An Analysis of Errors in Feature-Preserving Mesh Simplification Based on Edge Contraction

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Abstract

The Quadric Error Metric (QEM) [1] criterion has been widely applied in mesh simplification procedures. Related criteria—such as the Quasi-Covariance Error Metric (QCEM) [2], which may produce superior results to the QEM—have also been reported in recent years. In this paper, the underlying reasons that criteria such as QEM and QCEM are so successful in mesh simplification processes are considered through analysis of error in the simplification process. Focus is on the use of the QEM and QCEM criteria in edge contraction-based mesh simplification.

1 Introduction

Triangle mesh datasets are used in many application domains to represent various structures. Due to increased use of CAD data in virtual engineering and computational science and the increased use of 3D scanning to capture geometry for animation, cultural heritage (e.g., [6]), and other applications, and the increase in scanning resolution, the number and the size of such datasets have greatly increased. Transmitting, storing, and rendering the large mesh datasets can present challenges due to limits in memory, network, bus, and computational capabilities. Simplification of the mesh can allow a smaller model to be handled. Thus, mesh simplification algorithms that simplify a triangulation to an appropriate level of detail before rendering (or before data transferring) can be beneficial, especially if they can maintain a good fidelity to the original mesh.

Mesh simplification mechanisms have been particularly beneficial for level-of-detail processing in gaming, virtual reality, and other animation applications. Mesh simplification can also benefit some finite element analysis processes.

1.1 Edge Contraction and Optimality

The goal of the mesh dataset simplification is to reduce the mesh size, ideally while preserving as many features of the original mesh as possible. One of the typical features to be maintained is geometric shape. Several simplification strategies, such as the edge contraction, are in use. Edge contraction proceeds in an iterative fashion in which each step involves selection of one edge for removal. Each removed edge (and the vertex end points of the removed edge) is replaced by a vertex. The portion of the mesh about the contracted edge is re-triangulated after the replacement. Edge contraction often seeks to achieve preservation of shape by incorporating an error metric to determine which edge's removal produces the minimum change in shape. An optimal error metric would allow finding the edge with minimum error (i.e., the edge, which when removed, produces the mesh that differs least in shape from the original mesh). An alternative is to use a metric that quickly finds a near-optimal edge to remove.

Since edge contraction approaches for mesh simplification operate by iterative edge removal, for an error metric to be practical, it must be able to be computed quickly. Further, it should allow reasonable preservation of the original mesh's geometric shape.

Although many mesh simplification algorithms have been proposed and are in use (e.g., [3, 4, 7, 8, 9]), little work has been done to evaluate the error bound of algorithms relative to optimal edge removal. Two examples of simplification algorithms for which there are bounded errors are the algorithms of Cohen et al. [3]. However, in the Cohen et al. algorithms, the number of triangles is not bounded (i.e., it is not possible to control how many triangles are produced for the simplified mesh) [5]. Heckbert and Garland [5] have used differential geometry and approximation theory to prove that one error metric used in edge contraction-based mesh simplification, the Quadric Error Metric (QEM), is directly related to the surface curvature when triangle areas tend to zero on a differentiable sur-

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face and that in this limit, a triangulation that minimizes the QEM gives an optimal triangle aspect ratio in that it minimizes the L_2 geometric error. While their work showed that QEM was "on the right track" [5], it did not address why QEM is better than the other error metrics that are also based on squared distance. Thus, [5] cannot qualify QEM from other distance-based algorithms. One feature of the current paper is to qualify QEM from the other algorithms.

1.2 Paper Aim and Organization

Error metrics related to the QEM have also been developed. For example, the quasi-covariance error metric (QCEM) [2] has been used for mesh simplification. Computation of this error metric has the same complexity as QEM, which is $O(n \log n)$ for a manifold surface model [5], where n is the number of edges to be removed. Use of the QCEM has been reported to achieve higher fidelity to the original mesh than the QEM. Illustrations showing the flat-shaded renderings of a mesh simplified by removal of 95% of the edges using QEM and QCEM are shown in Figures 8 and 9, respectively. The original mesh is shown in Figure 7.

In this paper, the source of error in QEM and QCEM is analyzed. The remainder of this paper is organized as follows. In Section 2, iterative edge contraction, QEM, and QCEM are reviewed. In Section 3, previous work is described. In Section 4, the analysis of QEM and QCEM is presented. The paper concludes in Section 5.

