

Problem 1:

Error formula is $-(b-a)/12 \cdot h^2 \cdot f''(\zeta)$

$$F'(x) = 2 \cdot \pi \cdot x \cdot \cos(\pi \cdot x^2/3)/3$$

$$F''(x) = 2 \cdot \pi/3 \cdot (\cos(\pi \cdot x^2/3) - 2 \cdot \pi \cdot x^2 \cdot \sin(\pi \cdot x^2/3)/3)$$

The largest value that $|F''(x)|$ can achieve in the $[0,1]$ interval is 2.752

So if we wanted an error of magnitude of less than $1e-8$, we want to solve for n that makes the absolute value of the error less than $1e-8$, we have:

$$1/12 \cdot (1/n)^2 \cdot 2.752 \leq 1e-8$$

Solving this inequality, we get $n \geq 4788.8$, so we need at least 4789 subintervals to achieve this accuracy. Therefore, 4790 points are needed.

Problem 2:

$$r = 4, h = (1+1)/4 = 0.5$$

$$I_{\text{simpson}} = h/3 \cdot (f(1) + f(-1) + 2 \cdot f(-1+2 \cdot 0.5) + 4 \cdot (f(-1+0.5) + f(-1+1.5)))$$

Therefore, we get $I_{\text{simpson}} = 1.1667$

For error, we first have $f^{(4)}(x) = 0$

Since error term is multiplied by $f^{(4)}(x)$, if $f^{(4)}(x)$ is zero then the entire expression is zero. So we have zero error in this approximation.

Problem 3:

We have

A:	x:				b:
1	-1	0	0	X1	1.23
1	0	0	-1	X2	1.61
0	1	0	-1	X3	0.45
1	0	-1	0	X4	4.45
0	1	-1	0		3.21
0	0	1	-1		-2.75
1	0	0	0		2.95
0	1	0	0		1.74
0	0	1	0		-1.45
0	0	0	1		1.32

Solving $A \backslash b$ in matlab, we get $x1=2.9600$ $x2=1.7460$ $x3=-1.4600$ $x4=1.3140$

These values appear to be slightly different than the direct measurements. But they tend to have more nonzero digits than the original ones (like $x2$ and $x4$), which implies that they are more precise than the original ones. Also, because we solved in the least square sense to smooth out the error, we do expect it to have less errors than the direct measurements.

Problem 4:

My function: $f(x) = \sqrt{x}$

the errors of composite and adaptive Simpson, and C1 and C2

```
>> main_simpson
```

```
comp_simpson_error =
```

```
1.8225e-12
```

```
C1 =
```

```
68400001
```

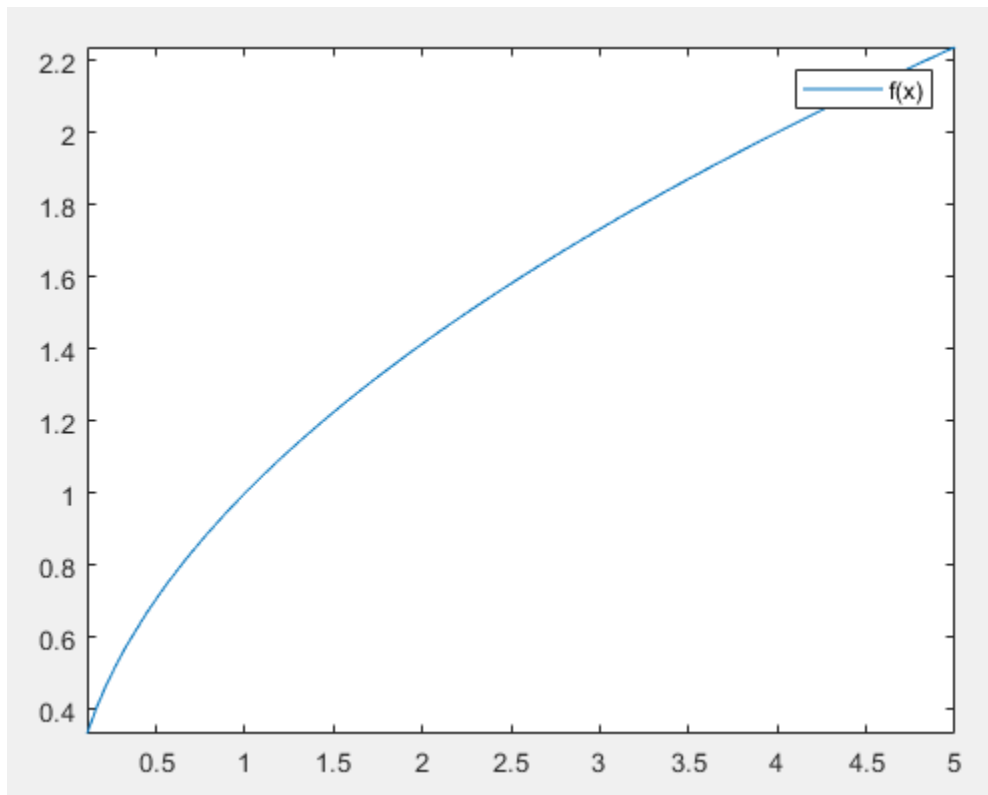
```
adp_simpson_error =
```

```
3.0553e-13
```

```
C2 =
```

