TRIGONOMETRIA

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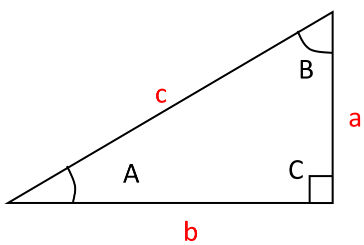
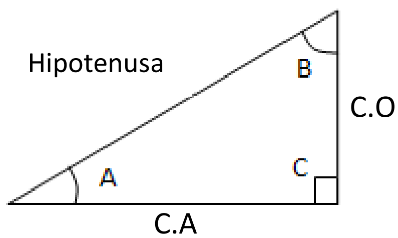
TRIGONOMETRIA

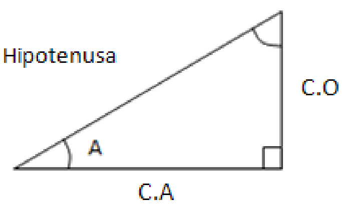
Parte de las matemáticas que relaciona la geometría con el algebra, para el estudio de triángulos, la relación que existe los lados y ángulos de triángulos.

RAZONES TRIGONOMETRICAS

Supongamos un triangulo rectángulo con un ángulo recto y dos ángulos agudos, cada uno de estos ángulos contendrá una relación con cada uno de los lados que contenga dicho triangulo a esas relaciones, las conocemos como: **razones trigonométricas**







$$\text{SenA}=\frac{C.O}{H}$$

$$\text{CosA}=\frac{C.A}{H}$$

$$\text{TanA}=\frac{C.O}{C.A}$$

$$\text{CotA}=\frac{C.A}{C.O}$$

$$\text{SecA}=\frac{H}{C.A}$$

$$\text{CscA}=\frac{H}{C.O}$$

$$\text{SenA}=\frac{A}{C} \qquad \text{SenB}=\frac{B}{C}$$

$$\text{CosA}=\frac{B}{C} \qquad \text{CosB}=\frac{A}{C}$$

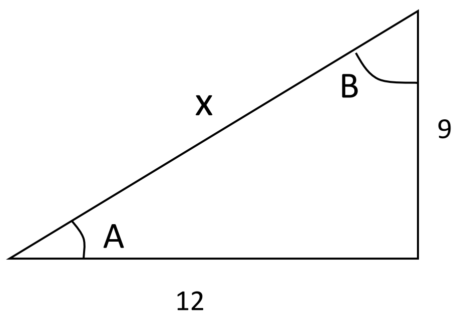
$$\text{TanA}=\frac{A}{B} \qquad \text{TanB}=\frac{B}{C}$$

$$\text{CotA}=\frac{B}{A} \qquad \text{CotB}=\frac{A}{B}$$

$$\text{SecA}=\frac{A}{B} \qquad \text{SecB}=\frac{C}{A}$$

$$\text{CscA}=\frac{C}{A} \qquad \text{CscB}=\frac{C}{B}$$

EJERCICIO:



$$x^2 = 12^2 + 9^2$$

$$x^2 = 144 + 81$$

$$x^2 = \sqrt{225}$$

$$x = \quad 15$$

Angulo A

SenA= $\frac{9}{15}$

CosA= $\frac{12}{15}$

TanA= $\frac{9}{12}$

CotA= $\frac{12}{9}$

SecA= $\frac{15}{12}$

CscA= $\frac{15}{9}$

Angulo B

SenB= $\frac{12}{15}$

CosB= $\frac{9}{15}$

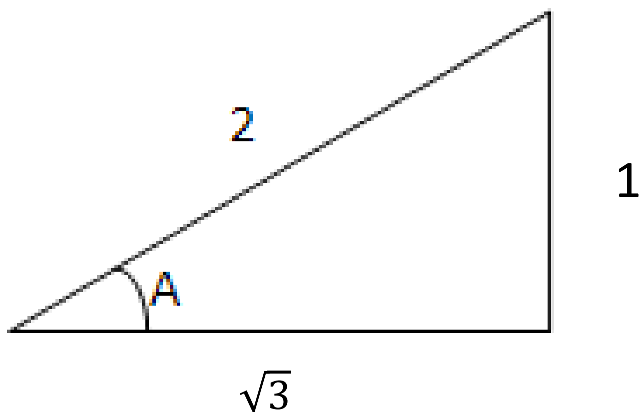
TanB= $\frac{12}{9}$

CotB= $\frac{9}{12}$

SecB= $\frac{15}{9}$

CscB= $\frac{15}{12}$

En el siguiente triangulo el valor de la Csc=2 calcular el valor de las demás razones (tarea).



$$C^2 = b^2 + a^2$$
$$2^2 = b^2 + 1^2$$
$$4 = b^2 + 1$$
$$b^2 = 4 - 1$$
$$b^2 = 3$$
$$b = \sqrt{3}$$

$$c^2 = b^2 + a^2$$
$$2^2 = b^2 + 1^2$$
$$4 = b^2 + 1$$
$$b^2 = 4 - 1$$
$$b^2 = 3$$
$$b^2 = \sqrt{3}$$

SenA= $\mathbf{\frac{1}{2}}$

CosA= $\mathbf{\frac{\sqrt{3}}{2}}$

TanA= $\frac{1}{\sqrt{3}}$

CotA= $\frac{\sqrt{3}}{1}$

SecA= $\frac{2}{\sqrt{3}}$

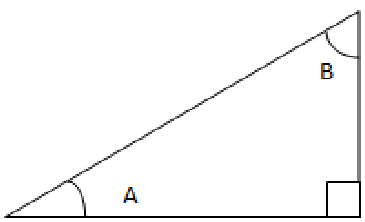
CscA= $\mathbf{\frac{2}{1}}$

COFUNCIONES

Como podemos observar en trigonometría solo existen tres funciones básicas las cuales son **seno, tangente** y **secante,** las otras tres funciones son derivadas de estas tres y están acompañadas por el prefijo **CO.**

- El coseno es cofuncion del seno
- La cotangente es cofuncion de la tangente
- La cosecante es cofuncion de la secante.

