University of BRISTOL

Bristol Robotics Laboratory

Aerial Robotics
Path Planning IV

Prof. Arthur Richards

Optimal Control

- Construct a mathematical model of my problem

- Need functions that define:
  - How drone responds to controls (dynamics)
  - Limits on drone flight envelope
  - Limits on flight regions (inc. obstacles)
  - Quality of a chosen path

- Hand all of these to an ***optimizer*** to find an answer

- Transcription problem: how to encode the control?

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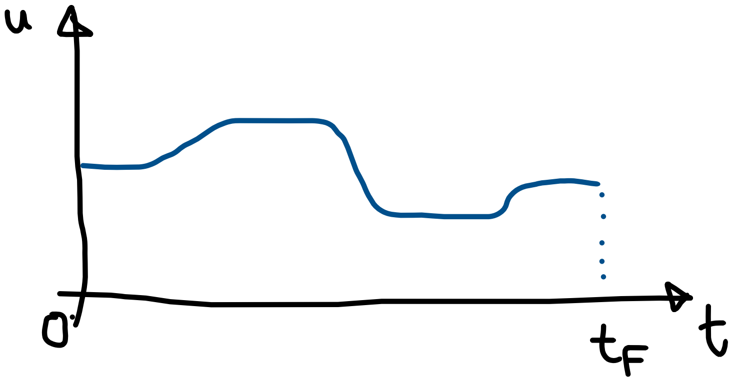
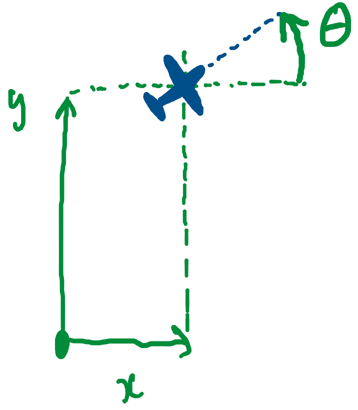


Typical Dynamics



$$\dot{x} = v\cos\theta$$

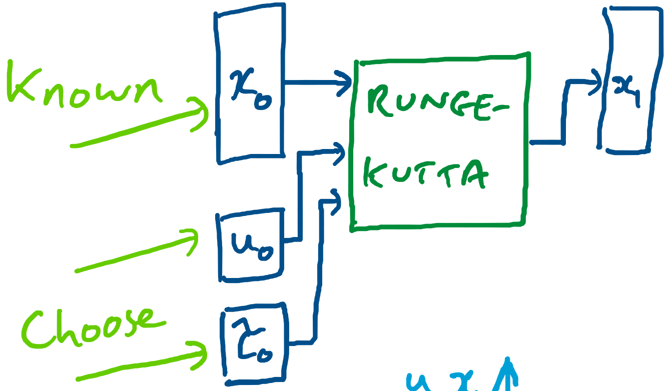
$$\dot{y} = v\sin\theta$$

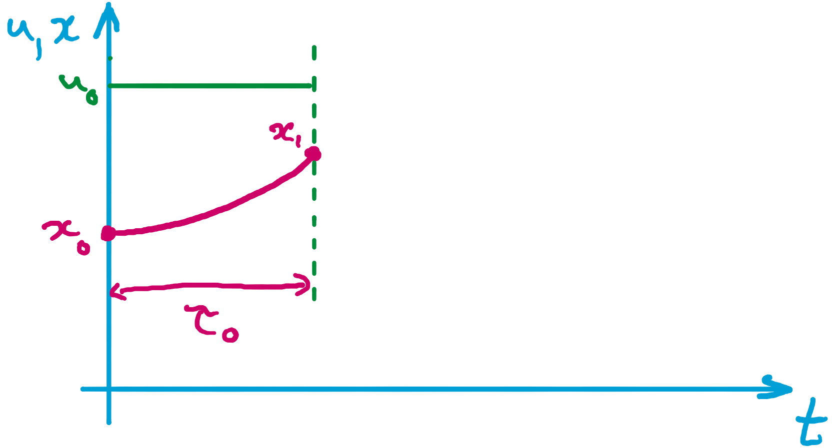
$$\dot{\theta} = v\kappa$$

curvature

$$\underline{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \qquad \underline{u} = \begin{pmatrix} v \\ \kappa \end{pmatrix}$$

