

# **Introduction to programming for data science**

**STAT 201**

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# Preface

This book is currently being written for the course STAT201.

# **Part I**

# **Python**

# 1 Introduction to Jupyter Notebooks and programming in python

This chapter is a very brief introduction to python and Jupyter notebooks. We only discuss the content relevant for applying python to analyze data.

## 1.1 Installation

**Anaconda:** If you are new to python, we recommend downloading the [Anaconda installer](#) and following the instructions for installation. Once installed, we'll use the Jupyter Notebook interface to write code.

**Quarto:** We'll use Quarto to publish the `*.ipynb*` file containing text, python code, and the output. Download and install Quarto from [here](#).

## 1.2 Jupyter notebook

### 1.2.1 Introduction

Jupyter notebook is an interactive platform, where you can write code and text, and make visualizations. You can access Jupyter notebook from the Anaconda Navigator, or directly open the Jupyter Notebook application itself. It should automatically open up in your default browser. The figure below shows a Jupyter Notebook opened with Google Chrome. This page is called the *landing page* of the notebook.

<IPython.core.display.Image object>

To create a new notebook, click on the **New** button and select the **Python 3** option. You should see a blank notebook as in the figure below.

<IPython.core.display.Image object>



### 1.2.2 Writing and executing code

**Code cell:** By default, a cell is of type *Code*, i.e., for typing code, as seen as the default choice in the dropdown menu below the *Widgets* tab. Try typing a line of python code (say, `2+3`) in an empty code cell and execute it by pressing *Shift+Enter*. This should execute the code, and create an new code cell. Pressing *Ctrl+Enter* for *Windows* (or *Cmd+Enter* for *Mac*) will execute the code without creating a new cell.

**Commenting code in a code cell:** Comments should be made while writing the code to explain the purpose of the code or a brief explanation of the tasks being performed by the code. A comment can be added in a code cell by preceding it with a `#` sign. For example, see the comment in the code below.

Writing comments will help other users understand your code. It is also useful for the coder to keep track of the tasks being performed by their code.

```
#This code adds 3 and 5  
3+5
```

8

**Markdown cell:** Although a comment can be written in a code cell, a code cell cannot be used for writing headings/sub-headings, and is not appropriate for writing lengthy chunks of text. In such cases, change the cell type to *Markdown* from the dropdown menu below the *Widgets* tab. Use any markdown cheat sheet found online, for example, [this one](#) to format text in the markdown cells.

Give a name to the notebook by clicking on the text, which says ‘Untitled’.

### 1.2.3 Saving and loading notebooks

Save the notebook by clicking on **File**, and selecting **Save as**, or clicking on the **Save and Checkpoint** icon (below the **File** tab). Your notebook will be saved as a file with an extension *ipynb*. This file will contain all the code as well as the outputs, and can be loaded and edited by a Jupyter user. To load an existing Jupyter notebook, navigate to the folder of the notebook on the *landing page*, and then click on the file to open it.

### 1.2.4 Rendering notebook as HTML

We'll use Quarto to print the `**ipynb*` file as HTML. Check the procedure for rendering a notebook as HTML [here](#). You have several options to format the file.

You will need to open the command prompt, navigate to the directory containing the file, and use the command: `quarto render filename.ipynb --to html`.

## 1.3 In-class exercise

1. Create a new notebook.
2. Save the file as `In_class_exercise_1`.
3. Give a heading to the file - `First HTML file`.
4. Print `Today is day 1 of my programming course`.
5. Compute and print the number of seconds in a day.

The HTML file should look like the picture below.

<IPython.core.display.Image object>

## 1.4 Python libraries

There are several [built-in functions](#) in python like `print()`, `abs()`, `max()`, `sum()` etc., which do not require importing any library. However, these functions will typically be insufficient for analyzing data. Some of the popular libraries in data science and their primary purposes are as follows:

1. NumPy: Performing numerical operations and efficiently storing numerical data.
2. Pandas: Reading, cleaning and manipulating data.
3. Matplotlib, Seaborn: Visualizing data.
4. SciPy: Performing scientific computing such as solving differential equations, optimization, statistical tests, etc.
5. Scikit-learn: Data pre-processing and machine learning, with a focus on prediction.
6. Statsmodels: Developing statistical models with a focus on inference

A library can be imported using the `import` keyword. For example, a NumPy library can be imported as:

```
import numpy as np
```

Using the `as` keyword, the NumPy library has been given the name `np`. All the functions and attributes of the library can be called using the `'np.'` prefix. For example, let us generate a sequence of whole numbers upto 10 using the NumPy function `arange()`:

```
np.arange(8)
```

```
array([0, 1, 2, 3, 4, 5, 6, 7])
```

Generating random numbers is very useful in python for performing simulations (we'll see in later chapters). The library `random` is used to generate random numbers such as integers, real numbers based on different probability distributions, etc.

Below is an example of using the `randint()` function of the library for generating random numbers in `[a, b]`, where `a` and `b` are integers.

```
import random as rm
rm.randint(5,10) #This will generate a random number in [5,10]
```

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## 1.5 Debugging and errors

Read sections 1.3 - 1.6 from [http://openbookproject.net/thinkcs/python/english3e/way\\_of\\_the\\_program.html](http://openbookproject.net/thinkcs/python/english3e/way_of_the_program.html)

## 1.6 Terms used in programming

Read section 1.11 from [http://openbookproject.net/thinkcs/python/english3e/way\\_of\\_the\\_program.html](http://openbookproject.net/thinkcs/python/english3e/way_of_the_program.html)

## 2 Variables, expressions and statements

### 2.1 Data types

Python has several built-in data types for storing different kinds of information in variables.

#### 2.1.1 Primitive

Integer, float, boolean, None, and string are *primitive data types* because they represent a single value.

#### 2.1.2 Containers

Other data types like list, tuple, and dictionary are often called *data structures* or *containers* because they hold multiple pieces of data together. We'll discuss these datatypes in chapter 5.

The data type of the object can be identified using the in-built python function `type()`. For example, see the following objects and their types:

```
type(4)
```

```
int
```

```
type(4.4)
```

```
float
```

```
type('4')
```

```
str
```

```
type(True)
```

bool

### 2.1.3 Practice exercise 1

What is the datatype of the following objects?

1. 'This is False'
2. "This is a number"
3. 1000
4. 65.65
5. False

## 2.2 Constants and Variables

A constant is a value that cannot be changed. It may be a number, string or any other datatype. Below are some examples of printing a constant:

```
print(4)
```

4

```
print("This is a string and also a constant")
```

This is a string and also a constant

```
print(False)
```

False

A variable is an object whose value can be changed. For example, consider the object below:

```
x = 2
```

In the above code the variable `x` has been assigned a value of 2. However, the value of `x` can be changed:

```
x = 3
print("x =", x)
```

```
x = 3
```

Thus, the object `x` in the above code is a variable that refers to a memory location storing the constant value of 3.

### 2.2.1 Variable names

There are a some rules for naming variables:

1. A variable name must start with a letter or underscore `_`
2. A variable name may consist of letters, numbers, and underscores only

For example, some of the valid variable names are `*salary`, `text10`, `__varname`. *Some of the invalid variable names are `salary% 10text`, `varname)`\**

3. Variable names are case-sensitive. For example, the variable `Varname` will be different from `varname`.
4. There are certain *reserved words* in python that have some meaning, and cannot be used as variable names. These reserved words are:

```
<IPython.core.display.Image object>
```

**Best coding practice:** Variables should be named such that they are informative of the value they are storing. For example, suppose we wish to compute the income tax a person has to pay based on their income and tax rate. Below are two ways of naming variables to do this computation:

```
income = 80000
tax_rate = 0.15
print("Income tax = ",income*tax_rate)
```

```
Income tax = 12000.0
```

```
a = 80000  
b = 0.15  
print("Income tax = ",a*b)
```

```
Income tax = 12000.0
```

The former code chunk is better than the latter one as it makes the code easy to read and understand.

## 2.2.2 Practice exercise 2

### 2.2.2.1 Variables or constants?

In the statements below, classify the objects as variables or constants?

1. value = "name"
2. constant = 7
3. another\_const = "variable"
4. True\_False = True

### 2.2.2.2 Valid variable names?

Which of the following variable names are valid?

1. var.name
2. var9name
3. \_\_varname
4. varname\*

## 2.3 Assignment statements

Values are assigned to variables with the assignment statement ( $=$ ). An assignment statement may have a constant or an expression on the right hand side of the ( $=$ ) sign, and a variable name on the left hand side.

For example, the code lines below are assignment statements

```
var = 2
var = var + 3
```

## 2.4 Expressions

The mathematical operations and their corresponding operators are as follows:

1. Exponent:  $**$
2. Remainder:  $\%$
3. Multiplication:  $*$
4. Division:  $/$
5. Addition:  $+$
6. Subtraction:  $-$

The operators above are in decreasing order of precedence, i.e., an exponent will be evaluated before a remainder, a remainder will be evaluated before a multiplication, and so on.

For example, check the precedence of operators in the computation of the following expression:

```
2+3%4*2
```

8

In case an expression becomes too complicated, use of parenthesis may help clarify the precedence of operators. Parenthesis takes precedence over all the operators listed above. For example, in the expression below, the terms within parenthesis are evaluated first:

```
2+3%(4*2)
```

5



### 2.4.1 Practice exercise 3

Which of the following statements is an assignment statement:

1. `x = 5`
2. `print(x)`
3. `type(x)`
4. `x + 4`

What will be the result of the following expression:

```
1%2**3*2+1
```

## 2.5 Converting datatypes

Sometimes a value may have a datatype that is not suitable for using it. For example, consider the variable called *annual\_income* in the code below:

```
annual_income = "80000"
```

Suppose we wish to divide *annual\_income* by 12 to get the monthly income. We cannot use the variable *monthly\_income* directly as its datatype is a string and not a number. Thus, numerical operations cannot be performed on the variable *annual\_income*.

We'll need to convert *annual\_income* to an integer. For that we will use the python's in-built `int()` function:

```
annual_income = int(annual_income)
monthly_income = annual_income/12
print("monthly income = ", monthly_income)
```

```
monthly income = 6666.666666666667
```

Similarly, datatypes can be converted from one type to another using in-built python functions as shown below:

```
#Converting integer to string
str(9)
```

```
'9'
```

```
#Converting string to float  
float("4.5")
```

```
4.5
```

```
#Converting bool to integer  
int(True)
```

```
1
```

Sometimes, conversion of a value may not be possible. For example, it is not possible to convert the variable *greeting* defined below to a number:

```
greeting = "hello"
```

However, in some cases, mathematical operators such as `+` and `*` can be applied on strings. The operator `+` concatenates multiple strings, while the operator `*` can be used to concatenate a string to itself multiple times:

```
"Hi" + " there!"
```

```
'Hi there!'
```

```
"5" + '3'
```

```
'53'
```

```
"5"*8
```

```
'55555555'
```

## 2.6 User input

Python's in-built `input()` function can be used to accept an input from the user. For example, suppose we wish the user to onput their age:

```
age = input("Enter your age:")
```

Enter your age:34

The entered value is stored in the variable **age** and can be used for computation.

### 2.6.1 Practice exercise 4

Ask the user to input their year of birth, and print their age.

## 2.7 Commenting code

The **#** symbol can be used to comment the code. Anything after the **#** sign is ignored by python. Commenting a code may have several purposes, such as:

- Describe what is going to happen in a sequence of code
- Document who wrote the code or other ancillary information
- Turn off a line of code - perhaps temporarily

For example, below is code with a comment to describe the purpose of the code:

```
#Computing number of hours of lecture in this course  
print("Total lecture hours of STAT201=",10*3*(5/6))
```

Total lecture hours of STAT201= 25.0

### 2.7.1 Practice exercise 5

Which of the following lines is a comment:

1. `#this is a comment`
2. `##this may be a comment`
3. `A comment#`

## 2.8 Programming errors

There are 3 types of errors that can occur in a program - syntax errors, run-time errors, and semantic errors.

### 2.8.1 Syntax errors

Syntax errors occur if the code is written in a way that it does not comply with the rules / standards / laws of the language (python in this case). For example, suppose a values is assigned to a variable as follows:

```
9value = 2
```

The above code when executed will indicate a syntax error as it violates the rule that a variable name must not start with a number.

### 2.8.2 Run-time errors

Run-time errors occur when a code is syntactically correct, but there are other issues with the code such as:

- Misspelled or incorrectly capitalized variable and function names
- Attempts to perform operations (such as math operations) on data of the wrong type (ex. attempting to subtract two variables that hold string values)
- Dividing by zero
- Attempts to use a type conversion function such as int on a value that can't be converted to an int

For example, suppose a number is multiplied as follows:

```
multiplication_result = x * 4
```

The above code is syntactically correct. However, it will generate an error as the variable x has not been defined as a number.

### 2.8.3 Semantic errors

Semantic errors occur when the code executes without an error being indicated by the compiler. However, it does not work as intended by the user. For example, consider the following code of multiplying the number 6 by 3:

```
x = '6'
x * 3
```

'666'

If it was intended to multiply the number 6, then the variable `x` should have been defined as `x=6` so that `x` has a value of type `integer`. However, in the above code `6` is a `string` type value. When a `string` is multiplied by an integer, say  $n$ , it concatenates with itself  $n$  times.

## 2.8.4 Practice exercise 6

Suppose we wish to compute tax using the income and the tax rate. Identify the type of error from amongst syntax error, semantic error and run-time error in the following pieces of code.

```
income = 2000
tax = .08 * Income
print("tax on", income, "is:", tax)
```

```
income = 2000
tax = .08 x income
print("tax on", income, "is:", tax)
```

```
income = 2000
tax = .08 ** income
print("tax on", income, "is:", tax)
```

## 2.9 Practice exercise 7

The formula for computing final amount if one is earning compound interest is given by:

$$A = P \left( 1 + \frac{r}{n} \right)^{nt},$$

where:

$P$  = Principal amount (initial investment),

$r$  = annual nominal interest rate,

$n$  = number of times the interest is computed per year,

$t$  = number of years

Write a Python program that assigns the principal amount of \$10000 to variable  $P$ , assign to  $n$  the value 12, and assign to  $r$  the interest rate of 8%. Then have the program prompt the user for the number of years  $t$  that the money will be compounded for. Calculate and print the final amount after  $t$  years.

What is the amount if the user enters  $t$  as 4 years?

## 3 Control flow statements

A [control flow statement](#) in a computer program determines the individual lines of code to be executed and/or the order in which they will be executed. In this chapter, we'll learn about 3 types of control flow statements:

1. if-elif-else
2. for loop
3. while loop

### 3.1 Conditional execution

The first type of control flow statement is `if-elif-else`. This statement helps with conditional execution of code, i.e., the piece of code to be executed is selected based on certain condition(s).

#### 3.1.1 Comparison operators

For testing if conditions are true or false, first we need to learn the operators that can be used for comparison. For example, suppose we want to check if two objects are equal, we use the `==` operator:

```
5==6
```

False

```
x = "hi"  
y = "hi"  
x==y
```

True

Below are the python comparison operators and their meanings.

