Supplementary Material: Problem Formulation of `run_Case1`

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December 10, 2025

1 Problem Formulation

This document details the problem formulation of `run_Case1` [1], which is formulated as follows:

$$\begin{aligned} \min_{u(t)} \quad & \int_0^1 \left(x_1(t)\right)^2 + x_2(t)^2 + 0.005u(t)^2)\mathrm{d}t \\ \text{s.t.} \quad & \dot{x}_1(t) = x_2(t) \\ & \dot{x}_2(t) = -x_2(t) + u(t) \\ & x(t_0) = [0, -1] \\ & x_2(t) - 8(t - 0.5)^2 + 0.5 \leq 0 \end{aligned}$$

with $t_0 = 0, t_f = 1$. The path constraint is:

$$x_2(t) - 8(t - 0.5)^2 + 0.5 \leq 0.$$

2 Numerical Setup and Observation

This problem is solved by the proposed method using a piecewise-constant control vector parameterization (CVP) with 1 interval. The initial time subintervals for both path constraints are set to coincide with these control discretization intervals. Despite the number of decision variables (i.e., 1) matching the total number of constraints generated by the two path constraints (i.e., 1), the algorithm converges to a KKT point within the specified tolerance after two iterations.

References

[1] Li, B., Yu, C. J., Teo, K. L., & Duan, G. R. (2011). An exact penalty function method for continuous inequality constrained optimal control problem. *Journal of Optimization Theory and Applications, 151*, 260–291.