A Dive into Simpson's Paradox

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The name in Simpson's paradox arises from Edward H. Simpson, a statistician who was perplexed by this phenomenon, there had also been other statisticians who tried defining it before him. Judea Pearl defines a paradox as entailing a conflict between two deeply held convictions. Simpson's paradox is not as easily defined. In this paper I will summarize two different views of Simpson's paradox, from Judea Pearl and Aris Spanos. I then will define go over some popular examples of Simpson's paradox. Finally, I will present the solutions of both authors, and reason why they are both valid solutions to solving this paradox.

1 Pearl's Definition

The Encyclopedia of Philosophy describes Simpson's Paradox as "... a statistical phenomenon where an association between variables in a population emerges, disappears or reverses when the population divided into sub-populations." While Pearl claims that this paradox is due to missing causal inference, Spanos argues that it is not a paradox at all. I will first go through Pearl's argument.

In The Book of Why, Judea Pearl offers an example to try to convey what Simpson's Paradox is. The idea is that there is a 'BBG' drug that is bad for women, bad for men, and good for people overall. This is where the paradox arises from. This seems impossible, and it is. A drug cannot exist that is bad for men and women but good for people. Pearl's proposed solution is to look at a causal model, "If we use the causal notion and diagrams, we can clearly and unambiguously decide whether Drug D prevents or causes heart attacks"(Pearl). He claims we have to make a decision between two statements, one should be true and the other false, in this example, either the drug is good for people or bad for men and women, it cannot be both. To go further, we must define Simpson's reversal, which can get confused for being a paradox. It is indeed, not a paradox, just a numerical concept. Simpson's reversal refers to a reversal in frequencies when pooling samples together. Mathematically we can write this as:

$(A/B) > (a/b)$, $(C/D) > (c/d) \neq (A+C)/(B+D) > (a+c)/(b+d)$ (1)

This applies to the BBG drug example. The exact numbers would be $(3+8)/(40+20) < (1+12)/(20+40) =$ men and women who take the drug and have a heart attack is less than men and women who do not take the drug and have a heart attack even though 3/40 women had a heart attack who took the drug which is a larger percentage than men (1/20).

	Control Group (No Drug)		Treatment Group (Took Drug)	
	Heart Attack	No Heart Attack	Heart Attack	No Heart Attack
Low Blood Pressure	1	19	3	37
High Blood Pressure	12	28	8	12
Total	13	47	11	49

Figure 1: Fictious Table for BBG Drug

Pearl goes on to claim that this type of drug cannot and does not exist, and to prove this, you need to use causal logic. Again, only one of the two logical sentences can be true in this instance, either this drug increases heart attacks for both men and women or it decreases for everyone. One issue we can look at is that it seems women preferred taking the drug to men. This test included 60 men and 60 women. Out of the 60 women, 40 took the drug versus only 20 of the men who took the drug. Therefore, there is a correlation between gender and taking the drug. In this causal model, we would call gender a confounder. A confounder variable is described as a variable that impacts both the dependent and independent variable. Now we are able to control for the confounder. In the BBG drug example, we can separate the data for men and women and then take the average, which is easy in this example since men and women are equally separated in the general population (50/50). The average between men and women that get heart attacks without taking the drug is 17.5%, and the average of those who get a heart attack while taking the drug is 23.75%. Therefore, taking the BBG drug is not good for people.

Although combining the data is acceptable in this example, it is not always the answer to a "lurking third variable". A mediator is a variable that seems to cause the dependent variable Y, when in reality, it is a stand-in for the true independent variable that is causing Y (refer to Figure 3). He gave another

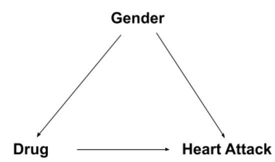

Figure 2: Causal Model for BBG Drug

example, this time instead of a drug causing heart attack, we look at what Drug B does to blood pressure. He gives exact opposite data, where people with low blood pressure have more heart attacks when taking the drug, people with high blood pressure have more heart attacks while taking the drug, but once added up, people in general have more heart attacks when not taking the drug. In this example, Blood Pressure is the mediator, but conditioning on it would cut off the actual dependent variable that we want. The mediator shows us that Drug B works, because aggregating the data wont work in regards to low and high blood pressure. Pearl believes that as long as we pay attention to the data and create causal models, Simpson's Paradox is solved; in which one of the two cases where the data does not match is indeed, false. In the end, he concludes that "Simpson's Paradox alerts us to cases where at least one of the statistical trends (either in the aggregated data, the partitioned data, or both) cannot represent the causal effects."

Figure 3: Causal Model 2

2 Spanos's Definition

Spanos had a different opinion about Simpson's Paradox. He claimed that the paradox stems from spurious results. Spurious refers to "statistically untrustworthy results, stemming from unreliable inference procedures"(Spanos). One example he gives is when there is positive correlation between a dependent variable and a independent variable, but the coefficient B_1 is negative, in a linear regression model. This can be viewed as a Paradox, but he proves that this is indeed possible. He believes that to assume a statistical paradox, the models must be statistically accurate. One major area where paradoxes can arise is when the assumptions for a statistically adequate model are violated. The five assumptions,

1. Normality
2. Linearity
3. Homosckedasticity
4. Independence
5. T-invariance

when invalidated, can result in spurious statistical results. "Statistical significance is more apparent than real, because it is just an untrustworthy result stemming from a statistically misspecified model" (Spanos). There are many instances where the statistics behind the data/model formed are not trustworthy. This can create what we call Simpson's Paradox.

3 Discussion and Conclusions

The issues pertaining to Pearl's argument is apparent in Spanos's paper 'Revisiting Simpson's Paradox: A statistical misspecification perspective', where he claims that relying on causal inference is not enough to prove either case of Simpson's Paradox is true. As he put it, "The modeler needs to account for the statistical information not accounted for by the original statistical model, with a view to ensure the trustworthiness of ensuring statistical results." I agree with argument against only using causal models and hidden variables to explain Simpson's Paradox, because it is very likely that small errors can change the outcome of data, and if we fail to look into violated assumptions, then the entire Paradox is not trustworthy. Causal models can help dive deeper into results of data, but should be done after knowing the results of your linear models or data are not spurious. On the other hand, Spanos does not give enough credit to how certain variables can give wrong correlations or coefficients due to being confounding or mediating. I believe that a mix of the two solutions is the best way to go about solving Simpson's Paradox. If the data comes out in a opposing way, first looking at possibly violated assumptions then using insight on the data to create causal models would be the best way to go about figuring out which case of the paradox is the truth. Simpson's Paradox is a result of using logic to realize that two opposing logical events cannot happen at the same time. Humans have long seemed to ponder on how to go about situational paradoxes like so. In regards to how much we know about causation, both solutions are of good quality.

4 Citations

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