

Data Science III with python (Class notes)

STAT 303-3

Arvind Krishna, Emre Besler, and Lizhen Shi

3/24/23

Table of contents

Preface	3
I Linear regression	4
1 Introduction to Scikit-learn	5
2 Regression splines	11
2.1 Polynomial regression vs Regression splines	12
2.1.1 Model of degree 1	12
2.1.2 Model of degree 2	14
2.1.3 Model of degree 3	15
2.2 Regression splines with knots at uniform quantiles of data	17
2.3 Natural cubic splines	18
2.4 Generalized additive model (GAM)	20
2.5 MARS (Multivariate Adaptive Regression Splines)	23
2.5.1 MARS of degree 1	23
2.5.2 MARS of degree 2	25
2.5.3 MARS including categorical variables	26
Appendices	31
A Datasets, assignment and project files	32
References	33

Preface

These are class notes for the course STAT303-3. This is not the course text-book. You are required to read the relevant sections of the book as mentioned on the course website.

The course notes are currently being written, and will continue to being developed as the course progresses (just like the class notes last quarter). Please report any typos / mistakes / inconsistencies / issues with the class notes / class presentations in your comments [here](#). Thank you!

Part I

Linear regression

1 Introduction to Scikit-learn

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

# sklearn has 100s of models - grouped in sublibraries, such as linear_model
from sklearn.linear_model import LogisticRegression, LinearRegression
# sklearn also has many tools for cleaning/processing data, also grouped in sublibraries
from sklearn.model_selection import train_test_split # splitting one dataset into train and test
from sklearn.metrics import accuracy_score, mean_absolute_error, mean_squared_error, r2_score
from sklearn.preprocessing import StandardScaler

data = pd.read_csv('./Datasets/diabetes.csv')

# Separating the predictors and response - THIS IS HOW ALL SKLEARN OBJECTS ACCEPT DATA (dict-like)
y = data.Outcome
X = data.drop("Outcome", axis = 1)

# Creating training and test data
# 80-20 split, which is usual - 70-30 split is also fine, 90-10 is fine if the dataset is small
# random_state to set a random seed for the splitting - reproducible results
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2, random_state = 42)
# stratify

# With linear/logistic regression in scikit-learn, especially when the predictors have different scales, scaling is necessary. This is to enable the training algorithm. which we did not cover yet
scaler = StandardScaler().fit(X_train)
X_train_scaled = scaler.transform(X_train)
X_test_scaled = scaler.transform(X_test) # Do NOT refit the scaler with the test data, just transform
X_train = X_train_scaled
```

```
X_test = X_test_scaled
```

```
# Create a model - not trained yet  
logreg = LogisticRegression()
```

```
# Train the model  
logreg.fit(X_train, y_train)
```

```
# Test the model - prediction - two ways to go  
y_pred = logreg.predict(X_test) # Get the predicted classes first  
print(accuracy_score(y_pred, y_test)*100) # Use the predicted and true classes for accuracy
```

73.37662337662337

```
print(logreg.score(X_test, y_test)*100) # Use .score with test predictors and response to  
# Implements the same thing under the hood
```

73.37662337662337

```
print(logreg.coef_) # Use coef_ to return the coefficients - only log reg inference you can
```

```
[[ 0.32572891  1.20110566 -0.32046591  0.06849882 -0.21727131  0.72619528  
  0.40088897  0.29698818]]
```

```
# all metrics exist in sklearn  
from sklearn.metrics import precision_score, recall_score, confusion_matrix
```

```
print(confusion_matrix(y_test, y_pred))  
print(precision_score(y_test, y_pred))  
print(recall_score(y_test, y_pred))
```

```
# What we covered today:
```

```
    # A recap of Log. Reg. with sklearn  
    # Separate the predictors and response (if necessary)  
    # Split the data into train and test (if necessary)
```

```
    # Create a model  
    # Train with .fit
```

```

# Predict with .predict or get the accuracy with .score
# Use sklearn metrics with y_pred and y_test

```

```

# Same idea with LinearRegression()
# .score returns r-squared by default
# use the appropriate metrics!

```

```

#####

```

```

[[87 17]
 [24 26]]
0.6046511627906976
0.52

```

```

# More details on the LogisticRegression model:
# Inputs - for regularization and
# prediction prob.s instead of classes - so we can change the thresholds

```

```

# .predict_proba returns the prob.s for both classes
# Two cols for two classes
# Apply your threshold to y_pred_probs[1] (second col)
y_pred_probs = logreg.predict_proba(X_test)

```

```

cutoff = 0.3

```

```

y_pred2 = y_pred_probs[:,1] > cutoff
y_pred2 = y_pred2.astype(int)

```

```

print(confusion_matrix(y_test, y_pred2))
print(precision_score(y_test, y_pred2))
print(recall_score(y_test, y_pred2))

```

```

[[78 26]
 [15 35]]
0.5737704918032787
0.7

```

```

# Throughout the course, we will create many different models in sklearn, all of them will
# .fit
# .predict
# .score
# .predict_proba
# methods, among others that are specific to them. Make sure how to use there 4 basic ones

# Let's take a look at the documentation - always do this for the other models we will see
# Log Reg model has default regularization
# Default (regularization) penalty is "l2" - this means Ridge
# C is 1/lambda - remember that lambda is the hyperparameter that is multiplied with t
# C is 1 by default

# Let's take away the regularization
logreg2 = LogisticRegression(C=1e10)
logreg2.fit(X_train, y_train)
y_pred = logreg2.predict(X_test) # Get the predicted classes first
print(accuracy_score(y_pred, y_test)*100)

```

73.37662337662337

```

# Test accuracy stayed the same - reg was not very necessary

# Too much reg
logreg2 = LogisticRegression(C=1e-10)
logreg2.fit(X_train, y_train)
y_pred = logreg2.predict(X_test) # Get the predicted classes first
print(accuracy_score(y_pred, y_test)*100)

# Test accuracy is even lower - too much reg caused underfitting

```

67.53246753246754

```

# The key to take full advantage of sklearn models is their inputs
# Always read the doc
# In Log Reg, you can switch to Lasso with penalty = 'l1' - for variable selection
# For no regression, besides what we did above, you can use penalty = None

# Recall that C, or lambda, is a hyperparameter, which is optimized with cross-validation
# There is LogisticRegressionCV, just like LassoCV and RidgeCV

```



```

# Works the exact same way - check LassoCV and RidgeCV notes

# For all the sklearn models we will create in this course, there will be hyperparameters.
# Mostly more than one for each model
# These hyperparameters will determine how much regularization the model will have
# These models will not have a CV version
# So, we need to use two sklearn tools that implement cross-validation
# cross_val_score - now
# GridSearchCV - later when we get to trees and tree-based models

from sklearn.model_selection import cross_val_score

val_scores = []

hyperparam_vals = 10**np.linspace(-5, 10)

for c_val in hyperparam_vals: # For each possible C value in your grid
    logreg_model = LogisticRegression(C=c_val) # Create a model with the C value

    val_scores.append(cross_val_score(logreg_model, X_train, y_train, scoring='accuracy',

import matplotlib.pyplot as plt

plt.plot(hyperparam_vals, np.mean(np.array(val_scores), axis=1))
plt.xlabel('possible C value')
plt.ylabel('avg 5-fold CV value')
plt.xscale('log')
plt.show()