2 Background

In this section, background on error metric-based iterative edge contraction is presented. The error metrics considered in this paper are also described.

2.1 Error Metric-based Iterative Edge Contraction

The first use of an iterative edge contraction strategy for mesh simplification was by Hoppe et al. [4]. Other works that also use this strategy include [7, 8, 9]. In the Hoppe et al. scheme, each edge in a given mesh is assigned a cost value that reflects how significant the edge is in maintaining shape features. A larger cost value means the edge is more critical to maintaining shape. Thus, the edge with the lowest cost is eliminated first since it maintains the least degree of shape. In an edge contraction algorithm, when an edge is eliminated, a new vertex is introduced as a replacement for the removed edge. This replacement is illustrated in Figure 1. The new vertex is often positioned at an end of the edge or at the midpoint of the edge. Its position can also be obtained with an optimization scheme that finds the location that has the minimum cost—that is, which best maintains shape. When an edge is contracted, a hole results in the mesh. As mentioned before, this hole can be filled with triangles by re-triangulation. The contraction process repeats until a specified number of edges has been eliminated.

In error metric-based mesh simplification schemes, edge contraction cost is calculated with a cost (i.e., error) metric, such as QEM or QCEM, which we describe next. While the QEM and QCEM can be applied to both manifold and non-manifold meshes, we only consider and analyze manifold meshes here.

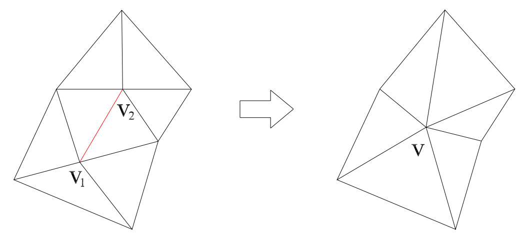

Figure 1: Edge (v_1, v_2) is contracted into vertex v

2.2 Quadric Error Metric (QEM)

The QEM was developed by Garland and Heckbert [1]. In QEM-based simplification, a squared distance sum at each internal mesh vertex is computed. These sums express distances from vertices to the original mesh's facets. In QEM, each sum is an error metric. A metric value is computed as follows. Assume that vertex v is the new vertex resulting from an edge contraction for edge (v_1, v_2). Then, the geometric error $D(v)$ (in the simplified mesh) at v is defined as the sum of squared distances $D_i^2(v)$ of a vertex v to certain facets of the original mesh. This error, D, is given by:

$$D(v) = \sum_i D_i^2(v),$$

where $D_i^2(v)$ is the squared distance between the vertex v and the plane of the i-th facet, where the plane's unitized normal is $n_i = [a_i \; b_i \; c_i]^T$ and its equation is $a_i x + b_i y + c_i z + d_i = 0$. The $D_i^2(v)$ can be written as:

$$D_i^2(v) = (n_i^T v + d_i)^2.$$

By using a homogenous coordinate system, this formulation can be expressed as:

$$D_i^2(v) = (n_i^T v + d_i)^2 = \left(\begin{bmatrix} a_i & b_i & c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \right)^2$$

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$$= (\tilde{n_i}^T \tilde{v})^2 = (\tilde{v}^T \tilde{n_i})(\tilde{n_i}^T \tilde{v}) = \tilde{v}^T (\tilde{n_i}\tilde{n_i}^T)\tilde{v} = \tilde{v}^T K_{pi} \tilde{v},$$

where $\tilde{v}^T = \begin{bmatrix} x & y & z & 1 \end{bmatrix}$ is a homogeneous representation for v, $\tilde{n_i}^T$ is $\begin{bmatrix} a_i & b_i & c_i & d_i \end{bmatrix}$, and K_{pi} is:

$$K_{pi} = \begin{bmatrix} a_i a_i & a_i b_i & a_i c_i & a_i d_i \\ b_i a_i & b_i b_i & b_i c_i & b_i d_i \\ c_i a_i & c_i b_i & c_i c_i & c_i d_i \\ d_i a_i & d_i b_i & d_i c_i & d_i d_i \end{bmatrix}. \quad (1)$$

Garland and Heckbert call matrix K_{pi} the fundamental error quadric. Thus, the error $D(v)$ can be written as

$$D(v) = \sum_{i=1}^{m} \tilde{v}^T K_{pi} \tilde{v} = \tilde{v}^T Q_v \tilde{v}, \text{ where}$$

$$Q_v = \sum_{i=1}^{m} K_{pi}, \quad (2)$$

and m is the number of facets considered.