Shooting Method





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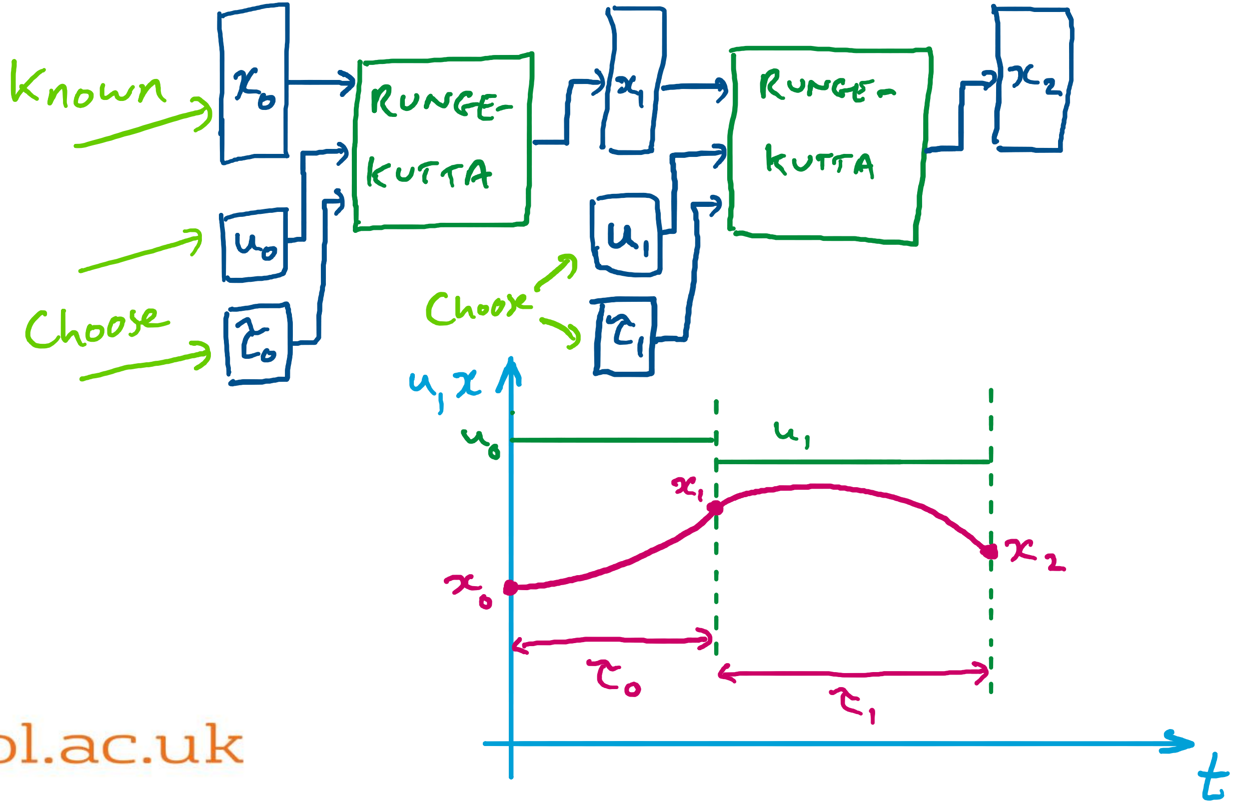

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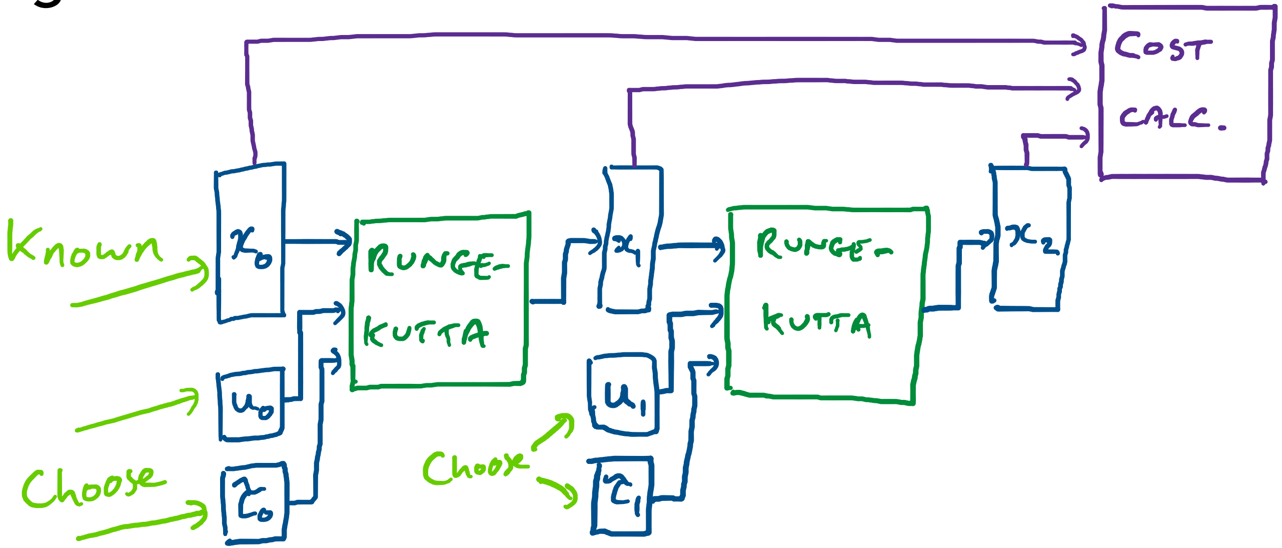

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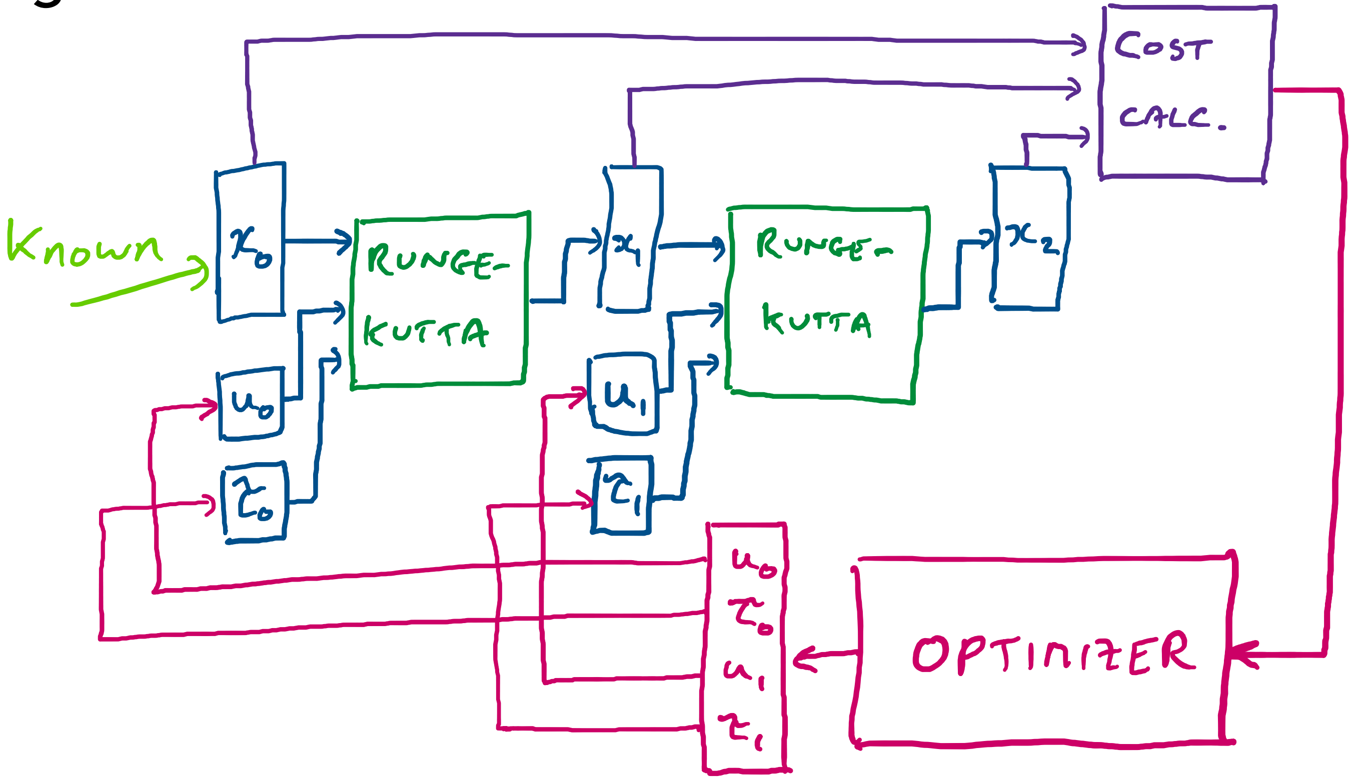