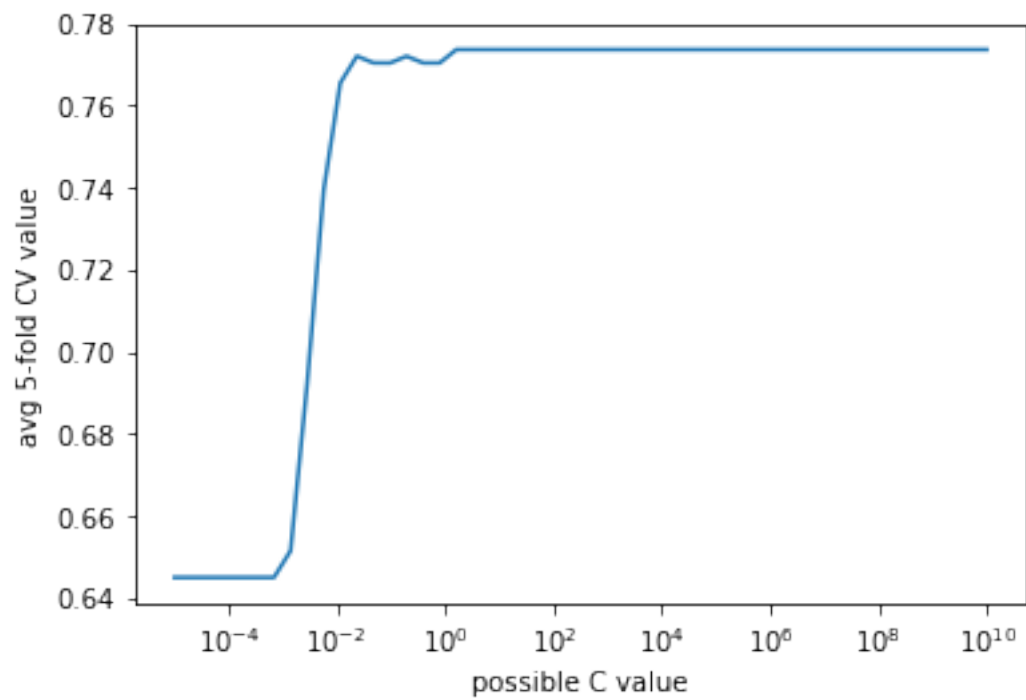
# Train the best model with the hyperparam val that returns the highest average accuracy
logreg_model_best = LogisticRegression(C=hyperparam_vals[np.argmax(np.mean(np.array(val_scores))

# .fit
# .predict & .predict_proba
# .score
# ...

# Log. reg. has one hyperparameter - C.
# More complex models will have more

```

```
# If we have two hyperparams - we can use a nested loop and cross_val_score
# or we can use GridSearchCV - more on that when we get to trees and tree-based models
```



2 Regression splines

Read sections 7.1-7.4 of the book before using these notes.

Note that in this course, lecture notes are not sufficient, you must read the book for better understanding. Lecture notes are just implementing the concepts of the book on a dataset, but not explaining the concepts elaborately.

```
import pandas as pd
import numpy as np
import statsmodels.formula.api as smf
import statsmodels.api as sm
import seaborn as sns
import matplotlib.pyplot as plt
from patsy import dmatrix
from sklearn.metrics import mean_squared_error
from pyearth import Earth
```

```
#Using the same datasets as used for linear regression in STAT303-2,
#so that we can compare the non-linear models with linear regression
trainf = pd.read_csv('Car_features_train.csv')
trainp = pd.read_csv('Car_prices_train.csv')
testf = pd.read_csv('Car_features_test.csv')
testp = pd.read_csv('Car_prices_test.csv')
train = pd.merge(trainf, trainp)
test = pd.merge(testf, testp)
train.head()
```

	carID	brand	model	year	transmission	mileage	fuelType	tax	mpg	engineSize	price
0	18473	bmw	6 Series	2020	Semi-Auto	11	Diesel	145	53.3282	3.0	37980
1	15064	bmw	6 Series	2019	Semi-Auto	10813	Diesel	145	53.0430	3.0	33980
2	18268	bmw	6 Series	2020	Semi-Auto	6	Diesel	145	53.4379	3.0	36850
3	18480	bmw	6 Series	2017	Semi-Auto	18895	Diesel	145	51.5140	3.0	25998
4	18492	bmw	6 Series	2015	Automatic	62953	Diesel	160	51.4903	3.0	18990

2.1 Polynomial regression vs Regression splines

2.1.1 Model of degree 1

```
ols_object = smf.ols(formula = 'price~mileage', data = train)
lr_model = ols_object.fit()

#Regression spline of degree 1

#Creating basis functions for splines of degree 1
transformed_x = dmatrix("bs(mileage , knots=(33000,66000,100000), degree = 1, include_intercept=True,
                        data = {'mileage':train['mileage']},return_type = 'dataframe')

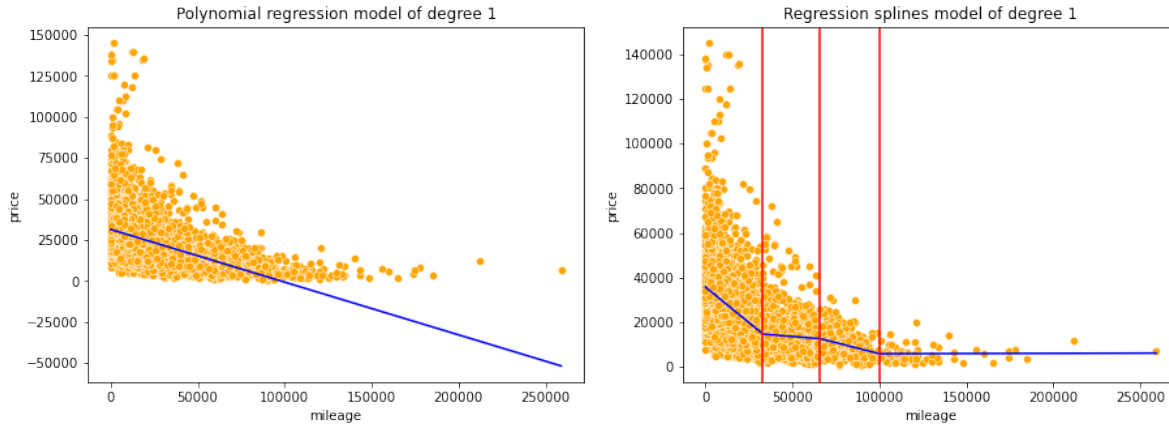
#Developing a linear regression model on the spline basis functions - this is the regression spline model
reg_spline_model = sm.OLS(train['price'], transformed_x).fit()

