CSE-250A-SP25-Practice-Final-Solutions

June 1, 2025

1 T/F

1. False
2. True
3. False
4. False
5. False
6. True
7. False
8. False
9. True
10. False

2 Viterbi Algorithm

C, D, E

The sequence of states returned by the Viterbi Algorithm maximizes the conditional $P(S_{1:T} \mid O_{1:T})$. Since $P(O_{1:T})$ is just a constant, $P(S_{1:T}, O_{1:T}) \propto P(S_{1:T} \mid O_{1:T})$. Hence, the full joint $P(S_{1:T}, O_{1:T})$ is also maximized. The third option $P(S_1)P(O_1 \mid S_1)\prod_{t=2}^{T} P(O_t \mid S_t)P(S_t \mid S_{t-1})$ is equal to $P(S_{1:T}, O_{1:T})$ due to the conditional independences implied by the HMM structure; therefore, this is also maximized.

3 D-Separation

1. All answers get full credit.
2. This is a Polytree
3.

$$
\begin{aligned}
P(D=0) &= \Sigma_{a,f} P(A=a, F=f, D=0) && \boxed{\textbf{Marginalization}} \\
&= \Sigma_{a,f} P(A=a) P(F=f|A=a) P(D=0|F=f, A=a) && \boxed{\textbf{Product Rule}} \\
&= \Sigma_{a,f} P(A=a) P(F=f) P(D=0|F=f, A=a) && \boxed{\textbf{CI}} \\
&= \frac{1}{2}^2 \times \frac{1}{3} + \frac{1}{2}^2 \times \frac{1}{3} + \frac{1}{2}^2 \times 0 + \frac{1}{2}^2 \times 0 \\
&= \frac{2}{12} = \frac{1}{6}
\end{aligned}
$$

4 Expectation Maximization

Question 1

Correct Answer: Option C

Question 2

Correct Answer: Option B

Question 3

1. Correct Answer: Option A
2. Correct Answer: Option A
3. Correct Answer: Option A

5 HMM's

Problem 1: $P(O_1 = 1)$

For the first observation $O_1 = 1$:

$$\alpha_1(1) = P(S_1 = 1) \cdot P(O_1 = 1|S_1 = 1) = 0.9 \cdot \frac{1}{6} = 0.15$$

$$\alpha_1(2) = P(S_1 = 2) \cdot P(O_1 = 1|S_1 = 2) = 0.1 \cdot 0.1 = 0.01$$

$$\alpha_1 = [0.15, 0.01]$$

$$P(O_1 = 1) = \Sigma_i \alpha_i = 0.15 + 0.01 = 0.16$$

Problem 2 - Algorithm for remaining entries

The questions asks us to compute $P(O_1, O_2, S_2 = 1)$. Applying the forward algorithm, we obtain that this is equivalent to:

$$\Sigma_{i=1}^{n} \alpha_{it} \times a_{ij} \times b_j(O_{t+1})$$

Where t=1 (denoting the previous time step in the forward algorithm) and j=1 (our current state at time step 2). Plugging these values in, we get:

$$\Sigma_{i=1}^{n} \alpha_{i1} \times a_{i1} \times b_1(O_2) =$$

$$\alpha_{11} \times a_{11} \times b_1(O_2) + \alpha_{21} \times a_{21} \times b_1(O_2)$$

. Simplifying further, we get:

$$\alpha_{11} \times 0.8 \times \frac{1}{6} + \alpha_{21} \times 0.1 \times \frac{1}{6}$$

. The student need not simplify further.

Problem 3: Extracting Final Solution

Sum up over the final column of the matrix α