Python code	Meaning
<code>x==y</code>	Produce True if ... x is equal to y
<code>x != y</code>	... x is not equal to y
<code>x &gt; y</code>	... x is greater than y
<code>x &lt; y</code>	... x is less than y
<code>x &gt;= y</code>	... x is greater than or equal to y
<code>x &lt;= y</code>	... x is less than or equal to y

### 3.1.2 Logical operators

Sometimes we may need to check multiple conditions simultaneously. The logical operator **and** is used to check if all the conditions are true, while the logical operator **or** is used to check if either of the conditions is true.

```
#Checking if both the conditions are true using 'and'
5==5 and 67==68
```

False

```
#Checking if either condition is true using 'or'
x = 6; y = 90
x<0 or y>50
```

True

### 3.1.3 if-elif-else statement

The **if-elif-else** statements can check several conditions, and execute the code corresponding to the condition that is true. Note that there can be as many **elif** statements as required.

**Syntax:** Python uses indentation to identify the code to be executed if a condition is true. All the code indented within a condition is executed if the condition is true.

**Example:** Input an integer. Print whether it is positive or negative.

```
number = input("Enter a number:") #Input an integer
number_integer = int(number)      #Convert the integer to 'int' datatype
```



```

if number_integer>0:                #Check if the integer is positive
    print("Number is positive")
else:
    print("Number is negative")

```

Enter a number:-9  
Number is negative

In the above code, note that anything entered by the user is taken as a string datatype by python. However, a string cannot be positive or negative. So, we converted the number input by the user to integer to check if it was positive or negative.

There may be multiple statements to be executed if a condition is true. See the example below.

**Example:** Input a number. Print whether it is positive, negative or zero. If it is negative, print its absolute value.

```

number = input("Enter a number:")
number_integer = int(number)
if number_integer>0:
    print("Number is positive")
elif number_integer==0:
    print("Number is zero")
else:
    print("Number is negative")
    print("Absolute value of number = ", abs(number_integer))

```

Enter a number:0  
Number is zero

### 3.1.4 Practice exercise 1

Input a number. Print whether its odd or even.

**Solution:**

```

num = int(input("Enter a number: "))
if num%2==0:                #Checking if the number is divisible by 2
    print("Number is even")
else:

```

```
print("Number is odd")
```

Enter a number: 5

Number is odd

### 3.1.5 Try-except

If we suspect that some lines of code may produce an error, we can put them in a **try** block, and if an error does occur, we can use the **except** block to instead execute an alternative piece of code. This way the program will not stop if an error occurs within the **try** block, and instead will be directed to execute the code within the **except** block.

**Example:** Input an integer from the user. If the user inputs a valid integer, print whether it is a multiple of 3. However, if the user does not input a valid integer, print a message saying that the input is invalid.

```
num = input("Enter an integer:")

#The code lines within the 'try' block will execute as long as they run without error
try:
    #Converting the input to integer, as user input is a string
    num_int = int(num)

    #checking if the integer is a multiple of 3
    if num_int%3==0:
        print("Number is a multiple of 3")
    else:
        print("Number is not a multiple of 3")

#The code lines within the 'except' block will execute only if the code lines within the '
except:
    print("Input must be an integer")
```

Enter an integer:hi

Input must be an integer

### 3.1.6 Practice exercise 2

#### 3.1.6.1

Ask the user to enter their exam score. Print the grade based on their score as follows:

Score	Grade
(90,100]	A
(80,90]	B
[0,80]	C

If the user inputs a score which is not a number in [0,100], print invalid entry.

**Solution:**

```
score = input("Enter exam score:")
try:

    #As exam score can be a floating point number (such as 90.65), we need to use 'float'
    score_num = float(score)
    if score_num>90 and score_num<=100:
        print("Grade: A")
    elif score_num>80 and score_num<=90:
        print("Grade: B")
    elif score_num>=0 and score_num<=80:
        print("Grade: C")
    else:
        print("Invalid score")           #If a number is less than 0 or more than 100
except:
    print("Invalid input")              #If the input is not a number
```

Enter exam score:90

Grade: B

#### 3.1.6.2

**Nested if-elif-else statements:** This question will lead you to create nested if statements, i.e., an if statement within another if statement.

Think of a number in [1,5]. Ask the user to guess the number.

- If the user guesses the number correctly, print “Correct in the first attempt!”, and stop the program. Otherwise, print “Incorrect! Try again” and give them another chance to guess the number.
- If the user guesses the number correctly in the second attempt, print “Correct in the second attempt”, otherwise print “Incorrect in both the attempts, the correct number is:”, and print the correct number.

#### Solution:

```
#Let us say we think of the number. Now the user has to guess the number in two attempts.
rand_no = 3
guess = input("Guess the number:")
if int(guess)==rand_no:
    print("Correct in the first attempt!")

#If the guess is incorrect, the program will execute the code block below
else:
    guess = input("Incorrect! Try again:")
    if int(guess)==rand_no:
        print("Correct in the second attempt")
    else:
        print("Incorrect in the both the attempts, the correct number was:", rand_no)
```

## 3.2 Loops

With loops, a piece of code can be executed repeatedly for a fixed number of times or until a condition is satisfied.

### 3.2.1 for loop

With a **for** loop, a piece of code is executed a fixed number of times.

We typically use **for** loops with an in-built python function called **range()** that supports **for** loops. Below is its description.

**range():** The **range()** function returns a sequence of evenly-spaced integer values. It is commonly used in **for** loops to define the sequence of elements over which the iterations are performed.

Below is an example where the **range()** function is used to create a sequence of whole numbers upto 10. Ignore the **list()** function in the code below, as it will be introduced later.

```
print(list(range(10)))
```

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

Note that the last element is one less than the integer specified in the `range()` function.

Using the `range()` function, the `for` loop can iterate over a sequence of numbers. See the example below.

**Example:** Print the first  $n$  elements of the [Fibonacci sequence](#), where  $n$  is an integer input by the user, such that  $n > 2$ . In a fibonacci sequence, each number is the sum of the preceding two numbers, and the sequence starts from 0,1. The sequence is as follows:

0,1,1,2,3,5,8,13,....

```
n=int(input("Enter number of elements:"))

#Initializing the sequence to start from 0, 1
n1=0;n2=1

#Printing the first two numbers of the sequence
print(n1)
print(n2)

for i in range(n-2): #Since two numbers of the sequence are already printed, n-2 numbers

    #Computing the next number of the sequence as the summation of the previous two number
    n3 = n1+n2
    print(n3)

    #As 'n3' is already printed, it is no longer the next number of the sequence.
    #Thus, we move the values of the variables n1 and n2 one place to the right to compute
    n1 = n2
    n2 = n3

print("These are the first", n, "elements of the fibonacci series")
```

Enter number of elements:6

0  
1  
1  
2

3

5

These are the first 6 elements of the fibonacci series

As in the `if-elif-else` statement, the `for` loop uses indentation to indicate the piece of code to be run repeatedly.

Note that we have used an in-built python function

### 3.2.2 while loop

With a `while` loops, a piece of code is executed repeatedly until certain condition(s) hold.

**Example:** Print all the elements of the [Fibonacci sequence](#) less than  $n$ , where  $n$  is an integer input by the user, such that  $n > 2$ . In a fibonacci sequence, each number is the sum of the preceding two numbers, and the sequence starts from 0,1. The sequence is as follows:

0,1,1,2,3,5,8,13,....

```
n=int(input("Enter the value of n:"))

#Initializing the sequence to start from 0, 1
n1=0;n2=1

#Printing the first number of the sequence
print(n1)

while n2<n:

    #Print the next number of the sequence
    print(n2)

    #Computing the next number of the sequence as the summation of the previous two number
    n3 = n1+n2

    #As n2 is already printed, assigning n2 to n3, so that the next number of the sequence
    #Assigning n1 to n2 as n1 has already been used to compute the next number of the sequence
    n1 = n2
    n2 = n3

print("These are all the elements of the fibonacci series less than", n)
```

Enter the value of n:50

0  
1  
1  
2  
3  
5  
8  
13  
21  
34

These are all the elements of the fibonacci series less than 50

### 3.2.3 Practice exercise 3

#### 3.2.3.1

Write a program that identifies whether a number input by the user is prime or not.

**Solution:**

```
number = int(input("Enter a positive integer:"))

#Defining a variable that will have a value of 0 if there are no divisors
num_divisors=0

#Checking if the number has any divisors from 2 to half of the number
for divisor in range(2,int(number/2+1)):
    if number%divisor==0:

        #If the number has a divisor, setting num_divisors to 1, to indicate that the
        num_divisors = 1

        #If a divisor has been found, there is no need to check if the number has more
        #Even if the number has a single divisor, it is not prime. Thus, we 'break' out
        #If you don't 'break', your code will still be correct, it will just do some u
        break

#If there are no divisors of the number, it is prime, else not prime
if num_divisors==0:
    print("Prime")
else:
```

```
print("Not prime")
```

Enter a positive integer:97

Prime

### 3.2.3.2

Update the program above to print the prime numbers starting from 2, and less than  $n$  where  $n$  is a positive integer input by the user.

**Solution:**

```
n = int(input("Enter a positive integer:"))

#Defining a variable - number_iterator. We will use this variable to iterate over all integers
#While iterating over each integer from 2 to n, we will check if the integer is prime or not
number_iterator = 2

print(number_iterator) #Since '2' is a prime number, we can print it directly (without checking)

#Continue to check for prime numbers until n (but not including n)
while(number_iterator<n):

    #After each check, increment the number_iterator to check if the next integer is prime
    number_iterator=number_iterator+1

    #Defining a variable that will have a value of 0 if there are no divisors
    num_divisors=0

    #Checking if the integer has any divisors from 2 to half of the integer being checked
    for divisor in range(2,int(number_iterator/2+1)):
        if number_iterator%divisor==0:

            #If the integer has a divisor, setting num_divisors to 1, to indicate that the integer is not prime
            num_divisors=1

    #If a divisor has been found, there is no need to check if the integer has more divisors
    #Even if the integer has a single divisor, it is not prime.
    #Thus, we 'break' out of the loop that checks for divisors
    break
```



```
#If there are no divisors of the integer being checked, the integer is a prime number,  
if num_divisors==0:  
    print(number_iterator)
```

Enter a positive integer:100

2  
3  
5  
7  
11  
13  
17  
19  
23  
29  
31  
37  
41  
43  
47  
53  
59  
61  
67  
71  
73  
79  
83  
89  
97

### 3.3 break statement

The `break` statement is used to unconditionally exit the innermost loop.

For example, suppose we need to keep asking the user to input year of birth and compute the corresponding age, until the user enters 1900 as the year of birth.

```
#The loop will continue to run indefinitely as the condition 'True' is always true  
while True:
```

```

year = int(input("Enter year of birth:"))
if year==1900:
    break          #If the user inputs 1900, then break out of the loop
else:
    print("Age = ",2022-year)    #Otherwise compute and print the age

```

```

Enter year of birth:1987
Age = 35
Enter year of birth:1995
Age = 27
Enter year of birth:2001
Age = 21
Enter year of birth:1900

```

### 3.3.1 Practice exercise 4

Write a program that finds and prints the largest factor of a number input by the user. Check the output if the user inputs 133.

**Solution:**

```

num = int(input("Enter an integer:"))

#Looping from the half the integer to 0 as the highest factor is likely to be closer to half
for i in range(int(num/2)+1,0,-1):
    if num%i==0:
        print("Largest factor = ",i)

        #Exiting the loop if the largest integer is found
        break

```

```

Enter an integer:133
Largest factor = 19

```

## 3.4 continue statement

The `continue` statement is used to continue with the next iteration of the loop without executing the lines of code below it.

For example, consider the following code:

```

for i in range(10):
    if i%2==0:
        continue
    print(i)

```

1  
3  
5  
7  
9

When the control flow reads the statement `continue`, it goes back to the beginning of the `for` loop, and ignores the lines of code below the statement.

### 3.4.1 Practice exercise 5:

Write a program that asks the user the question, “How many stars are in the Milky Way (in billions)?”. If the user answers 100, the program should print correct, and stop. However, if the user answers incorrectly, the program should print “incorrect”, and ask them if they want to try again. The program should continue to run until the user answers correctly, or they want to stop trying.

```

#Defining an infinite while loop as the loop may need to run indefinitely if the user keeps
while True:
    answer = input("How many stars are there in the Milky Way? ")
    if answer=='100':
        print("Correct")

        #Exiting the loop if the user answers correctly
        break
    else:
        print("Incorrect")
        try_again = input("Do you want to try again? (Y/N) ")
        if try_again=='Y':

            #Continuing with the infinite loop if the user wants to try again
            continue
        else:

            #Exiting the infinite loop if the user wants to stop trying
            break

```

```
How many stars are there in the Milky Way? 101
Incorrect
Do you want to try again? (Y/N) Y
How many stars are there in the Milky Way? 7
Incorrect
Do you want to try again? (Y/N) Y
How many stars are there in the Milky Way? 5
Incorrect
Do you want to try again? (Y/N) Y
How many stars are there in the Milky Way? 100
Correct
```

### 3.5 Loops with strings

Loops can be used to iterate over a string, just like we used them to iterate over a sequence of integers.

Consider the following string:

```
sentence = "She sells sea shells on the sea shore"
```

The  $i^{th}$  character of the string can be retrieved by its index. For example, the first character of the string `sentence` is:

```
sentence[0]
```

```
'S'
```

#### Slicing a string:

A part of the string can be sliced by passing the starting index (say `start`) and the stopping index (say `stop`) as `start:stop` to the index operator `[]`. This is called slicing a string. For a string `S`, the characters starting from the index `start` upto the index `stop`, but not including `stop`, can be sliced as `S[start:stop]`.

For example, the slice of the string `sentence` from index 4 to index 9, but not including 9 is:

```
sentence[4:9]
```

```
'sells'
```

### Example:

Input a string, and count and print the number of “t”s.

```
string = input("Enter a sentence:")

#Initializing a variable 'count_t' which will store the number of 't's in the string
count_t = 0

#Iterating over the entire length of the string.
#The length of the string is given by the len() function
for i in range(len(string)):

    #If the ith character of the string is 't', then we count it
    if string[i]=='t':
        count_t = count_t+1

print("Number of 't's in the string = ",count_t)
```

Enter a sentence:Getting a tatto is not a nice experience  
Number of 't's in the string = 6

### 3.5.1 Practice exercise 6

Write a program that asks the user to input a string, and print the number of “the”s in the string.

```
string = input("Enter a sentence:")

#Defining a variable to store the count of the word 'the'
count_the = 0

#Looping through the entire length of the string except the last 3 letters.
#As we are checking three letters at a time starting from the index 'i', the last 3 letters
for i in range(len(string)-3):

    #Slicing 3 letters of the string and checking if they are 'the'
    if string[i:(i+3)]=='the':

        #Counting the words that are 'the'
        count_the = count_the+1
```

```
print("Number of 'the's in the string = ",count_the)
```

Enter a sentence:She sells the sea shells on the sea shore in the spring  
Number of 'the's in the string = 3

# 4 Functions

## 4.1 Introduction

As the words suggests, *functions* are a piece of code that have a specific function or purpose. As an analogy, if a human is a computer program, then the mind can be considered to be a function, which has purpose of thinking, eyes can be another function, which have a purpose of seeing. These functions are called upon by the human when needed.