#Visualizing polynomial model and the regression spline model of degree 1

knots = [33000,66000,100000] #Knots for the spline
d=1 #Degree of predictor in the model
#Writing a function to visualize polynomial model and the regression spline model of degree d
def viz_models():
    fig, axes = plt.subplots(1,2,figsize = (15,5))
    plt.subplots_adjust(wspace=0.2)

    #Visualizing the linear regression model
    pred_price = lr_model.predict(train)
    sns.scatterplot(ax = axes[0],x = 'mileage', y = 'price', data = train, color = 'orange')
    sns.lineplot(ax = axes[0],x = train.mileage, y = pred_price, color = 'blue')
    axes[0].set_title('Polynomial regression model of degree '+str(d))

    #Visualizing the regression splines model of degree 'd'
    axes[1].set_title('Regression splines model of degree '+ str(d))
    sns.scatterplot(ax=axes[1],x = 'mileage', y = 'price', data = train, color = 'orange')
    sns.lineplot(ax=axes[1],x = train.mileage, y = reg_spline_model.predict(), color = 'blue')
    for i in range(3):
        plt.axvline(knots[i], 0,100,color='red')
viz_models()
```



We observe the regression splines model better fits the data as compared to the polynomial regression model. This is because regression splines of degree 1 fit piecewise polynomials, or linear models on sub-sections of the predictor, which helps better capture the trend. However, this added flexibility may also lead to overfitting. Hence, one must be careful to check for overfitting when using splines. Overfitting may be checked by k-fold cross validation or comparing test and train errors.

The red lines in the plot on the right denote the position of knots. Knots separate distinct splines.

```
#Creating basis functions for test data for prediction
test_x = dmatrix("bs(mileage , knots=(33000,66000,100000), degree = 1, include_intercept =

#Function to compute RMSE (root mean squared error on train and test datasets)
def rmse():
    #Error on train data for the linear regression model
    print("RMSE on train data:")
    print("Linear regression:", np.sqrt(mean_squared_error(lr_model.predict(),train.price)

    #Error on train data for the regression spline model
    print("Regression splines:", np.sqrt(mean_squared_error(reg_spline_model.predict(),tra

    #Error on test data for the linear regression model
    print("\nRMSE on test data:")
    print("Linear regression:",np.sqrt(mean_squared_error(lr_model.predict(test),test.pric

    #Error on test data for the regression spline model
```

```
print("Regression splines:", np.sqrt(mean_squared_error(reg_spline_model.predict(test_x),
rmse())
```

RMSE on train data:

Linear regression: 14403.250083261853

Regression splines: 13859.640716531134

RMSE on test data:

Linear regression: 14370.94086395544

Regression splines: 13770.133025694666

2.1.2 Model of degree 2

A higher degree model will lead to additional flexibility for both polynomial and regression splines models.

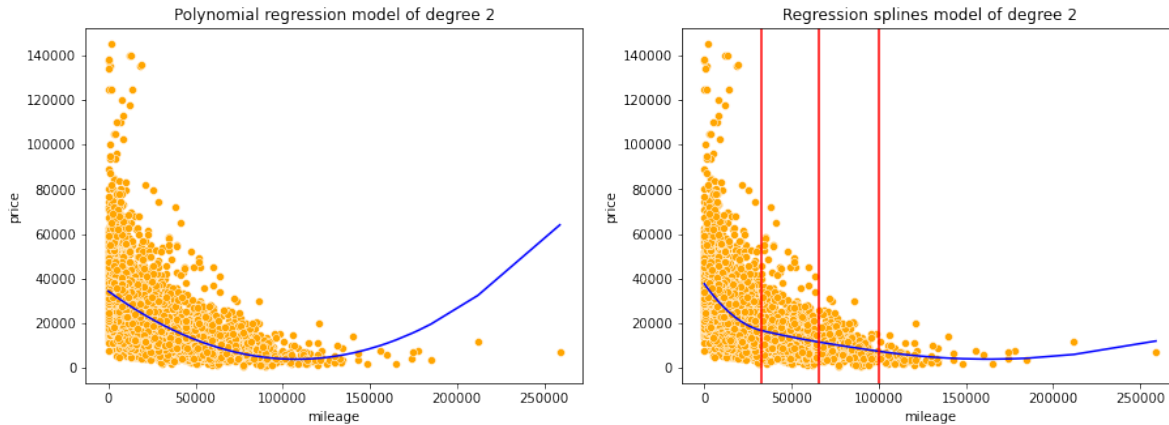
```
#Including mileage squared as a predictor and developing the model
ols_object = smf.ols(formula = 'price~mileage+I(mileage**2)', data = train)
lr_model = ols_object.fit()

#Regression spline of degree 2

#Creating basis functions for splines of degree 2
transformed_x = dmatrix("bs(mileage , knots=(33000,66000,100000), degree = 2, include_intercept=True",
                        data = {'mileage':train['mileage']},return_type = 'dataframe')

#Developing a linear regression model on the spline basis functions - this is the regression spline model
reg_spline_model = sm.OLS(train['price'], transformed_x).fit()

d=2
viz_models()
```



Unlike polynomial regression, splines functions avoid imposing a global structure on the non-linear function of X . This provides a better local fit to the data.

```
#Creating basis functions for test data for prediction
test_x = dmatrix("bs(mileage , knots=(33000,66000,100000), degree = 2, include_intercept = 1)", data = test_data)

rmse()
```

RMSE on train data:

Linear regression: 14403.250083261853

Regression splines: 13859.640716531134

RMSE on test data:

Linear regression: 14370.94086395544

Regression splines: 13770.133025694666

2.1.3 Model of degree 3

```
ols_object = smf.ols(formula = 'price~mileage+I(mileage**2)+I(mileage**3)', data = train)
lr_model = ols_object.fit()

#Regression spline of degree 3

#Creating basis functions for splines of degree 3
transformed_x = dmatrix("bs(mileage , knots=(20000,40000,80000), degree = 3, include_intercept = 1)", data = test_data)
```

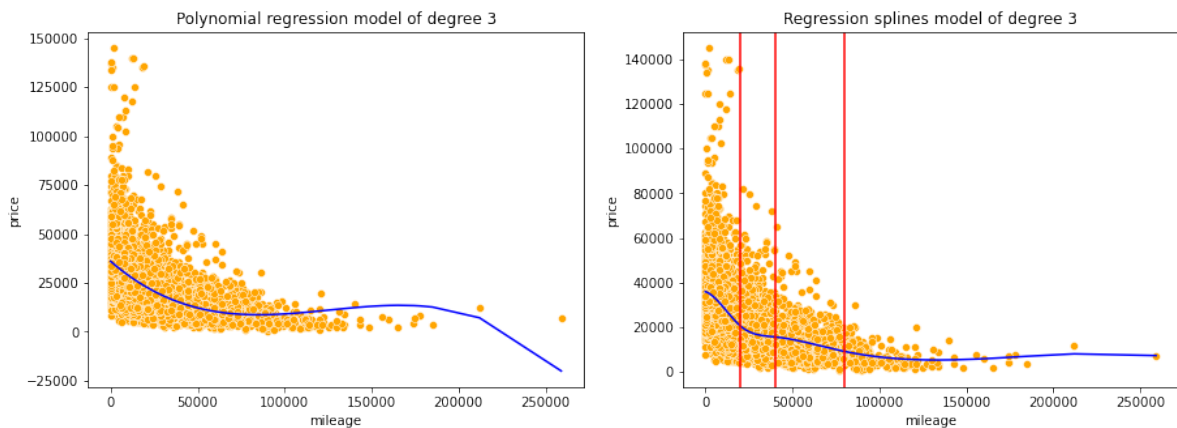
```
data = {'mileage':train['mileage']},return_type = 'dataframe')