(SOLO SE APLICA EN UN TRIANGULO RECTANGULO)



$$< A + < B = 90°$$

$$< A = 90° - < B$$

$$< B = 90° - < A$$

SenA= CosB

SenA= Cos (90-∆A)

CosA= Sen (90-∆A)

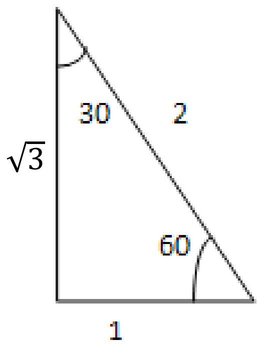
TanA= Cos (90- <A=)

CotA= tan (90-A)

SecA= Csc (90-A)

CscA= (90-A)

FUNCIONES DE ANGULOS NOTABLES (30°, 60° y 45°)



$$sen30 = \frac{1}{2} \qquad sen60 = \frac{\sqrt{3}}{2}$$

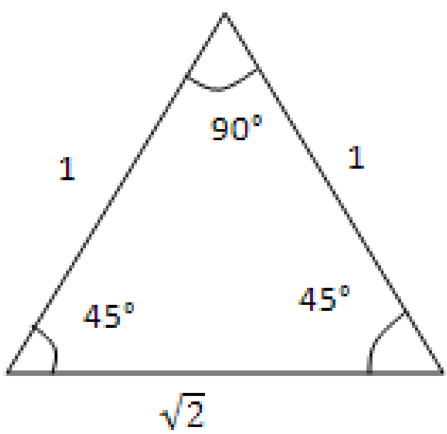
$$cos30 = \frac{\sqrt{3}}{2} \qquad cos60 = \frac{1}{2}$$

$$tan30 = \frac{1}{\sqrt{3}} \qquad tan60 = \frac{\sqrt{3}}{1}$$

$$cot30 = \frac{\sqrt{3}}{1} \qquad cot60 = \frac{1}{\sqrt{3}}$$

$$sec30 = \frac{2}{\sqrt{3}} \qquad sec60 = \frac{2}{1}$$

$$csc30 = \frac{2}{1} \qquad csc60 = \frac{2}{\sqrt{3}}$$



$$sen45 = \frac{1}{\sqrt{2}}$$

$$cos45 = \frac{1}{\sqrt{2}}$$

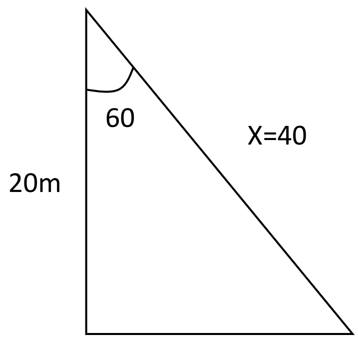
$$tan45 = 1$$

$$cot45 = 1$$

$$sec45 = \frac{\sqrt{2}}{1}$$

$$csc45 = \frac{\sqrt{2}}{1}$$

Resolución de triángulos rectángulos.

