# Nonlinear System Identification

This example shows how to use `anfis` command for nonlinear dynamic system identification.

This example requires System Identification Toolbox™, as a comparison is made between a nonlinear ANFIS and a linear ARX model.

## Problem Setup

Exit if System Identification Toolbox is not available.

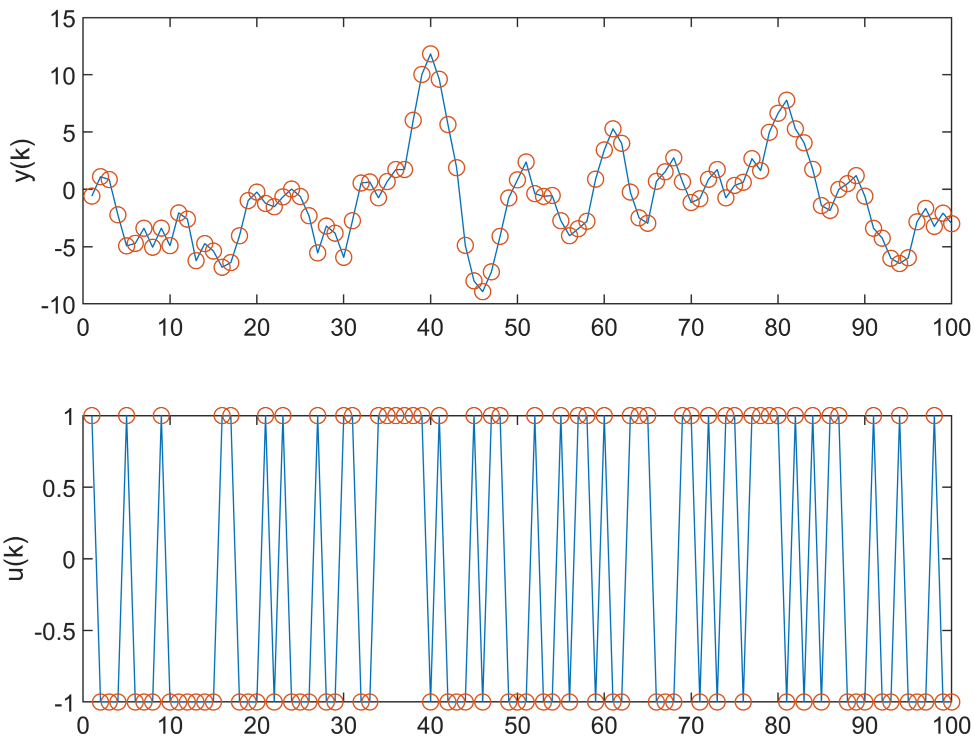
```
if ~fuzzychecktoolboxinstalled('ident')
    errordlg('DRYDEMO needs the System Identification Toolbox.');
    return;
end
```

The data set for ANFIS and ARX modeling was obtained from a laboratory device called Feedback's Process Trainer PT 326, as described in Chapter 17 of Prof. Lennart Ljung's book "System Identification, Theory for the User", Prentice-Hall, 1987. The device functions like a hair dryer: air is fanned through a tube and heated at the inlet. The air temperature is measured by a thermocouple at the outlet. The input u(k) is the voltage over a mesh of resistor wires to heat incoming air; the output y(k) is the outlet air temperature.


Here are the results of the test.

```
load iddata1 z1
y2=z1.OutputData;
u2=z1.InputData;
data_n = length(y2);
output = y2;
input = [[0; y2(1:data_n-1)] ...
        [0; 0; y2(1:data_n-2)] ...
        [0; 0; 0; y2(1:data_n-3)] ...
        [0; 0; 0; 0; y2(1:data_n-4)] ...
        [0; u2(1:data_n-1)] ...
        [0; 0; u2(1:data_n-2)] ...
        [0; 0; 0; u2(1:data_n-3)] ...
        [0; 0; 0; 0; u2(1:data_n-4)] ...
        [0; 0; 0; 0; 0; u2(1:data_n-5)] ...
        [0; 0; 0; 0; 0; 0; u2(1:data_n-6)]];
data = [input output];
data(1:6, :) = [];
input_name = char('y(k-1)','y(k-2)','y(k-3)','y(k-4)','u(k-1)','u(k-2)','u(k-3)','u(k-4)','u(k-
index = 1:100;
subplot(2,1,1)
```

```
plot(index,y2(index),'-',index,y2(index),'o')
ylabel('y(k)','fontsize',10)
subplot(2,1,2)
plot(index,u2(index),'-',index,u2(index),'o')
ylabel('u(k)','fontsize',10)
```



The data points were collected at a sampling time of 0.08 seconds. One thousand input-output data points were collected from the process as the input u(k) was chosen to be a binary random signal shifting between 3.41 and 6.41 V. The probability of shifting the input at each sample was 0.2. The data set is available from the System Identification Toolbox, and the above plots show the output temperature y(k) and input voltage u(t) for the first 100 time steps.