#Developing a linear regression model on the spline basis functions - this is the regression
reg_spline_model = sm.OLS(train['price'], transformed_x).fit()
```

```
transformed_x
```

	Intercept	bs(mileage, knots=(20000, 40000, 80000), degree=3, include__intercept=False)[0]	bs(mileage
0	1.0	0.001499	3.749187
1	1.0	0.583162	3.001491
2	1.0	0.000750	9.374336
3	1.0	0.293446	6.009875
4	1.0	0.000000	2.580169
...
4955	1.0	0.005441	4.824519
4956	1.0	0.206763	6.438755
4957	1.0	0.000000	0.000000
4958	1.0	0.198162	6.468919
4959	1.0	0.000000	2.233101

```
d=3
knots=[20000,40000,80000]
viz_models()
```



Unlike polynomial regression, splines functions avoid imposing a global structure on the non-linear function of X. This provides a better local fit to the data.


```
#Creating basis functions for test data for prediction
test_x = dmatrix("bs(mileage , knots=(20000,40000,80000), degree = 3, include_intercept =
```

```
rmse()
```

RMSE on train data:

Linear regression: 13891.962447594644

Regression splines: 13792.371446327243

RMSE on test data:

Linear regression: 13789.708418357186

Regression splines: 13651.288965905529

2.2 Regression splines with knots at uniform quantiles of data

If degrees of freedom are provided instead of knots, the knots are by default chosen at uniform quantiles of data. For example if there are 7 degrees of freedom (including the intercept), then there will be $7-4 = 3$ knots. These knots will be chosen at the 25th, 50th and 75th quantiles of the data.

```
#Regression spline of degree 3
```

```
#Creating basis functions for splines of degree 3
```

```
transformed_x = dmatrix("bs(mileage , df=6, degree = 3, include_intercept = False)",
                        data = {'mileage':train['mileage']},return_type = 'dataframe')
```

```
#Developing a linear regression model on the spline basis functions - this is the regression
```

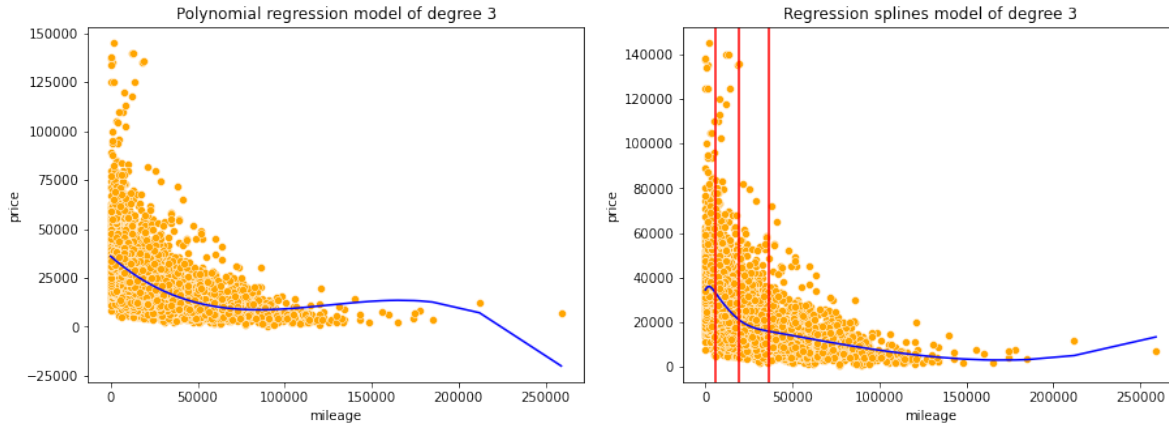
```
reg_spline_model = sm.OLS(train['price'], transformed_x).fit()
```

```
d=3
```

```
unif_knots = pd.qcut(train.mileage,4,retbins=True)[1][1:4]
```

```
knots=unif_knots
```

```
viz_models()
```



Splines can be unstable at the outer range of predictors. In the figure (on the right), the left-most spline may be overfitting.

```
#Creating basis functions for test data for prediction
test_x = dmatrix("bs(mileage , knots="+str(tuple(unif_knots))) + ", degree = 3, include_in

rmse()
```

RMSE on train data:
 Linear regression: 13891.962447594644
 Regression splines: 13781.79102252679

RMSE on test data:
 Linear regression: 13789.708418357186
 Regression splines: 13676.271829882426

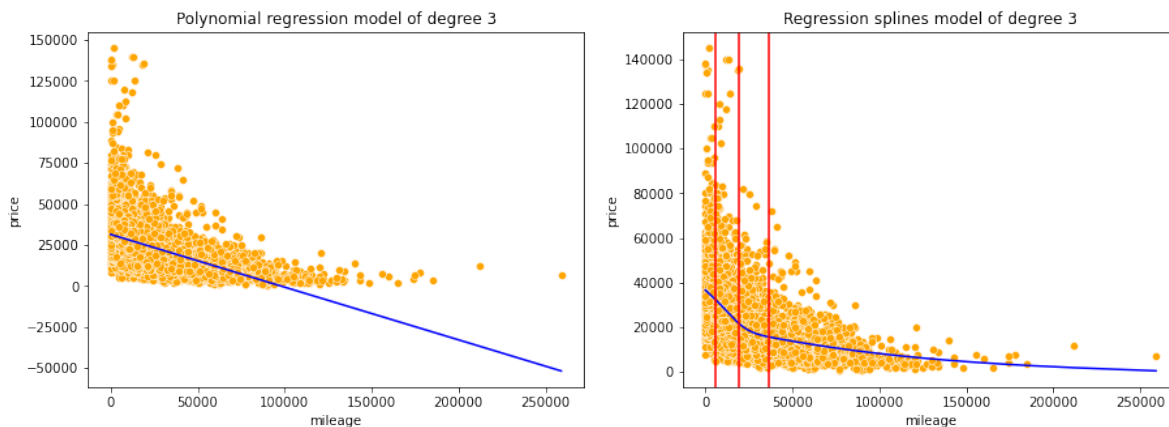
2.3 Natural cubic splines

Page 298: “A natural spline is a regression spline with additional boundary constraints: the function is required to be linear at the boundary (in the region where X is smaller than the smallest knot, or larger than the largest knot). This additional constraint means that natural splines generally produce more stable estimates at the boundaries.”