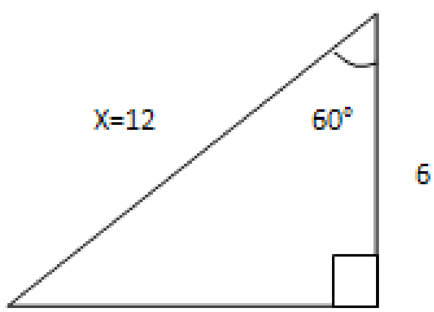


$$cos60° = \frac{1}{2}$$

$$cos60° = \frac{20}{x}$$

$$\frac{20}{x} = \frac{1}{2}$$

$$(20)(2)$$
$$= (1)(x)$$



$$cos60° = \frac{1}{2}$$

Cos60°= $\frac{6}{x}$

$$\frac{6}{x} = \frac{1}{2}$$

$$(6)(2) = (1)(x)$$

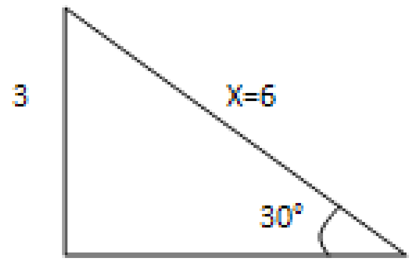
$$x = 12$$

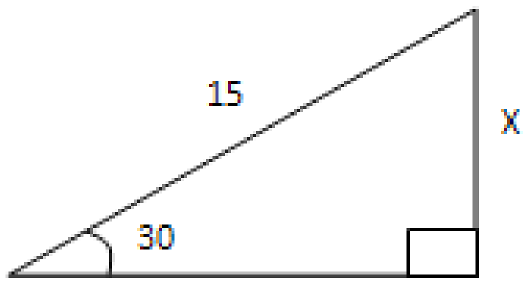
$$sen30° = \frac{1}{2}$$

$$sen30° = \frac{3}{x}$$

$$(3)(2) = (1)(x)$$

$$x = 6$$





$$sen30° = \frac{1}{2}$$

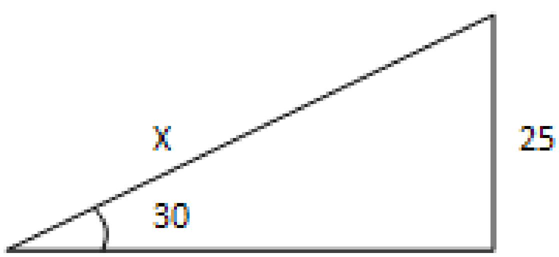
$$sen30° = \frac{15}{x}$$

$$\frac{15}{x} = \frac{1}{2}$$

$$(15)(2) = (1)(x)$$

$$x = 30$$

-



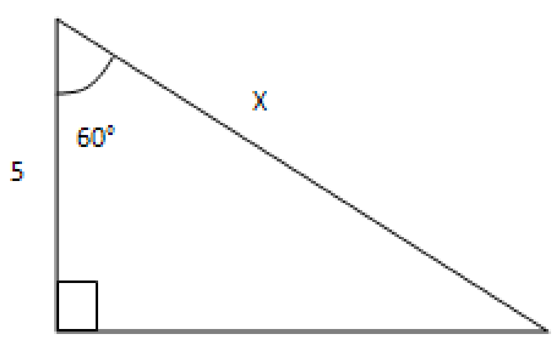
$$sen30° = \frac{1}{2}$$

$$sen30 = \frac{15}{x}$$

$$\frac{15}{x} = \frac{1}{2}$$

$$(15)(2) = (1)(x)$$

$$x = 50$$



$$cos60° = \frac{1}{2}$$

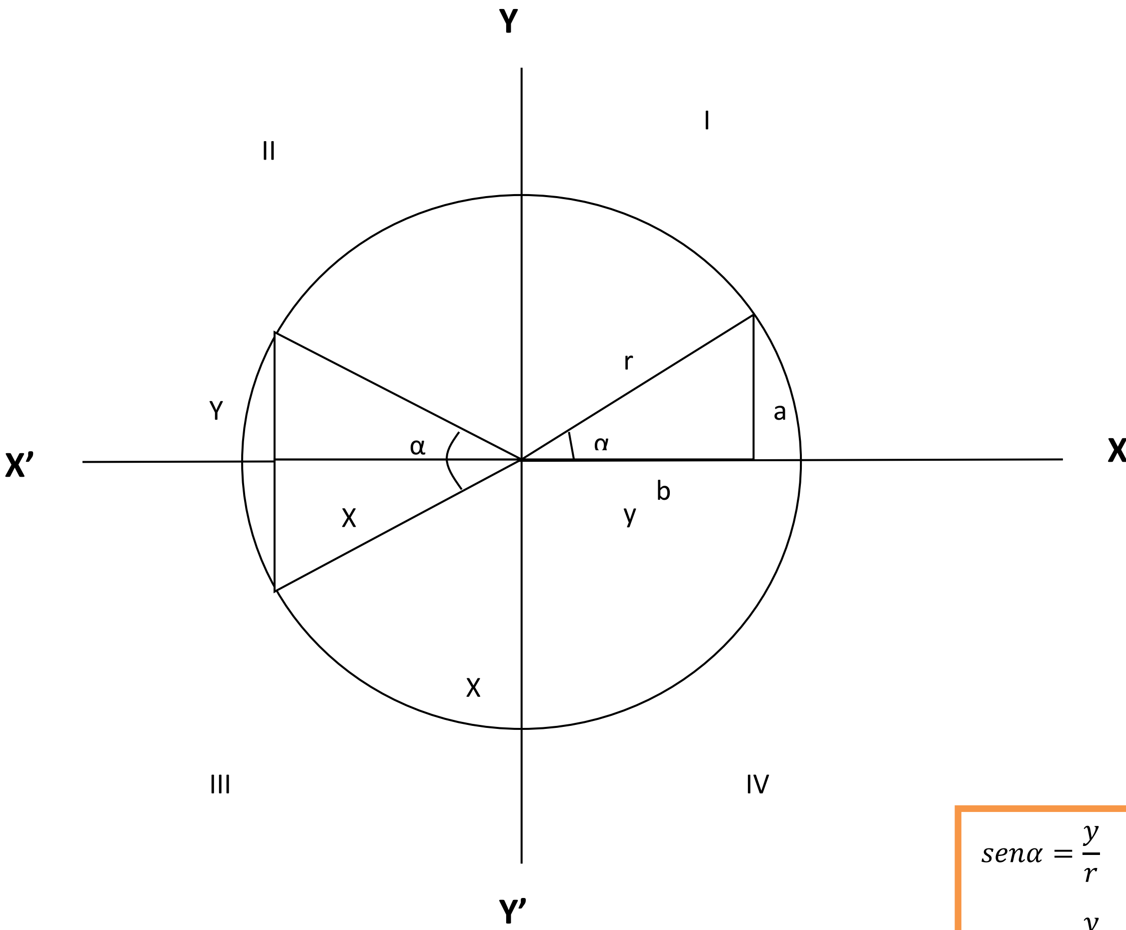
$$cos60° = \frac{5}{x}$$

$$\frac{5}{x} = \frac{1}{2}$$

$$(5)(2) = (1)(x)$$

$$x = 10$$

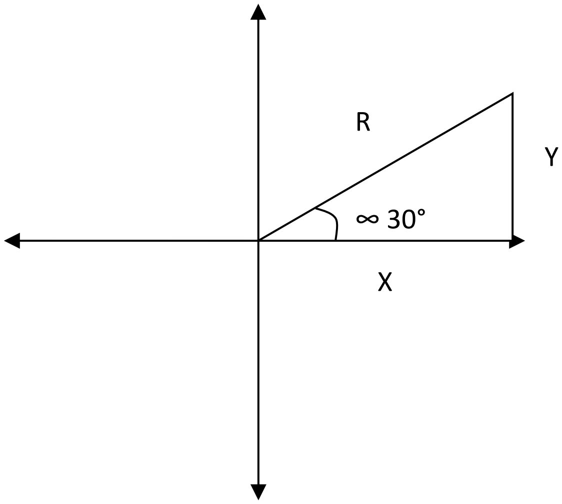
FUNCIONES TRIGONOMETRICAS DE CUALQUIER ANGULO.