## ARX Model Identification

A conventional method is to remove the means from the data and assume a linear model of the form:

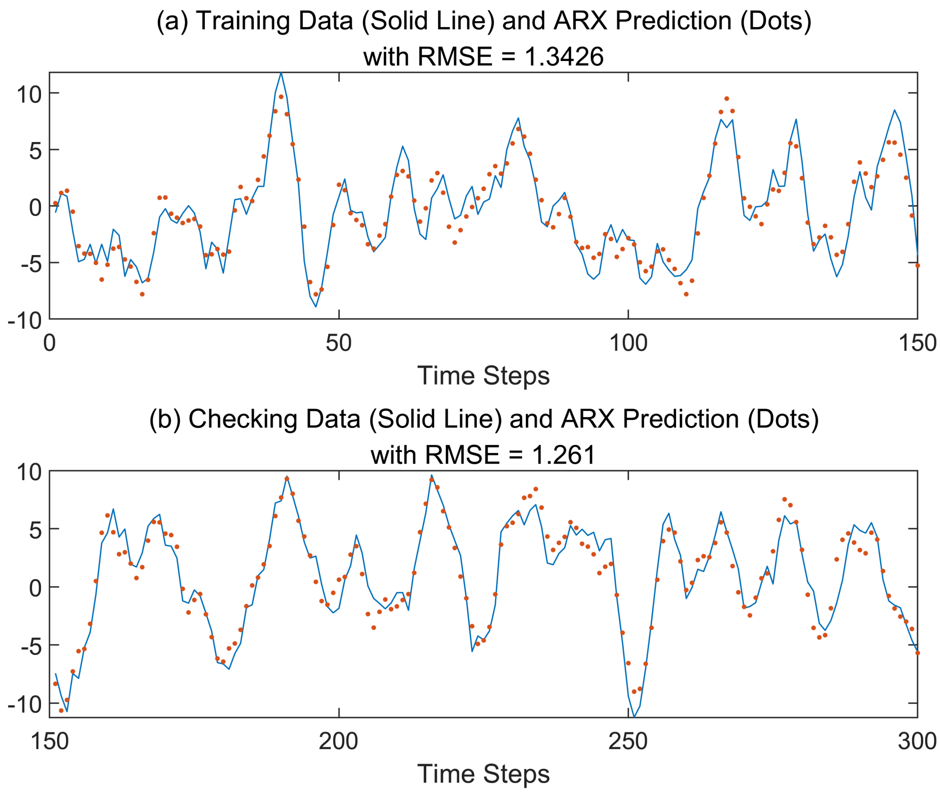
y(k)+a1*y(k-1)+...+am*y(k-m)=b1*u(k-d)+...+bn*u(k-d-n+1)

where ai (i = 1 to m) and bj (j = 1 to n) are linear parameters to be determined by least-squares methods. This structure is called the ARX model and it is exactly specified by three integers [m, n, d]. To find an ARX model for the dryer device, the data set was divided into a training (k = 1 to 300) and a checking (k = 301 to 600) set. An exhaustive search was performed to find the best combination of [m, n, d], where each of the integer is allowed to changed from 1 to 10 independently. The best ARX model thus found is specified by [m, n, d] = [5, 10, 2], with a training RMSE of 0.1122 and a checking RMSE of 0.0749. The above figure shows the fitting results of the best ARX model.

```
trn_data_n = 150;
total_data_n = 300;
z = [y2 u2];
z = dtrend(z);
ave = mean(y2);
ze = z(1:trn_data_n,:);
zv = z(trn_data_n+1:total_data_n,:);
T = 0.08;

% Run through all different models
V = arxstruc(ze,zv,struc(1:10,1:10,1:10));
% Find the best model
nn = selstruc(V,0);
% Time domain plot
th = arx(ze,nn);
th.Ts = 0.08;
u = z(:,2);
y = z(:,1)+ave;
yp = sim(u,th)+ave;

xlbl = 'Time Steps';

subplot(2,1,1)
index = 1:trn_data_n;
plot(index, y(index), index, yp(index), '.')
rmse = norm(y(index)-yp(index))/sqrt(length(index));
title(sprintf(['(a) Training Data (Solid Line) and ARX Prediction (Dots)\nwith RMSE = ' num2str
disp(['[na nb d] = ' num2str(nn)])
```

```
[na nb d] = 10    9    1
```

```
xlabel(xlbl,'fontsize',10)

subplot(2,1,2)
index = (trn_data_n+1):(total_data_n);
plot(index,y(index),index,yp(index),'.')
rmse = norm(y(index)-yp(index))/sqrt(length(index));
title(sprintf(['(b) Checking Data (Solid Line) and ARX Prediction (Dots)\nwith RMSE = ' num2str
xlabel(xlbl,'fontsize',10)
```