```
#Natural cubic spline
```

```
#Creating basis functions for the natural cubic spline
transformed_x = dmatrix("cr(mileage , df=4,constraints='center')",
                        data = {'mileage':train['mileage']},return_type = 'dataframe')
reg_spline_model = sm.GLM(train['price'], transformed_x).fit()
```

```
d=3;
unif_knots = pd.qcut(train.mileage,4,retbins=True)[1][1:4]
knots=unif_knots
viz_models()
```



Note that the natural cubic spline is more stable than a cubic splines with knots at uniformly distributed quantiles.

```
#Creating basis functions for test data for prediction
test_x = dmatrix("cr(mileage , knots="+str(tuple(unif_knots))+",constraints='center')",data=test_data)

rmse()
```

RMSE on train data:

Linear regression: 13891.962447594644

Regression splines: 13805.022189679756

RMSE on test data:

Linear regression: 13789.708418357186

Regression splines: 13666.943224268975

2.4 Generalized additive model (GAM)

GAM allow for flexible nonlinearities in several variables, but retain the additive structure of linear models. In a GAM, non-linear basis functions of predictors can be used as predictors of a linear regression model. For example,

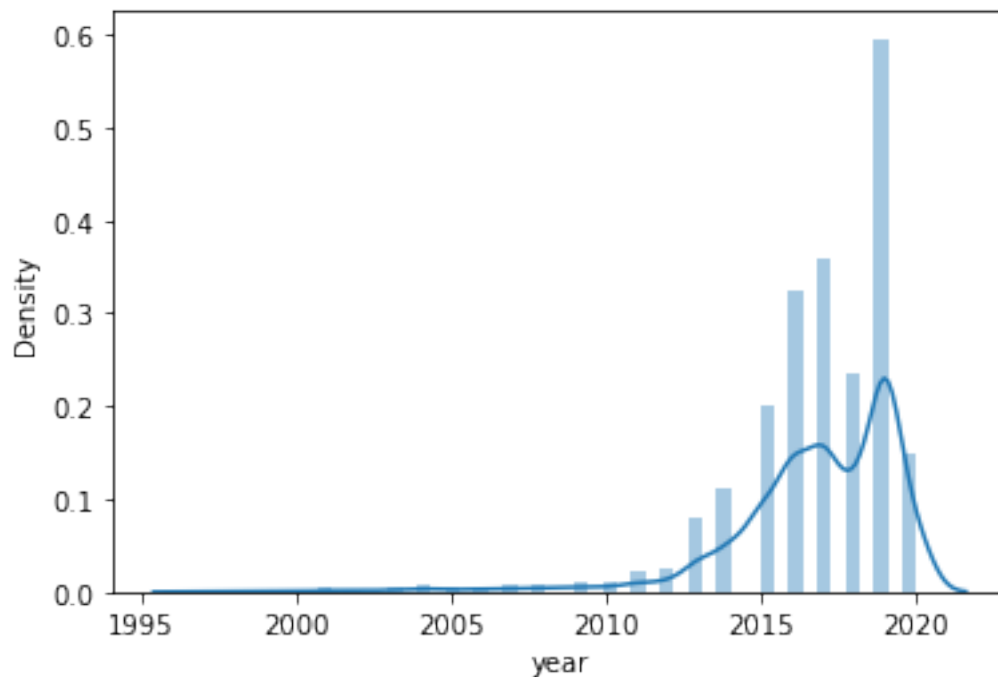
$$y = f_1(X_1) + f_2(X_2) + \epsilon$$

is a GAM, where $f_1(\cdot)$ may be a cubic spline based on the predictor X_1 , and $f_2(\cdot)$ may be a step function based on the predictor X_2 .

```
sns.distplot(train.year)
```

```
C:\Users\akl0407\Anaconda3\lib\site-packages\seaborn\distributions.py:2557: FutureWarning: `warnings.warn(msg, FutureWarning)`
```

```
<AxesSubplot:xlabel='year', ylabel='Density'>
```



```

#GAM
#GAM includes cubic splines for mileage. Other predictors are year, engineSize, mpg, mileage
X_transformed = dmatrix('bs(mileage,df=6,degree = 3)+year*engineSize*mpg*mileage',
                        data = {'year':train['year'],'engineSize':train['engineSize'],'mpg':train['mpg']},
                        return_type = 'dataframe')

# fit the model
model_gam = sm.OLS(train['price'],X_transformed).fit()

#Creating basis functions for test data for prediction
X_test = dmatrix('bs(mileage,df=6,degree = 3, include_intercept = False)+year*engineSize*mpg*mileage',
                 data = {'year':test['year'],'engineSize':test['engineSize'],'mpg':test['mpg']},
                 return_type = 'dataframe')

preds = model_gam.predict(X_test)
np.sqrt(mean_squared_error(preds,test.price))

```

8434.756663328963

```

#GAM
#GAM includes cubic splines for mileage, year, engineSize, mpg, and interactions of all predictors
X_transformed = dmatrix('bs(mileage,df=6,degree = 3)+bs(mpg,df=6,degree = 3)+bs(engineSize,df=6,degree = 3)+year*engineSize*mpg*mileage',
                        data = {'year':train['year'],'engineSize':train['engineSize'],'mpg':train['mpg']},
                        return_type = 'dataframe')

# fit the model
model_gam = sm.OLS(train['price'],X_transformed).fit()

#Creating basis functions for test data for prediction
X_test = dmatrix('bs(mileage,df=6,degree = 3, include_intercept = False)+bs(mpg,df=6,degree = 3)+bs(engineSize,df=6,degree = 3)+year*engineSize*mpg*mileage',
                 data = {'year':test['year'],'engineSize':test['engineSize'],'mpg':test['mpg']},
                 return_type = 'dataframe')

preds = model_gam.predict(X_test)
np.sqrt(mean_squared_error(preds,test.price))

```

7997.325718841729