$$sen\alpha = \frac{y}{r}$$

$$sen\alpha = \frac{y}{r}$$

$$sen\alpha = \frac{-y}{r}$$



$$sen\infty = \frac{y}{r}$$

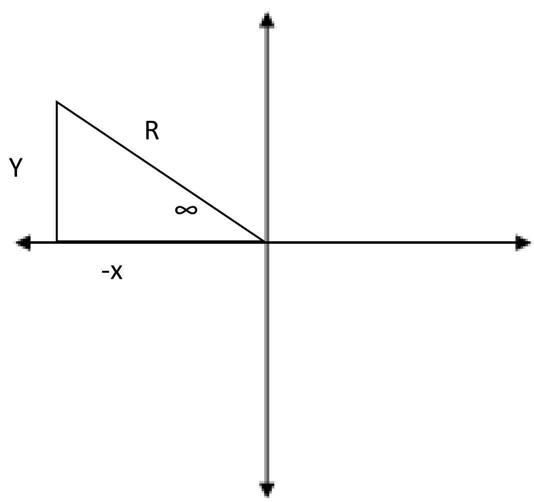
$$cos\infty = \frac{x}{y}$$

$$tan\infty = \frac{y}{x}$$

$$cot\infty = \frac{x}{y}$$

$$sec\infty = \frac{y}{x}$$

$$csc\infty = \frac{r}{y}$$



$$sen\infty = \frac{y}{r}$$

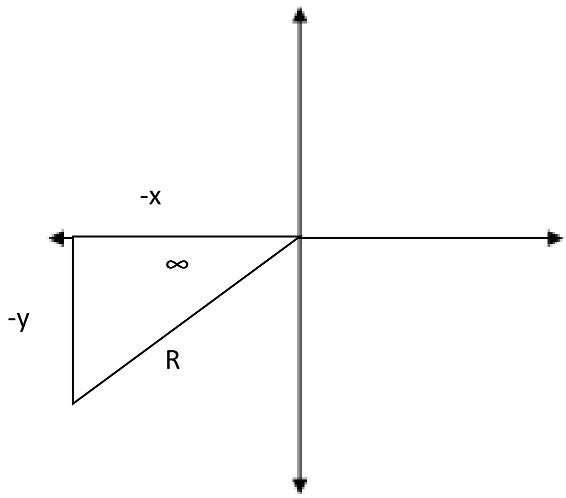
$$cos\infty = \frac{-x}{y}$$

$$tan\infty = \frac{y}{-x}$$

$$cot\infty = \frac{x}{y}$$

$$sec\infty = \frac{y}{-x}$$

$$csc\infty = \frac{r}{y}$$



$$sen\infty = \frac{-y}{r}$$

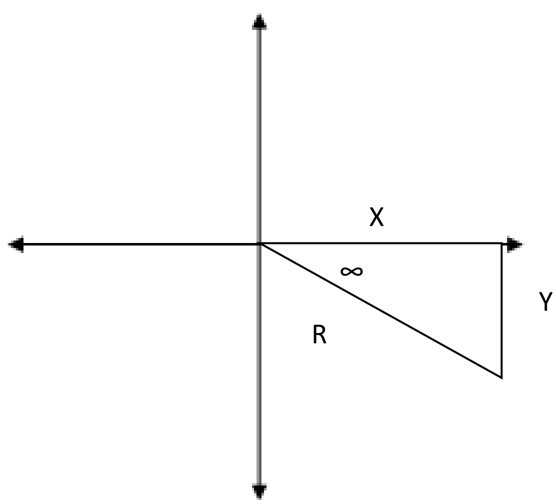
$$cos\infty = \frac{-x}{r}$$

$$tan\infty = \frac{-y}{-x}$$

$$cot\infty = \frac{-x}{y}$$

$$sec\infty = \frac{r}{-x}$$

$$csc\infty = \frac{r}{-y}$$



$$sen\infty = \frac{-y}{r}$$

$$cos\infty = \frac{x}{y}$$

$$tan\infty = \frac{-y}{-x}$$

$$cot\infty = \frac{r}{-y}$$

$$sec\infty = \frac{y}{x}$$

$$csc\infty = \frac{r}{-y}$$

funciones	I	II	III	IV
$\frac{\boldsymbol{seno}}{\boldsymbol{cosecante}}$	+	+	-	-
$\frac{\boldsymbol{coseno}}{\boldsymbol{secante}}$	+	-	-	+
$\frac{\boldsymbol{tangente}}{\boldsymbol{cotangente}}$	+	-	+	-

$$sen30 = \frac{1}{2}$$

$$sen150 = \frac{1}{2}$$

$$sen210 = -\frac{1}{2}$$

$$sen330 = -\frac{1}{2}$$

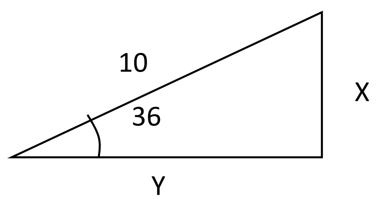
SOLUCION DE TRIANGULOS RECTANGULOS.

Una de las aplicaciones mas usada para las razones trigonométricas, es la solución de triángulos rectángulos, lo cual se usa relacionando, los datos, teniendo uno de los ángulos agudos.