## ANFIS Model Identification

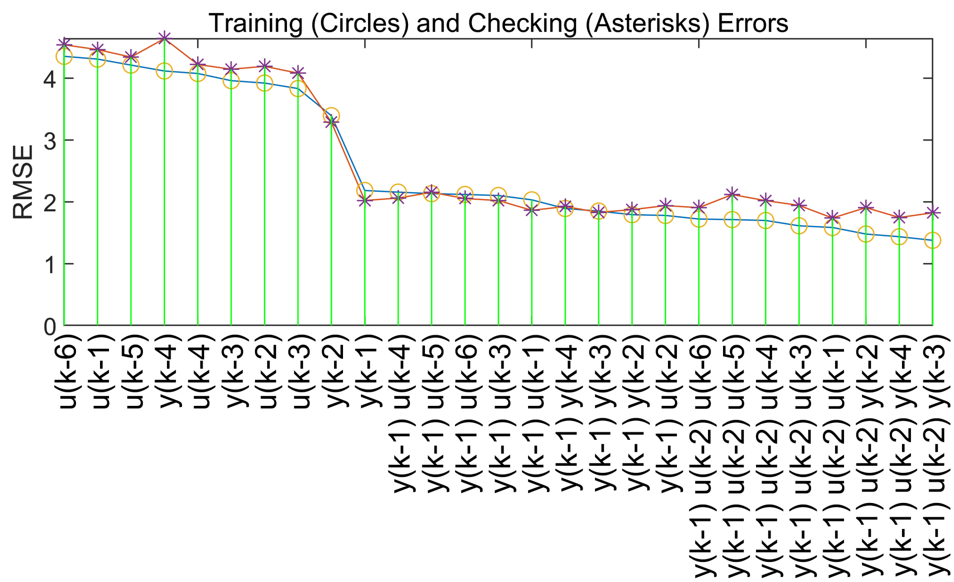
The ARX model is inherently linear and the most significant advantage is that we can perform model structure and parameter identification rapidly. The performance in the above plots appears to be satisfactory. However, if a better performance level is desired, we might want to resort to a nonlinear model. In particular, we are going to use a neuro-fuzzy modeling approach, ANFIS, to see if we can push the performance level with a fuzzy inference system.