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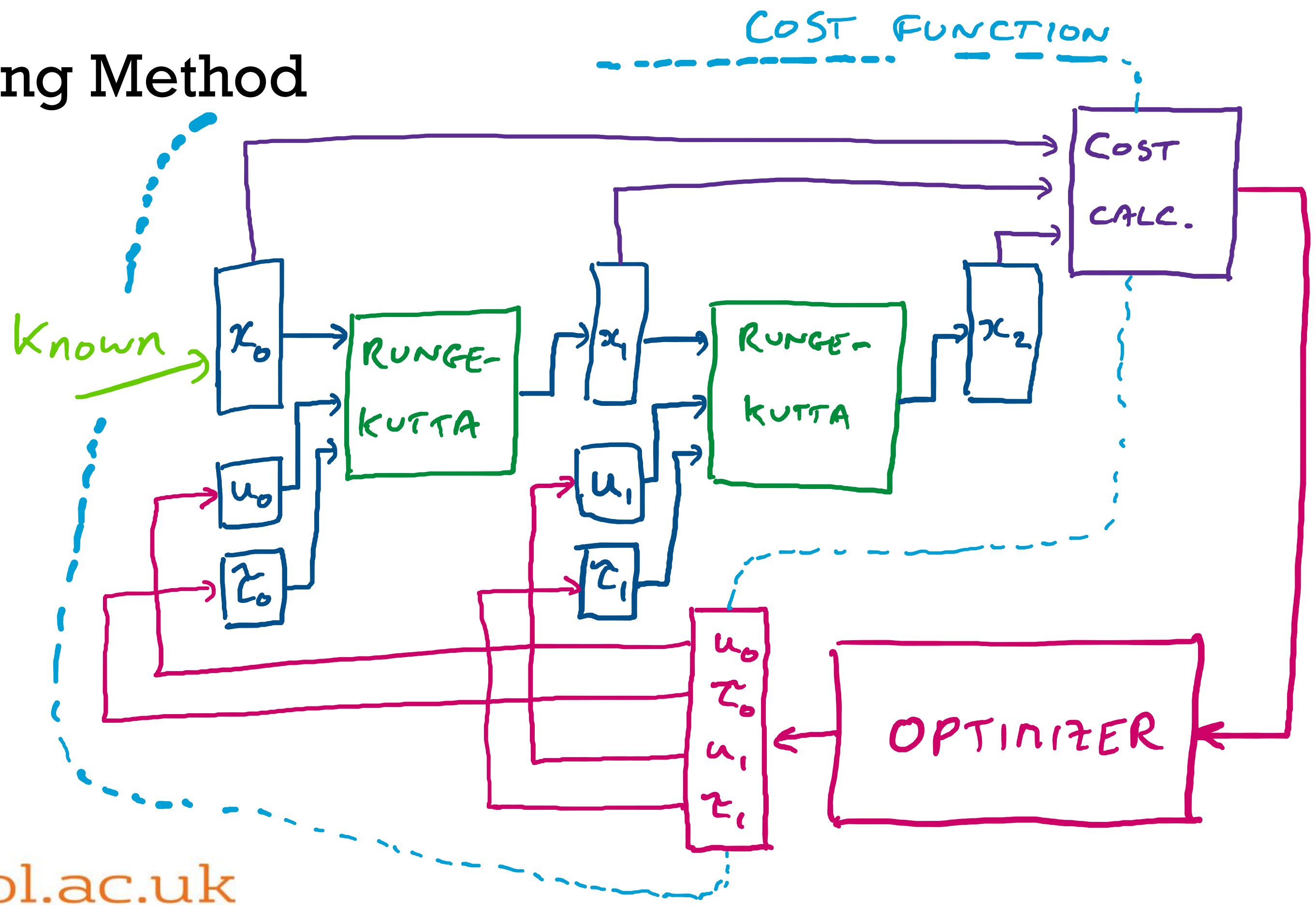


