Supplementary Material: Penicillin Fed-Batch Fermentor Problem with Two Path Constraints

Lizhong Jiang

November 29, 2025

1 Problem Formulation

This document details the penicillin fed-batch fermentation (PFBF) problem with two path constraints, originally based on [1]. The problem is formulated as follows:

$$
\begin{aligned}
\min_{u(t)} \quad & -x_3(t_f) \\
\text{s.t. } \dot{x}_1(t) &= \frac{\mu x_1(t)x_2(t)}{K_l x_1(t) + x_2(t)} - \frac{u(t)x_1(t)}{x_4(t)}, \\
\dot{x}_2(t) &= -\frac{\mu x_1(t)x_2(t)}{Y_{xs}(K_l x_1(t) + x_2(t))} - M_x x_1(t) \\
&\quad - \frac{\theta_m x_1(t)x_2(t)}{Y_p(x_2(t) + K_p + x_2(t)^2/K_i)} + u(t)\frac{S_0 - x_2(t)}{x_4(t)}, \\
\dot{x}_3(t) &= \frac{\theta_m x_1(t)x_2(t)}{x_2(t) + K_p + x_2(t)^2/K_i} - K_{xp}x_3(t) - \frac{u(t)x_3(t)}{x_4(t)}, \\
\dot{x}_4(t) &= u(t),
\end{aligned}
\tag{1}
$$

with the time horizon extended to $t_0 = 0$ and $t_f = 150$. The initial state is given by:

$$x(0) = [1, 0.2, 0.001, 250]^T.$$

The problem includes one control input, $u(t)$, and two path constraints:

$$g_1(x(t)) = x_1(t) - 40 \leq 0, \quad \forall t \in [t_0, t_f], \tag{2}$$

$$g_2(x(t)) = x_2(t) - 0.5 \leq 0, \quad \forall t \in [t_0, t_f]. \tag{3}$$

The control input is bounded as follows:

$$0 \leq u(t) \leq 10, \quad \forall t.$$

The values for the model parameters are provided in Table 1.

Table 1: Parameter values .

K_l	μ	Y_{xs}	θ_m	Y_p	K_i	M_x	K_{xp}	K_p	S_0
6e−3	0.11	0.47	4e−3	1.2	0.1	0.029	0.01	1e−4	400

2 Numerical Setup and Observation

This problem is solved by the proposed method using a piecewise-constant control vector parameterization (CVP) with 75 equal-width intervals. The initial time subintervals for both path constraints are set to coincide with these control discretization intervals. Despite the number of decision variables (75) not matching the total number of constraints generated by the two path constraints (150), the algorithm converges to a KKT point within the specified tolerance after one iteration.

References

[1] Visser, E., Srinivasan, B., Palanki, S., & Bonvin, D. (2000). A feedback-based implementation scheme for batch process optimization. *Journal of Process Control, 10*(5), 399–410.