In applying QEM in edge contraction with v' the replacement for (v_1, v_2), $D(v')$ is taken as:

$$D(v') = \tilde{v'}^T [Q_{v_1} + Q_{v_2}] \tilde{v'}.$$

For a vertex v of the original mesh, the K_{pi}'s are for facets meeting at v. In simplification, as edges are contracted, sums of Q's are aggregated for each replacement vertex.

2.3 Quasi-Covariance Error Metric (QCEM)

QCEM [2], which is a generalization of QEM, is derived from the concept of covariance. Since covariance expresses the strength of the relation between variables, it is appropriate to use covariance to find the features that are embodied in the relationship between variables. The QCEM considers not only the relationship of one vertex with the individual triangles, which is the case in QEM, but it also incorporates relationships between triangles. Based on this concept, the QCEM formulation uses as replacements for each Q (Eqn. 2) the expression:

$$\sum_{i=j} C_{ij} + k \sum_{i \neq j} C_{ij},$$

where i and j are triangle indices, k is a constant, and C_{ij} is as follows:

$$C_{ij} = \begin{bmatrix} a_i a_j & a_i b_j & a_i c_j & a_i d_j \\ b_i a_j & b_i b_j & b_i c_j & b_i d_j \\ c_i a_j & c_i b_j & c_i c_j & c_i d_j \\ d_i a_j & d_i b_j & d_i c_j & d_i d_j \end{bmatrix}.$$

3 Previous Work

In Heckbert and Garland's paper [5], they considered why the QEM-based edge contraction mesh simplification works as well as it does and how the approximations generated using QEM can be close to optimal. Specifically, differential geometry and approximation theory was used to show that as triangle area tends to zero on a differentiable surface, the QEM will be directly related to surface curvature. When the area limit is approached, a triangulation that minimizes the QEM achieves the optimal triangle aspect ratio due to the metric's minimization of geometric error. Their work attempted to relate practical surface simplification methods from computer graphics with theoretical, asymptotic results from approximation theory. However, as Heckbert and Garland mentioned [5], their analysis does not address why the QEM-based algorithm yields better approximations for a finite polygonal mesh than many other approaches.

Although their analysis was presented only as a validation that QEM is theoretically "on the right track" in the sense that as the original mesh becomes finer and finer, the resulting approximation will become more nearly optimal, however, in practical applications, the mesh is usually a finite polygonal mesh. Furthermore, since the goal of simplification is to reduce the number of facets in a given polygonal model, we always get a model with a mesh that really cannot be said to be a fine mesh.

The paper here presents a new analysis that can explain why QEM and QCEM work for mesh simplification. Our focus is on the analysis of finite polygonal meshes, instead of the case when triangle area tends to zero.

4 Contributing Factors for the optimality of QEM and QCEM

In this section, we will discuss in detail the factors that contribute to the optimality of the QEM and QCEM. These factors explain why quadric-related approaches are better than other approaches. Some QEM characteristics that are improved on by QCEM are also discussed.

4.1 Min Curvature Change

In this section, we show a new development relating QEM to curvature when triangle areas tend to zero. We also explain that QEM is actually the mean of curvature change. The relation of ACEM with curvature is also shown.

4.1.1 Relation between QEM and Curvature

In essence, QEM is extracted from the sum of the distances between the new vertex and certain triangles of the original mesh (initially, the triangles that meet at the end points of the edge the new vertex replaces). QEM establishes the error associated with each edge deletion as was shown as in Equation 2. In fact, Equation 2 is a reformulation of the following relation:

$$f = \sum_{i=1}^{m} D_i^2, \tag{3}$$

where D_i is the distance between the new vertex and the i-th facet of the m facets meeting at an old vertex.

We can denote the area of the i-th triangle that contains the vertex to be deleted as A_i. When A_i tends to zero, the implication is that the triangle has been refined so that it almost exactly matches the true underlying surface. Therefore, all distances D_i will tend toward being equal for such a mesh. Thus, from Equation 3 we will have

$$f \doteq k D_t^2, \tag{4}$$

where D_t is the distance between the new vertex and the tangent plane of the edge end point vertex that is to be eliminated.

Comparing Equation 4 with Equation 2, without considering any relations in differential geometry, we know that when A_i tends to zero, the Q is only related to the number of planes linked to the vertex and the distances from the newly generated vertex to the tangent plane that passes though the vertex to be eliminated.