Typical Cost

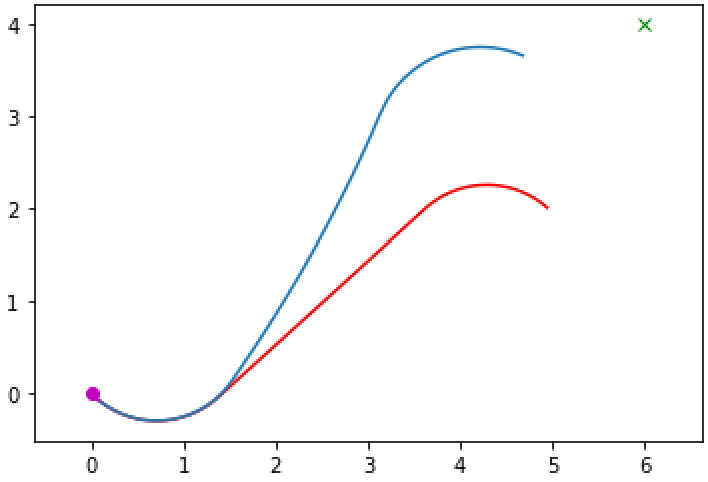
$$J = \underbrace{\tau_0 + \tau_1}_{\text{TIME OF FLIGHT}} + \underbrace{W_f}\,\underbrace{\| x_2 - x_{GOAL} \|}_{\text{DISTANCE FROM GOAL AT END}}$$

$W_f >> 1$ (i.e. big!)

$\Rightarrow$ TOP PRIORITY TO REACH GOAL

Shooting Method Results

- Disappointing at first…



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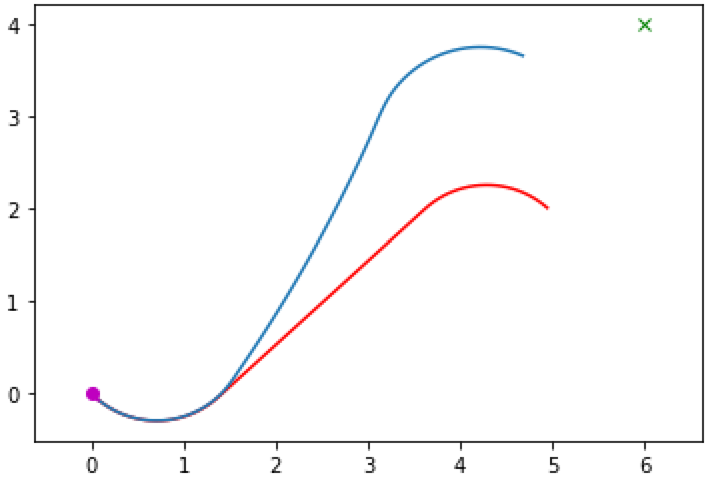
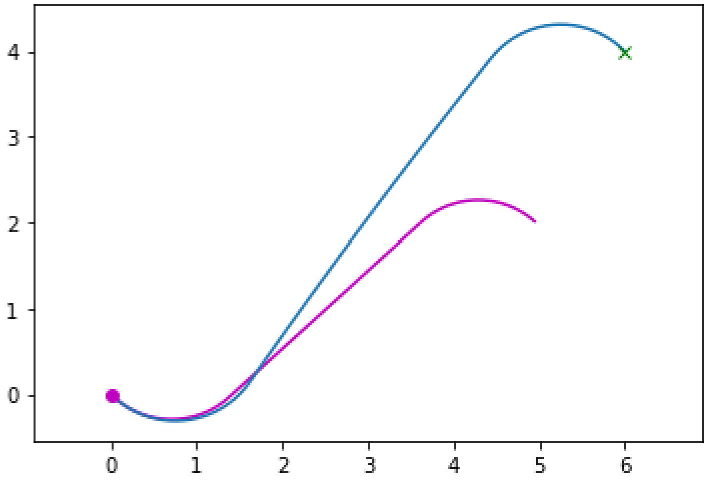
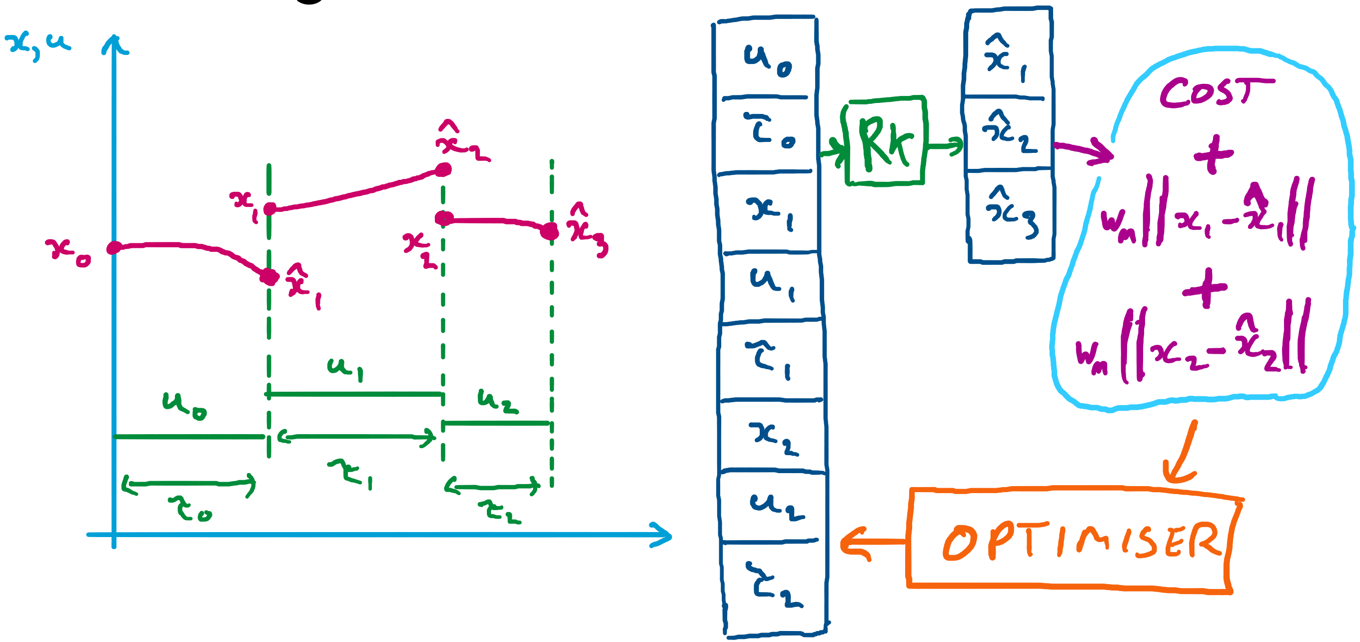


