The FINAL PROOF IS AS FOLLOWS

```
theorem PExample {a b c : ℝ} (ha : a > 0) (hb : b > 0) (hc : c > 0) (h : a + b + c = 1) :
1 / (a + 2) + 1 / (b + 2) + 1 / (c + 2) ≤ 1 / (6 * sqrt (a * b) + c) + 1 / (6 * sqrt (b *
c) + a) + 1 / (6 * sqrt (c * a) + b) := by

  make_homogeneous 1 / (2 * a + 2 * b + 3 * c) + 1 / (2 * a + 3 * b + 2 * c) + 1 / (3 * a
+ 2 * b + 2 * c) - (1 / (a + 6 * sqrt (b * c))) - (1 / (b + 6 * sqrt (a * c))) - (1 / (c
+ 6 * sqrt (a * b))) ≤ 0

  scale AM_GM_div_cycle_normal_right_2vars (u1 := a) (u2 := c) (u3 := c) (v1 := b) (v2 :=
a) (v3 := b) (i1 := 6) (i2 := 6) (i3 := 6) (j1 := c) (j2 := b) (j3 := a) (k := 1) (l :=
0) (left := 1 / (2 * a + 2 * b + 3 * c) + 1 / (2 * a + 3 * b + 2 * c) + 1 / (3 * a + 2 *
b + 2 * c))

  llm_simplify 1 / (2 * a + 2 * b + 3 * c) + 1 / (2 * a + 3 * b + 2 * c) + 1 / (3 * a + 2
* b + 2 * c) - (1 / (6 * (a + b) / 2 + c) + 1 / (6 * (c + a) / 2 + b) + 1 / (6 * (c + b)
/ 2 + a)) = 1 / ((a + 2)) + 1 / ((b + 2)) + 1 / ((c + 2)) - (1 / ((-(2 * a) + 3))) - (1 /
((-(2 * b) + 3))) - (1 / ((-(2 * c) + 3)))

  sym_tangent_line 27 * a / 49 + 27 * b / 49 + 27 * c / 49 - (27 / 49) ≤ (1 / (-(2 * c) +
3) + 1 / (-(2 * b) + 3) + 1 / (-(2 * a) + 3)) - (1 / (a + 2) + 1 / (b + 2) + 1 / (c + 2))

  try close
```

Translate the following proof into natural language format. (DeepSeek)

Natural Language Translation of the Proof

Theorem:
Let a, b, c be positive real numbers such that $a + b + c = 1$. Then:

$$\frac{1}{a+2} + \frac{1}{b+2} + \frac{1}{c+2} \leq \frac{1}{6\sqrt{ab}+c} + \frac{1}{6\sqrt{bc}+a} + \frac{1}{6\sqrt{ca}+b}. \tag{1}$$

Step-by-Step Proof in Natural Language

Step 1: Homogenization (Rewriting the Inequality in a Symmetric Form)

The original inequality compares two sums of fractions. To make it easier to analyze, we first rewrite it in a **homogeneous** (balanced) form.

- The left side has denominators like $(a + 2)$, which can be seen as $(a + 2(a + b + c))$ because $a + b + c = 1$.
- The right side has denominators involving $6\sqrt{ab} + c$, which we relate to arithmetic and geometric means.

This step ensures that all terms scale properly when we manipulate them.

Step 2: Applying the AM-GM Inequality (Bounding the Square Roots)

The right-hand side (RHS) contains terms like $6\sqrt{ab}$. Since $\sqrt{ab}$ is the **geometric mean** of a and b, we recall the **Arithmetic Mean-Geometric Mean (AM-GM) Inequality**, which states:

$$\sqrt{ab} \leq \frac{a+b}{2}. \tag{2}$$

- We apply this to each square root term, replacing $6\sqrt{ab}$ with $6 \cdot \frac{a+b}{2} = 3(a+b)$.
- This gives us a **weaker but simpler** inequality to work with.

Effect:
The RHS becomes larger (since $\sqrt{ab} \leq \frac{a+b}{2}$, making denominators smaller and fractions bigger). If we can prove the inequality for this relaxed version, the original one must also hold.

Step 3: Simplifying the Expression

After applying AM-GM, we substitute and simplify:

$$\frac{1}{6\sqrt{ab}+c} \approx \frac{1}{3(a+b)+c}. \tag{3}$$

But since $a+b+c=1$, we can rewrite the denominator:

$$3(a+b)+c = 2(a+b)+(a+b+c) = 2(a+b)+1. \tag{4}$$

This leads to expressions like $\frac{1}{2(a+b)+1}$.

Key Insight:
We now compare the original left-hand side (LHS) $\frac{1}{a+2}$ with the simplified RHS terms.

Step 4: Tangent Line Method (Approximating the Difference)

To compare the two sides, we use **tangent line approximations** (a technique from calculus).

- Consider the function $f(x) = \frac{1}{x+2}$.
- Its tangent line at $x = \frac{1}{3}$ (since a, b, c average to $\frac{1}{3}$) gives a linear approximation.
- We use this to estimate how much the LHS and RHS differ.

Result:
We find that the difference between the two sides can be controlled by a linear function, and due to convexity/concavity properties, the inequality holds.

Step 5: Final Verification (Ensuring Non-Positivity of the Remainder)

After all substitutions and approximations, we are left with an expression of the form:

$$(\text{LHS - RHS}) \leq 0. \tag{5}$$

- By carefully checking coefficients and using $a + b + c = 1$, we confirm that this difference is indeed non-positive.
- Thus, the original inequality is proven.

Conclusion

The proof combines:

1. **Homogenization** (balancing terms using $a + b + c = 1$),
2. **AM-GM Inequality** (to relax square root terms),
3. **Algebraic Simplification** (rewriting denominators),
4. **Tangent Line Approximation** (for comparison),
5. **Final Inequality Verification** (ensuring correctness).

This structured approach ensures that the inequality holds for all positive a, b, c summing to 1.

Final Answer:
The inequality is true, and the proof is complete. ✅