Similarly, in case of a computer program, functions are a piece of code, that perform a specific task, when called upon by the program. Instead of being defined as a function, the piece of code can also be used directly whenever it is needed in a program. However, defining a frequently-used piece of code as a function has the following benefits:

1. It reduces the number of lines of code, as the lines of code need to be written just once in the function definition. Thereafter, the function is called by its name, wherever needed in the program. This makes the code compact, and enhances readability.
2. It makes the process of writing code easier, as the user needs to just type the name of the function, wherever it is needed, instead of pasting lines of code.
3. It can be used in different programs, thereby saving time in writing other programs.

To put it more formally, a function is a piece of code that takes arguments (if any) as input, performs computations or tasks, and then returns a result or results.

## 4.2 Defining a function

Look at the function defined below. It asks the user to input a number, and prints whether the number is odd or even.

```
#This is an example of a function definition
```

```
#A function definition begins with the 'def' keyword followed by the name of the function.
```

```
#Note that 'odd_even()' is the name of the function below.
```

```
def odd_even():
```

```

num = int(input("Enter an integer:"))
if num%2==0:
    print("Even")
else:
    print("Odd")    #Function definition ends here

print("This line is not a part of the function as it is not indented") #This line is not a

```

This line is not a part of the function as it is not indented

Note that the function is defined using the **def** keyword. All the lines within the function definition are indented. The indentation shows the lines of code that belong to the function. When the indentation stops, the function definition is considered to have ended.

Whenever the user wishes to input a number and print whether it is odd or even, they can call the function defined above by its name as follows:

```
odd_even()
```

```

Enter an integer:5
Odd

```

## 4.3 Parameters and arguments of a function

Note that the function defined above needs no input when called. However, sometimes we may wish to define a function that takes input(s), and performs computations on the inputs to produce an output. These input(s) are called parameter(s) of a function. When a function is called, the value(s) of these parameter(s) must be specified as argument(s) to the function.

### 4.3.1 Function with a parameter

Let us change the previous example to write a function that takes an integer as an input argument, and prints whether it is odd or even:

```

#This is an example of a function definition that has an argument
def odd_even(num):
    if num%2==0:
        print("Even")
    else:

```



```
print("Odd")
```

We can use the function whenever we wish to find a number is odd or even. For example, if we wish to find that a number input by the user is odd or even, we can call the function with the user input as its argument.

```
number = int(input("Enter an integer:"))  
odd_even(number)
```

```
Enter an integer:6
```

```
Even
```

Note that the above function needs an argument as per the function definition. It will produce an error if called without an argument:

```
odd_even()
```

```
TypeError: odd_even() missing 1 required positional argument: 'num'
```

#### 4.3.2 Function with a parameter having a default value

To avoid errors as above, sometimes is a good idea to assign a default value to the parameter in the function definition:

```
#This is an example of a function definition that has an argument with a default value  
def odd_even(num=0):  
    if num%2==0:  
        print("Even")  
    else:  
        print("Odd")
```

Now, we can call the function without an argument. The function will use the default value of the parameter specified in the function definition.

```
odd_even()
```

```
Even
```

### 4.3.3 Function with multiple parameters

A function can have as many parameters as needed. Multiple parameters/arguments are separated by commas. For example, below is a function that inputs two strings, concatenates them with a space in between, and prints the output:

```
def concat_string(string1, string2):  
    print(string1+' '+string2)
```

```
concat_string("Hi", "there")
```

Hi there

### 4.3.4 Practice exercise 1

Write a function that prints prime numbers between two real numbers - **a** and **b**, where **a** and **b** are the parameters of the function. Call the function and check the output with **a = 60**, **b = 80**.

**Solution:**

```
def prime_numbers (a,b=100):  
    num_prime_nos = 0  
  
    #Iterating over all numbers between a and b  
    for i in range(a,b):  
        num_divisors=0  
  
        #Checking if the ith number has any factors  
        for j in range(2, i):  
            if i%j == 0:  
                num_divisors=1;break;  
  
        #If there are no factors, then printing and counting the number as prime  
        if num_divisors==0:  
            print(i)  
prime_numbers(60,80)
```

61

67

71  
73  
79

## 4.4 Functions that return objects

Until now, we saw functions that print text. However, the functions did not **return** any object. For example, the function `odd_even` prints whether the number is odd or even. However, we did not save this information. In future, we may need to use the information that whether the number was odd or even. Thus, typically, we return an object from the function definition, which consists of the information we may need in the future.

The example `odd_even` can be updated to return the text “odd” or “even” as shown below:

```
#This is an example of a function definition that has an argument with a default value, and
def odd_even(num=0):
    if num%2==0:
        return("Even")
    else:
        return("Odd")
```

The function above returns a string “Odd” or “Even”, depending on whether the number is odd or even. This result can be stored in a variable, which can be used later.

```
response=odd_even(3)
response
```

'Odd'

The variable `response` now refers to the object where the string “Odd” or “Even” is stored. Thus, the result of the computation is stored, and the variable can be used later on in the program. Note that the control flow exits the function as soon as the first **return** statement is executed.

Figure 4.1 below shows the terminology associated with functions.

<IPython.core.display.Image object>

Figure 4.1: Terminology associated with functions

## 4.5 Global and local variables with respect to a function

A variable defined within a function is local to that function, while a variable defined outside the function is global with respect to that function. In case a variable with the same name is defined both outside and inside a function, it will refer to its global value outside the function and local value within the function.

The example below shows a variable with the name `var` referring to its local value when called within the function, and global value when called outside the function.

```
var = 5
def sample_function(var):
    print("Local value of 'var' within 'sample_function()' = ",var)

sample_function(4)
print("Global value of 'var' outside 'sample_function()' = ",var)
```

```
Local value of 'var' within 'sample_function()' = 4
Global value of 'var' outside 'sample_function()' = 5
```

## 4.6 Built-in python functions

So far we have seen user-defined functions in this chapter. These functions were defined by us, and are not stored permanently in the python compiler. However, there are some functions that come built-in with python and we can use them directly without defining them. These built-in functions can be seen [here](#). For example the built-in function `max()` computes the max of numeric values:

```
max(1,2,3)
```

3

Another example is the `round()` function that rounds up floating point numbers:

```
round(3.7)
```

4

## 4.7 Python libraries

Other than the built-in functions, python has hundreds of thousands of libraries that contain several useful functions. These libraries are contributed by people around the world as python is an open-source platform. Some of the libraries popular in data science, and their purposes are the following:

1. NumPy: Performing numerical operations and efficiently storing numerical data.
2. Pandas: Reading, cleaning and manipulating data.
3. Matplotlib, Seaborn: Visualizing data.
4. SciPy: Performing scientific computing such as solving differential equations, optimization, statistical tests, etc.
5. Scikit-learn: Data pre-processing and machine learning, with a focus on prediction.
6. Statsmodels: Developing statistical models with a focus on inference

A library can be imported using the `import` keyword. For example, a NumPy library can be imported as:

```
import numpy as np
```

Using the `as` keyword, the NumPy library has been given the name `np`. All the functions and attributes of the library can be called using the `'np.'` prefix. For example, let us generate a sequence of whole numbers upto 10 using the NumPy function `arange()`:

```
np.arange(8)
```

```
array([0, 1, 2, 3, 4, 5, 6, 7])
```

Generating random numbers is very useful in python for performing simulations (we'll see in later chapters). The library `random` is used to generate random numbers such as integers, real numbers based on different probability distributions, etc.

Below is an example of using the `randint()` function of the library for generating random numbers in `[a, b]`, where `a` and `b` are integers.

```
import random as rm
rm.randint(5,10) #This will generate a random number in [5,10]
```

### 4.7.1 Practice exercise 2

Generate a random number between  $[-5,5]$ . Do this 10,000 times. Find the mean of all the 10,000 random numbers generated.

**Solution:**

```
import random as rm
counter = 0
for i in range(10000):
    counter = counter + rm.uniform(-5,5)
print("Mean is:", counter/10000)
```

Mean is: 0.061433810226516616

## 5 Data structures

In this chapter we'll learn about the python data structures that are often used or appear while analyzing data.

### 5.1 Tuple

Tuple is a sequence of python objects, with two key characteristics: (1) the number of objects are fixed, and (2) the objects are immutable, i.e., they cannot be changed.

Tuple can be defined as a sequence of python objects separated by commas, and enclosed in rounded brackets (). For example, below is a tuple containing three integers.

```
tuple_example = (2,7,4)
```

We can check the data type of a python object using the *type()* function. Let us check the data type of the object *tuple\_example*.

```
type(tuple_example)
```

tuple

Elements of a tuple can be extracted using their index within square brackets. For example the second element of the tuple *tuple\_example* can be extracted as follows:

```
tuple_example[1]
```

7

Note that an element of a tuple cannot be modified. For example, consider the following attempt in changing the second element of the tuple *tuple\_example*.

```
tuple_example[1] = 8
```

`TypeError: 'tuple' object does not support item assignment`

The above code results in an error as tuple elements cannot be modified.

### 5.1.1 Practice exercise 1

USA's GDP per capita from 1960 to 2021 is given by the tuple `T` in the code cell below. The values are arranged in ascending order of the year, i.e., the first value is for 1960, the second value is for 1961, and so on. Print the years in which the GDP per capita of the US increased by more than 10%.

```
T = (3007, 3067, 3244, 3375, 3574, 3828, 4146, 4336, 4696, 5032, 5234, 5609, 6094, 6726, 7226, 78
```

**Solution:**

```
#Iterating over each element of the tuple
for i in range(len(T)-1):

    #Computing percentage increase in GDP per capita in the (i+1)th year
    increase = (T[i+1]-T[i])/T[i]

    #Printing the year if the increase in GDP per capita is more than 10%
    if increase>0.1:
        print(i+1961)
```

```
1973
1976
1977
1978
1979
1981
1984
```

### 5.1.2 Concatenating tuples

Tuples can be concatenated using the `+` operator to produce a longer tuple:

```
(2,7,4) + ("another", "tuple") + ("mixed","datatypes",5)
```

```
(2, 7, 4, 'another', 'tuple', 'mixed', 'datatypes', 5)
```



Multiplying a tuple by an integer results in repetition of the tuple:

```
(2,7,"hi") * 3
```

```
(2, 7, 'hi', 2, 7, 'hi', 2, 7, 'hi')
```

### 5.1.3 Unpacking tuples

If tuples are assigned to an expression containing multiple variables, the tuple will be unpacked and each variable will be assigned a value as per the order in which it appears. See the example below.

```
x,y,z = (4.5, "this is a string", (("Nested tuple",5)))
```

```
x
```

```
4.5
```

```
y
```

```
'this is a string'
```

```
z
```

```
('Nested tuple', 5)
```

If we are interested in retrieving only some values of the tuple, the expression `*_` can be used to discard the other values. Let's say we are interested in retrieving only the first and the last two values of the tuple:

```
x,*_,y,z = (4.5, "this is a string", (("Nested tuple",5)), "99", 99)
```

```
x
```

```
4.5
```

```
y
```

```
'99'
```

```
z
```

```
99
```

### 5.1.4 Practice exercise 2

USA's GDP per capita from 1960 to 2021 is given by the tuple T in the code cell below. The values are arranged in ascending order of the year, i.e., the first value is for 1960, the second value is for 1961, and so on.

Write a function that has two parameters:

1. Year : which indicates the year from which the GDP per capita are available in the second parameter
2. Tuple of GDP per capita's: Tuple consisting of GDP per capita for consecutive years starting from the year mentioned in the first parameter.

The function should return a tuple of length two, where the first element of the tuple is the number of years when the increase in GDP per capita was more than 5%, and the second element is the most recent year in which the GDP per capita increase was more than 5%.

Call the function to find the number of years, and the most recent year in which the GDP per capita increased by more than 5%, since the year 2000. Assign the **number of years** returned by the function to a variable named **num\_years**, and assign the most recent year to a variable named **recent\_year**. Print the values of **num\_years** and **recent\_year**.

```
T = (3007, 3067, 3244, 3375,3574, 3828, 4146, 4336, 4696, 5032,5234,5609,6094,6726,7226,78
```

```
def gdp_inc(year,gdp_tuple):
    count=0
    for i in range(len(gdp_tuple)-1):

        #Computing the increase in GDP per capita for the (i+1)th year
        increase = (gdp_tuple[i+1]-gdp_tuple[i])/gdp_tuple[i]
        if increase>0.05:
            print(year+i)
```

```

        #Over-writing the value of recent_year if the increase in GDP per capita for a
        recent_year = year+i+1

        #Counting the number of years for which the increase in GDP per capita is more
        count = count+1
    return((count,recent_year))

num_years, recent_year = gdp_inc(2000,T[40:])
print("Number of years when increase in GDP per capita was more than 5% = ", num_years)
print("The most recent year in which the increase in GDP per capita was more than 5% = ",

```

2003

2004

2020

Number of years when increase in GDP per capita was more than 5% = 3

The most recent year in which the increase in GDP per capita was more than 5% = 2021

### 5.1.5 Tuple methods

A couple of useful tuple methods are `count`, which counts the occurrences of an element in the tuple and `index`, which returns the position of the first occurrence of an element in the tuple:

```
tuple_example = (2,5,64,7,2,2)
```

```
tuple_example.count(2)
```

3

```
tuple_example.index(2)
```

0

Now that we have an idea about tuple, let us try to think where it can be used.

<IPython.core.display.HTML object>

## 5.2 List

List is a sequence of python objects, with two key characteristics that differentiates it from tuple: (1) the number of objects are variable, i.e., objects can be added or removed from a list, and (2) the objects are mutable, i.e., they can be changed.

List can be defined as a sequence of python objects separated by commas, and enclosed in square brackets []. For example, below is a list consisting of three integers.

```
list_example = [2,7,4]
```

### 5.2.1 Adding and removing elements in a list

We can add elements at the end of the list using the *append* method. For example, we append the string 'red' to the list *list\_example* below.

```
list_example.append('red')
```

```
list_example
```

```
[2, 7, 4, 'red']
```

Note that the objects of a list or a tuple can be of different datatypes.

An element can be added at a specific location of the list using the *insert* method. For example, if we wish to insert the number 2.32 as the second element of the list *list\_example*, we can do it as follows:

```
list_example.insert(1,2.32)
```

```
list_example
```

```
[2, 2.32, 7, 4, 'red']
```

For removing an element from the list, the *pop* and *remove* methods may be used. The *pop* method removes an element at a particular index, while the *remove* method removes the element's first occurrence in the list by its value. See the examples below.