To use ANFIS for system identification, the first thing we need to do is select the input. That is, to determine which variables should be the input arguments to an ANFIS model. For simplicity, we suppose that there are 10 input candidates (y(k-1), y(k-2), y(k-3), y(k-4), u(k-1), u(k-2), u(k-3), u(k-4), u(k-5), u(k-6)), and the output to be predicted is y(k). A heuristic approach to input selection is called sequential forward search, in which each input is selected sequentially to optimize the total squared error. This can be done by the function seqsrch; the result is shown in the above plot, where 3 inputs (y(k-1), u(k-3), and u(k-4)) are selected with a training RMSE of 0.0609 and checking RMSE of 0.0604.

```
trn_data_n = 150;
trn_data = data(1:trn_data_n,:);
chk_data = data(trn_data_n+1:trn_data_n+144,:);
[~,elapsed_time] = seqsrch(3,trn_data,chk_data,input_name); % #ok<*ASGLU>
```

```
Selecting input 1 ...
ANFIS model 1: y(k-1) --> trn=2.1817, chk=2.0203
ANFIS model 2: y(k-2) --> trn=3.3909, chk=3.2868
ANFIS model 3: y(k-3) --> trn=3.9578, chk=4.1426
ANFIS model 4: y(k-4) --> trn=4.1142, chk=4.6379
```

```
ANFIS model 5: u(k-1) --> trn=4.3065, chk=4.4606
ANFIS model 6: u(k-2) --> trn=3.9191, chk=4.1899
ANFIS model 7: u(k-3) --> trn=3.8305, chk=4.0790
ANFIS model 8: u(k-4) --> trn=4.0739, chk=4.2200
ANFIS model 9: u(k-5) --> trn=4.2096, chk=4.3389
ANFIS model 10: u(k-6) --> trn=4.3515, chk=4.5373
Currently selected inputs: y(k-1)

Selecting input 2 ...
ANFIS model 11: y(k-1) y(k-2) --> trn=1.7906, chk=1.8719
ANFIS model 12: y(k-1) y(k-3) --> trn=1.8489, chk=1.8282
ANFIS model 13: y(k-1) y(k-4) --> trn=1.8908, chk=1.9225
ANFIS model 14: y(k-1) u(k-1) --> trn=2.0303, chk=1.8613
ANFIS model 15: y(k-1) u(k-2) --> trn=1.7787, chk=1.9375
ANFIS model 16: y(k-1) u(k-3) --> trn=2.1009, chk=2.0192
ANFIS model 17: y(k-1) u(k-4) --> trn=2.1553, chk=2.0596
ANFIS model 18: y(k-1) u(k-5) --> trn=2.1312, chk=2.1524
ANFIS model 19: y(k-1) u(k-6) --> trn=2.1176, chk=2.0521
Currently selected inputs: y(k-1) u(k-2)

Selecting input 3 ...
ANFIS model 20: y(k-1) u(k-2) y(k-2) --> trn=1.4778, chk=1.9053
ANFIS model 21: y(k-1) u(k-2) y(k-3) --> trn=1.3762, chk=1.8211
ANFIS model 22: y(k-1) u(k-2) y(k-4) --> trn=1.4347, chk=1.7455
ANFIS model 23: y(k-1) u(k-2) u(k-1) --> trn=1.5835, chk=1.7368
ANFIS model 24: y(k-1) u(k-2) u(k-3) --> trn=1.6113, chk=1.9411
ANFIS model 25: y(k-1) u(k-2) u(k-4) --> trn=1.6969, chk=2.0180
ANFIS model 26: y(k-1) u(k-2) u(k-5) --> trn=1.7116, chk=2.1160
ANFIS model 27: y(k-1) u(k-2) u(k-6) --> trn=1.7204, chk=1.9045
Currently selected inputs: y(k-1) y(k-3) u(k-2)
```




```
fprintf('\nElapsed time = %f\n',elapsed_time);
```

```
Elapsed time = 9.920000
```

```
winH1 = gcf;
```

For input selection, another more computationally intensive approach is to do an exhaustive search on all possible combinations of the input candidates. The function that performs exhaustive search is exhsrch, which selects 3 inputs from 10 candidates. However, exhsrch usually involves a significant amount of computation if all combinations are tried. For instance, if 3 is selected out of 10, the total number of ANFIS models is C(10, 3) = 120.

Fortunately, for dynamic system identification, we do know that the inputs should not come from either of the following two sets of input candidates exclusively:

Y = {y(k-1), y(k-2), y(k-3), y(k-4)}

U = {u(k-1), u(k-2), u(k-3), u(k-4), u(k-5), u(k-6)}

A reasonable guess would be to take two inputs from Y and one from U to form the inputs to ANFIS; the total number of ANFIS models is then C(4,2)*6=36, which is much less. The above plot shows that the selected inputs are y(k-1), y(k-2) and u(k-3), with a training RMSE of 0.0474 and checking RMSE of 0.0485, which are better than ARX models and ANFIS via sequential forward search.

```
group1 = [1 2 3 4]; % y(k-1), y(k-2), y(k-3), y(k-4)
group2 = [1 2 3 4]; % y(k-1), y(k-2), y(k-3), y(k-4)
group3 = [5 6 7 8 9 10]; % u(k-1) through y(k-6)

anfis_n = 6*length(group3);
index = zeros(anfis_n,3);
trn_error = zeros(anfis_n,1);
chk_error = zeros(anfis_n,1);
% ======= Training options
% Create option set for generating initial FIS.
genOpt = genfisOptions('GridPartition','NumMembershipFunctions',2, ...
                       'InputMembershipFunctionType','gbellmf');
% Create option set for |anfis| command and set options that remain constant
% for different training scenarios.
anfisOpt = anfisOptions('EpochNumber',1,...
                        'InitialStepSize',0.1,...
                        'StepSizeDecreaseRate',0.5,...
                        'StepSizeIncreaseRate',1.5,...
                        'DisplayANFISInformation',0,...
                        'DisplayErrorValues',0,...
                        'DisplayStepSize',0,...
                        'DisplayFinalResults',0);
% ====== Train ANFIS with different input variables
fprintf('\nTrain %d ANFIS models, each with 3 inputs selected from 10 candidates...\n\n',...
    anfis_n);
```

```
Train 36 ANFIS models, each with 3 inputs selected from 10 candidates...
```

```
model = 1;
for i = 1:length(group1)
    for j = i+1:length(group2)
        for k = 1:length(group3)
```