However, when A_i tends to zero, there is indeed a differential relationship between the tangent plane of the vertex to be eliminated and the new vertex. If we assume that the signed distance between the i-th facet plane and the new vertex is D_i, we have, from the Equation 2 development:

$$D_i = a_i x + b_i y + c_i z + d_i,$$

assuming (a_i, b_i, c_i) is a unit length normal.

For a given vertex, when each i-th triangle is refined so that A_i tends to zero, we have the following relationship:

$$\begin{aligned} d(D_i) &= x\frac{\partial D_i}{\partial a_i} d(a_i) + y\frac{\partial D_i}{\partial b_i} d(b_i) + \\ &\quad z\frac{\partial D_i}{\partial c_i} d(c_i) + \frac{\partial D_i}{\partial d_i} d(d_i) \\ &= x^2 d(a_i) + y^2 d(b_i) + z^2 d(c_i) + d(d_i). \end{aligned}$$

Clearly, when

$$d(a_i) = d(b_i) = d(c_i) = d(d_i) = 0,$$

which is the case when $A_i = 0$, we have

$$d(D_i) = 0.$$

Since D_i equals the distance between the new vertex and the tangent plane of the vertex to be deleted, this proves again the correctness of Equation 4, proceding only from the terms in the Eqn. 3.

Next, we consider the relationship between curvature and QEM. The Gaussian curvature of a point v on a surface is the product of the principal curvatures, k_1 and k_2, of the given point. We discuss the effect on QEM of each of the principal curvatures separately for the case where triangle areas tend to zero. For k_1, we can consider the projection of a curve that is on the surface along the principal direction indicated by principal curvature k_1 onto a 2D plane. Thus, the curvatures at the projected point v' of point v and the point v are the same, and they are all k_1. For this 2D curve, the curvature at point v' can be expressed as follows:

$$k_1' = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]} = \frac{1}{\alpha}\,, \tag{5}$$

where (x, y) is the coordinate and α is the radius of curvature, as shown in Figure 2.

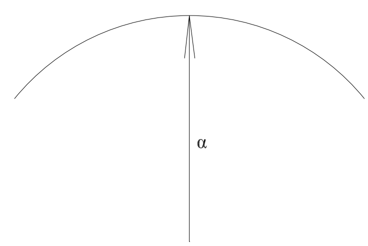

Figure 2: α is the radius of curvature

We can derive a similar relationship for principal curvature k_2. Note that it is possible for the radius of curvature to be different in the two cases.

From Equations 4 and 5, we can see that both f and k_1' are related to distances. In the next section, will see that f embodies the change of the radius of curvature α.

4.1.2 QEM as Mean of Curvature Change

Although the Gaussian curvature of a given surface point depends on the two principle curvatures at the point, its intrinsic definition at a point v is the following: for a short line segment ab of length r with its end a tied to v, then, if the end b runs around v while the line segment is completely stretched and we measure the length $P(r)$ of one complete trip around v, the Gaussian curvature is defined as $K = (2\pi r - P(r))3/\pi r^3$.

If we consider a circle with perimeter of $2\pi r$ on a given plane, then, we can see that the above relation can be transformed so that the variable K is a function of the sum of distances from each point of the curve $P(r)$ to the plane. Thus, we can measure the sum of distances to calculate the

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curvature at the point v. Furthermore, we can also use the relation to calculate the change of curvature. For example, for a given curvature, if only n points have changed the distances to the circle on a plane, then, the sum of the changes of these n distances will give us the change of curvature.

Based on the above reasoning, for a polygonal mesh where A_i does not necessarily tend to zero, while it is precise to use an infinite number of triangles to find a good estimation of curvature at a point v, we can actually use the available finite number of triangle patches that connects the point v to find a good estimation. Thus, the best estimation of curvature change at a given candidate replacement vertex comes when we consider all the distances from that vertex to the planes that meet at the edge to be deleted.

Since QEM and QCEM use a sum of distances, each of which approximately measures the change of some curvature at a vertex v, QCEM and QEM are appropriate metrics. The sum of the distances is a measure of the change of the curvatures in the given set of triangles connected to the vertex to be deleted. Thus, a smaller QEM (or QCEM) means smaller change in curvature; deleting the minimum error edge changes shape (curvature) the least.