Let us say, we need to remove the third element of the list.

```
list_example.pop(2)
```

7

```
list_example
```

```
[2, 2.32, 4, 'red']
```

Let us say, we need to remove the element 'red'.

```
list_example.remove('red')
```

```
list_example
```

```
[2, 2.32, 4]
```

```
#If there are multiple occurrences of an element in the list, the first occurrence will be removed
list_example2 = [2,3,2,4,4]
list_example2.remove(2)
list_example2
```

```
[3, 2, 4, 4]
```

For removing multiple elements in a list, either `pop` or `remove` can be used in a `for` loop, or a `for` loop can be used with a condition. See the examples below.

Let's say we need to remove integers less than 100 from the following list.

```
list_example3 = list(range(95,106))
list_example3
```

```
[95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105]
```

```
#Method 1: For loop with remove
list_example3_filtered = list(list_example3) #
```

```

for element in list_example3:
    if element<100:
        list_example3_filtered.remove(element)
print(list_example3_filtered)

```

[100, 101, 102, 103, 104, 105]

**Q1:** What's the need to define a new variable `list\_example3\_filtered` in the above code?

**A1:** Replace `list_example3_filtered` with `list_example3` and identify the issue.

```

#Method 2: Check this method after reading Section 5.2.6 on slicing a list
list_example3 = list(range(95,106))

#Slicing a list using ':' creates a copy of the list, and so
for element in list_example3[:]:
    if element<100:
        list_example3.remove(element)
print(list_example3)

```

[100, 101, 102, 103, 104, 105]

```

#Method 3: For loop with condition
[element for element in list_example3 if element>100]

```

[101, 102, 103, 104, 105]

## 5.2.2 List comprehensions

List comprehension is a compact way to create new lists based on elements of an existing list or other objects.

**Example:** Create a list that has squares of natural numbers from 5 to 15.

```

sqrt_natural_no_5_15 = [(x**2) for x in range(5,16)]
print(sqrt_natural_no_5_15)

```

[25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225]

**Example:** Create a list of tuples, where each tuple consists of a natural number and its square, for natural numbers ranging from 5 to 15.

```
sqrt_natural_no_5_15 = [(x,x**2) for x in range(5,16)]
print(sqrt_natural_no_5_15)
```

```
[(5, 25), (6, 36), (7, 49), (8, 64), (9, 81), (10, 100), (11, 121), (12, 144), (13, 169), (14, 196)]
```

### 5.2.3 Practice exercise 3

Below is a list consisting of responses to the question: “At what age do you think you will marry?” from students of the STAT303-1 Fall 2022 class.

```
exp_marriage_age=['24','30','28','29','30','27','26','28','30+', '26','28','30','30','30','30',
```

Use list comprehension to:

#### 5.2.3.1

Remove the elements that are not integers - such as ‘*probably never*’, ‘30+’, etc. What is the length of the new list?

**Hint:** The built-in python function of the `str` class - `isdigit()` may be useful to check if the string contains only digits.

```
exp_marriage_age_num = [x for x in exp_marriage_age if x.isdigit()==True]
print("Length of the new list = ",len(exp_marriage_age_num))
```

```
Length of the new list = 181
```

#### 5.2.3.2

Cap the values greater than 80 to 80, in the clean list obtained in (1). What is the mean age when people expect to marry in the new list?

```
exp_marriage_age_capped = [min(int(x),80) for x in exp_marriage_age_num]
print("Mean age when people expect to marry = ", sum(exp_marriage_age_capped)/len(exp_marriage_age_capped))
```

```
Mean age when people expect to marry = 28.955801104972377
```

### 5.2.3.3

Determine the percentage of people who expect to marry at an age of 30 or more.

```
print("Percentage of people who expect to marry at an age of 30 or more =", str(100*sum([1
```

Percentage of people who expect to marry at an age of 30 or more = 37.01657458563536 %

### 5.2.4 Concatenating lists

As in tuples, lists can be concatenated using the + operator:

```
import time as tm

list_example4 = [5,'hi',4]
list_example4 = list_example4 + [None,'7',9]
list_example4
```

[5, 'hi', 4, None, '7', 9]

For adding elements to a list, the **extend** method is preferred over the + operator. This is because the + operator creates a new list, while the **extend** method adds elements to an existing list. Thus, the **extend** operator is more memory efficient.

```
list_example4 = [5,'hi',4]
list_example4.extend([None, '7', 9])
list_example4
```

[5, 'hi', 4, None, '7', 9]

### 5.2.5 Sorting a list

A list can be sorted using the **sort** method:

```
list_example5 = [6,78,9]
list_example5.sort(reverse=True) #the reverse argument is used to specify if the sorting i
list_example5
```

[78, 9, 6]



## 5.2.6 Slicing a list

We may extract or update a section of the list by passing the starting index (say **start**) and the stopping index (say **stop**) as **start:stop** to the index operator []. This is called *slicing* a list. For example, see the following example.

```
list_example6 = [4,7,3,5,7,1,5,87,5]
```

Let us extract a slice containing all the elements from the 3rd position to the 7th position.

```
list_example6[2:7]
```

```
[3, 5, 7, 1, 5]
```

Note that while the element at the **start** index is included, the element with the **stop** index is excluded in the above slice.

If either the **start** or **stop** index is not mentioned, the slicing will be done from the beginning or until the end of the list, respectively.

```
list_example6[:7]
```

```
[4, 7, 3, 5, 7, 1, 5]
```

```
list_example6[2:]
```

```
[3, 5, 7, 1, 5, 87, 5]
```

To slice the list relative to the end, we can use negative indices:

```
list_example6[-4:]
```

```
[1, 5, 87, 5]
```

```
list_example6[-4:-2:]
```

```
[1, 5]
```

An extra colon (':') can be used to slice every *i*th element of a list.

```
#Selecting every 3rd element of a list  
list_example6[::3]
```

[4, 5, 5]

```
#Selecting every 3rd element of a list from the end  
list_example6[::-3]
```

[5, 1, 3]

```
#Selecting every element of a list from the end or reversing a list  
list_example6[::-1]
```

[5, 87, 5, 1, 7, 5, 3, 7, 4]

### 5.2.7 Practice exercise 4

Start with the list [8,9,10]. Do the following:

#### 5.2.7.1

Set the second entry (index 1) to 17

```
L = [8,9,10]  
L[1]=17
```

#### 5.2.7.2

Add 4, 5, and 6 to the end of the list

```
L = L+[4,5,6]
```

#### 5.2.7.3

Remove the first entry from the list

```
L.pop(0)
```

8

#### 5.2.7.4

Sort the list

```
L.sort()
```

#### 5.2.7.5

Double the list (concatenate the list to itself)

```
L=L+L
```

#### 5.2.7.6

Insert 25 at index 3

The final list should equal [4,5,6,25,10,17,4,5,6,10,17]

```
L.insert(3,25)  
L
```

```
[4, 5, 6, 25, 10, 17, 4, 5, 6, 10, 17]
```

Now that we have an idea about lists, let us try to think where it can be used.

<IPython.core.display.HTML object>

Now that we have learned about lists and tuples, let us compare them.

**Q2:** A list seems to be much more flexible than tuple, and can replace a tuple almost everywhere. Then why use tuple at all?

**A2:** The additional flexibility of a list comes at the cost of efficiency. Some of the advantages of a tuple over a list are as follows:

1. Since a list can be extended, space is over-allocated when creating a list. A tuple takes less storage space as compared to a list of the same length.
2. Tuples are not copied. If a tuple is assigned to another tuple, both tuples point to the same memory location. However, if a list is assigned to another list, a new list is created consuming the same memory space as the original list.
3. Tuples refer to their element directly, while in a list, there is an extra layer of pointers that refers to their elements. Thus it is faster to retrieve elements from a tuple.

The examples below illustrate the above advantages of a tuple.

```
#Example showing tuples take less storage space than lists for the same elements
tuple_ex = (1, 2, 'Obama')
list_ex = [1, 2, 'Obama']
print("Space taken by tuple =",tuple_ex.__sizeof__()," bytes")
print("Space taken by list =",list_ex.__sizeof__()," bytes")
```

Space taken by tuple = 48 bytes  
Space taken by list = 64 bytes

```
#Examples showing that a tuples are not copied, while lists can be copied
tuple_copy = tuple(tuple_ex)
print("Is tuple_copy same as tuple_ex?", tuple_ex is tuple_copy)
list_copy = list(list_ex)
print("Is list_copy same as list_ex?",list_ex is list_copy)
```

Is tuple\_copy same as tuple\_ex? True  
Is list\_copy same as list\_ex? False

```
#Examples showing tuples takes lesser time to retrieve elements
import time as tm
tt = tm.time()
list_ex = list(range(1000000)) #List containinig whole numbers upto 1 million
a=(list_ex[::-2])
print("Time take to retrieve every 2nd element from a list = ", tm.time()-tt)

tt = tm.time()
tuple_ex = tuple(range(1000000)) #tuple containinig whole numbers upto 1 million
a=(tuple_ex[::-2])
print("Time take to retrieve every 2nd element from a tuple = ", tm.time()-tt)
```

```
Time take to retrieve every 2nd element from a list = 0.03579902648925781
Time take to retrieve every 2nd element from a tuple = 0.02684164047241211
```

## 5.3 Dictionary

A dictionary consists of key-value pairs, where the keys and values are python objects. While values can be any python object, keys need to be immutable python objects, like strings, integers, tuples, etc. Thus, a list can be a value, but not a key, as elements of list can be changed. A dictionary is defined using the keyword `dict` along with curly braces, colons to separate keys and values, and commas to separate elements of a dictionary:

```
dict_example = {'USA': 'Joe Biden', 'India': 'Narendra Modi', 'China': 'Xi Jinping'}
```

Elements of a dictionary can be retrieved by using the corresponding key.

```
dict_example['India']
```

```
'Narendra Modi'
```

### 5.3.1 Adding and removing elements in a dictionary

New elements can be added to a dictionary by defining a key in square brackets and assigning it to a value:

```
dict_example['Japan'] = 'Fumio Kishida'
dict_example['Countries'] = 4
dict_example
```

```
{'USA': 'Joe Biden',
 'India': 'Narendra Modi',
 'China': 'Xi Jinping',
 'Japan': 'Fumio Kishida',
 'Countries': 4}
```

Elements can be removed from the dictionary using the `del` method or the `pop` method:

```
#Removing the element having key as 'Countries'
del dict_example['Countries']
```

```
dict_example
```

```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Japan': 'Fumio Kishida'}
```

```
#Removing the element having key as 'USA'  
dict_example.pop('USA')
```

```
'Joe Biden'
```

```
dict_example
```

```
{'India': 'Narendra Modi', 'China': 'Xi Jinping', 'Japan': 'Fumio Kishida'}
```

New elements can be added, and values of existing keys can be changed using the `update` method:

```
dict_example = {'USA': 'Joe Biden', 'India': 'Narendra Modi', 'China': 'Xi Jinping', 'Countries': 3}  
dict_example
```

```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Countries': 3}
```

```
dict_example.update({'Countries': 4, 'Japan': 'Fumio Kishida'})
```

```
dict_example
```

```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Countries': 4,  
 'Japan': 'Fumio Kishida'}
```

### 5.3.2 Iterating over elements of a dictionary

The `items()` attribute of a dictionary can be used to iterate over elements of a dictionary.

```
for key,value in dict_example.items():  
    print("The Head of State of",key,"is",value)
```

```
The Head of State of USA is Joe Biden  
The Head of State of India is Narendra Modi  
The Head of State of China is Xi Jinping  
The Head of State of Countries is 4  
The Head of State of Japan is Fumio Kishida
```

### 5.3.3 Practice exercise 5

The GDP per capita of USA for most years from 1960 to 2021 is given by the dictionary D given in the code cell below.

Find:

1. The GDP per capita in 2015
2. The GDP per capita of 2014 is missing. Update the dictionary to include the GDP per capita of 2014 as the average of the GDP per capita of 2013 and 2015.
3. Impute the GDP per capita of other missing years in the same manner as in (2), i.e., as the average GDP per capita of the previous year and the next year. Note that the GDP per capita is not missing for any two consecutive years.
4. Print the years and the imputed GDP per capita for the years having a missing value of GDP per capita in (3).

```
D = {'1960':3007,'1961':3067,'1962':3244,'1963':3375,'1964':3574,'1965':3828,'1966':4146,'
```

**Solution:**

```
print("GDP per capita in 2015 =", D['2015'])  
D['2014'] = (D['2013']+D['2015'])/2  
  
#Iterating over all years from 1960 to 2021  
for i in range(1960,2021):  
  
    #Imputing the GDP of the year if it is missing  
    if str(i) not in D.keys():  
        D[str(i)] = (D[str(i-1)]+D[str(i+1)])/2
```

```
print("Imputed GDP per capita for the year",i,"is $",D[str(i)])
```

```
GDP per capita in 2015 = 56763
Imputed GDP per capita for the year 1969 is $ 4965.0
Imputed GDP per capita for the year 1977 is $ 9578.5
Imputed GDP per capita for the year 1999 is $ 34592.0
```

## 5.4 Functions

If an algorithm or block of code is being used several times in a code, then it can be separately defined as a function. This makes the code more organized and readable. For example, let us define a function that prints prime numbers between **a** and **b**, and returns the number of prime numbers found.

```
#Function definition
def prime_numbers (a,b=100):
    num_prime_nos = 0

    #Iterating over all numbers between a and b
    for i in range(a,b):
        num_divisors=0

        #Checking if the ith number has any factors
        for j in range(2, i):
            if i%j == 0:
                num_divisors=1;break;

        #If there are no factors, then printing and counting the number as prime
        if num_divisors==0:
            print(i)
            num_prime_nos = num_prime_nos+1

    #Return count of the number of prime numbers
    return num_prime_nos
```

In the above function, the keyword **def** is used to define the function, **prime\_numbers** is the name of the function, **a** and **b** are the arguments that the function uses to compute the output.

Let us use the defined function to print and count the prime numbers between 40 and 60.



```
#Printing prime numbers between 40 and 60
num_prime_nos_found = prime_numbers(40,60)
```

```
41
43
47
53
59
```

```
num_prime_nos_found
```

```
5
```

If the user calls the function without specifying the value of the argument **b**, then it will take the default value of 100, as mentioned in the function definition. However, for the argument **a**, the user will need to specify a value, as there is no value defined as a default value in the function definition.

#### 5.4.1 Global and local variables with respect to a function

A variable defined within a function is local to that function, while a variable defined outside the function is global to that function. In case a variable with the same name is defined both outside and inside a function, it will refer to its global value outside the function and local value within the function.

The example below shows a variable with the name **var** referring to its local value when called within the function, and global value when called outside the function.

```
var = 5
def sample_function(var):
    print("Local value of 'var' within 'sample_function()' = ",var)

sample_function(4)
print("Global value of 'var' outside 'sample_function()' = ",var)
```

```
Local value of 'var' within 'sample_function()' = 4
Global value of 'var' outside 'sample_function()' = 5
```

### 5.4.2 Practice exercise 6

The object `deck` defined below corresponds to a deck of cards. Estimate the probability that a five card hand will be a [flush](#), as follows:

1. Write a function that accepts a hand of 5 cards as argument, and returns whether the hand is a flush or not.
2. Randomly pull a hand of 5 cards from the deck. Call the function developed in (1) to determine if the hand is a flush.
3. Repeat (2) 10,000 times.
4. Estimate the probability of the hand being a flush from the results of the 10,000 simulations.