$$a^2 + b^2 = c$$

$$b^2 = c^2 - a^2$$

$$a^2 = c^2 - b^2$$



$$sen36° = \frac{c.o}{h}$$

$$sen36° = \frac{x}{10}$$

$$10(sen36) = x$$

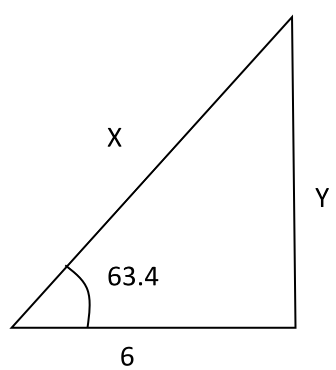
$$10^2 = 5.87^2 = y^2$$

$$100 - 34.45 = y^2$$

$$65.45 = y^2$$

$$\sqrt{65.45}$$

$$y = 8.09$$



$$a^2 = 6^2 - 2.68^2$$

$$a^2 = 36 - 7.18$$

$$a^2 = 28.82$$

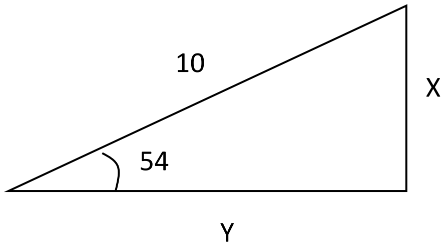
$$\sqrt{28.82}$$

$$a = 5.36$$

$$cos63 = \frac{6}{x}$$

$$x(\cos 63.4) = \mathrm{x}$$

$$x = \frac{6}{0.44}$$



$$sen54 = \frac{x}{10}$$

$$10(sen54) = 6$$

$$x = 8.09$$

$$10^2 - 8.09^2 = y^2$$

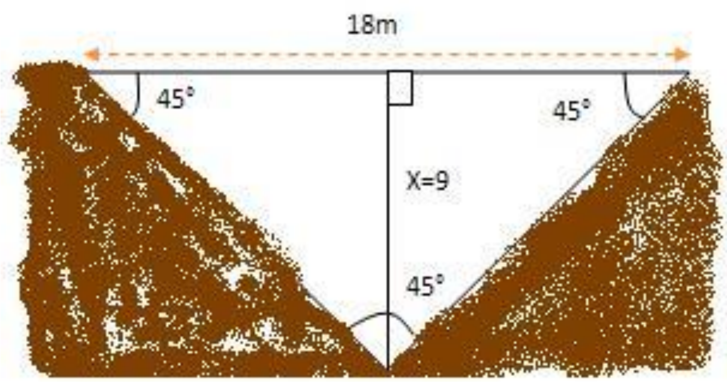
$$100 - 65.44 = y^2$$

$$35.56 = y^2$$

$$\sqrt{35.56}$$

$$y = 5.90$$

Calcular la profundidad del barranco



$$cot45° = 1$$

$$cot45 = \frac{9}{x}$$

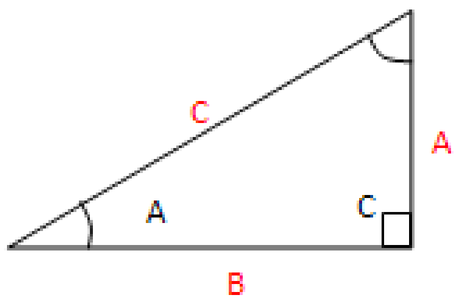
$$\frac{9}{x} = \frac{1}{1}$$

$$(9)(1) = (1)(x)$$

$$x = 9$$

IDENTIDADES TRIGONOMETRICAS.

Una identidad trigonométrica es una igualdad que contiene funciones trigonométricas y que es verdadero para todos los valores de los ángulos para los cuales están definidas estas funciones.



$$\frac{sen\ A}{cosA} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{ac}{bc} = \frac{a}{b} = tanA$$

$$\frac{cosA}{senA} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{bc}{ac} = \frac{b}{a} = cotA$$

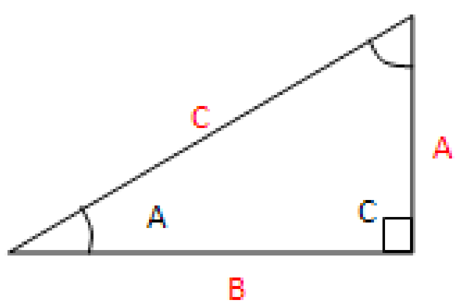
$$(\frac{2}{3})^2 = \frac{4}{9} = \frac{2^2}{3^2}$$

$$senA = \frac{a}{c}$$

$$(senA)^2 = (\frac{a}{c})^2 = \frac{a^2}{c^2}$$

$$\left.\begin{array}{l} senA = \frac{A}{csc}A \\[1em] cosA = \frac{1}{secA} \\[1em] tanA = \frac{1}{cotA} \\[1em] \frac{1}{secA} = cscA \end{array}\right\} \text{Reciprocas}$$

$$\left.\begin{array}{l} senAx\ cscA = 1 \\[1em] cosAx\ secA = 1 \\[1em] tanAx\ cotA = 1 \end{array}\right\} \text{Producto}$$



$$senA = \frac{a}{c}$$

$$cscA = \frac{c}{a}$$

$$senAxcscA = \frac{a}{c} = \frac{c}{a} = \frac{a}{c}\frac{c}{a} = 1$$

$$cosA = \frac{b}{c}$$

$$secA = \frac{c}{b}$$

$$cosAxsecA = \frac{b}{c}x\frac{c}{b} = \frac{b}{c}\frac{c}{b} = 1$$

$$tanA = \frac{a}{b}$$

$$cotA = \frac{b}{c}$$

$$tanAxcotA = \frac{a}{b}x\frac{b}{a} = \frac{a}{b}\frac{b}{a} = 1$$

$$\frac{c^2}{a^2} = \frac{a^2}{a^2} + \frac{b^2}{a^2} \qquad csc^2A = 1 + cot = A$$

$$\frac{c^2}{a^2} = 1 + \frac{b^2}{a^2}$$

$$\frac{c^2}{b^2} = \frac{a^2}{b^2} + \frac{b^2}{b^2} \qquad sec^2A = tan^2A + 1$$

$$\frac{c^2}{b^2} = \frac{a^2}{b^2} + 1$$

ANGULO DOBLE.

$$sen2A = 2senA\,cosA$$

$$cos2A = \ cos^2A - sen^2A$$

$$tan2A = \frac{2tanA}{1 - tan^2A}$$

ANGULO MITAD.

$$sen^2A = \frac{1}{2}(1 - cos2A)$$

$$cos^2A = \frac{1}{2}(1cos2A)$$

DEMOSTRAR QUE $secA - tanA - secA = cosA$

$secA = \frac{1}{cosA}$ → Reciprocas

$tanA = \frac{senA}{cosA}$ → Cociente

$$\frac{1}{cosA} - \frac{(senA)}{(cosA)} - senA = cosA$$

$$\frac{1}{cosA} - \frac{sen^2A}{cosA} = cosA$$

$$\frac{1 - sen^2A}{1cosA} = cosA$$

$$\frac{cos^2A}{cosA} = cosA$$

$$cosA = cosA$$

DEMOSTRAR QUE

$$\frac{senA + cscA}{\text{cscA}\, senA} = senA + cscA$$

$$\frac{senA}{cscA\ senA} + \frac{cscA}{cscA\ senA} = senA + cscA$$

$$\frac{1}{cscA} + \frac{1}{senA} = senA\ x\ cscA$$

$$senA + cscA = senA\ x\ cscA$$

$$\boldsymbol{senx\,(senx + cscx) - cosx\,(secx - cscx) = secx \times cscx}$$

$$senx\,\left(\frac{1}{cos}x + \frac{1}{sen}x\right) - cosx\,\left(\frac{1}{cos}x - \frac{1}{sen}x\right) = secx - cscx$$

$$\frac{senx}{cosx} + \frac{senx}{senx} - \frac{cosx}{cosx} + \frac{cosx}{senx} = secx\ \times cscx$$