- …but better with a bit of tuning



Multiple Shooting

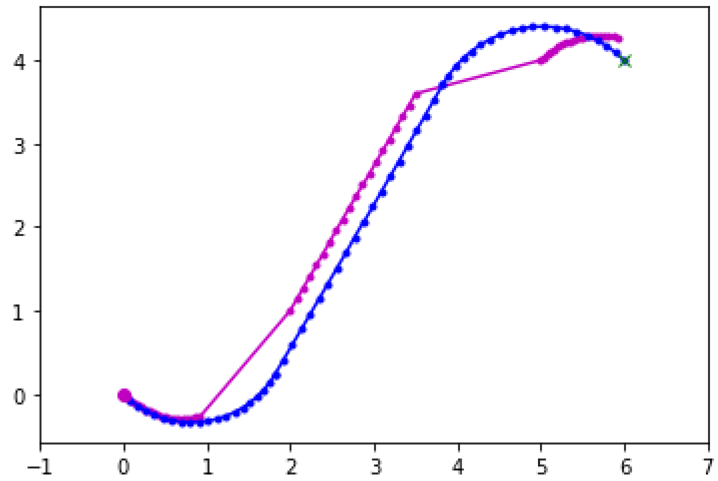


PRIORITIES:

1. CLOSE $x - \hat{x}$ GAPS
2. REACH GOAL
3. MINIMIZE FLIGHT TIME

Multiple Shooting Results

- Much better!



- Weird! More decision variables yet better outcome
  – Structure in the problem helps

Multiple Shooting with Avoidance

$$\text{Cost} = \sum \tau_i \longleftarrow \text{minimize time}$$

$$+\, w_f \| \hat{x}_3 - x_G \| \longleftarrow \text{reach goal}$$

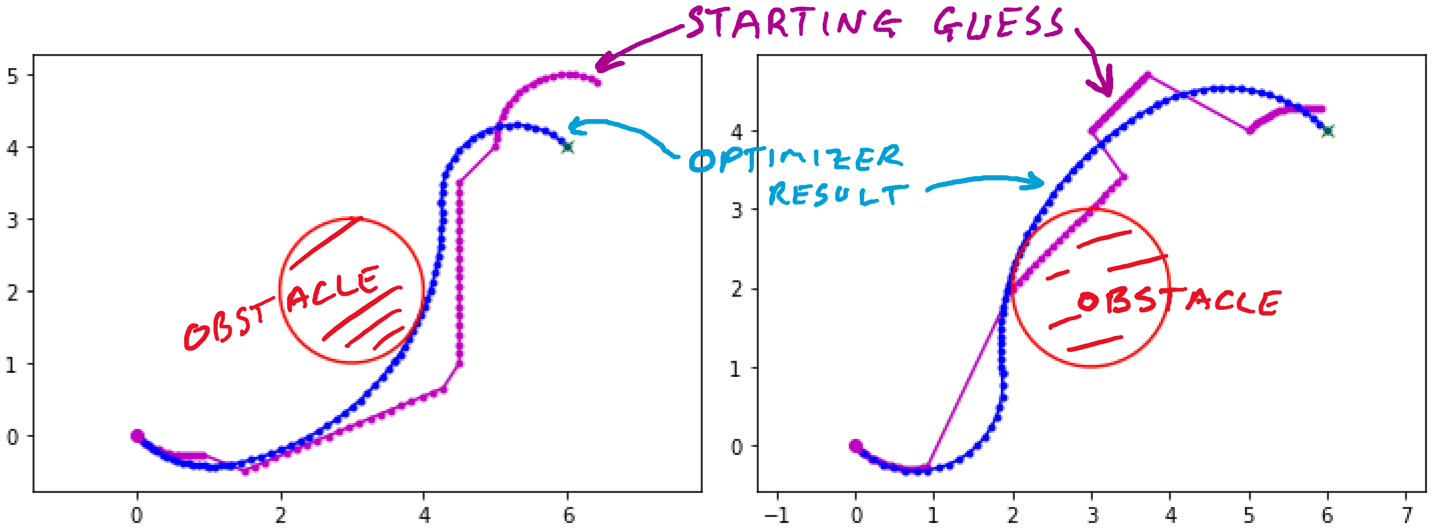
$$+\, w_m \| \hat{x}_2 - x_2 \| + w_m \| \hat{x}_1 - x_1 \| \longleftarrow \text{no gaps}$$

$$+\, w_a \sum_i \max \{ R_o - \| x_i - C_o \|, 0 \} \longleftarrow \text{stay } R_o \text{ away from } C_o$$