You may use the function [shuffle\(\)](#) from the `random` library to shuffle the deck everytime before pulling a hand of 5 cards.

```
deck = [{'value':i, 'suit':c}
for c in ['spades', 'clubs', 'hearts', 'diamonds']
for i in range(2,15)]
```

**Solution:**

```
import random as rm

#Function to check if a 5-card hand is a flush
def chk_flush(hands):

    #Assuming that the hand is a flush, before checking the cards
    yes_flush = 1

    #Storing the suit of the first card in 'first_suit'
    first_suit = hands[0]['suit']

    #Iterating over the remaining 4 cards of the hand
    for j in range(1,len(hands)):

        #If the suit of any of the cards does not match the suit of the first card, the ha
        if first_suit!=hands[j]['suit']:
            yes_flush = 0;

        #As soon as a card with a different suit is found, the hand is not a flush and
        break;
```

```

    return yes_flush

flush=0
for i in range(10000):

    #Shuffling the deck
    rm.shuffle(deck)

    #Picking out the first 5 cards of the deck as a hand and checking if they are a flush
    #If the hand is a flush it is counted
    flush=flush+chck_flush(deck[0:5])

print("Probability of obtaining a flush=", 100*(flush/10000),"%")

```

Probability of obtaining a flush= 0.2 %

## 5.5 Practice exercise 7

The code cell below defines an object having the nutrition information of drinks in starbucks. Assume that the manner in which the information is structured is consistent throughout the object.

```
, 'value': 1}, {'starbucks_types_nutrition':{'value':10},'starbucks_types_nutrition':{'value':10}]}]
```

Use the object above to answer the following questions:

### 5.5.1

What is the datatype of the object?

```
print("Datatype=",type(starbucks_drinks_nutrition))
```

```
Datatype= <class 'dict'>
```

### 5.5.1.1

If the object in (1) is a dictionary, what is the datatype of the values of the dictionary?

```
print("Datatype=",type(starbucks_drinks_nutrition[list(starbucks_drinks_nutrition.keys())])
```

Datatype= <class 'list'>

#### 5.5.1.2

If the object in (1) is a dictionary, what is the datatype of the elements within the values of the dictionary?

```
print("Datatype=",type(starbucks_drinks_nutrition[list(starbucks_drinks_nutrition.keys())])
```

Datatype= <class 'dict'>

#### 5.5.1.3

How many calories are there in Iced Coffee?

```
print("Calories = ",starbucks_drinks_nutrition['Iced Coffee'][0]['value'])
```

Calories = 5

#### 5.5.1.4

Which drink(s) have the highest amount of protein in them, and what is that protein amount?

```
#Defining an empty dictionary that will be used to store the protein of each drink
protein={}
```

```
for key,value in starbucks_drinks_nutrition.items():
    for nutrition in value:
        if nutrition['Nutrition_type']=='Protein':
            protein[key]=(nutrition['value'])
```

```
#Using dictionary comprehension to find the key-value pair having the maximum value in the
{key:value for key, value in protein.items() if value == max(protein.values())}
```

```
{'Starbucks® Doubleshot Protein Dark Chocolate': 20,  
 'Starbucks® Doubleshot Protein Vanilla': 20,  
 'Chocolate Smoothie': 20}
```

#### 5.5.1.5

Which drink(s) have a fat content of more than 10g, and what is their fat content?

```
#Defining an empty dictionary that will be used to store the fat of each drink  
fat={}  
for key,value in starbucks_drinks_nutrition.items():  
    for nutrition in value:  
        if nutrition['Nutrition_type']=='Fat':  
            fat[key]=(nutrition['value'])  
  
#Using dictionary comprehension to find the key-value pair having the value more than 10  
{key:value for key, value in fat.items() if value>=10}
```

```
{'Starbucks® Signature Hot Chocolate': 26.0, 'White Chocolate Mocha': 11.0}
```

## 6 Object-Oriented Programming

In Python, everything is an object, which makes it an object-oriented programming language.

Object-oriented programming is the one in which a program is based on *objects*. An object is an independent entity within the program and can cooperatively work with other objects. A program can be made up of one or more objects, which can leverage the functionality and information contained in other objects.

### 6.1 Object

An object consists of two items:

1. **Attributes** - Attributes are the data stored within the object.
2. **Methods** - Methods are the functions defined within the object. Methods can use the object attributes (*or data stored within the object*) as well as accept additional data as arguments.

We have already seen several in-built python objects such as string objects, integer objects, float objects, list objects, tuple objects and dictionary objects, in previous chapters. Each of these objects have attributes and methods associated with them.

For example, consider a *integer* object named as `integer_example`.

```
integer_example = 5
```

The attributes and methods of this *integer* object can be seen by putting a `.` next to its name, and pressing the *tab* key. A dropdown menu consisting of the attributes and methods will appear as shown below.

```
<IPython.core.display.Image object>
```

A list of all attributes and methods associated with an object can be obtained with the `dir()` function. Ignore the ones with underscores - these are used by Python itself. The rest of them can be used to perform operations.

```
#This code is not executed to avoid printing a long list
dir(integer_example)
```

For example, an attribute of `integer_example` is `real`, which contains the real part of the number:

```
integer_example.real
```

5

A example of a method of `integer_example` is `as_integer_ratio()`, which returns a tuple containing the numerator and denominator of the integer when it is expressed as a fraction.

```
integer_example.as_integer_ratio()
```

(5, 1)

Note that attributes do not have any parenthesis after them as they are just data, and cannot accept arguments. On the other hand methods have parenthesis after them as they are functions that may or may not have arguments.

## 6.2 Class

A *class* is a template for objects. It contains the attributes and methods associated with the object of the class. As an analogy, the *class* `Cat` will consist of characteristics (or *attributes*) shared by all cats such as breed, fur color, etc., as well as capability to perform functions (or *methods*) such as run, meow, etc.

**Instance:** An *instance* is a specific realization of the object of a particular class. Continuing with the `Cat` analogy of a class, a particular cat is an *instance* of the class `Cat`. Similarly, in the example above, the object `integer_example` is an instance of the class `integer`. The words *object* and *instance* are often used interchangeably.

Creating an *instance* of a class is called **Instantiation**.

### 6.2.1 Creating your own class

Until now we saw examples of in-built Python classes, such as *integer*, *List*, etc. Now, we'll learn to create our own class that serves our purpose.

Below is a toy example of a class.

```
class ToyClass:
    def __init__(self, x, y):
        self.x = x
        self.y = y

    def add(self):
        return self.x + self.y

    def multiply(self):
        return self.x*self.y
```

We'll use the example above to explain the following terms:

- **The class statement:** We use the `class` statement to create a class. The [Python style guide](#) recommends to use CamelCase for class names.
- **The constructor (or the `__init__()` method):** A class typically has a method called `__init__`. This method is called a constructor and is automatically called when an object or instance of the class is created. The constructor initializes the attributes of the class. In the above example, the constructor accepts two values as arguments, and initializes its attributes `x` and `y` with those values.
- **The `self` argument:** This is the first argument to every method in the class. Whenever the class refers to one of its attributes or methods, it must precede them by `self`. The purpose of `self` is to distinguish the class's attributes and methods from other variables and functions in the program.

The class `ToyClass` consists of two attributes `x` and `y`, a constructor `__init__()`, and two methods `add()` and `multiply()`.

To create an *object* or *instance* of the class `ToyClass`, we'll use the class name with the values to be passed as argument to the constructor for initializing the *object* / *instance*.

```
toy_instance = ToyClass(6,12)
```

The `x` *attribute* of the class `ToyClass` can be called using the `.` operator with the object name:



```
toy_instance.x
```

6

To use the `multiply()` *method* of the class `ToyClass`, we'll use the `.` operator with the object name:

```
toy_instance.multiply()
```

72

## 6.2.2 Example: A class that analyzes a string

Let us create a class that analyzes a string.

```
class AnalyzeString:

    #Constructor
    def __init__(self, s):
        s = s.lower()
        self.words = s.split()

    #This method counts the numebr of words
    def number_of_words(self):
        return (len(self.words))

    #This method counts the number of words starting with the string s
    def starts_with(self,s):
        return len([x for x in self.words if x[:len(s)]==s])

    #This method counts the number of words of length n
    def words_with_length(self,n):
        return len([x for x in self.words if len(x)==n])

    #This method returns the frequency of the word w
    def word_frequency(self,w):
        return self.words.count(w)
```

Let us create an instance of the class `AnalyzeString()` to analyze a sentence.

```
#Defining a string
sentence = 'This sentence in an example of a string that we will analyse using a class we

#Creating an instance of class AnalyzeString()
sentence_analysis = AnalyzeString(sentence)

#The attribute 'word' contains the list of words in the sentence
sentence_analysis.words
```

```
['this',
 'sentence',
 'in',
 'an',
 'example',
 'of',
 'a',
 'string',
 'that',
 'we',
 'will',
 'analyse',
 'using',
 'a',
 'class',
 'we',
 'have',
 'defined']
```

```
#The method 'word_frequency()' provides the frequency of a word in the sentence
sentence_analysis.word_frequency('we')
```

2

```
#The method 'starts_with()' provides the frequency of number of words starting with a part
sentence_analysis.starts_with('th')
```

2

### 6.2.3 Practice exercise 1

Write a class called `PasswordManager`. The class should have a list called `old_passwords` that holds all of the user's past passwords. The last item of the list is the user's current password. There should be a method called `get_password` that returns the current password and a method called `set_password` that sets the user's password. The `set_password` method should only change the password if the attempted password is different from all the user's past passwords. It should either print *'Password changed successfully!'*, or *'Old password cannot be reused, try again.'* Finally, create a method called `is_correct` that receives a string and returns a boolean `True` or `False` depending on whether the string is equal to the current password or not.

To initialize the object of the class, use the list below.

After defining the class:

1. Check the attribute `old_passwords`
2. Check the method `get_password()`
3. Try re-setting the password to `'ibiza1972'`, and then check the current password.
4. Try re-setting the password to `'oktoberfest2022'`, and then check the current password.
5. Check the `is_correct()` method

```
past_passwords = ['titanic1911','ibiza1972','montecarlo799']
```

```
class PasswordManager:
    def __init__(self,past_passwords):
        self.old_passwords = past_passwords

    def get_password(self):
        return self.old_passwords[len(self.old_passwords)-1]

    def set_password(self,new_password):
        if new_password not in self.old_passwords:
            self.old_passwords.append(new_password)
            print("Password changed!")
        else:
            print("Old password cannot be reused, try again.")

    def is_correct(self,password):
        if password == self.old_passwords[len(self.old_passwords)-1]:
            return True
```

```
        return False

passwd = PasswordManager(past_passwords)
```

## 6.3 Inheritance

In object-oriented programming there is a concept called *inheritance* where we can create a new class that builds off of another class. The new class gets all of the variables and methods of the class it is inheriting from (called the base class). It can then define additional variables and methods that are not present in the base class, and it can also override some of the methods of the base class. That is, it can rewrite them to suit its own purposes. Here is a simple example:

```
class Parent:

    def __init__(self, a, b):
        self.a = a

    def method1(self):
        return self.a+' should study!'

    def method2(self):
        return self.a+' does not study enough '

class Child(Parent):

    def __init__(self, a,b):
        self.a = a
        self.b = b

    def method1(self):
        return self.a+' should play with ' + self.b

    def method3(self):
        return self.a + ' does not play enough'
```

Note that when inheriting from a class, we indicate the parent class in parentheses in the `class` statement.

We see that `method1` is present in both the `Parent` and `Child` classes, while `method2` is only present in the `Parent` class. Let us understand how does the `Child` class use the methods of

the Parent class, and what happens if a method with the same name is present in both the parent and child classes.

```
p = Parent('Sam', 'John')
c = Child('Sam', 'Pam')
print('Parent method 1: ', p.method1())
print('Parent method 2: ', p.method2())
print()
print('Child method 1: ', c.method1())
print('Child method 2: ', c.method2())
print('Child method 3: ', c.method3())
```

```
Parent method 1: Sam should study!
Parent method 2: Sam does not study enough
```

```
Child method 1: Sam should play with Pam
Child method 2: Sam does not study enough
Child method 3: Sam does not play enough
```

We see in the example above that the child has overridden the parent's `method1`. The child has inherited the parent's `method2`, so it can use it without having to define it. The child also adds some features to the parent class, namely a new variable `b` and a new method, `method3`.

### 6.3.1 Practice exercise 2

Define a class that inherits the in-built Python class `list`, and adds a new method to the class called `nunique()` which returns the number of unique elements in the list.

Define the following list as an object of the class you created. Then:

1. Find the number of unique elements in the object using the method `nunique()` of the inherited class.
2. Check if the `pop()` method of the parent class works to pop an element out of the object.

```
list_ex = [1,2,5,3,6,5,5,5,12]

class list_v2(list):
    def nunique(self):
        unique_elements = []
        for x in self:
            if x not in unique_elements:
```

```

        unique_elements.append(x)
    return len(unique_elements)

list_ex = list_v2(list_ex)
print("Number of unique elements = ", list_ex.nunique())
print("Checking the pop() method, the popped out element is", list_ex.pop())

```

Number of unique elements = 6  
 Checking the pop() method, the popped out element is 12

### 6.3.2 Practice exercise 3

Define a class named `PasswordManagerUpdated` that inherits the class `PasswordManager` defined in Practice exercise 1. The class `PasswordManagerUpdated` should have two methods, other than the *constructor*:

1. The method `set_password()` that sets a new password. The new password must only be accepted if it does not have any punctuations in it, and if it is not the same as one of the old passwords. If the new password is not acceptable, then one of the appropriate messages should be printed - (a) *Cannot have punctuation in password, try again*, or (b) *Old password cannot be reused, try again*.
2. The method `suggest_password()` that randomly sets and returns a password as a string comprising of 15 randomly chosen letters. Letters may be repeated as well.

```

from string import punctuation
import random as rm
import string as st
class PasswordManager_updated(PasswordManager):
    def __init__(self, past_passwords):
        self.old_passwords = past_passwords.copy()

    def set_password(self, new_password):
        if new_password not in self.old_passwords:
            for punc in punctuation:
                if punc in new_password:
                    print("Cannot have punctuation in password, try again")
                    break
            self.old_passwords.append(new_password)
            print("password changed!")
        else:

```

```
        print("Old password cannot be reused, try again.")

def suggest_password(self):
    suggested_passwd = ''
    for i in range(15):
        suggested_passwd = suggested_passwd + st.ascii_lowercase[rm.randint(0,26)]
    self.old_passwords.append(suggested_passwd)
    return suggested_passwd
```

## Part II

### R



## 7 R: Variables, expression and statements

### 7.1 Data Types

Some of the commonly used objects in R are numbers - integer and double (or numeric), character and logical (TRUE/FALSE). The data type of the object can be identified using the in-built R function `class()` or `typeof()`. For example, see the following objects and their types:

```
class(4)
```

```
[1] "numeric"
```

```
typeof(4)
```

```
[1] "double"
```

```
class(4.4)
```

```
[1] "numeric"
```

```
typeof(4.4)
```

```
[1] "double"
```

```
class(4L)
```

```
[1] "integer"
```

```
typeof(4L)
```

```
[1] "integer"
```

```
class('4')
```

```
[1] "character"
```

```
typeof('4')
```

```
[1] "character"
```

```
class(TRUE)
```

```
[1] "logical"
```

```
typeof(FALSE)
```

```
[1] "logical"
```

## 7.2 Variable names

We have the following rules for a R variable name:

- A variable name must start with a letter and can be a combination of letters, digits, period(.) and underscore(\_). If it starts with period(.), it cannot be followed by a digit.
- A variable name cannot start with a number or underscore (\_)
- Variable names are case-sensitive (age, Age and AGE are three different variables)
- Reserved words cannot be used as variables (TRUE, FALSE, NULL, if...)