$$\frac{senx}{cosx} + x - x + \frac{cosx}{senx} = secx\ \times cscx$$

$$\frac{senx}{cosx} + \frac{cosx}{senx} = secx\ \times cscx$$

$$\boldsymbol{\frac{cosA - secA}{secA\ \ cosA} = cosA - secA}$$

$$\frac{cosA}{secA\ cosA} - \frac{secA}{secA\ cosA} = cosA - secA$$

$$\frac{1}{secA} - \frac{1}{cosA} = cosA - secA$$

$$cosA = \frac{1}{secA}\ \ secA = \frac{1}{cosA}$$

$$cosA - secA = cosA - secA$$

$$\boldsymbol{cotx\ senx = cosx}$$

$$\frac{cosx}{senx}senx = cosx$$

$$\frac{cosx\ senx}{senx = cosx}$$

$$cosx = cosx$$

$$\mathbf{1 = 2cosx\ secx - tanx\ cotx}$$

$$1 = 2(1)$$

$$1 = 2 - 1$$

$$1 = 1$$

$$\boldsymbol{tanx + cotx = \frac{1}{secx\ cosx}}$$

$$\frac{senx}{cosx} + \frac{cosx}{senx} = \frac{1}{senx\ cosx}$$

$$\frac{sen^2x + cos^2x}{cosx\ senx}$$

$$\frac{1}{cosx\ senx} = \frac{1}{senx\ cosx}$$

$$\boldsymbol{tanA + cotA = secA\ cscA}$$

$$\frac{senA}{cosA} + \frac{cotA}{senA}$$

$$\frac{sen^2 + cos^2A}{cosA\ senA}$$

$$\frac{1}{cosA\ senA}$$

$$\frac{1}{cosA} \times \frac{1}{senA}$$

$$senA\ \times cscA$$

$$\frac{\boldsymbol{senx + cosx\,tanx}}{\boldsymbol{cosx}} = \boldsymbol{2tan}$$

$$\frac{senx + cosx\ \left(\frac{senx}{cosx}\right)}{cosx} = 2tanx$$

$$\frac{senx + \frac{cosx\ senx}{cosx}}{cosx} = 2tanx$$

$$\frac{senx + senx}{cosx} = 2tanx$$

$$\frac{2senx}{cox} = 2tanx$$

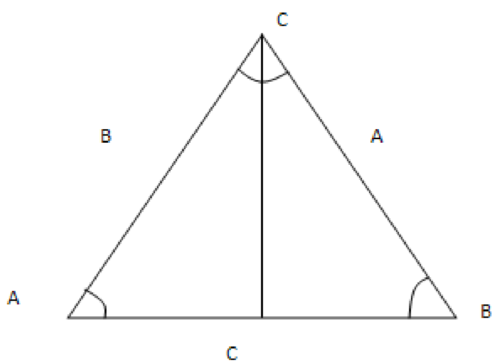
$$2(tanx) = 2tanx$$

SOLUCION DE TRIANGULOS OBLICUANGULOS.

Un triangulo oblicuángulo es aquel que contiene un ángulo obtuso o sus tres ángulos agudos llamándose oblicuángulo y acutángulo.

LEY DE SENO.

Teorema: en todo triangulo los lados tienen una proporción correspondiente a los senos que los ángulos opuesta.



$$senA = \frac{h}{b}$$

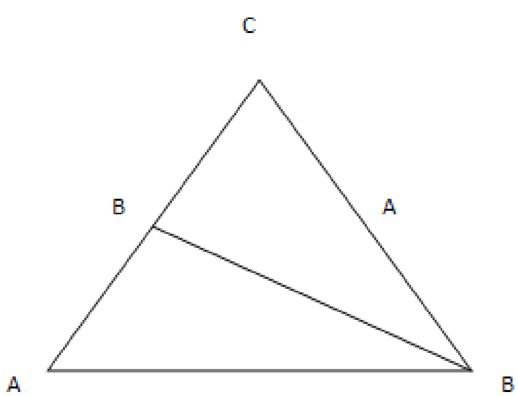
$$senB = \frac{h}{a}$$

$$\frac{senA}{senB} = \frac{\frac{h}{b}}{\frac{h}{a}} = \frac{ha}{hb} = \frac{a}{b}$$

$$\frac{senA}{senB} = \frac{a}{b}$$

$$senB(a) = senA(b)$$

LEY DE SENOS $\dashrightarrow$ $\frac{a}{senA} = \frac{b}{senB} = \frac{c}{senC}$



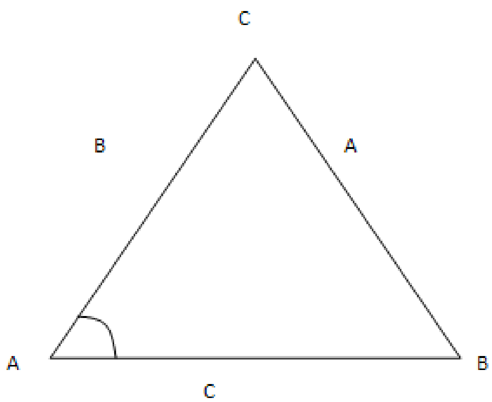
$$senA = \frac{h}{c} \qquad \frac{a}{senA} = \frac{c}{senC}$$

$$senC = \frac{h}{a} \qquad \frac{senA}{secC} = \frac{\frac{h}{c}}{\frac{h}{a}} = \frac{ha}{hc} = \frac{a}{c}$$

$$\frac{senA}{senC} = \frac{a}{c} \qquad secC = \frac{h}{a}$$

LEY DE COSENO.

En un triangulo cualquiera el cuadrado de un lado es igual a la suma de los cuadrados de los otros dos menos el doble producto de estos dos lados por el coseno del ángulo que forman.

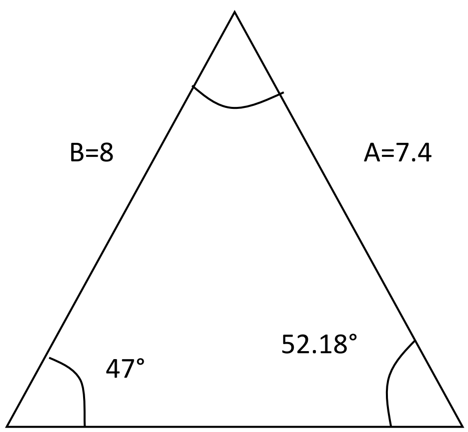


$$a^2 = b^2 + c^2 - 2bc\ cosA$$

$$b^2 = a^2 + c^2 - 2ac\ cosB$$

$$c^2 = a^2 + b^2 - 2ab\ cosC$$

$$\frac{a}{senA} = \frac{b}{senB} = \frac{c}{senC}$$



$$a^2 = 8^2 + 10^2 - 2(8)(10)cos47°$$

$$a^2 = 64 + 100 - 160\ (0.6820)$$

$$a^2 = 54.88$$

$$\sqrt{54.88}$$

$$a = 7.4$$

PARA B.