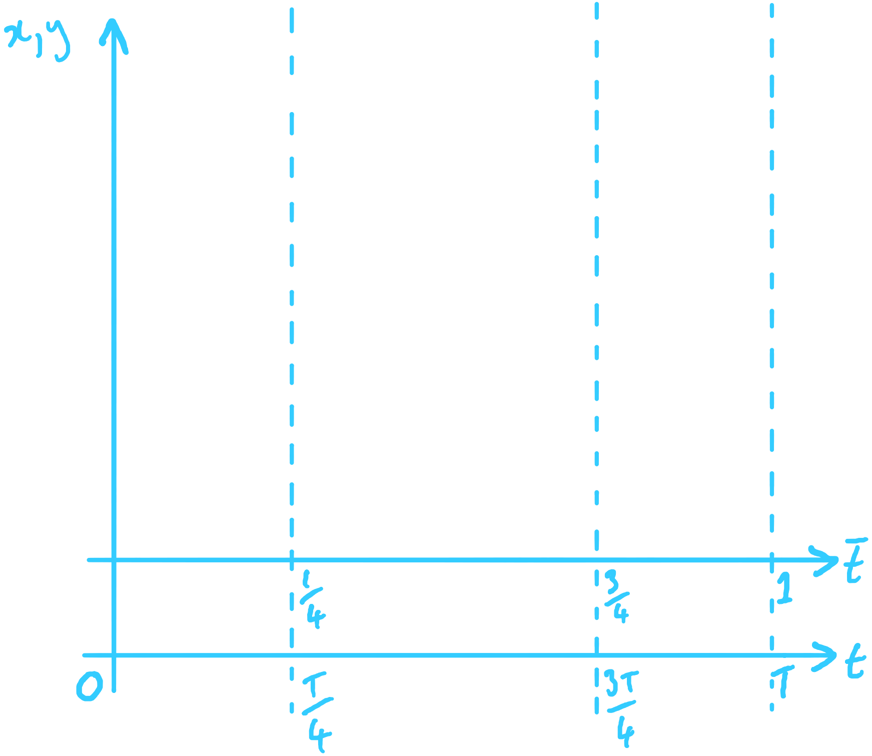
all the little points from RK output

Multiple Shooting with Avoidance

- Works OK. Notice that optimization is only ***local***.



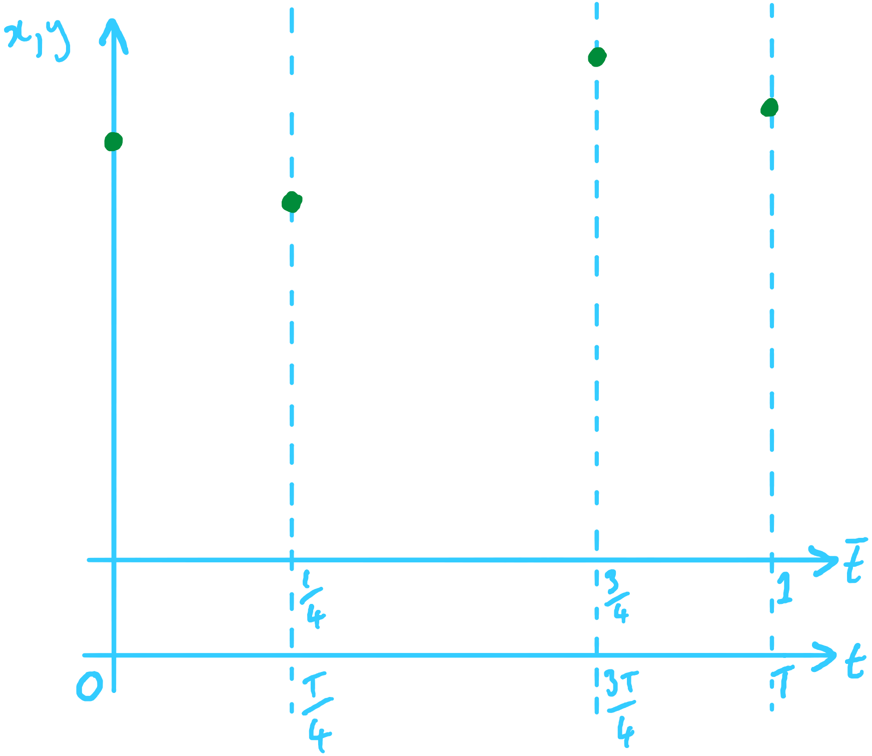
Collocation



<u>Choose</u>:

- Time scale T

Collocation



Choose:

- Time scale T
- Four 'x' points

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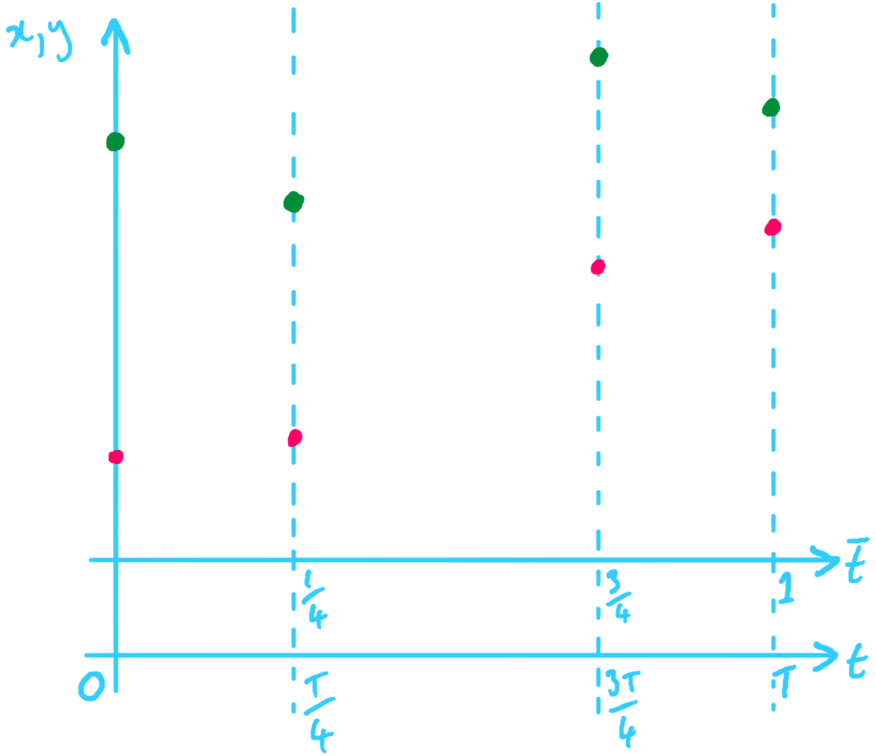
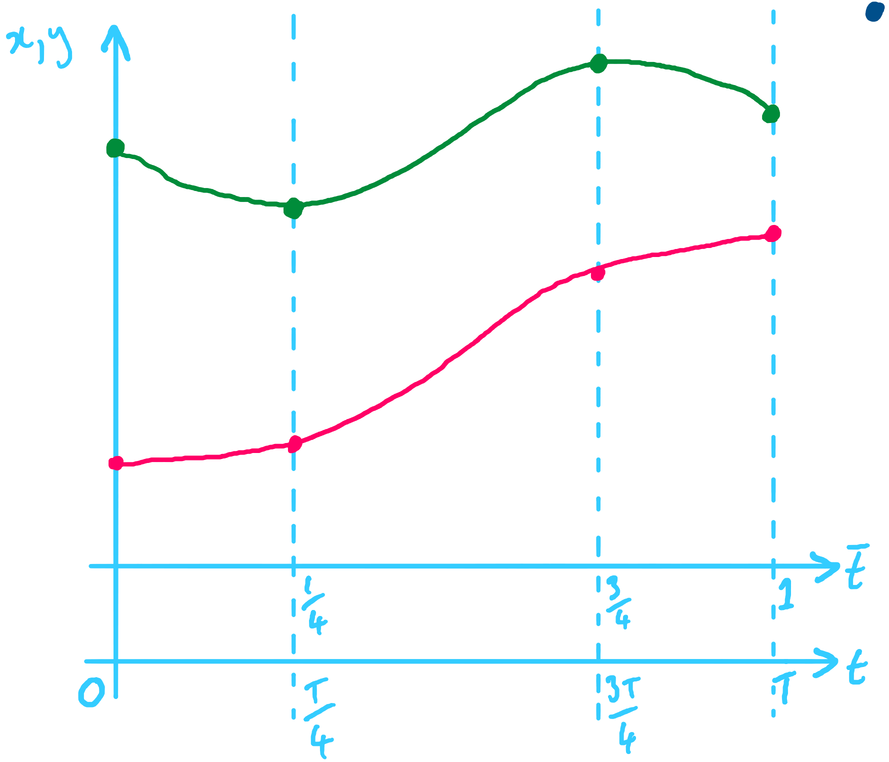
Collocation