## 7.3 Converting datatypes

Sometimes a value may have a datatype that is not suitable for using it. For example, consider the variable called `annual_income` in the code below:

```
annual_income = "80000"
```

Suppose we wish to divide `annual_income` by 12 to get the monthly income. We cannot use the variable `annual_income` directly as its datatype is a string and not a number. Thus, numerical operations cannot be performed on the variable `annual_income`.

We'll need to convert `annual_income` to an integer. For that we will use the R's in-built `as.integer()` function:

```
annual_income = as.integer(annual_income)
monthly_income = annual_income/12
print(paste0("monthly income = ", monthly_income))
```

```
[1] "monthly income = 6666.66666666667"
```

Similarly, datatypes can be converted from one type to another using in-built R functions as shown below:

```
#Converting integer to character
as.character(9)
```

```
[1] "9"
```

```
#Converting character to numeric
as.numeric('9.4')
```

```
[1] 9.4
```

```
#Converting logical to integer
as.numeric(FALSE)
```

```
[1] 0
```

Sometimes, conversion of a value may not be possible. For example, it is not possible to convert the variable `greeting` defined below to a number:

```
greeting = "hello"
```

However, strings can be concatenated using the `paste0()` function:

```
paste0("hello", " there!")
```

```
[1] "hello there!"
```

R's in-built `readline()` function can be used to accept an input from the user. For example, suppose we wish the user to input their age:

```
age = readline("Enter your age:")
```

Enter your age:

The entered value is stored in the variable `age` and can be used for computation.

## 8 R: Control flow statements

### 8.1 The if - else if - else statement

The if - else if - else statements can check several conditions, and execute the code corresponding to the condition that is true. Note that there can be as many **else if** statements as required.

**Syntax:** R uses curly braces {} to identify the code to be executed if a condition is true. All the code in the curly braces within a condition is executed if the condition is true.

**Example:** Input a number. Print whether it is positive, negative or zero. If it is negative, print its absolute value.

```
number = readline("Enter a number:")
```

Enter a number:

```
number = '3'

number_integer = as.integer(number)
if(number_integer>0)
{
  print("Number is positive")
}else if(number_integer==0)
{
  print("Number is zero")
}else
{
  print("Number is negative")
}
```

```
[1] "Number is positive"
```

## 8.2 TryCatch

If we suspect that some lines of code may produce an error or a warning, we can put them in a `tryCatch()` statement, and if an error does occur, we can use the *warning* or the *error* argument to instead execute an alternative piece of code. Both the *warning* and *error* arguments have a function that is executed in case of warnings and errors respectively. The argument to this function is the warning / error message. The program will direct the code to the relevant function if an error or warning occurs.

**Example:** Input an integer from the user. If the user inputs a valid integer, print whether it is a multiple of 3. However, if the user does not input a valid integer, print a message saying that the input is invalid.

```
num = readline("Enter a number:")
```

Enter a number:

```
num = '3r'

tryCatch(
  { num_int = as.integer(num) ;
    if(num_int%%3==0)
    {
      print("Number is a multiple of 3")
    }else{
      print("Number is not a multiple of 3")
    }
  },warning = function(w) {print("Input must be an integer")})
```

```
[1] "Input must be an integer"
```

## 8.3 Loops

### 8.3.1 for loop

We use the `:` operator to define a vector of consecutive integers. For example, the sequence of integers from 1 to 10 can be generated with the code `1:10`. Usually, we generate a sequence in this manner to iterate over the sequence with a `for` loop.

**Example:** Print the first  $n$  elements of the Fibonacci sequence, where  $n$  is an integer input by the user, such that  $n > 2$ . In a fibonacci sequence, each number is the sum of the preceding two numbers, and the sequence starts from 0,1. The sequence is as follows:

0, 1, 1, 2, 3, 5, 8, 13, ....

```
n = readline("Enter a number:")
```

Enter a number:

```
n = 6
```

```
#Initializing the sequence to start from 0, 1
n1=0;n2=1
```

```
#Printing the first two numbers of the sequence
elements<-c(n1,n2)
```

```
for(i in 1:(n-2)) #Since two numbers of the sequence are already printed,n-2 numbers are
{
  #Computing the next number of the sequence as the summation of the previous two numbers
  n3 = n1+n2
  elements<-c(elements,n3)

  #As 'n3' is already printed, it is no longer the next number of the sequence.
  #Thus, we move the values of the variables n1 and n2 one place to the right to compute t
  n1 = n2
  n2 = n3
}
print(elements)
```

```
[1] 0 1 1 2 3 5
```

```
print(paste0("These are the first ", n, " elements of the fibonacci series"))
```

```
[1] "These are the first 6 elements of the fibonacci series"
```

### 8.3.2 while loop

**Example:** Print all the elements of the Fibonacci sequence less than  $n$ , where  $n$  is an integer input by the user, such that  $n > 2$ . In a fibonacci sequence, each number is the sum of the preceding two numbers, and the sequence starts from 0,1. The sequence is as follows:

0,1,1,2,3,5,8,13,..

```
n = readline("Enter a number:")
```

Enter a number:

```
n = 50
```

```
#Initializing the sequence to start from 0, 1  
n1=0;n2=1
```

```
#Printing the first number of the sequence  
elements<-n1  
while(n2<n)  
{
```

```
  #Print the next number of the sequence  
  elements<-c(elements,n2)
```

```
  #Computing the next number of the sequence as the summation of the previous two numbers  
  n3 = n1+n2
```

```
  #As n2 is already printed, assigning n2 to n3, so that the next number of the sequence (n3)  
  #Assigning n1 to n2 as n1 has already been used to compute the next number of the sequence  
  n1 = n2  
  n2 = n3
```

```
}  
print(elements)
```

```
[1] 0 1 1 2 3 5 8 13 21 34
```

```
print(paste("These are all the elements of the fibonacci series less than", n))
```

```
[1] "These are all the elements of the fibonacci series less than 50"
```



## 8.4 break statement

The `break` statement is used to unconditionally exit the innermost loop.

For example, suppose we need to keep asking the user to input year of birth and compute the corresponding age, until the user enters 1900 as the year of birth.

```
#The loop will continue to run indefinitely as the condition 'True' is always true
while(TRUE)
{
  year = (readline("Enter year of birth:"))
  year = as.integer(year)
  if(year==1900){break}else{print(paste("Age = ",2022-year))}
}
```

## 8.5 next statement

The `next` statement is used to continue with the next iteration of the loop without executing the lines of code below it.

For example, consider the following code:

```
for(i in 1:10)
{
  if(i%%2==0){next}
  print(i)
}
```

```
[1] 1
[1] 3
[1] 5
[1] 7
[1] 9
```

When the control flow reads the statement `next`, it goes back to the beginning of the `for` loop, and ignores the lines of code below the statement.

### 8.5.1 Practice exercise

Write a program that asks the user the question, “How many stars are in the Milky Way (in billions)?”. If the user answers 100, the program should print correct, and stop. However, if the user answers incorrectly, the program should print “incorrect”, and ask them if they want to try again. The program should continue to run until the user answers correctly, or they want to stop trying.

## 8.6 Loops with the character vector

Loops can be used to iterate over the elements in a **character** vector, just like we used them to iterate over a sequence of integers. The function **nchar** is used to get the number of characters in the **character** vector/

Consider the following **character** vector:

```
sentence = 'She sells sea shells on the sea shore'
```

The length of the **character** vector is:

```
nchar(sentence)
```

```
[1] 37
```

**Slicing a character data:** The function **substr()** is used to slice a character vector from the **start** index to the **stop** index. Note that the indices in R start from 1, instead of 0. For example, let us slice the object **sentence** from the 5<sup>th</sup> index to the 9<sup>th</sup> index:

```
substr(sentence,5,9)
```

```
[1] "sells"
```

**Example:** Define a **character** vector and count the number of *ts*.

```
char_vec = 'Getting a tatto is not a nice experience'
#Initializing a variable 'count_t' which will store the number of 't's in the string
count_t = 0

#Iterating over the entire length of the string.
#The length of the string is given by the len() function
```

```
for(i in 1:nchar(char_vec))
{
  if(substr(char_vec,i,i)=='t')
  {
    count_t<-count_t+1
  }
}
print(paste("Number of 't's in the string = ",count_t))
```

```
[1] "Number of 't's in the string = 6"
```

### 8.6.1 Practice exercise

Write a program that prints the number of 'the's found in sentence

## 9 R: Functions

### 9.1 Defining a function

Functions in R are defined using the keyword `function()`. All the statements within a function are enclosed with `{}` braces. Look at the function defined below. It takes an integer as an argument, and prints whether the integer is odd or even.

```
odd_even<-function(intgr)
{
  if(intgr%%2==0)
  {
    print("even")}else{
      print("odd")
    }
}

odd_even(3)
```

```
[1] "odd"
```

#### 9.1.1 Practice exercise

Write a function that returns all prime numbers between  $a$  and  $b$ , where  $a$  and  $b$  are parameters of the function.

```
prime<-function(a,b)
{
  prime_numbers<-c()
  for(number in a:b)
  {
    prime = 1
    for(factor in 2:(number-1))
    {
      if(number%%factor==0){
```

```
        prime = 0
    }
}
if(prime==1){prime_numbers<-c(prime_numbers,number)}
}
return(prime_numbers)
}
prime(40,60)
```

```
[1] 41 43 47 53 59
```

# 10 R: Objects

## 10.1 Atomic vectors

An atomic vector in R is a vector containing objects of the same datatype. If the objects are not of the same datatype, then they are coerced to be of the same datatype. It is defined using the keyword `c()`.

```
numbers = c(1,2,67)
```

The in-built R function `length()` is used to find the length of an atomic vector.

```
length(numbers)
```

```
[1] 3
```

### 10.1.1 Slicing the atomic vector

An atomic vector can be sliced using the indices of the elements within `[]` brackets.

For example, consider the vector:

```
vec<-1:40
```

Suppose, we wish to get the  $3^{rd}$  element of the vector. We can get it using the index 3:

```
vec[3]
```

```
[1] 3
```

A sequence of consecutive elements can be sliced using the indices of the first element and the last element around the `:` operator. For example, let us slice elements from the  $3^{rd}$  index to the  $10^{th}$  element of the vector `vec`:

```
vec[3:10]
```

```
[1] 3 4 5 6 7 8 9 10
```

We can slice elements at different indices by putting the indices in an atomic vector within the `[]` brackets. Let us slice the 4<sup>th</sup>, 7<sup>th</sup>, and 18<sup>th</sup> elements of the vector `vec`:

```
vec[c(4,7,18)]
```

```
[1] 4 7 18
```

We can slice consecutive elements, and non-consecutive elements simultaneously. Let us slice the elements from the 4<sup>th</sup> index to the 9<sup>th</sup> index and the 30<sup>th</sup> and 36<sup>th</sup> element.

```
vec[c(4:9,30,36)]
```

```
[1] 4 5 6 7 8 9 30 36
```

### 10.1.2 Removing elements from atomic vector

Elements can be removed from the vector using the negative sign within `[]` brackets.

Remove the 2nd element from the vector:

```
vec<-1:5  
vec[-2]
```

```
[1] 1 3 4 5
```

If multiple elements need to be removed, the indices of the elements to be removed can be given as an atomic vector.

Remove elements 2 to 6 and element 10 from the vector:

```
vec<-1:20  
vec[-c(2:6,10)]
```

```
[1] 1 7 8 9 11 12 13 14 15 16 17 18 19 20
```