$$\frac{7.4}{sen47} = \frac{8}{senB}$$

$$senB = \frac{8(sen47°)}{7.4}$$

$$senB = 0.79$$

$$B = 52.18$$

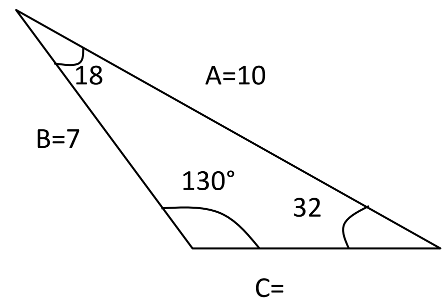
PARA C.

$$\frac{7.4}{sen47°} = \frac{10}{senC}$$

$$senC = \frac{10(sen47°)}{7.4}$$

$$senC = 0.98$$

$$C = 78.52$$



$c^2 = 10^2 + 7^2 - 2(10)(7)cos18$

$c^2 = 100 + 49 - 140(0.9510$

$c^2 = 149 - 133.14$

$c = \sqrt{15.86}$

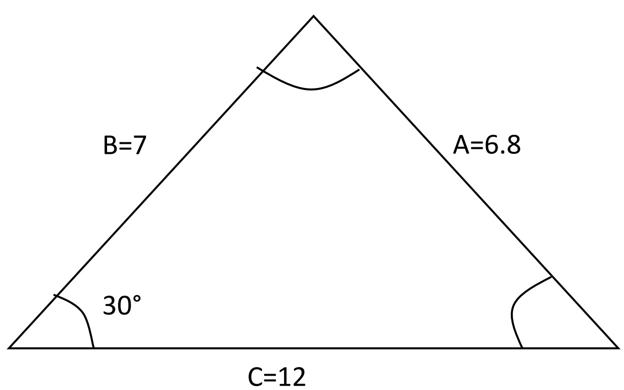
$C = 6.89$

$$\frac{10}{sen130} = \frac{7}{senb}$$

$$senB = \frac{7(sen130)}{10}$$

$senB = 0.53$

$B = 32.00$



$a^2 = 7^2 + 12^2 - 2(7)(12)cos30$

$a^2 = 49 + 144 - 168\ (0.8660)$

$a^2 = 193 - 145.48$

$a = \sqrt{47.52}$

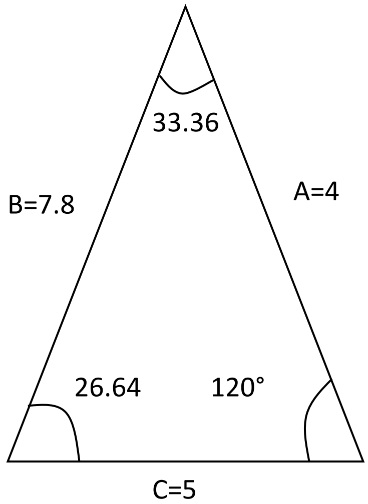
$a = 6.89$

$$\frac{6.8}{sen30°} = \frac{7}{senB}$$

$$senB = \frac{7(sen30)}{6.8}$$

$senB = 0.5$

$B = 20.48$



$$b^2 = 4^2 + 5^2 - 2(4)(5)\cos 120$$

$$b^2 = 8 + 25 - 40(-0.5)$$

$$b^2 = 61$$

$$b = \sqrt{61}$$

$$b = 7.8$$

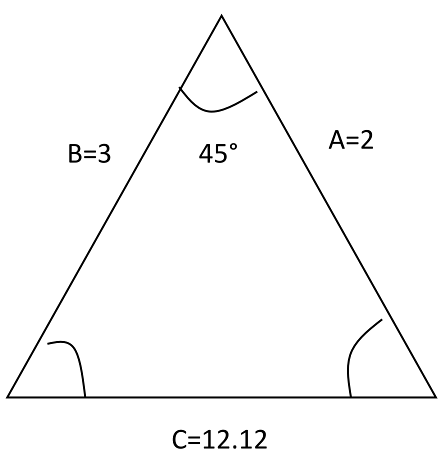
$$\frac{7.8}{sen120°} = \frac{5}{senC}$$

$$senC = \frac{5(sen120)}{7.8}$$

$$senC = 0.55$$

$$C = 33.36$$

$\boldsymbol{a = 2}$, $\boldsymbol{b = 3}$, $\boldsymbol{C = 45°}$



$$c^2 = 2^2 + 3^2 - 2(2)(3)\cos 45°$$

$$c^2 = 4 + 9 - 12(0.7071)$$

$$c^2 = 13 - 8.48$$

$$c = \sqrt{4.52}$$

$$c = 2.12$$

Ecuaciones trigonométricas

Son aquellas las cuales la incógnita aparece como un ángulo de funciones trigonométricas.

Pare resolver una ecuación trigonométrica aremos las transformaciones necesarias para trabajar una sola función