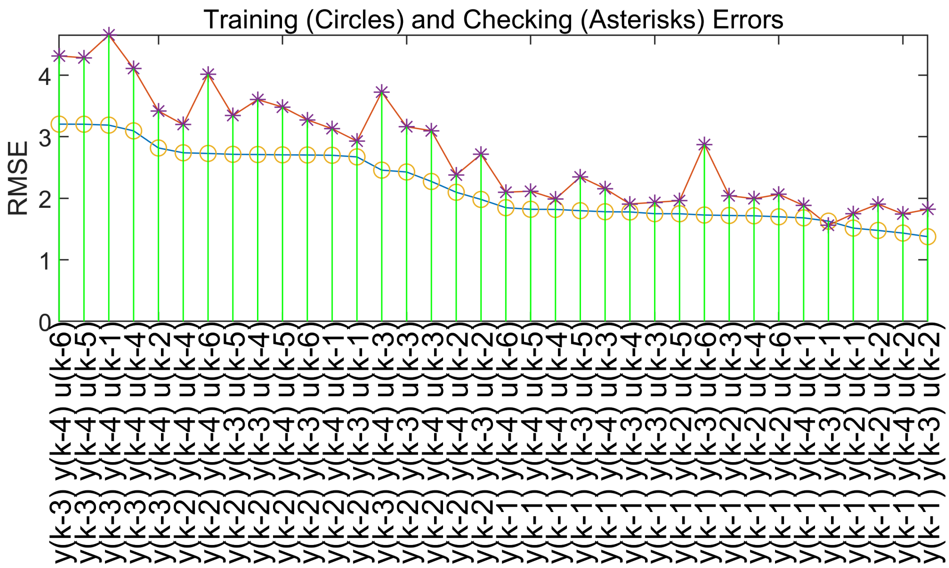
```
                in1 = deblank(input_name(group1(i),:));
                in2 = deblank(input_name(group2(j),:));
                in3 = deblank(input_name(group3(k),:));
                index(model, :) = [group1(i) group2(j) group3(k)];
                trn_data = data(1:trn_data_n, [group1(i) group2(j) group3(k) size(data,2)]);
                chk_data = data(trn_data_n+1:trn_data_n+144, [group1(i) group2(j) group3(k) size(da
                in_fismat = genfis(trn_data(:,1:end-1),trn_data(:,end),genOpt);
                % Set initial FIS and validation data in option set for ANFIS training.
                anfisOpt.InitialFIS = in_fismat;
                anfisOpt.ValidationData = chk_data;
                [~, t_err, ~, ~, c_err] = anfis(trn_data,anfisOpt);
                trn_error(model) = min(t_err);
                chk_error(model) = min(c_err);
                fprintf('ANFIS model = %d: %s %s %s',model,in1,in2,in3);
                fprintf(' --> trn=%.4f,',trn_error(model));
                fprintf(' chk=%.4f',chk_error(model));
                fprintf('\n');
                model = model+1;
            end
        end
    end
```

```
ANFIS model = 1: y(k-1) y(k-2) u(k-1)
 --> trn=1.5155,
 chk=1.7472
ANFIS model = 2: y(k-1) y(k-2) u(k-2)
 --> trn=1.4778,
 chk=1.9053
ANFIS model = 3: y(k-1) y(k-2) u(k-3)
 --> trn=1.7208,
 chk=2.0421
ANFIS model = 4: y(k-1) y(k-2) u(k-4)
 --> trn=1.7155,
 chk=1.9942
ANFIS model = 5: y(k-1) y(k-2) u(k-5)
 --> trn=1.7487,
 chk=1.9616
ANFIS model = 6: y(k-1) y(k-2) u(k-6)
 --> trn=1.6993,
 chk=2.0648
ANFIS model = 7: y(k-1) y(k-3) u(k-1)
 --> trn=1.6282,
 chk=1.5623
ANFIS model = 8: y(k-1) y(k-3) u(k-2)
 --> trn=1.3762,
 chk=1.8211
ANFIS model = 9: y(k-1) y(k-3) u(k-3)
 --> trn=1.7498,
 chk=1.9335
ANFIS model = 10: y(k-1) y(k-3) u(k-4)
 --> trn=1.7778,
 chk=1.9078
ANFIS model = 11: y(k-1) y(k-3) u(k-5)
 --> trn=1.7982,
 chk=2.3476
ANFIS model = 12: y(k-1) y(k-3) u(k-6)
 --> trn=1.7279,
 chk=2.8742
ANFIS model = 13: y(k-1) y(k-4) u(k-1)
 --> trn=1.6823,
 chk=1.8829
```

```
ANFIS model = 14: y(k-1) y(k-4) u(k-2)
 --> trn=1.4347,
 chk=1.7455
ANFIS model = 15: y(k-1) y(k-4) u(k-3)
 --> trn=1.7811,
 chk=2.1563
ANFIS model = 16: y(k-1) y(k-4) u(k-4)
 --> trn=1.8196,
 chk=1.9877
ANFIS model = 17: y(k-1) y(k-4) u(k-5)
 --> trn=1.8210,
 chk=2.1154
