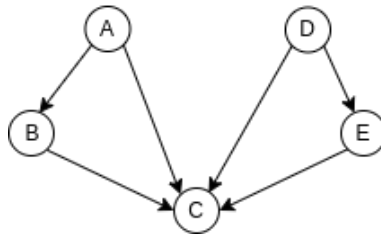


Figure 4.1: Toy topology for the initial qubit placement

And circuit interacting on this topology illustrated in Figure 4.2.

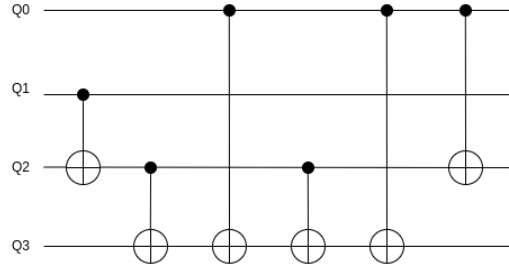


Figure 4.2: Toy circuit model of CNOT gates for the initial qubit placement

## 4.2 Weighted Directed Graph

To devise a way to arrange the logical qubits in the topology, in such a way that all CNOT-gate operations are possible, it is necessary to look at what kind of topology is expected. This is done by creating a directed graph based on, based on the out-degree of the qubits. An out-degree is the number of edges going out from a node. So it is counted how many interactions each qubit has to engage in. Using the example of circuit 4.2, the Directed Graph 4.3 can be made, illustrating the operations that need to be performed.

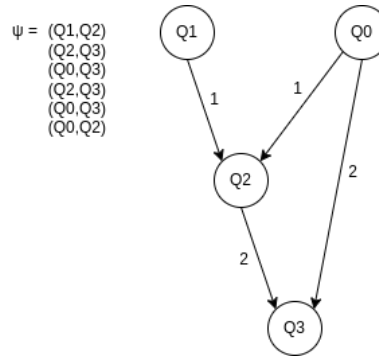


Figure 4.3: Coupling Directed Graph of the toy circuit model

In the list  $\psi$  all CNOT-gate operations are written out in a set with the interacting qubits. What can be inferred from this graph is that when all the out-degrees of every qubit in the graph (the number of arrows) are added up, a logical order of most out-degree would be  $Q0 - Q2 - Q1 - Q3$ . Another thing that can be made out is the different interactions that each qubit has. For example, it is clear that  $Q0$  undergoes two different interactions, with  $Q2$  and  $Q3$ . From this we can already say that  $Q0$  must come to a position where the physical qubit interacts with two other physical qubits. It can also be established that  $Q3$  itself has no out-degrees, so  $Q3$  must be allocated in a position in where there are no out-degrees either.

If the out-degree criterion is used as stimulus to allocate the logical qubits to physical qubits, it becomes possible to make the CNOT-gate operations executable. The table below shows an overview of the number of out-degrees of each node in the topology.

Node	number of out-degrees
A	2
B	1
C	0
D	2
E	1

When the topology of the physical qubits and the Weighted Directed Graph are combined, an allocation like the one shown in Figure 4.4 would yield based on the matching on their number of out-degrees.

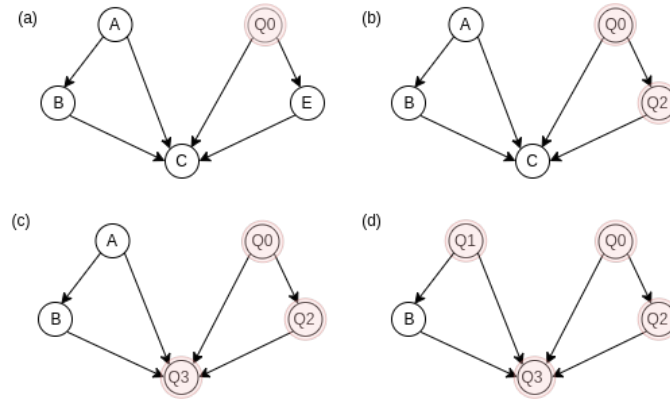


Figure 4.4: talk about how we start with the highest out-degree and match them on the highest out-degree of the actual topology, we do this recursively. starting from (a) and going from there.

If we

## Chapter 5

# Monte Carlo Tree Search with Reinforcement Learning

This chapter will describe how the Monte Carlo Tree Search (MCTS) in combination with Reinforcement Learning (RL) has been applied to optimize quantum circuit routing.

### 5.1 Implementation of the MCTS

In 2016, the computer program AlphaGo was nominated as the breakthrough of the year. The program consists of a combination of advanced MCTS with deep neural networks and is made to play the Asian game Go [11]. The idea for using the MCTS for the quantum circuit routing is inspired by AlphaGo, where all possible actions are simulated, until a best answer comes out. MCTS is used to determine subsequent actions. To give an illustration of how it works, imagine the game Tic-Tac-Toe, where the best actions for crosses must be decided 5.1.

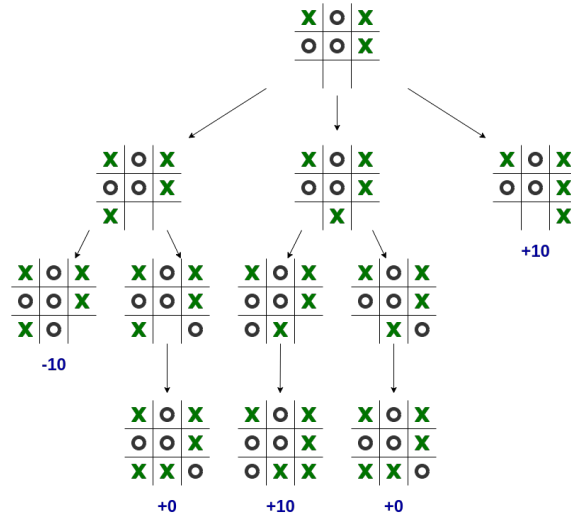


Figure 5.1: Simple illustration of simulation of Tic-Tac-Toe. Rewards being add when crosses wins, subtracted when noughts wins and no reward will be given when it is a tie.

