Supplementary Material: Problem Formulation of `run_Rayleigh_problem`

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1 Problem Formulation

This document details the problem formulation of `run_Rayleigh_problem` [1], which is formulated as follows:

$$\begin{array}{ll} \min\limits_{u(t)} & \int_0^{4.5} (u^2(t) + x_1^2(t))\mathrm{d}t \\ \text{s.t.} & \dot{x}_1(t) = x_2(t) \\ & \dot{x}_2(t) = -x_1(t) + x_2(t)(1.4 - 0.14x_2^2(t)) + 4u(t) \\ & x(0) = [-5, -5] \\ & u(t) + \frac{1}{6}x_1(t) \leq 0, \forall t \in [t_0, t_f] \end{array}$$

with $t_0 = 0, t_f = 4.5$. The path constraint is:

$$u(t) + \frac{1}{6}x_1(t) \leq 0.$$

2 Numerical Setup and Observation

This problem is solved by the proposed method using a piecewise-constant control vector parameterization (CVP) with 20 equal-width intervals. Since the control trajectory $u_{\mathrm{traj}}(t)$ is piecewise-constant, $g(t)$ exhibits discontinuities at the control grid points. By setting the initial constraint subintervals to coincide with the control discretization intervals, the proposed algorithm successfully solves this problem and guarantees the strict satisfaction of the path constraint.

References

[1] Li, B., Yu, C. J., Teo, K. L., & Duan, G. R. (2011). An exact penalty function method for continuous inequality constrained optimal control problem. *Journal of Optimization Theory and Applications, 151*, 260–291.