An alternative to using distance sums as QEM and QCEM do is to use maximum distances. An error metric that considers only maximum distance as a measure of the change of the curvatures in the given set of triangles connected to the vertex to be deleted will be worse than quadric-related metrics. In a maximum distance metric, only the change of one curvature is approximated, which may not even be the most representative curvature. Clearly, maximum distance-based metrics cannot express the true change of curvature at the vertex compared with QEM. This explains why QEM is better than an error criterion based on maximum distance.

However, since QEM didn't consider the relation between triangles that connect the vertex v, QEM is, in fact, the mean of the curvature change. QCEM solved this problem by considering the variation between triangles. This issue is further discussed in Section 4.3.2.

Now we discuss the effect of triangle aspect ratio on mesh simplification results. Aspect ratio (AR) measures the regularity of the triangle. The goal of QEM is to help simplify a mesh while keeping as many features of the original mesh as possible. In effect, QEM is changing ARs to realize this goal. When a feature that is represented with many triangles is greatly simplified, many triangles will be deleted. Thus, only a limited number of triangles that best fit the feature should be kept in order to best describe the feature. The number of triangles should be minimum relative to the given simplification requirement, such as to reduce the mesh size by some percentage while keeping as many features as possible. The way these triangles are made to embody the features is by being stretched or compressed (i.e., having their AR changed) based on the nature of the feature. QEM can automatically adjust the AR of triangles based on the sum of distances between the new vertex and the planes that are linked to the vertex to be deleted. The adjustment is how feature preservation occurs. Thus, triangle AR adjustment through edge contraction is an important factor in the success of QEM.

4.2 Min Error Propagation

In this section, we discuss the error minimization that exists in two parts of QEM-based mesh simplification: edge contraction and overall edge simplification.

4.2.1 Error minimization in single edge contraction

As stated before, QEM takes the new vertex's Q as the sum of the Q 's at both ends of the edge (v_1, v_2):

$$Q_v = Q_{v1} + Q_{v2}.$$

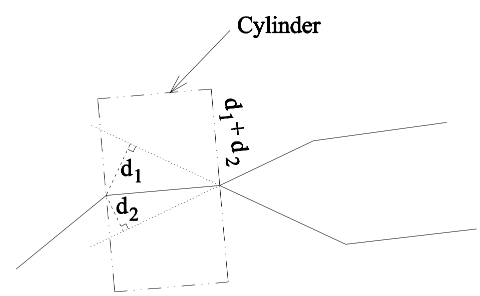

Figure 3: Illustration of line segments before edge contraction, with distances as error measure

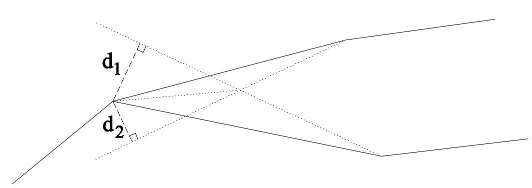

Figure 4: Illustration of line segments after edge contraction, with distances as error measure

Thus, the distance sum Q_v at a new vertex is the sum of all the distances between the new vertex and all the original mesh triangles that are directly linked to the edge to be contracted or which are linked to some edge that has been contracted into a predecessor of the edge. We call this process of iterative contraction a *clustering*. The distance sum after clustering can be viewed as defining a cylinder about the

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edge. The cylinder for a point v has the radius of $D(v)$, with the contracted edge as its axis. Figure 3 illustrates the idea for a 2D curve edge contraction example. In the figure, dashed-dotted lines form a cylinder of radius $d_1 + d_2$. In this example, the new vertex's position is at an end point of the edge that is contracted. Solid lines represent curve segments. Dotted lines represent extensions of segments. Dashed lines represent the distances from the new vertex to the segment extensions. In this case, the sum $d_1 + d_2$ is the Q for the new vertex. Figure 4 illustrates the line segments after edge contraction.

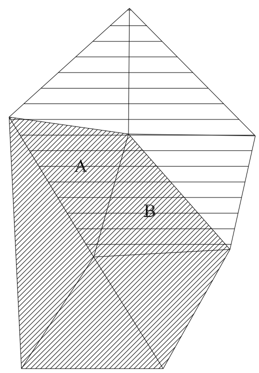

Figure 5: Shadings showing triangles considered by error metric for (v_1, v_2)

Although the above example uses the existing end of the edge to be deleted as the new vertex, it also applies to the cases where the new vertex is in any appropriate location. In such a case, the distances between the new vertex and all the triangles that connect both ends of the edge to be deleted (and distances to one of the predecessor contracted into this edge) should be considered. It is easily seen that this consideration does not change the cylindrical bound on error in position.