To determine the best action, a tree is made consisting of all possible actions, from which all branches arise, each with its own branch. When the game is over,

rewards are given out. When crosses has won, that whole branch gets a reward of 10. When crosses has lost, the reward is -10 and when no one has won, nothing will be added or subtracted. The way in which the MCTS chooses the best action based on the step-by-step plan shown in figure 5.2.

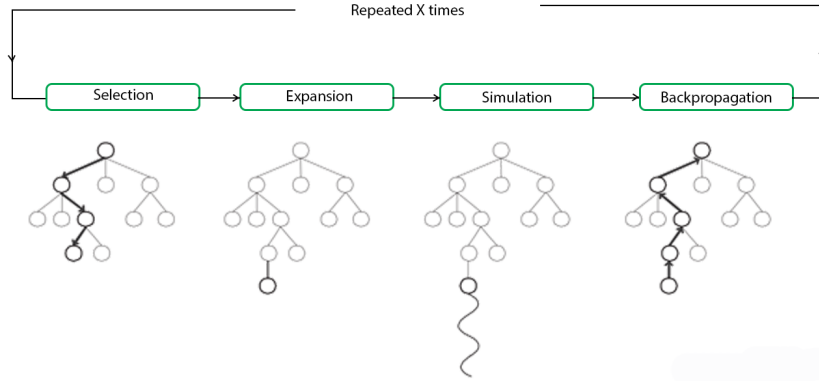


Figure 5.2: Monte Carlo Tree Search roadmap for choosing the best action.

The tree search first starts with a state that by simulating a number of random nodes. A node represents a possible action. A node can be a parent or a child. The moment a node is added, the parent and child nodes move down. So the bottom nodes become the child nodes and the node above it becomes the parent node. Each node consists of two values, the Upper Confidence Bound (UCB) and a heuristic value that comes from the neural network. UCB is the factor that decides which node evaluate next in order to maximize probability of victory from the given state. 5.3.

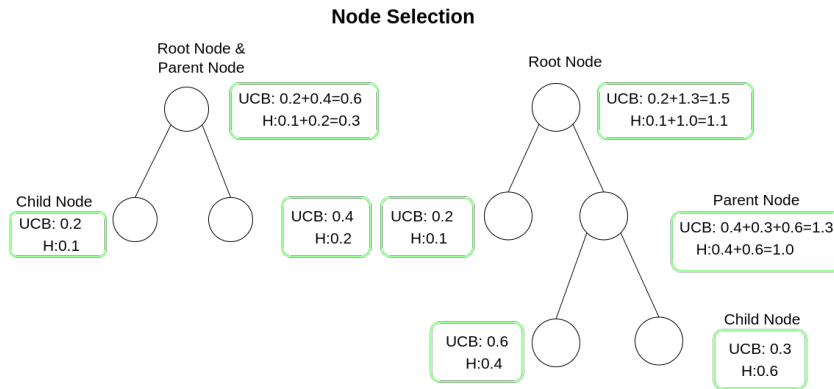


Figure 5.3: Selection of nodes with Upper Bound Confidence (UCB) and a heuristic value H given by the neural network.

The selection is guided by the UCB, which treats the problem as the multi-armed bandit problem. Imagine you are in a casino facing multiple slot machines and each is configured with an unknown probability of how likely you can get a reward at one play. To figure out the best strategy with highest reward, the exploration vs exploitation dilemma is demonstrated. Say, one is looking for a new restaurant, it is very likely to eat unpleasant food from time to time at these new restaurants. If all the information about the environment is known, it is possible to find the best restaurant by even just simulating brute-force, so by checking every restaurant. The dilemma comes from the incomplete information, it is necessary to gather enough information to make the best decisions while keeping the risk under control. With exploitation, advantage is taken from the best known option. When exploring, we take some risk to gather information about unknown options. For calculating the UCB applied on trees, the standard formula is stated as:

$$UCB(node_i) = \bar{x} + c\sqrt{\frac{\log N}{n_i}},$$

Where  $\bar{x}$  denoted the mean node value,  $N$  number of times the parent has been visited,  $n_i$  number of times the child has been visited and  $c$  being a constant.

For the quantum circuit routing optimization, this formula has been built up to the following formula:

$$UCB(node_i) = reward + 2\sqrt{\frac{\log N + e + 10^{-6}}{n_i + 10^{-1}}},$$

Where the previous  $\bar{x}$  is depicted as the reward of that node and the number of times the parent and child have been visited is multiplied by some factors, to avoid big numbers.

## 5.2 Implementation of the Reinforcement Learning Model

### 5.3 Training of the Model

# Chapter 6

## Results and Discussion

### 6.1 Results and Analyses

What did you find?

### 6.2 Discussion

Did your findings support your hypothesis? Why? Why not?

# Chapter 7

## Conclusions and Future Work

This Chapter concludes the thesis by summarizing the findings from the study, the contributions and possible limitations of the approach. It can also identify issues that were not solved, or new problems that came up during the work, and suggests possible directions going forward. [? ]



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