ANFIS model = 18: y(k-1) y(k-4) u(k-6)
 --> trn=1.8472,
 chk=2.0988
ANFIS model = 19: y(k-2) y(k-3) u(k-1)
 --> trn=2.6726,
 chk=2.9291
ANFIS model = 20: y(k-2) y(k-3) u(k-2)
 --> trn=1.9812,
 chk=2.7147
ANFIS model = 21: y(k-2) y(k-3) u(k-3)
 --> trn=2.4273,
 chk=3.1624
ANFIS model = 22: y(k-2) y(k-3) u(k-4)
 --> trn=2.7109,
 chk=3.6038
ANFIS model = 23: y(k-2) y(k-3) u(k-5)
 --> trn=2.7142,
 chk=3.3435
ANFIS model = 24: y(k-2) y(k-3) u(k-6)
 --> trn=2.7036,
 chk=3.2746
ANFIS model = 25: y(k-2) y(k-4) u(k-1)
 --> trn=2.6986,
 chk=3.1303
ANFIS model = 26: y(k-2) y(k-4) u(k-2)
 --> trn=2.0978,
 chk=2.3785
ANFIS model = 27: y(k-2) y(k-4) u(k-3)
 --> trn=2.2733,
 chk=3.0979
ANFIS model = 28: y(k-2) y(k-4) u(k-4)
 --> trn=2.7395,
 chk=3.1976
ANFIS model = 29: y(k-2) y(k-4) u(k-5)
 --> trn=2.7059,
 chk=3.4822
ANFIS model = 30: y(k-2) y(k-4) u(k-6)
 --> trn=2.7285,
 chk=4.0173
ANFIS model = 31: y(k-3) y(k-4) u(k-1)
 --> trn=3.1888,
 chk=4.6527
ANFIS model = 32: y(k-3) y(k-4) u(k-2)
 --> trn=2.8177,
 chk=3.4184
ANFIS model = 33: y(k-3) y(k-4) u(k-3)
 --> trn=2.4572,
 chk=3.7243
ANFIS model = 34: y(k-3) y(k-4) u(k-4)
 --> trn=3.0958,
 chk=4.1090
ANFIS model = 35: y(k-3) y(k-4) u(k-5)
 --> trn=3.2035,
```

```
 chk=4.2832
ANFIS model = 36: y(k-3) y(k-4) u(k-6)
 --> trn=3.2049,
 chk=4.3130
```

```
% ====== Reordering according to training error
[~, b] = sort(trn_error);
b = flipud(b);  % List according to decreasing trn error
trn_error = trn_error(b);
chk_error = chk_error(b);
index = index(b,:);

% ====== Display training and checking errors
x = (1:anfis_n)';
subplot(2,1,1)
plot(x, trn_error,'-',x,chk_error,'-', ...
     x,trn_error,'o',x,chk_error,'*')
tmp = x(:, ones(1,3))';
X = tmp(:);
tmp = [zeros(anfis_n,1) max(trn_error,chk_error) nan*ones(anfis_n,1)]';
Y = tmp(:);
hold on
plot(X,Y,'g')
hold off
axis([1 anfis_n -inf inf])
h_gca = gca;
h_gca.XTickLabel = [];

% ====== Add text of input variables
for k = 1:anfis_n
    text(x(k), 0, ...
        [input_name(index(k,1),:) ' ' ...
         input_name(index(k,2),:) ' ' ...
         input_name(index(k,3),:)]);
end
h = findobj(gcf,'type','text');
set(h,'rot',90,'fontsize',11,'hori','right');

drawnow

% ====== Generate input_index for bjtrain.m
[a, b] = min(trn_error);
input_index = index(b,:);
title('Training (Circles) and Checking (Asterisks) Errors','fontsize',10)
ylabel('RMSE','fontsize',10)
```