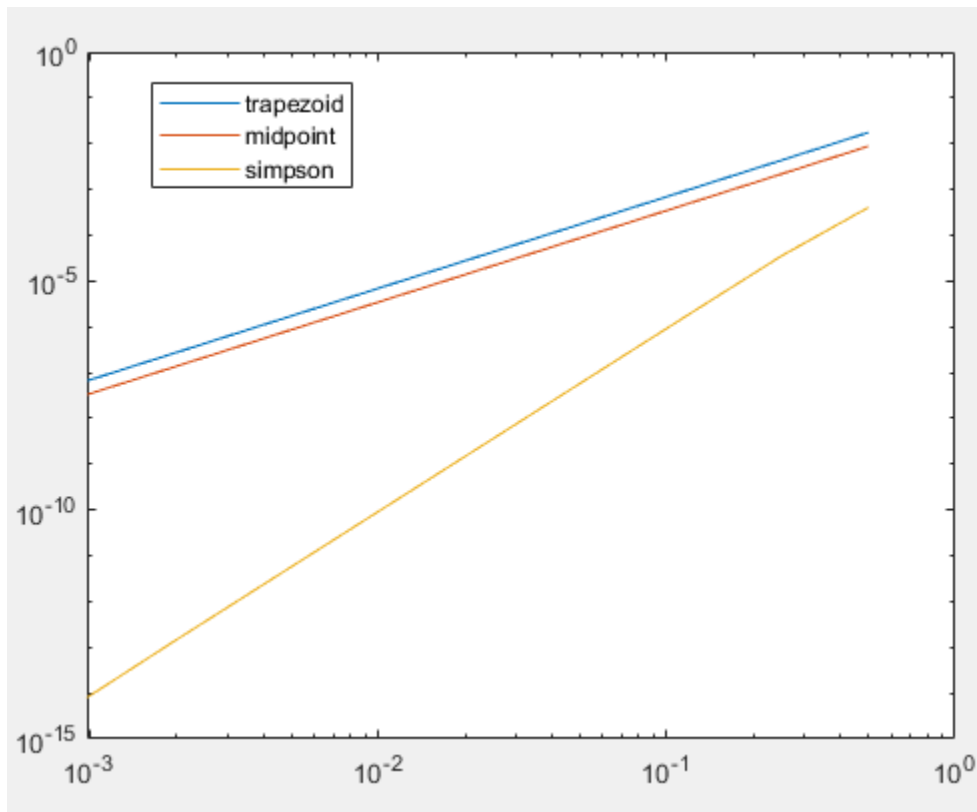
```
6840
```

the plot



Problem 5:

The plot



The computed constants

```
>> main_integration_error
trapezoid 6.95e-02*h^2.00
midpoint 3.48e-02*h^2.00
simpson 7.99e-03*h^3.97
..
```

Problem 6:

```
>> main_orbit
```

PlanetName	a	b	c	d	e
"Jupiter"	-1.1854	0.022029	-0.49504	-0.14505	26.982
"Saturn"	-1.1667	0.035963	0.11673	-1.0899	90.382
"Uranus"	-1.1941	0.011627	1.8271	-0.25926	367.27
"Neptune"	-1.1671	0.020704	-0.39269	-0.42316	903.81
"Pluto"	-1.0033	0.23883	11.847	12.717	1290.7