Choose:

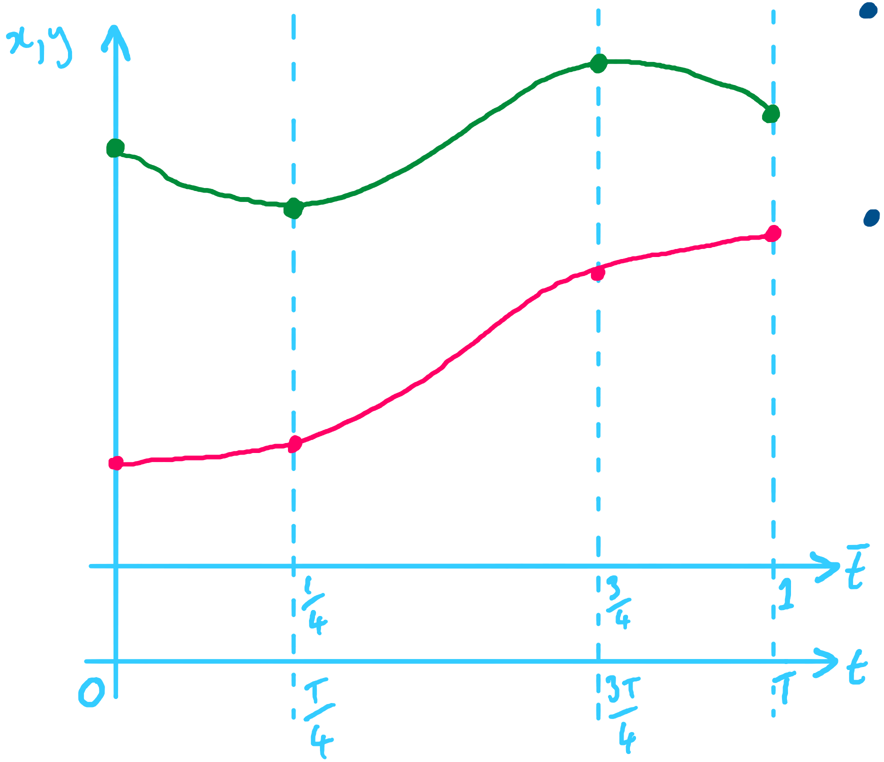
- Time scale T
- Four 'x' points
- Four 'y' points