This window shows ANFIS predictions on both training and checking data sets. Obviously the performance is better than those of the ARX model.

```
if ishghandle(winH1),delete(winH1);
end

trn_data = data(1:trn_data_n,[input_index, size(data,2)]);
chk_data = data(trn_data_n+1:294,[input_index, size(data,2)]);

% generate FIS matrix
in_fismat = genfis(trn_data(:,1:end-1),trn_data(:,end));
anfisOpt = anfisOptions('InitialFIS',in_fismat,...
                        'EpochNumber',1,...
                        'InitialStepSize',0.01,...
                        'StepSizeDecreaseRate',0.5,...
                        'StepSizeIncreaseRate',1.5,...
                        'ValidationData',chk_data);
[trn_out_fismat,trn_error,step_size,chk_out_fismat,chk_error] = ...
    anfis(trn_data,anfisOpt);
```

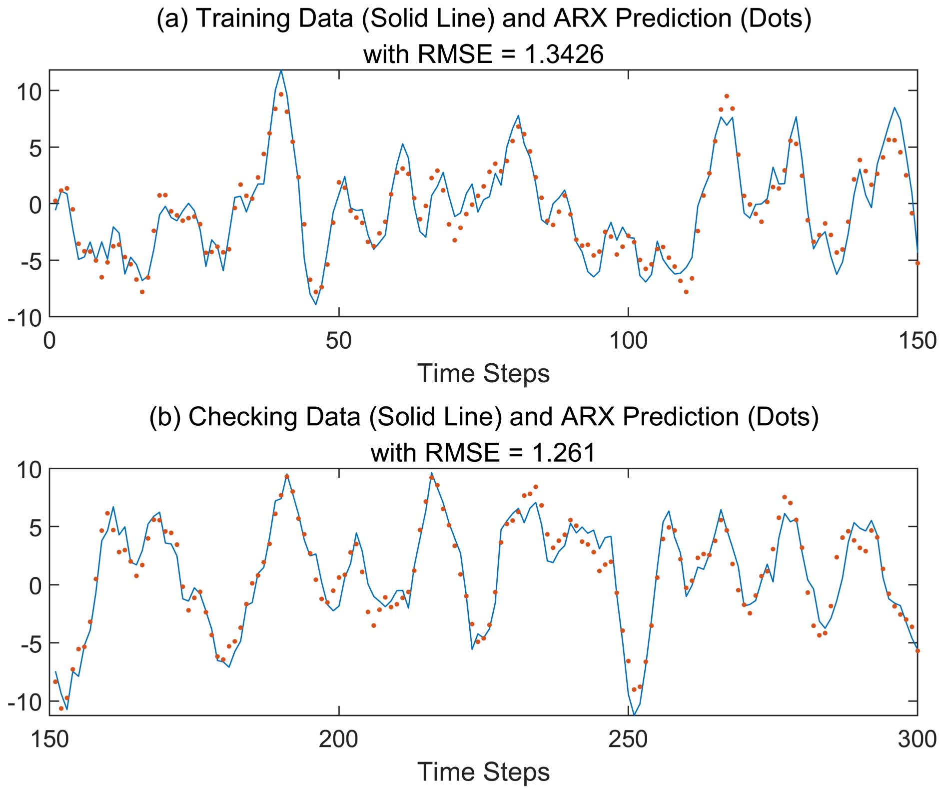
```
ANFIS info:
 Number of nodes: 34
 Number of linear parameters: 32
 Number of nonlinear parameters: 18
 Total number of parameters: 50
 Number of training data pairs: 150
 Number of checking data pairs: 144
 Number of fuzzy rules: 8
```

```
Start training ANFIS ...

   1    1.37623    1.82108

Designated epoch number reached --> ANFIS training completed at epoch 1.

Minimal training RMSE = 1.376230
Minimal checking RMSE = 1.82108
```

```
subplot(2,1,1)
index = 1:trn_data_n;
plot(index,y(index),index,yp(index),'.')
rmse = norm(y(index)-yp(index))/sqrt(length(index));
title(sprintf(['(a) Training Data (Solid Line) and ARX Prediction (Dots)\nwith RMSE = ' num2str
disp(['[na nb d] = ' num2str(nn)])
```

