CSE-250A-SP25-Final-Solutions

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1 T/F

1. True
2. False
3. False
4. False
5. True
6. True
7. True
8. False
9. True
10. False

2 Naive Bayes

1. C
2. B,C

3 Variable Elimination

$$
\begin{aligned}
P(D) &= \sum_{a,b,c,e} P(D, e, c, b, a) && \text{(Marginalization)} \\
&= \sum_{a,b,c,e} P(D \mid c) P(e \mid c) P(c \mid a) P(b \mid a) P(a) && \text{(CI)} \\
&= \sum_{c} P(D \mid c) \sum_{e} P(e \mid c) \sum_{a} P(c \mid a) P(a) \sum_{b} P(b \mid a) \\
&= \sum_{c} P(D \mid c) \sum_{e} P(e \mid c) \sum_{a} P(c \mid a) P(a) f_1(a) && (f_1(a) \equiv \textstyle\sum_b P(b \mid a)) \\
&= \sum_{c} P(D \mid c) \sum_{e} P(e \mid c) f_2(c) && (f_2(c) \equiv \textstyle\sum_a P(c \mid a) P(a) f_1(a)) \\
&= \sum_{c} P(D \mid c) f_3(c) && (f_3(c) \equiv \textstyle\sum_e P(e \mid c) f_2(c)) \\
&= f_4(D) && (f_4(D) \equiv \textstyle\sum_c P(D \mid c) f_3(c))
\end{aligned}
$$

6 pts total: 1 pts awarded for each correct factor (f_1 through f_4) and the remaining 2 pts awarded for correct math

4 Expectation Maximization

The formula from the M-step of the EM algorithm (for a single observation sequence) is:

$$
b_{m3} = P(O_t = 3 \mid S_t = m) = \frac{\sum_t P(S_t = m \mid O_1, \ldots, O_T) \cdot \mathbb{I}(O_t = 3)}{\sum_t P(S_t = m \mid O_1, \ldots, O_T)}
$$

where $\mathbb{I}(O_t = 3)$ is an indicator function that is 1 when $O_t = 3$ and 0 otherwise.

For each t, we have:

- $P(S_1 = m | O_1 \ldots, O_T) = \frac{0.04}{0.12} = \frac{1}{3}$
- $P(S_2 = m | O_1 \ldots, O_T) = \frac{0.04}{0.12} = \frac{1}{3}$
- $P(S_3 = m | O_1 \ldots, O_T) = \frac{0.04}{0.12} = \frac{1}{3}$
- $P(S_4 = m | O_1 \ldots, O_T) = \frac{0.05}{0.12} = \frac{5}{12}$
- $P(S_5 = m | O_1 \ldots, O_T) = \frac{0.08}{0.12} = \frac{2}{3}$

From the observation, $\mathbb{I}(O_t = 3) = 1$ for $t = 2$ and $t = 5$. Thus,

$$
b_{m3} = P(O_t = 3 \mid S_t = m) = \frac{\frac{2}{3} + \frac{1}{3}}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{5}{12} + \frac{2}{3}} = \frac{\frac{12}{12}}{\frac{25}{12}} = \frac{12}{25}
$$

5 Inference in HMMs

Let the forward variable be defined as:

$$\alpha_t(j) = P(O_{1:t}, S_t = j)$$

1. **Initial step ($t = 1$):**

$$\alpha_1(1) = \pi_1 \cdot b_1(\text{Yes})$$
$$\alpha_1(2) = \pi_2 \cdot b_2(\text{Yes})$$

2. **Induction step ($t = 2$):**

$$\alpha_2(1) = [\alpha_1(1) \cdot a_{11} + \alpha_1(2) \cdot a_{21}] \cdot b_1(\text{Yes})$$
$$\alpha_2(2) = [\alpha_1(1) \cdot a_{12} + \alpha_1(2) \cdot a_{22}] \cdot b_2(\text{Yes})$$

3. **Induction step ($t = 3$):**

$$\alpha_3(1) = [\alpha_2(1) \cdot a_{11} + \alpha_2(2) \cdot a_{21}] \cdot b_1(\text{No})$$
$$\alpha_3(2) = [\alpha_2(1) \cdot a_{12} + \alpha_2(2) \cdot a_{22}] \cdot b_2(\text{No})$$

4. **Final step (total probability):**

$$P(O = [\text{Yes},\ \text{Yes},\ \text{No}]) = \alpha_3(1) + \alpha_3(2)$$

6 Two-state MDP - Policy Evaluation

From the Bellman equation:

$$\begin{aligned}
V^\pi(0) &= R(0) + \gamma\left[P(s'{=}0|s{=}0,a{=}\uparrow)V^\pi(0) + P(s'{=}1|s{=}0,a{=}\uparrow)V^\pi(1)\right] \\
&= -2 + \tfrac{1}{2}\left[\tfrac{1}{4}V^\pi(0) + \tfrac{3}{4}V^\pi(1)\right] \\
&= -2 + \tfrac{1}{8}V^\pi(0) + \tfrac{3}{8}V^\pi(1)
\end{aligned}$$

$$\begin{aligned}
V^\pi(1) &= R(1) + \gamma\left[P(s'{=}0|s{=}1,a{=}\downarrow)V^\pi(0) + P(s'{=}1|s{=}1,a{=}\downarrow)V^\pi(1)\right] \\
&= 2 + \tfrac{1}{2}\left[\tfrac{1}{4}V^\pi(0) + \tfrac{3}{4}V^\pi(1)\right] \\
&= 2 + \tfrac{1}{8}V^\pi(0) + \tfrac{3}{8}V^\pi(1)
\end{aligned}$$

Solving the equations give: $V^\pi(0) = -1$ and $V^\pi(1) = 3$.