```

ols_object = smf.ols(formula = 'price~(year+engineSize+mileage+mpg)**2+I(mileage**2)+I(mileage*year)+I(mileage*engineSize)+I(mileage*mpg)+I(year*engineSize)+I(year*mpg)+I(engineSize*mpg)')
model = ols_object.fit()
model.summary()

```

Table 2.3: OLS Regression Results

Dep. Variable:	price	R-squared:	0.704
Model:	OLS	Adj. R-squared:	0.703
Method:	Least Squares	F-statistic:	1308.
Date:	Sun, 27 Mar 2022	Prob (F-statistic):	0.00
Time:	01:08:50	Log-Likelihood:	-52157.
No. Observations:	4960	AIC:	1.043e+05
Df Residuals:	4950	BIC:	1.044e+05
Df Model:	9		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-0.0009	0.000	-2.752	0.006	-0.002	-0.000
year	-1.1470	0.664	-1.728	0.084	-2.448	0.154
engineSize	0.0052	0.000	17.419	0.000	0.005	0.006
mileage	-31.4751	2.621	-12.010	0.000	-36.613	-26.337
mpg	-0.0201	0.002	-13.019	0.000	-0.023	-0.017
year:engineSize	9.5957	0.254	37.790	0.000	9.098	10.094
year:mileage	0.0154	0.001	11.816	0.000	0.013	0.018
year:mpg	0.0572	0.013	4.348	0.000	0.031	0.083
engineSize:mileage	-0.1453	0.008	-18.070	0.000	-0.161	-0.130
engineSize:mpg	-98.9062	11.832	-8.359	0.000	-122.102	-75.710
mileage:mpg	0.0011	0.000	2.432	0.015	0.000	0.002
I(mileage ** 2)	7.713e-06	3.75e-07	20.586	0.000	6.98e-06	8.45e-06
I(mileage ** 3)	-1.867e-11	1.43e-12	-13.077	0.000	-2.15e-11	-1.59e-11

Omnibus:	1830.457	Durbin-Watson:	0.634
Prob(Omnibus):	0.000	Jarque-Bera (JB):	34927.811
Skew:	1.276	Prob(JB):	0.00
Kurtosis:	15.747	Cond. No.	2.50e+18

```
np.sqrt(mean_squared_error(model.predict(test),test.price))
```

9026.775740000594

Note the RMSE with GAM that includes regression splines for mileage is lesser than that of the linear regression model, indicating a better fit.

2.5 MARS (Multivariate Adaptive Regression Splines)

```
X=train['mileage']
y=train['price']
```

2.5.1 MARS of degree 1

```
model = Earth(max_terms=500, max_degree=1) # note, terms in brackets are the hyperparameters
model.fit(X,y)
```

```
C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:813: FutureWarning: `rcond` parameter will be removed from
To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old default use
pruning_passer.run()
C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:1066: FutureWarning: `rcond` parameter will be removed from
To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old default use
coef, resid = np.linalg.lstsq(B, weighted_y[:, i])[0:2]
```

```
Earth(max_degree=1, max_terms=500)
```

```
print(model.summary())
```

Earth Model

Basis Function	Pruned	Coefficient
(Intercept)	No	-553155
h(x0-22141)	Yes	None
h(22141-x0)	Yes	None
h(x0-3354)	No	-6.23571
h(3354-x0)	Yes	None
h(x0-15413)	No	-36.9613
h(15413-x0)	No	38.167
h(x0-106800)	Yes	None
h(106800-x0)	No	0.221844
h(x0-500)	No	170.039
h(500-x0)	Yes	None
h(x0-741)	Yes	None

h(741-x0)	No	-54.5265
h(x0-375)	No	-126.804
h(375-x0)	Yes	None
h(x0-2456)	Yes	None
h(2456-x0)	No	7.04609

MSE: 188429705.7549, GCV: 190035470.5664, RSQ: 0.2998, GRSQ: 0.2942

Model equation:

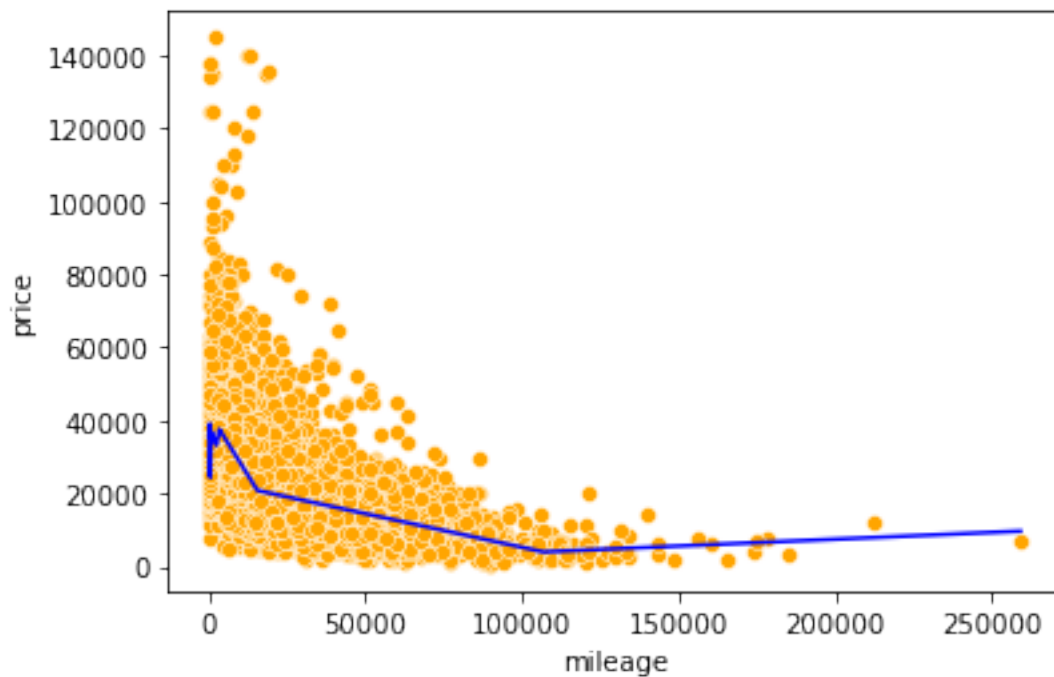
$$-553155 - 6.23(h(x0 - 3354)) - 36.96(h(x0 - 15413) + \dots - 7.04(h(2456 - x0))$$

```
pred = model.predict(test.mileage)
np.sqrt(mean_squared_error(pred, test.price))
```

13650.2113154515

```
sns.scatterplot(x = 'mileage', y = 'price', data = train, color = 'orange')
sns.lineplot(x = train.mileage, y = model.predict(train.mileage), color = 'blue')
```

<AxesSubplot:xlabel='mileage', ylabel='price'>



2.5.2 MARS of degree 2