**Example:** USA's GDP per capita from 1960 to 2021 is given by the vector `G` in the code chunk below. The values are arranged in ascending order of the year, i.e., the first value is for 1960, the second value is for 1961, and so on. Store the years in which the GDP per capita of the US increased by more than 10%, in a vector.

```
G = c(3007, 3067, 3244, 3375, 3574, 3828, 4146, 4336, 4696, 5032, 5234, 5609, 6094, 6726, 7226, 7500, 7700, 7900, 8100, 8300, 8500, 8700, 8900, 9100, 9300, 9500, 9700, 9900, 10100, 10300, 10500, 10700, 10900, 11100, 11300, 11500, 11700, 11900, 12100, 12300, 12500, 12700, 12900, 13100, 13300, 13500, 13700, 13900, 14100, 14300, 14500, 14700, 14900, 15100, 15300, 15500, 15700, 15900, 16100, 16300, 16500, 16700, 16900, 17100, 17300, 17500, 17700, 17900, 18100, 18300, 18500, 18700, 18900, 19100, 19300, 19500, 19700, 19900, 20100, 20300, 20500, 20700, 20900, 21100, 21300, 21500, 21700, 21900, 22100, 22300, 22500, 22700, 22900, 23100, 23300, 23500, 23700, 23900, 24100, 24300, 24500, 24700, 24900, 25100, 25300, 25500, 25700, 25900, 26100, 26300, 26500, 26700, 26900, 27100, 27300, 27500, 27700, 27900, 28100, 28300, 28500, 28700, 28900, 29100, 29300, 29500, 29700, 29900, 30100, 30300, 30500, 30700, 30900, 31100, 31300, 31500, 31700, 31900, 32100, 32300, 32500, 32700, 32900, 33100, 33300, 33500, 33700, 33900, 34100, 34300, 34500, 34700, 34900, 35100, 35300, 35500, 35700, 35900, 36100, 36300, 36500, 36700, 36900, 37100, 37300, 37500, 37700, 37900, 38100, 38300, 38500, 38700, 38900, 39100, 39300, 39500, 39700, 39900, 40100, 40300, 40500, 40700, 40900, 41100, 41300, 41500, 41700, 41900, 42100, 42300, 42500, 42700, 42900, 43100, 43300, 43500, 43700, 43900, 44100, 44300, 44500, 44700, 44900, 45100, 45300, 45500, 45700, 45900, 46100, 46300, 46500, 46700, 46900, 47100, 47300, 47500, 47700, 47900, 48100, 48300, 48500, 48700, 48900, 49100, 49300, 49500, 49700, 49900, 50100, 50300, 50500, 50700, 50900, 51100, 51300, 51500, 51700, 51900, 52100, 52300, 52500, 52700, 52900, 53100, 53300, 53500, 53700, 53900, 54100, 54300, 54500, 54700, 54900, 55100, 55300, 55500, 55700, 55900, 56100, 56300, 56500, 56700, 56900, 57100, 57300, 57500, 57700, 57900, 58100, 58300, 58500, 58700, 58900, 59100, 59300, 59500, 59700, 59900, 60100, 60300, 60500, 60700, 60900, 61100, 61300, 61500, 61700, 61900, 62100, 62300, 62500, 62700, 62900, 63100, 63300, 63500, 63700, 63900, 64100, 64300, 64500, 64700, 64900, 65100, 65300, 65500, 65700, 65900, 66100, 66300, 66500, 66700, 66900, 67100, 67300, 67500, 67700, 67900, 68100, 68300, 68500, 68700, 68900, 69100, 69300, 69500, 69700, 69900, 70100, 70300, 70500, 70700, 70900, 71100, 71300, 71500, 71700, 71900, 72100, 72300, 72500, 72700, 72900, 73100, 73300, 73500, 73700, 73900, 74100, 74300, 74500, 74700, 74900, 75100, 75300, 75500, 75700, 75900, 76100, 76300, 76500, 76700, 76900, 77100, 77300, 77500, 77700, 77900, 78100, 78300, 78500, 78700, 78900, 79100, 79300, 79500, 79700, 79900, 80100, 80300, 80500, 80700, 80900, 81100, 81300, 81500, 81700, 81900, 82100, 82300, 82500, 82700, 82900, 83100, 83300, 83500, 83700, 83900, 84100, 84300, 84500, 84700, 84900, 85100, 85300, 85500, 85700, 85900, 86100, 86300, 86500, 86700, 86900, 87100, 87300, 87500, 87700, 87900, 88100, 88300, 88500, 88700, 88900, 89100, 89300, 89500, 89700, 89900, 90100, 90300, 90500, 90700, 90900, 91100, 91300, 91500, 91700, 91900, 92100, 92300, 92500, 92700, 92900, 93100, 93300, 93500, 93700, 93900, 94100, 94300, 94500, 94700, 94900, 95100, 95300, 95500, 95700, 95900, 96100, 96300, 96500, 96700, 96900, 97100, 97300, 97500, 97700, 97900, 98100, 98300, 98500, 98700, 98900, 99100, 99300, 99500, 99700, 99900, 100100, 100300, 100500, 100700, 100900, 101100, 101300, 101500, 101700, 101900, 102100, 102300, 102500, 102700, 102900, 103100, 103300, 103500, 103700, 103900, 104100, 104300, 104500, 104700, 104900, 105100, 105300, 105500, 105700, 105900, 106100, 106300, 106500, 106700, 106900, 107100, 107300, 107500, 107700, 107900, 108100, 108300, 108500, 108700, 108900, 109100, 109300, 109500, 109700, 109900, 110100, 110300, 110500, 110700, 110900, 111100, 111300, 111500, 111700, 111900, 112100, 112300, 112500, 112700, 112900, 113100, 113300, 113500, 113700, 113900, 114100, 114300, 114500, 114700, 114900, 115100, 115300, 115500, 115700, 115900, 116100, 116300, 116500, 116700, 116900, 117100, 117300, 117500, 117700, 117900, 118100, 118300, 118500, 118700, 118900, 119100, 119300, 119500, 119700, 119900, 120100, 120300, 120500, 120700, 120900, 121100, 121300, 121500, 121700, 121900, 122100, 122300, 122500, 122700, 122900, 123100, 123300, 123500, 123700, 123900, 124100, 124300, 124500, 124700, 124900, 125100, 125300, 125500, 125700, 125900, 126100, 126300, 126500, 126700, 126900, 127100, 127300, 127500, 127700, 127900, 128100, 128300, 128500, 128700, 128900, 129100, 129300, 129500, 129700, 129900, 130100, 130300, 130500, 130700, 130900, 131100, 131300, 131500, 131700, 131900, 132100, 132300, 132500, 132700, 132900, 133100, 133300, 133500, 133700, 133900, 134100, 134300, 134500, 134700, 134900, 135100, 135300, 135500, 135700, 135900, 136100, 136300, 136500, 136700, 136900, 137100, 137300, 137500, 137700, 137900, 138100, 138300, 138500, 138700, 138900, 139100, 139300, 139500, 139700, 139900, 140100, 140300, 140500, 140700, 140900, 141100, 141300, 141500, 141700, 141900, 142100, 142300, 142500, 142700, 142900, 143100, 143300, 143500, 143700, 143900, 144100, 144300, 144500, 144700, 144900, 145100, 145300, 145500, 145700, 145900, 146100, 146300, 146500, 146700, 146900, 147100, 147300, 147500, 147700, 147900, 148100, 148300, 148500, 148700, 148900, 149100, 149300, 149500, 149700, 149900, 150100, 150300, 150500, 150700, 150900, 151100, 151300, 151500, 151700, 151900, 152100, 152300, 152500, 152700, 152900, 153100, 153300, 153500, 153700, 153900, 154100, 154300, 154500, 154700, 154900, 155100, 155300, 155500, 155700, 155900, 156100, 156300, 156500, 156700, 156900, 157100, 157300, 157500, 157700, 157900, 158100, 158300, 158500, 158700, 158900, 159100, 159300, 159500, 159700, 159900, 160100, 160300, 160500, 160700, 160900, 161100, 161300, 161500, 161700, 161900, 162100, 162300, 162500, 162700, 162900, 163100, 163300, 163500, 163700, 163900, 164100, 164300, 164500, 164700, 164900, 165100, 165300, 165500, 165700, 165900, 166100, 166300, 166500, 166700, 166900, 167100, 167300, 167500, 167700, 167900, 168100, 168300, 168500, 168700, 168900, 169100, 169300, 169500, 169700, 169900, 170100, 170300, 170500, 170700, 170900, 171100, 171300, 171500, 171700, 171900, 172100, 172300, 172500, 172700, 172900, 173100, 173300, 173500, 173700, 173900, 174100, 174300, 174500, 174700, 174900, 175100, 175300, 175500, 175700, 175900, 176100, 176300, 176500, 176700, 176900, 177100, 177300, 177500, 177700, 177900, 178100, 178300, 178500, 178700, 178900, 179100, 179300, 179500, 179700, 179900, 180100, 180300, 180500, 180700, 180900, 181100, 181300, 181500, 181700, 181900, 182100, 182300, 182500, 182700, 182900, 183100, 183300, 183500, 183700, 183900, 184100, 184300, 184500, 184700, 184900, 185100, 185300, 185500, 185700, 185900, 186100, 186300, 186500, 186700, 186900, 187100, 187300, 187500, 187700, 187900, 188100, 188300, 188500, 188700, 188900, 189100, 189300, 189500, 189700, 189900, 190100, 190300, 190500, 190700, 190900, 191100, 191300, 191500, 191700, 191900, 192100, 192300, 192500, 192700, 192900, 193100, 193300, 193500, 193700, 193900, 194100, 194300, 194500, 194700, 194900, 195100, 195300, 195500, 195700, 195900, 196100, 196300, 196500, 196700, 196900, 197100, 197300, 197500, 197700, 197900, 198100, 198300, 198500, 198700, 198900, 199100, 199300, 199500, 199700, 199900, 200100, 200300, 200500, 200700, 200900, 201100, 201300, 201500, 201700, 201900, 202100)
```

```
years<-c()
for(i in 1:(length(G)-1))
{
  diff = (G[i+1]-G[i])/G[i]
  if(diff>0.1){years<-c(years,1960+i)}
}
print(years)
```

```
[1] 1973 1976 1977 1978 1979 1981 1984
```

### 10.1.3 The `seq()` function

The `seq()` function is used to generate an atomic vector consisting of a sequence of integers with a constant gap. For example, the code below generates a sequence of integers starting from 20 upto 60 with gaps of 5.

```
seq(20,60,5)
```

```
[1] 20 25 30 35 40 45 50 55 60
```

### 10.1.4 The `rep()` function

The `rep()` function is used to repeat an object a fixed number of times.

```
rep(4,10)
```

```
[1] 4 4 4 4 4 4 4 4 4 4
```

```
rep(c(2,3),10)
```

```
[1] 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3
```



### 10.1.5 The `which()` function

The `which()` function is used to find the index of an object in an atomic vector.

```
vec<-c(8,3,4,7,9,7,5)
```

```
which(vec==8)
```

```
[1] 1
```

The index of the maximum and minimum values can be found using `which.max()` and `which.min()` respectively.

```
which.max(vec)
```

```
[1] 5
```

```
which.min(vec)
```

```
[1] 2
```

### 10.1.6 Practice exercise 1

Below is a vector consisting of responses to the question: “At what age do you think you will marry?” from students of the STAT303-1 Fall 2022 class.

```
exp_marriage_age=c('24','30','28','29','30','27','26','28','30+', '26','28','30','30','30',
```

#### 10.1.6.1 Cleaning data

Remove the elements that are not integers - such as ‘probably never’, ‘30+’, etc. Convert the remaining elements to integer. What is the length of the new vector?

```
new_vector<-as.integer(exp_marriage_age)
```

Warning: NAs introduced by coercion

```
numeric_values<-new_vector[!is.na(new_vector)]  
length(numeric_values)
```

```
[1] 181
```

### 10.1.6.2 Capping unreasonably high values

Cap the values greater than 80 to 80, in the clean vector obtained above. What is the mean age when people expect to marry in the new vector?

```
numeric_values[numeric_values>80]<-80  
mean(numeric_values)
```

```
[1] 28.9558
```

### 10.1.6.3 People marrying at 30 or more

Determine the percentage of people who expect to marry at an age of 30 or more.

```
sum(numeric_values>=30)/length(numeric_values)
```

```
[1] 0.3701657
```

## 10.1.7 The `sapply()` function

The `sapply()` function is used to apply a function on all the elements of a list, atomic vector or matrix.

For example, consider the vector below:

```
vec<-1:6  
vec
```

```
[1] 1 2 3 4 5 6
```

Suppose, we wish to square each element of the vector. We can use the `sapply()` function as below:

```
sapply(vec, FUN = function(x) x**2)
```

```
[1] 1 4 9 16 25 36
```

### 10.1.8 Practice exercise 2

Write a function that identifies if a word is a **palindrome** (*A palindrome is a word that reads the same both backwards and forwards, for example, peep, rotator, madam, etc.*). Apply the function to the vector of words below to count the number of palindrome words.

```
words_vec<-c('fat', 'civic', 'radar', 'mountain', 'noon','papa')
```

```
palindrome<-function(word)
{
  for(i in 1:as.integer(nchar(word)/2))
  {
    if(substr(word,i,i)!=substr(word,nchar(word)-(i-1),nchar(word)-(i-1)))
    {
      return(FALSE)
    }
  }
  return(TRUE)
}
sum(sapply(words_vec,palindrome))
```

```
[1] 3
```

## 10.2 Matrix

Matrices are two-dimensional arrays. The in-built function `matrix()` is used to define a matrix. An atomic vector can be organized as a matrix by specifying the number of rows and columns.

For example, let us define a 2x3 matrix (2 rows and 3 columns) consisting of consecutive integers from 1 to 6.

```
mat<-matrix(1:6,2,3)
mat
```

	[,1]	[,2]	[,3]
[1,]	1	3	5
[2,]	2	4	6

Note that the integers fill up column-wise in the matrix. If we wish to fill-up the matrix by row, we can use the `byrow` argument.

```
mat<-matrix(1:6,2,3, byrow = TRUE)
mat
```

	[,1]	[,2]	[,3]
[1,]	1	2	3
[2,]	4	5	6

The functions `nrow()` and `ncol()` can be used to get the number of rows and columns of the matrix respectively.

```
nrow(mat)
```

```
[1] 2
```

```
ncol(mat)
```

```
[1] 3
```

Matrices can be sliced using the indices of row and column separated by a `,` in box brackets. Suppose we wish to get the element in the  $2^{nd}$  row and  $3^{rd}$  column of the matrix:

```
mat[2,3]
```

```
[1] 6
```

For selecting all rows or columns of a matrix, the index for the row/column can be left blank. Suppose we wish to get all the elements of the  $1^{st}$  of the matrix:

```
mat[1,]
```

```
[1] 1 2 3
```

Row and columns of the matrix can be sliced using the `:` operator. Suppose we want to select a sub-matrix that has elements in the first two rows and columns 2 and 3 of the matrix `mat`:

```
mat[1:2,2:3]
```

```
      [,1] [,2]  
[1,]     2     3  
[2,]     5     6
```

Element-wise arithmetic operations can be performed between 2 matrices of the same shape.

```
mat1<-matrix(1:6,2,3)  
mat2<-matrix(c(9,2,6,5,1,0),2,3)  
mat1+mat2
```

```
      [,1] [,2] [,3]  
[1,]    10     9     6  
[2,]     4     9     6
```

```
mat1-mat2
```

```
      [,1] [,2] [,3]  
[1,]    -8    -3     4  
[2,]     0    -1     6
```

Suppose we need to sum up all the rows of the matrix. We can do it using a `for` loop as follows:

```
row_sum<-c(0,0)  
for(i in 1:nrow(mat))  
{  
  for(j in 1:ncol(mat))  
  {  
    row_sum[i]<-row_sum[i]+mat[i,j]  
  }  
}  
row_sum
```

```
[1]  6 15
```

Observe that in the above `for` loop, elements of each row are added one at a time. We can add all the elements of a row simultaneously using the `sum()` function. This will reduce a `for` loop from the above code:

```
row_sum<-c(0,0)
for(i in 1:nrow(mat))
{
  row_sum[i]<-sum(mat[i,])
}
row_sum
```

```
[1]  6 15
```

In the above code, we sum up all the elements of the row simultaneously. However, we still need to sum up the elements of each row one at a time.

### 10.2.1 The `apply()` function

The `apply()` function can be used to apply a function simultaneously on all rows or columns of a matrix. Thus, this function helps avoid a `for` loop to iterate over all the rows and columns of the matrix. This reduces the execution time of the code since operations are performed in-parallel, instead of one-at-a-time in a `for` loop.

Let us use the `apply()` function to sum up all the rows of the matrix `mat`.

```
apply(mat,1,sum)
```

```
[1]  6 15
```

Let us compare the time taken to sum up rows of a matrix using a `for` loop with the time taken using the `apply()` function.

```
options(digits.secs = 6)
start.time <- Sys.time()
row_sum<-c(0,0)
for(i in 1:nrow(mat))
{
  row_sum[i]<-sum(mat[i,])
}
row_sum
```

```
[1] 6 15
```

```
end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken
```

Time difference of 0.006519794 secs

```
start.time <- Sys.time()
apply(mat,1,sum)
```

```
[1] 6 15
```

```
end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken
```

Time difference of 0.001955271 secs

Observe that the `apply()` function takes much lesser time to sum up all the rows of the matrix as compared to the `for` loop.