Ejemplo

$2secx - 1 = 0$

$2secx - 1 = 0$

$2senx = 1$

$senx\frac{1}{2}$

$Arcsen\ senx = Arcseno\frac{1}{2}$

$x = Arcseno\frac{1}{2}$

$Arcsen\frac{1}{2} = 30$

$x1 = 30$

$x2 = 150$

Ejemplo

$\frac{sec^2x}{2} = 2$

$sec^2x = 4$

$secx = \sqrt{4}$

$sex = \pm 2$

$secx = 2$

$\frac{1}{cosx} = 2$

$1 = 2cosx$

$Arcos0.5 = 60°$

$x1 = 60°$

$x2 = 300$

$secx = -2$

$\frac{1}{cosx} = -2$

$1 = -2cosx$

$cosx = \frac{1}{2}$

$x3 = 120°$

$x4 = 240°$

Ejemplo

$4cos^2x = 3 - 4cosx$

$cosx = y$

$4y^2 = 3 - 4y$

$4y^2 + 4y - 3 = 0$

$(2y - 1)(2y + 3)$

$2y - 1 = 0$

$y = \frac{1}{2}$

$cosx = \frac{1}{2}$

$Arcos0.5 = 60°$

$x1 = 60 \quad x2 = 300$

$(2y + 3) = 0$

$y = \frac{-3}{2}$

$cosx = \frac{-3}{2}$

$cosx = \frac{-3}{2}$

$Arcos = \frac{-3}{2}\ no\ existe$

$8senx = 2 + \frac{4}{\frac{1}{senx}}$

$8senx = 2 + \frac{\frac{4}{1}}{\frac{1}{senA}}$

$8senx = 2 + 4senx$

$8senx - 4senx = 2$

$4senx = 2$

$senx = 2/4$

$senx = \frac{1}{2}$

$\frac{Arcsen1}{2} = x$

$x = 30° \quad x = 150$

$$2tanx - 3\left(\frac{1}{tanx}\right) - 1 = 0$$

$$2tanx - 3\left(\frac{1}{tanx}\right) - 1 = 0tanx$$

$$2tan^2x - \left(\frac{3(tanx)}{tanx}\right) - tanx = 0$$

$$2tan^2 - 3 - tanx = 0$$

$$tanx = y$$

$$2y^2 - y - 3 = 0$$

$$(2y - 3)(y + 1) = 0$$

$$2y - 3 = 0$$

$$y = \frac{2}{3} \quad y = 1$$

$$tanx = \frac{3}{2}$$

$$x1 = 56.30 \quad x2 = 236.30 \quad x3 = 45 \quad x4 = 225$$

Ejemplo

$$3cos^3x + sen^2x = 3$$

$$3(1 - sen^2) + sen^2x = 3$$

$$-3sen^2 + sen^2x = 3 - 3$$

$$sen^2x = 0$$

$$senx = \sqrt{0}$$

$$sen = 0$$

$$acrsen = 0$$

$x1 = 0\ x2 = 180$

Ejemplo.

$2sen^2x + senx = 0$

$senx(2senx + 1) = 0$

$senx = 0$

$arcsen = 0 = 0$

$x1 = 0 \quad x2 = 180°$

Ejemplo

$2senx + 1 = 0$

$2senx = -1$

$arcsen\frac{1}{2} = 30°$

$x1 = 210 \quad x4 = 330$

Ejemplo

$2tan^2x + sec^2x$=2

$2tan^2x + tan^2x + 1 = 2$

$2tan^2x + tan^2x = 1$

$3tan^2x = 1$

$tan^2x = \frac{1}{3}$

$tanx = \frac{1}{\sqrt{3}}$

$x1 = 30 \qquad x2 = 210$

Ejemplo

$4sen^2x - 3$ =0

$4y^2 = 3$

$y^2 = \frac{3}{4}$

$y = \sqrt{\frac{3}{4}}$

$y = \sqrt{\frac{3}{2}}$

$senx = \frac{\sqrt{3}}{2}$

$Arcsen\frac{\sqrt{3}}{2} = x$

$x1 = 60° \quad x2 = 120$

$4sen^2x - 3$ =0

$4sen^2x = 3$

$sen^2x = \sqrt{\frac{3}{4}} = \frac{\sqrt{5}}{2}$

$Arcsen\sqrt{\frac{3}{4}} = x$

$x1 = 60° \quad x2 = 120°$

Ejercicio

$1 + 2cosx + cos^2x = 3sen^2x$

$1 + 2cosx + cos^2x = 3(1 - cos^2x)$

$1 + 2cosx + cos^2x = 3 - 3cos^2x$

$3cos^2x + \ cos^2x + 2cosx = 3 - 1$

$4cos^2x + 2cosx = 2$

$4cos^2x + 2cosx - 2 = 0$

$y = cosx$

$(2y - 1)(2y + 2) = 0$

$(2y - 1)(2y + 2) = 0$

$2y - 1 = 0 \ 2y + 2 = 0$

$2y = 1 \quad 2y = -2$

$y = 2 \ \ y = 1$

$\text{arccos}\frac{1}{2} = 60°$ $\qquad\qquad\qquad Arcos1 = 0$

$x1 = 60 \ \ x2 = 300$ $\qquad\qquad\qquad x3 = 180°$

$cos^2x - 3sen^2x = 0$

$cos^2x - 3(1 - sen^2x) = 0$

$4cos^2x - 3 = 0$

$cosx = \pm\frac{\sqrt{3}}{2} = 0.86$

$Arceos0.86 = 30$

$x1 = 30 \quad x2 = 330$

$\cos \ \ x3 = 150 \quad x4 = 210$

$cosx(cosx + 5) = 2 + sen^2x$

$cos^2x + 5cosx = 2 - 1 - cos^2x$

$cosx = y$

$2y^2 + 5y - 3$

$2y - 1 = 0$

$2y = 1$

$y = \frac{1}{2}$

$1y + 3 = 0$

$1y = -\frac{3}{-3}$

Cosx=1 no existe

$arcosx = \frac{1}{2}$

$x1 = 60° \; x2 = 300$

Ejercicio

$4cos2x + 3cosx = 1$

$(4)(cos^2x - sen^2x) + 3cosx = 1$

$4cos^2x - 4sen^2x + 3cosx = 1$

$8cos^2x + 3cosx - 5 = 0$

$cosx = y$

$8y^2 + 3y - 5 = 0$

$(8y - 5)(1y + 1)3$

$8y - 5 = 0$

$y = \frac{5}{8}$

$arcosx = \frac{5}{8}$

$x1 \quad 51.32 \quad x2 \quad 308.69$

$1y + 1 = 0$

$1y = -1$

$cos - 1 = x$

$x3 = 80$

Ejemplo

$2sen^2x + \sqrt{3}cosx + 1 =$

$2 - 2cos^2x + \sqrt{3}cosx + 1 = 0$

$-2cos^2x + \sqrt{3}cosx + 1 = 0$

$y = cosx$

$-2y^2 + \sqrt{3y} + 3 = 0$

$\sqrt{3 = -\sqrt{3}}$

$\sqrt{3} = \frac{2\sqrt{3}}{\sqrt{3}}$

$(2y + \sqrt{3})(-y + \sqrt{3}) = 0$

$2y = -\sqrt{3} = 0 \quad -y = -\sqrt{3}$

$y = \frac{\sqrt{3}}{2}$

$y = \sqrt{3}$

$cosx = \frac{-\sqrt{3}}{2}$

$arc\frac{\sqrt{3}}{2} = 30°$

$x1 = 150° \quad x2 = 210$