4.2.2 Error minimization in overall mesh simplification

In edge contraction, the QEM at each vertex represents the sum of the distances that are measured from the current vertex to the original mesh, as its formulation derivation shows. In other words, the quadric error at each vertex, no matter how many edges have been contracted, is related to the sum of the distances to the original mesh's triangles whose elimination due to edge contraction leads to the current vertex. Figure 6 shows the clustering as a tree structure. In this figure, different levels of clustering are shown. The Level 0 represents the vertices in the original mesh. Each level corresponds to the vertices at a level of simplification. The advantage of the clustering process used in QEM is that it enables the error calculation in any simplified mesh to be always based on the original mesh, which avoids errors that could have resulted if there was no reference back to the original.

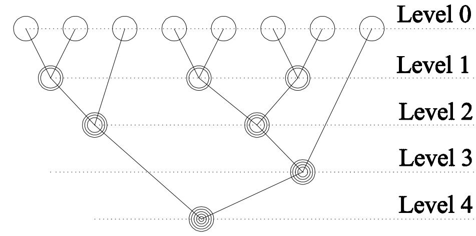

Figure 6: Clustering structure at different levels of simplification

4.3 Optimal Feature Preservation

Next, we consider some weakness in QEM-based edge contraction and some measures to improve on the weakness: redundancy and exclusion of the deviation of curvature change.

4.3.1 Redundancy in QEM

In the edge contraction process in QEM, the distances from the new vertex to the two triangles that share the edge to be contracted have been added twice, which introduces a bias in the sum. This source of bias has been mentioned in [1], and is illustrated in Figure 5. In the figure, the triangles counted for one end point of the edge to be contracted are shown in one shading and the triangles for the other end point are shown in another shading. In this example, factors for Triangles A and B have been added twice in the sum while other triangles were considered just once. Due to the existence of multiply added triangles, the QEM of each new vertex is not the actual error. As processing progresses, error can increasingly deviate from the true value. Thus error build-up can occur from the QEM formulation itself.

We propose remedy of this bias by removing the redundancy in counting. Our remedy increases memory consumption only slightly.

Figure 10 shows an apple dataset, a QEM simplified mesh that was based on the redundancy in counting, and a QEM simplified mesh that was not based on redundancy in counting. It is easy to see that over half of the concave part near the apple stem is gone in the QEM-based result while the concave region is retained in the modified one. Figure 11 shows similar comparison for a sphere dataset. The upper left result for original QEM is not as spherical as the modified QEM method. In both cases, removing the counting redundancy gives better visual results.

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Table 2 summarizes mesh goodness for these examples using the Attribute Deviation Metric (ADM) [10], which is a quality assessment measure based on appearance attributes. This assessment measure allows the measurement of local quality and the computation of global quality statistics of a simplified mesh. The metric values in this table show that mean values for the models are all smaller for non-redundant counting than when redundant counting is used, although the variance in the apple case is better for the redundant counting strategy. However, we are primarily concerned with mean values. Thus, the results for the formulation without repetitive counting seem to confirm the theoretical basis for avoiding redundancy.

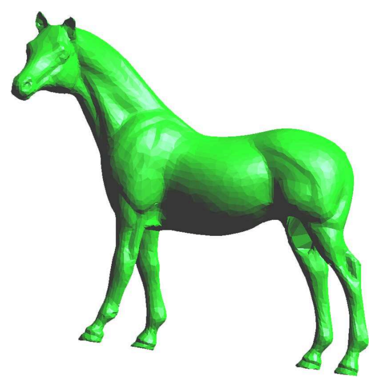

Figure 7: Original horse dataset

Figure 8: Simplified horse dataset with QEM; 95% reduction

Table 1: Attribute Deviation Measures for Horse Model

		QEM	QCEM
Horse	Mean	6.88E-04	6.38E-04
	Variance	3.79E-07	3.38E-07

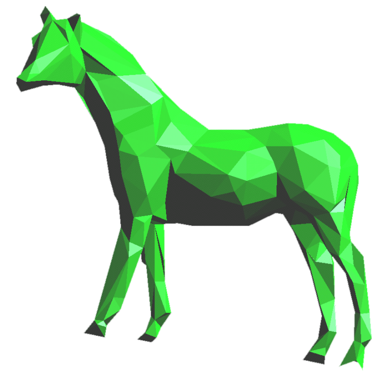

Figure 9: Simplified horse dataset with QCEM; 95% reduction

Table 2: Attribute Deviation Measures QEM-simplified for Models

		w/ redundancy	w/o redundancy
Apple	Mean	0.00103712	0.00102025
	Variance	8.20E-07	8.53E-07
Sphere	Mean	6.98642	6.71417
	Variance	2.12E+01	1.97E+01