```
model = Earth(max_terms=500, max_degree=2) # note, terms in brackets are the hyperparameters
model.fit(X,y)
print(model.summary())
```

Earth Model

Basis Function	Pruned	Coefficient
(Intercept)	No	19369.7
h(x0-22141)	Yes	None
h(22141-x0)	Yes	None
h(x0-7531)*h(22141-x0)	No	3.74934e-05
h(7531-x0)*h(22141-x0)	No	-6.74252e-05
x0*h(x0-22141)	No	-8.0703e-06
h(x0-15012)	Yes	None
h(15012-x0)	No	1.79813
h(x0-26311)*h(x0-22141)	No	8.85097e-06
h(26311-x0)*h(x0-22141)	Yes	None

MSE: 189264421.5682, GCV: 190298913.1652, RSQ: 0.2967, GRSQ: 0.2932

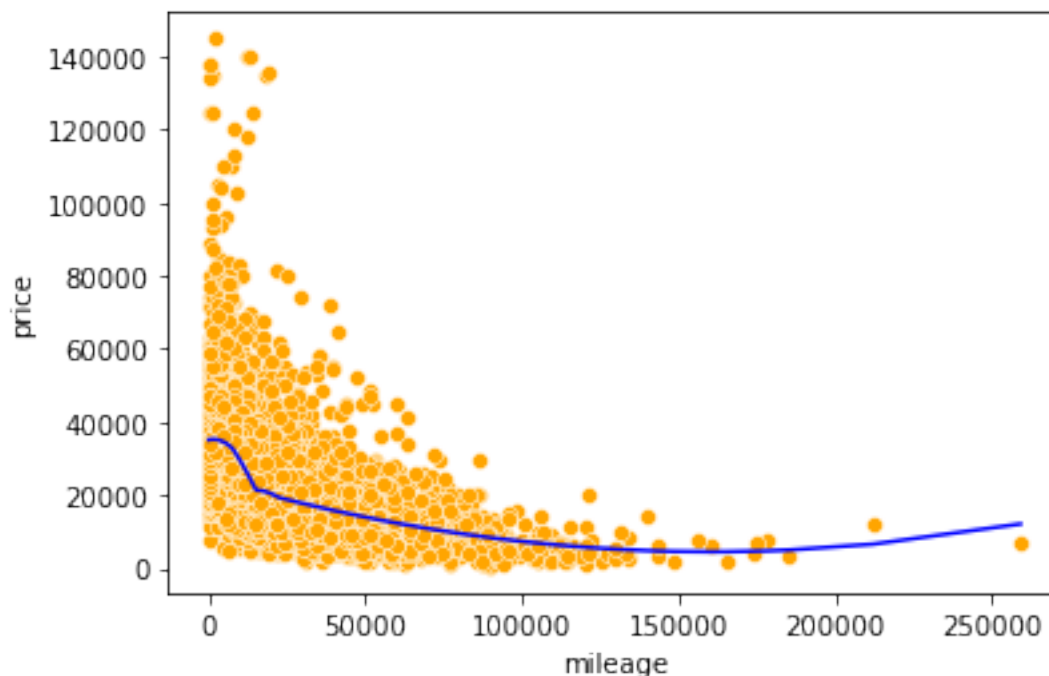
```
C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:813: FutureWarning: `rcond` parameter will be removed from
To use the future default and silence this warning we advise to pass `rcond=None`, to keep us
    pruning_passer.run()
C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:1066: FutureWarning: `rcond` parameter will be removed from
To use the future default and silence this warning we advise to pass `rcond=None`, to keep us
    coef, resid = np.linalg.lstsq(B, weighted_y[:, i])[0:2]
```

```
pred = model.predict(test.mileage)
np.sqrt(mean_squared_error(pred, test.price))
```

13590.995419204985

```
sns.scatterplot(x = 'mileage', y = 'price', data = train, color = 'orange')
sns.lineplot(x = train.mileage, y = model.predict(train.mileage), color = 'blue')
```

<AxesSubplot: xlabel='mileage', ylabel='price'>



MARS provides a better fit than the splines that we used above. This is because MARS tunes the positions of the knots and considers interactions (also with tuned knots) to improve the model fit. Tuning of knots may improve the fit of splines as well.

2.5.3 MARS including categorical variables

```
#A categorical variable can be turned to dummy variables to use the Earth package for fitting
train_cat = pd.concat([train,pd.get_dummies(train.fuelType)],axis=1)
test_cat = pd.concat([test,pd.get_dummies(test.fuelType)],axis=1)

train_cat.head()
```

	carID	brand	model	year	transmission	mileage	fuelType	tax	mpg	engineSize	price
0	18473	bmw	6 Series	2020	Semi-Auto	11	Diesel	145	53.3282	3.0	37980
1	15064	bmw	6 Series	2019	Semi-Auto	10813	Diesel	145	53.0430	3.0	33980
2	18268	bmw	6 Series	2020	Semi-Auto	6	Diesel	145	53.4379	3.0	36850
3	18480	bmw	6 Series	2017	Semi-Auto	18895	Diesel	145	51.5140	3.0	25998
4	18492	bmw	6 Series	2015	Automatic	62953	Diesel	160	51.4903	3.0	18990

```
X = train_cat[['mileage','mpg','engineSize','year','Diesel','Electric','Hybrid','Petrol']]
Xtest = test_cat[['mileage','mpg','engineSize','year','Diesel','Electric','Hybrid','Petrol']]

model = Earth(max_terms=500, max_degree=2) # note, terms in brackets are the hyperparameters
model.fit(X,y)
print(model.summary())
```

C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:813: FutureWarning: `rcond` parameter will be deprecated in the future. To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old default use `rcond=-1`.

```
pruning_passer.run()
```

Earth Model

Basis Function	Pruned	Coefficient
(Intercept)	No	2.17604e+06
h(engineSize-5.5)	No	9.80752e+06
h(5.5-engineSize)	No	1.92817e+06
h(mileage-21050)	No	18.687
h(21050-mileage)	No	-177.871
h(mileage-21050)*h(5.5-engineSize)	Yes	None
h(21050-mileage)*h(5.5-engineSize)	No	-0.224909
year	No	4126.41
h(mpg-53.3495)	No	344595
h(53.3495-mpg)	Yes	None
Hybrid*h(5.5-engineSize)	No	6124.34
h(mileage-21050)*year	No	-0.00930239
h(21050-mileage)*year	No	0.0886455
h(engineSize-5.5)*year	No	-4864.84
h(5.5-engineSize)*year	No	-952.92
h(mileage-1422)*h(53.3495-mpg)	No	-16.62
h(1422-mileage)*h(53.3495-mpg)	No	16.4306
Hybrid	No	-89090.6
h(mpg-21.1063)*h(53.3495-mpg)	Yes	None
h(21.1063-mpg)*h(53.3495-mpg)	No	-8815.99
h(mpg-23.4808)*h(5.5-engineSize)	No	-3649.97
h(23.4808-mpg)*h(5.5-engineSize)	Yes	None
h(mpg-20.5188)*year	No	31.7341
h(20.5188-mpg)*year	Yes	None
h(mpg-22.2566)*h(53.3495-mpg)	No	-52.2531
h(22.2566-mpg)*h(53.3495-mpg)	No	7916.19

h(mpg-22.6767)	No	7.56432e+06
h(22.6767-mpg)	Yes	None
h(mpg-23.9595)*h(mpg-22.6767)	Yes	None
h(23.9595-mpg)*h(mpg-22.6767)	No	-63225.4
h(mpg-21.4904)*h(22.6767-mpg)	No	-149055
h(21.4904-mpg)*h(22.6767-mpg)	Yes	None
h(mpg-21.1063)	No	-887098
h(21.1063-mpg)	Yes	None
h(mpg-29.5303)*h(mpg-22.6767)	No	-3028.87
h(29.5303-mpg)*h(mpg-22.6767)	Yes	None
h(mpg-28.0681)*h(5.5-engineSize)	No	3572.89
h(28.0681-mpg)*h(5.5-engineSize)	Yes	None
engineSize*h(5.5-engineSize)	No	-2952.65
h(mpg-25.3175)*h(mpg-21.1063)	No	-332551
h(25.3175-mpg)*h(mpg-21.1063)	No	324298
Petrol*year	No	-1.37031
h(mpg-68.9279)*Hybrid	No	-4087.9
h(68.9279-mpg)*Hybrid	Yes	None
h(mpg-31.5043)*h(5.5-engineSize)	Yes	None
h(31.5043-mpg)*h(5.5-engineSize)	No	3691.82
h(mpg-32.7011)*h(5.5-engineSize)	Yes	None
h(32.7011-mpg)*h(5.5-engineSize)	No	-2262.78
h(mpg-44.9122)*h(mpg-22.6767)	No	335577
h(44.9122-mpg)*h(mpg-22.6767)	No	-335623
h(engineSize-5.5)*h(mpg-21.1063)	No	27815
h(5.5-engineSize)*h(mpg-21.1063)	Yes	None
h(mpg-78.1907)*Hybrid	Yes	None
h(78.1907-mpg)*Hybrid	No	2221.49
h(mpg-63.1632)*h(mpg-22.6767)	Yes	None
h(63.1632-mpg)*h(mpg-22.6767)	No	21.0093
Hybrid*h(mpg-53.3495)	No	4121.91
h(mileage-22058)*h(53.3495-mpg)	No	16.6177
h(22058-mileage)*h(53.3495-mpg)	No	-16.6044
h(mpg-21.8985)	Yes	None
h(21.8985-mpg)	No	371659

MSE: 45859836.5623, GCV: 47884649.3622, RSQ: 0.8296, GRSQ: 0.8221

C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:1066: FutureWarning: `rcond` parameter will be removed from `np.linalg.lstsq` in the future. To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the current default use `rcond=-1`.
coef, resid = np.linalg.lstsq(B, weighted_y[:, i])[0:2]