Collocation



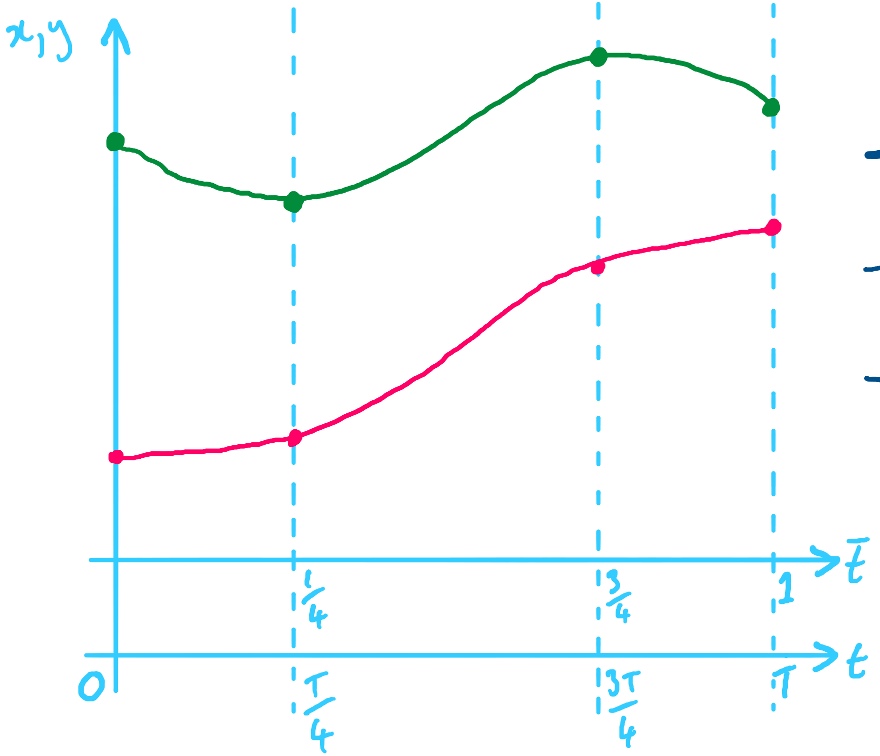
- Fit cubic curves to x, y

Collocation



- Fit cubic curves to x, y
- Calculate derivatives: $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}$

Collocation



- Get states + inputs :

→ $\theta = \tan^{-1}\left(\dot{y}/\dot{x}\right)$

→ $v = \sqrt{\dot{x}^2 + \dot{y}^2}$

→ $\kappa = \dfrac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{v^3}$

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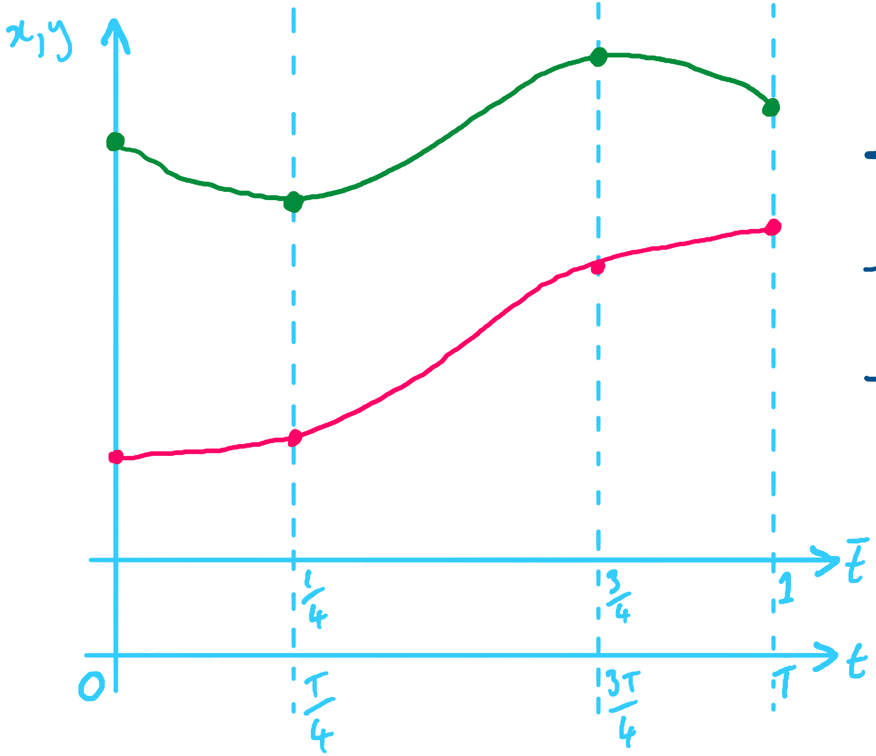

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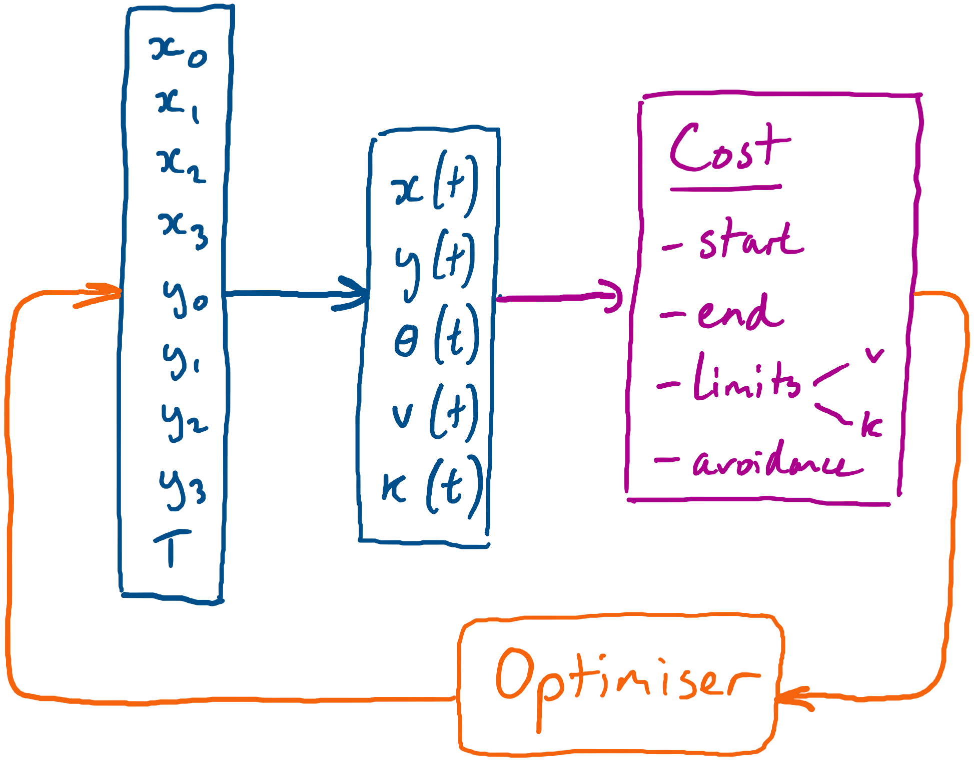
Know whole trajectory!

Collocation



Collocation

- Good results, fast
- Quite robust
- Not quite as flexible as shooting