4.3.2 Deviation of Curvature Change

One shortcoming of QEM is that it does not consider the variations between triangles. It is easily understood that a vertex with linked triangles that have a large variation in orientations contains more shape features than a vertex with linked triangles that have little variation in orientations. This shortcoming is solved by the QCEM, which considers the features that exist due to the variations among triangles.

Figures 8 and 9 show the difference between QEM and QCEM for the horse dataset shown in Figure 7. The ADM measures are shown in Table 1. The mesh simplified by QCEM is better than the one simplified by QEM, according to both visual and numerical assessment.

5 Conclusions

We have considered the errors in the QEM and the QCEM. By considering the geometric bound and the parameters that are related to the geometric features, we have shown that QEM is directly related to the geometric bound on error in edge contraction-based mesh simplification in real, finite-size problems. Furthermore, we have shown that QEM considers mean curvature change whereas the QCEM considers variation in the curvatures. We have also shown that the QEM doesn't consider all the errors that are used to de-

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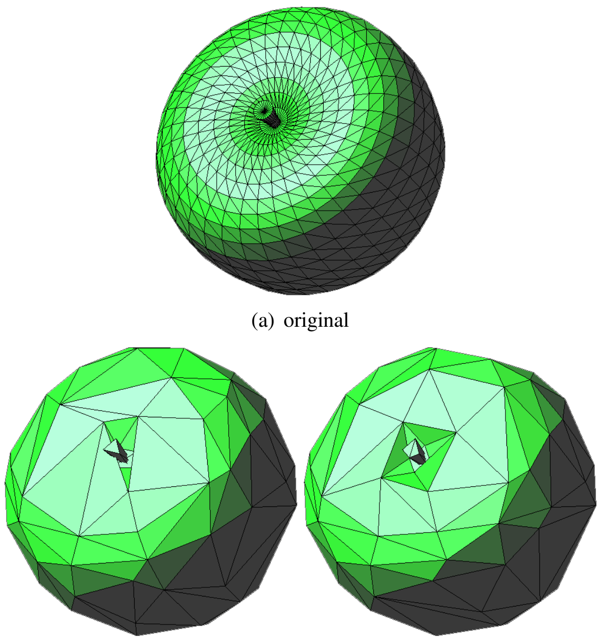
scribe features contained among planes that are connected to a vertex of interest. Improvement of QEM by avoiding redundancy in counting was also considered. Finally, metric relation to surface curvature was described.

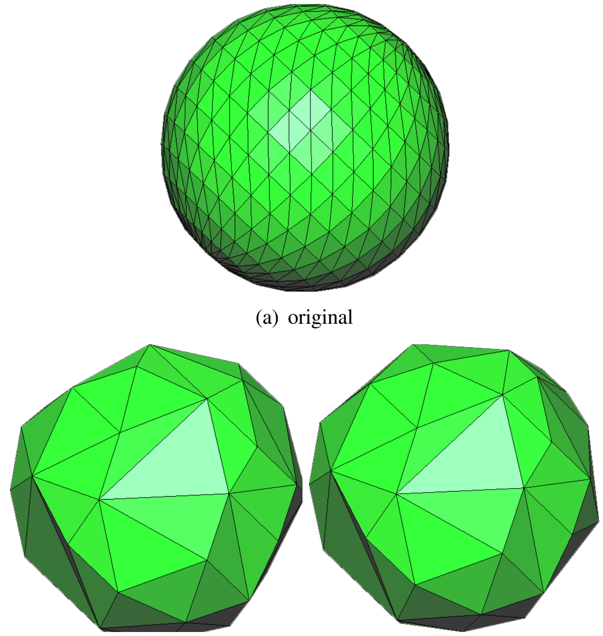


(a) original

(b) Original QEM

(c) Modified QEM

Figure 10: Apple data set with 95% reduction



(a) original

(b) Original QEM

(c) Modified QEM

Figure 11: Sphere data set with 95% reduction

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