Recall the earlier example where we computed year's in which the increase in GDP per capita was more than 10%. Let us use matrices to solve the problem. We'll also compare the time it takes using a matrix with the time it takes using `for` loops.

```
start.time <- Sys.time()

#Let the first column of the matrix be the GDP of all the years except 1960, and the second
GDP_mat<-matrix(c(G[-1],G[-length(G)]),length(G)-1,2)

#The percent increase in GDP can be computed by performing computations using the 2 column
inc<-(GDP_mat[,1]-GDP_mat[,2])/GDP_mat[,2]
years<-1961:2021
years<-years[inc>0.1]
years
```

```
[1] 1973 1976 1977 1978 1979 1981 1984
```

```
end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken
```

Time difference of 0.005506754 secs

Without matrices, the time taken to perform the same computation is measured with the code below.

```
start.time <- Sys.time()
years<-c()
for(i in 1:(length(G)-1))
{
  diff = (G[i+1]-G[i])/G[i]
  if(diff>0.1){years<-c(years,1960+i)}
}
print(years)
```

```
[1] 1973 1976 1977 1978 1979 1981 1984
```

```
#print(proc.time()[3]-start_time)
end.time <- Sys.time()
time.taken <- end.time - start.time
time.taken
```

Time difference of 0.009813547 secs

Observe that matrices reduce the execution time of the code as computations are performed simultaneously, in contrast to a `for` loop where computations are performed one at a time.

Sometimes, the computations on rows / columns of a matrix are not straightforward and we may need to use the `apply()` function to apply a function on each row / column of a matrix.

**Example:** Find the maximum GDP per capita of the US in each of the 5 year periods starting from 1961-1965, and upto 2015-2020.

```
GDP_5year<-matrix(G[-c(1,length(G))],12,5,byrow = TRUE)
GDP_max_5year<-apply(GDP_5year,1,max)
```



In the above code, we applied the in-built function `max` on all the rows. Sometimes, an in-built function may not be available for the computations to be performed. In such as case, we can write our own user-defined function within the `apply()` function. See the example below.

**Example:** Find the range (max-min) of GDP per capita of the US in each of the 5 year periods starting from 1961-1965, and upto 2015-2020.

```
GDP_5year<-matrix(G[-c(1,length(G))],12,5,byrow = TRUE)
GDP_range_5year<-apply(GDP_5year,1,function(x) max(x)-min(x))
GDP_range_5year
```

```
[1] 761 1088 2192 3983 4261 4818 4349 6362 6989 2349 6697 7228
```

In the code above we applied a user-defined function on each row of the matrix. However, if the function has multiple lines, it may be inconvenient to write the function within the `apply()` function. In that case, we can define the function outside the `apply()` function.

**Example:** Find the five year periods starting from 1961-1965, and upto 2016-2020, during which the GDP per capita decreased as compared to the previous year.

```
GDP_inc<-function(GDP_5yr)
{
  dec<-0
  for(i in 1:4)
  {
    if(GDP_5yr[i+1]<GDP_5yr[i]){dec<-1}
  }
  return(dec)
}

GDP_5year_mat<-matrix(G[-c(1,length(G))],12,5,byrow = TRUE)
years_inc_dec<-apply(GDP_5year_mat,1,GDP_inc)
five_year_periods<-seq(1960,2015,5)
print("Five year periods in which the GDP per capita decreased are those starting from the year")
```

```
[1] "Five year periods in which the GDP per capita decreased are those starting from the year"
```

```
print(five_year_periods[years_inc_dec==1]+1)
```

```
[1] 2006 2016
```

The 5 year periods during which the GDP per capita decreased as compared to the previous year are 2006-2010, and 2016-2020.

### 10.2.2 Practice exercise 3

Find the 5 year period in which the difference of the maximum GDP per capita and the minimum GDP per capita as a percentage of the minimum GDP per capita was the highest.

**Solution:**

```
five_year_periods[which.max(apply(GDP_5year_mat,1,function(x) (max(x)-min(x))/min(x)))]+1
```

```
[1] 1976
```

```
print("During 1976-1980 the difference of the maximum GDP per capita and the minimum GDP p
```

```
[1] "During 1976-1980 the difference of the maximum GDP per capita and the minimum GDP per c
```

### 10.2.3 Practice exercise 4

The object `country_names` is an atomic vector consisting of country names. The object `coordinates_capital_cities` is a matrix consisting of the latitude-longitude pair of the capital city of the respective country. The order of countries in `country_names` is the same as the order in which their capital city coordinates (latitude-longitude) appear in the matrix `coordinates_capital_cities`.

Download the file `capital_cities.csv` from [here](#). Make sure the file is in your current working directory. Execute the following code to obtain the objects `coordinates_capital_cities` and `country_names`.

```
capital_cities<-read.csv('capital_cities.csv')
coordinates_capital_cities<-as.matrix(capital_cities[,c(3,4)])
country_names<-capital_cities[,1]
```

### 10.2.3.1 Country with capital closest to DC

Print the name and coordinates of the country with the capital city closest to the US capital - Washington DC.

Note that:

1. The *Country Name* for US is given as *United States* in the data.
2. The 'closeness' of capital cities from the US capital is based on the Euclidean distance of their coordinates to those of the US capital.

**Hint:**

1. Get the coordinates of Washington DC from `coordinates_capital_cities`. The row that contains the coordinates of DC will have the same index as `United States` has in the vector `country_names`
2. Create a matrix that has coordinates of Washington DC in each row, and has the same number of rows as the matrix `coordinates_capital_cities`.
3. Subtract `coordinates_capital_cities` from the matrix created in (2). Element-wise subtraction will occur between the matrices.
4. Use the `apply()` function on the matrix obtained above to find the Euclidean distance of Washington DC from the rest of the capital cities.
5. Using the distances obtained above, find the country that has the closest capital to DC.

### 10.2.3.2 Top 10 countries closest to DC

1. Print the names of the countries of the top 10 capital cities closest to the US capital - Washington DC.
2. Create and print a matrix containing the coordinates of the top 10 capital cities closest to Washington DC.

```
US_index = which(country_names == 'United States')
dc_coord <- coordinates_capital_cities[US_index,]
distances_to_DC <- apply(coordinates_capital_cities, 1, function(x) sqrt(sum((x-dc_coord)*
num_of_countries <- length(country_names)
distances_to_DC_matrix <- cbind(1:num_of_countries, distances_to_DC)
sorted<-distances_to_DC_matrix[order(distances_to_DC_matrix[,2]),]
```

Top 10 countries with capitals closest to Washington DC are the following:

```
country_names[sorted[3:12, 1]]
```

The coordinates of the top 10 capital cities closest to Washington DC are:

```
coordinates_capital_cities[sorted[3:12, 1],]
```

## 10.3 Lists

Atomic vectors and matrices are quite useful in R. However, a constraint with them is that they can only contain objects of the same datatype. For example, an atomic vector can contain all numeric objects, all character objects, or all logical objects, but not a mixture of multiple types of objects. Thus, there arises a need for a `list` data structure that can store objects of multiple datatypes.

A list can be defined using the `list()` function. For example, consider the list below:

```
list_ex<-list(1, "apple", TRUE, list("another list", TRUE))
```

The list `list_ex` consists of objects of multiple datatypes. The length of the list can be obtained using the `length()` function:

```
length(list_ex)
```

```
[1] 4
```

A list is an ordered collection of objects. Each object of the list is associated with an index that corresponds to its order of occurrence in the list.

A single element can be sliced from the list by specifying its index within the `[[ ]]` operator. Let us slice the 2<sup>nd</sup> element of the list `list_ex`:

```
list_ex[[2]]
```

```
[1] "apple"
```

Multiple elements can be sliced from the list by specifying the indices as an atomic vector within the `[ ]` operator. Let us slice the 1<sup>st</sup> and 3<sup>rd</sup> elements from the list `list_ex`:

```
list_ex[c(1,3)]
```

```
[[1]]
[1] 1

[[2]]
[1] TRUE
```

Elements of a list can be named using the `names()` function. Let us name the elements of `list_ex`:

```
names(list_ex)<-c("Name1", "second_name", "3rd_element", "Number 4")
```

A single element can be sliced from the list using the name of the element with the `$` operator. Let us slice the element named as `second_name` from the list `list_ex`:

```
list_ex$second_name
```

```
[1] "apple"
```

Note that if the name of the element does not begin with a letter or has special characters such as a space, then it should be specified within single quotes after the `$` operator. For example, let us slice the element named as `3rd_element` from the list `list_ex`:

```
list_ex$`3rd_element`
```

```
[1] TRUE
```

Names of elements of a list can also be specified while defining the list, as in the example below:

```
list_ex_with_names<-list(movie = 'The Dark Knight', IMDB_rating = 9)
```

A list can be converted to an atomic vector using the `unlist()` function. For example, let us convert the list `list_ex` to a vector:

```
unlist(list_ex)
```

Name1	second_name	3rd_element	Number 41	Number 42
"1"	"apple"	"TRUE"	"another list"	"TRUE"

Since a vector can contain objects of a single datatype, note that all objects have been converted to the `character` datatype in the vector above.

### 10.3.1 Practice exercise 5

Download the dataset *movies.json*. Execute the following code to read the data into the object *movies*:

```
library(rjson)
movies<-fromJSON(file = 'movies.json')
```

#### 10.3.1.1

What is the datatype of the object *movies*?

```
class(movies)
```

The datatype of the object *movies* is list.

#### 10.3.1.2

Count the number movies having a negative profit, i.e., their production budget is higher than their worldwide gross.

Ignore the movies that:

1. Have missing values of production budget or worldwide gross. Use the `is.null()` function to identify missing or NULL values.
2. Have a zero worldwide gross (*A zero worldwide gross is probably an incorrect value*).

```
negative_profit<-c()
count<-0
for(i in 1:length(movies))
{
  pb<-movies[[i]]$`Production Budget`
  wg<-movies[[i]]$`Worldwide Gross`
  if(!(is.null(pb) | is.null(wg)))
  {
    if(pb>wg & wg>0)
    {
      count<-count+1
    }
  }
}
```

```
}
print(paste("Number of movies with negative profit =",count))
```

### 10.3.2 The lapply() function

The `lapply()` function is used to apply a function on each element of a list, and returns a list of the same length.

For example, consider the list below:

```
list_ex<-list(1, "apple", TRUE, list("another list", TRUE))
```

Let us use the `lapply()` function to find the class of each element of the list `list_ex`:

```
lapply(list_ex, function(x) class(x))
```

```
[[1]]
[1] "numeric"

[[2]]
[1] "character"

[[3]]
[1] "logical"

[[4]]
[1] "list"
```

### 10.3.3 Practice exercise 6

Solve [practice exercise 5](#) without using a for loop. Use the `lapply()` function.

```
profit<-lapply(movies, function(x) x$`Worldwide Gross`-x$`Production Budget`)
positive_wg<-lapply(movies, function(x) x$`Worldwide Gross`>0)
sum(profit<0 & positive_wg>0,na.rm = TRUE)
```

# 11 R: Recursion

## 11.1 Recursion

Recursion is a method of solving a problem by dividing it into smaller instances of the same problem. Recursion solves such problems by using functions that call themselves from within their own code. This forms a loop, where every time the function is called, it calls itself again and again. However, every time the function calls itself, it checks certain condition(s) which are the stopping condition(s). When such condition(s) are true the function will stop calling itself. These conditions are called the base case of the recursive function.

Every recursive function must have at least two cases:

- 1. Base case:** This is the simplest case that can be answered directly, and the function does not call itself.
- 2. Recursive case:** This is a relatively more complex case that cannot be answered directly, but can be described as a smaller instance of the same problem. In this case, the function calls itself to answer the smaller problem.

Below is an example, where we defined a function that computes the factorial of an integer by recursion.

```
factorial<-function(n)
{
  if(n==1)      #Base case
  {
    return(1)
  }
  return(n*factorial(n-1)) #Recursive case
}
factorial(5)
```

```
[1] 120
```



In the above example, the case  $n = 1$  is the base case, where the function does not need to call itself, and returns 1. All other cases, where  $n > 1$ , and  $n \in \mathbb{Z}$  are recursive cases, where the function calls itself with a smaller instance of the same problem.

A recursive function must satisfy the following conditions:

1. There must be a case for all valid inputs.
2. There must be a base case that makes no recursive calls.
3. When the function makes a recursive call, it should be to a simpler instance and make forward progress towards the base case.

**Example:** Write a recursive function that returns the  $n^{th}$  term of the Fibonacci sequence, where  $n$  is an integer, and  $n > 0$ . In a Fibonacci sequence, each number is the sum of the preceding two numbers, and the sequence starts from 0, 1. The sequence is as follows:

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

```
fibonacci<-function(n)
{
  if(n==0 | n==1){ #Base case
    return(n)
  }
  return(fibonacci(n-1)+fibonacci(n-2)) #Recursive case
}
#The function `fibonacci` prints the n+1th term of the fibonacci sequence when `n` is passed
nth_term<-function(N)
{
  fibonacci(N-1)
}
nth_term(7)
```

```
[1] 8
```

### 11.1.1 Practice exercise 1

Write a recursive function that computes the sum of squares of the first  $N$  natural numbers, where  $N$  is a parameter to the function.

```
squares<-function(N)
{
  if(N==1) #Base case
  {
```

```

    return(1)
  }else{    #Recursive case
    return(N**2+squares(N-1))
  }
}
squares(10)

```

### 11.1.2 Practice exercise 2

Write a function that counts the occurrence of digit  $k$  in a given integer  $n$  using recursion. The function has  $n$  and  $k$  as parameters.

```

freq_digits<-function(n,d)
{
  if(n==0)
  {
    return(0)
  }
  digit = n%%10
  n_int = as.integer(n/10)
  if(digit==d)
  {
    return(1+freq_digits(n_int,d))
  }
  return(freq_digits(n_int,d))
}
freq_digits(8670800,0)

```

### 11.1.3 Practice exercise 3

Use recursion to write a function that accepts a word as an argument, and returns TRUE if the word is a palindrome, otherwise returns FALSE.

```

word<-'racecar'
palindrome<-function(word)
{
  if(nchar(word)<=1)
  {
    return(TRUE)
  }
}

```

```

    }else if(substr(word,1,1)==substr(word,nchar(word),nchar(word)))
    {
        palindrome(substr(word,2,nchar(word)-1))
    }else{
        return(FALSE)
    }
}
palindrome(word)

```

## 11.2 Recursion vs iteration

Recursion is typically used when the problem is naturally recursive (for e.g., generating a Fibonacci sequence), or the data is naturally recursive (for e.g., filesystem). Recursive solutions can be easy to read and understand as compared to the corresponding iterative solution.

One downside of recursion is that it may take more space than an iterative solution. Building up a stack of recursive calls consumes memory temporarily, and the stack is limited in size, which may become a limit on the size of the problem that the recursive implementation can solve.

# A Assignment templates and Datasets

Assignment templates and datasets used in the book can be found [here](#)