```
pred = model.predict(Xtest)
np.sqrt(mean_squared_error(pred,test2.price))
```

7499.709075454322

Let us compare the RMSE of a MARS model with *mileage*, *mpg*, *engineSize* and *year* with a linear regression model having the same predictors.

```
X = train[['mileage','mpg','engineSize','year']]

model = Earth(max_terms=500, max_degree=2) # note, terms in brackets are the hyperparameters
model.fit(X,y)
print(model.summary())
```

C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:813: FutureWarning: `rcond` parameter will be removed from the future default and silence this warning we advise to pass `rcond=None`, to keep using the current default value of `1e-16`.
 pruning_passer.run()

Earth Model

Basis Function	Pruned	Coefficient
(Intercept)	No	-8.13682e+06
h(engineSize-5.5)	No	9.53908e+06
h(5.5-engineSize)	Yes	None
h(mileage-21050)	No	23.4448
h(21050-mileage)	No	-215.861
h(mileage-21050)*h(5.5-engineSize)	Yes	None
h(21050-mileage)*h(5.5-engineSize)	No	-0.278562
year	No	4125.85
h(mpg-53.3495)	Yes	None
h(53.3495-mpg)	Yes	None
h(mileage-21050)*year	No	-0.0116601
h(21050-mileage)*year	No	0.107624
h(mpg-53.2957)*h(5.5-engineSize)	No	-59801.3
h(53.2957-mpg)*h(5.5-engineSize)	No	59950.5
h(engineSize-5.5)*year	No	-4713.74
h(5.5-engineSize)*year	No	-755.742
h(mileage-1766)*h(53.3495-mpg)	No	-0.00337072
h(1766-mileage)*h(53.3495-mpg)	No	-0.144905

h(mpg-19.1277)*h(53.3495-mpg)	No	161.153
h(19.1277-mpg)*h(53.3495-mpg)	Yes	None
h(mpg-23.4808)*h(5.5-engineSize)	Yes	None
h(23.4808-mpg)*h(5.5-engineSize)	Yes	None
h(mpg-21.4971)*h(5.5-engineSize)	Yes	None
h(21.4971-mpg)*h(5.5-engineSize)	Yes	None
h(mpg-40.224)*h(5.5-engineSize)	Yes	None
h(40.224-mpg)*h(5.5-engineSize)	No	298.139
engineSize*h(5.5-engineSize)	No	-2553.17
h(mpg-22.2566)	Yes	None
h(22.2566-mpg)	No	29257.3
h(mpg-20.7712)*h(22.2566-mpg)	No	143796
h(20.7712-mpg)*h(22.2566-mpg)	No	-1249.17
h(mpg-21.4971)*h(22.2566-mpg)	No	-315486
h(21.4971-mpg)*h(22.2566-mpg)	Yes	None
h(mpg-27.0995)*h(mpg-22.2566)	No	3855.71
h(27.0995-mpg)*h(mpg-22.2566)	Yes	None
h(mpg-29.3902)*year	No	6.05449
h(29.3902-mpg)*year	No	-20.176
h(mpg-28.0681)*h(5.5-engineSize)	No	59901.6
h(28.0681-mpg)*h(5.5-engineSize)	No	-55502.2
h(mpg-23.2962)*h(mpg-22.2566)	No	-56126
h(23.2962-mpg)*h(mpg-22.2566)	No	73153.9
h(mpg-69.0719)*h(mpg-53.3495)	Yes	None
h(69.0719-mpg)*h(mpg-53.3495)	No	-124.847
h(engineSize-5.5)*h(22.2566-mpg)	No	-20955.8
h(5.5-engineSize)*h(22.2566-mpg)	No	-8336.23
h(mpg-23.9595)*h(mpg-22.2566)	No	-62983
h(23.9595-mpg)*h(mpg-22.2566)	Yes	None
h(mpg-23.6406)*h(mpg-22.2566)	No	115253
h(23.6406-mpg)*h(mpg-22.2566)	Yes	None
h(mpg-56.1908)	Yes	None
h(56.1908-mpg)	No	-2239.85
h(mpg-29.7993)*h(53.3495-mpg)	No	-139.61
h(29.7993-mpg)*h(53.3495-mpg)	No	788.756

MSE: 49704412.0771, GCV: 51526765.3943, RSQ: 0.8153, GRSQ: 0.8086

C:\Users\akl0407\Anaconda3\lib\site-packages\pyearth\earth.py:1066: FutureWarning: `rcond` parameter will be deprecated in the future. To use the future default and silence this warning we advise to pass `rcond=None`, to keep using the old default, pass `rcond=np.finfo(float).eps`
coef, resid = np.linalg.lstsq(B, weighted_y[:, i])[0:2]

```
Xtest = test[['mileage','mpg','engineSize','year']]
pred = model.predict(Xtest)
np.sqrt(mean_squared_error(pred,test.price))
```

7614.158359050244

```
ols_object = smf.ols(formula = 'price~(year+engineSize+mileage+mpg)**2', data = train)
model = ols_object.fit()
pred = model.predict(test)
np.sqrt(mean_squared_error(pred,test.price))
```

8729.912066822455

The RMSE for the MARS model is lesser than that of the linear regression model, as expected.

A Datasets, assignment and project files

Datasets used in the book, assignment files, project files, and prediction problems report template can be found [here](#)

References