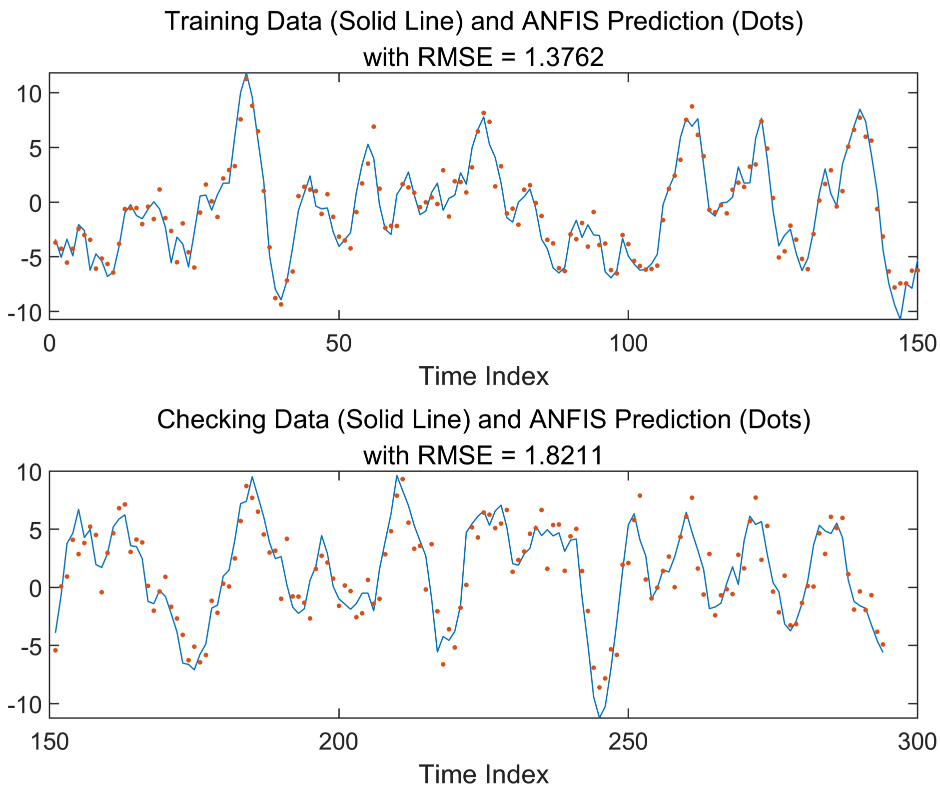
```
[na nb d] = 10    9    1
```

```
xlabel('Time Steps','fontsize',10)
subplot(2,1,2)
index = (trn_data_n+1):(total_data_n);
plot(index, y(index),index,yp(index),'.')
rmse = norm(y(index)-yp(index))/sqrt(length(index));
title(sprintf(['(b) Checking Data (Solid Line) and ARX Prediction (Dots)\nwith RMSE = ' num2str
xlabel('Time Steps','fontsize',10)
```




```
y_hat = evalfis(chk_out_fismat,data(1:294,input_index));
```

```
subplot(2,1,1)
index = 1:trn_data_n;
plot(index,data(index,size(data,2)),'-', ...
     index,y_hat(index),'.')
rmse = norm(y_hat(index)-data(index,size(data,2)))/sqrt(length(index));
title(sprintf(['Training Data (Solid Line) and ANFIS Prediction (Dots)\nwith RMSE = ' num2str(r
xlabel('Time Index','fontsize',10)
ylabel('')

subplot(2,1,2)
index = trn_data_n+1:294;
plot(index,data(index,size(data,2)),'-',index,y_hat(index),'.')
rmse = norm(y_hat(index)-data(index,size(data,2)))/sqrt(length(index));
title(sprintf(['Checking Data (Solid Line) and ANFIS Prediction (Dots)\nwith RMSE = ' num2str(r
xlabel('Time Index','fontsize',10)
ylabel('')
```




## Conclusion

The table above is a comparison among various modeling approaches. The ARX modeling spends the least amount of time to reach the worst precision, and the ANFIS modeling via exhaustive search takes the most amount of time to reach the best precision. In other words, if fast modeling is the goal, then ARX is the right choice. But if precision is the utmost concern, then we should go with ANFIS, which is designed for nonlinear modeling and higher precision.

| Models | ARX Model | ANFIS Model (Sequential Search) | ANFIS (Exhausti |
|---|---|---|---|
| . of Input Arguments | 14 | 3 | 3 |
| Training RMSE | 0.1122 | 0.0609 | 0.0 |
| Checking RMSE | 0.0749 | 0.0604 | 0.0 |
| of Linear Parameters | 15 | 32 | 3 |
| f Nonlinear Parameters | 0 | 18 | 1 |
| Computation Time<br>Pro 200 MHz, 64 MB RAM) | 1.6 S | 10.3 S | 21. |

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