https://github.com/LizzieSparling/magnetic_pendulum

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Euler’s Method to plot pendulum trajectory:

Set height of the pendulum bob above the x-y plane to be 0.5cm. In order for our derived equations to hold, we need to assume that small angle approximations can be used, and thus we are making the assumption that the length of the pendulum is always very large in comparison to the radius of the magnets. This allows us to plot the trajectory by adopting the plan view of the system, projecting the bob’s position onto the x-y plane.

Given initial point (0.9, -0.6). This point was chosen using the *numpy.random.uniform* method, selecting x and y from -1 to 1, increments of 0.1).

Initial velocity set to 0.0m/s, releasing the pendulum from rest.

Magnetic force (from each of the four magnets) is proportional to $1 / ((x_n - x)^2 + (y_n - y)^2 + h^2)$. Magnetic strength is kept at a constant such that the magnetic force constant of proportionality is 5.

Total magnetic force is the sum of the 4 magnetic forces provided by each magnet.

Gravitational restoring force acting on the pendulum is proportional to $-(x_0, y_0)$.

Damping force is proportional to chosen constant (b = 0.05) multiplied by the velocity of the pendulum at that point in time.

The total force acting on the pendulum is, therefore, given by the sum of these three force components.

Using Newton’s Second Law, assuming the pendulum has unit mass, we equate the acceleration of the pendulum to the total force acting upon it.

Euler’s Method gives us the following algorithm to compute the velocity of our pendulum with steps in time dt = 0.01 : $y_1 = y_0 + f(t_0, y_0)*(t - t_0) = y_0 + f(t_0, y_0)*(0.01)$

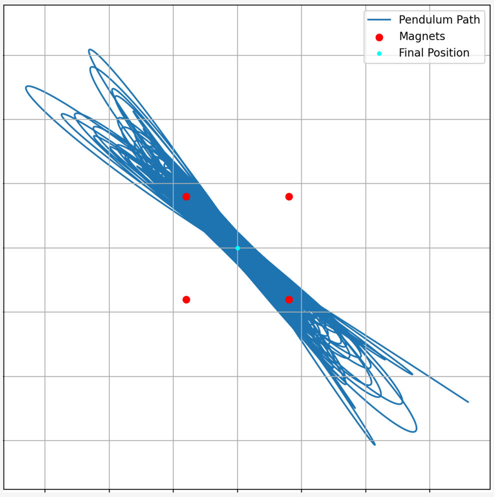
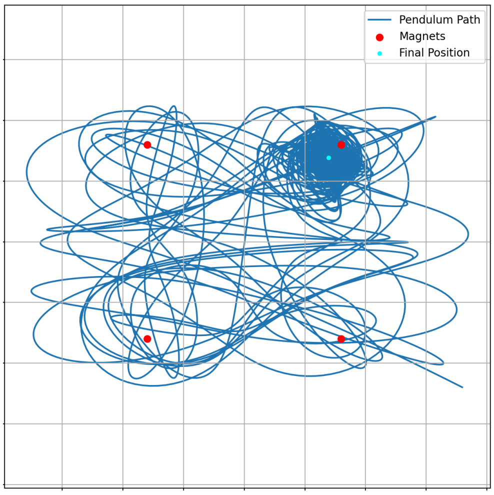
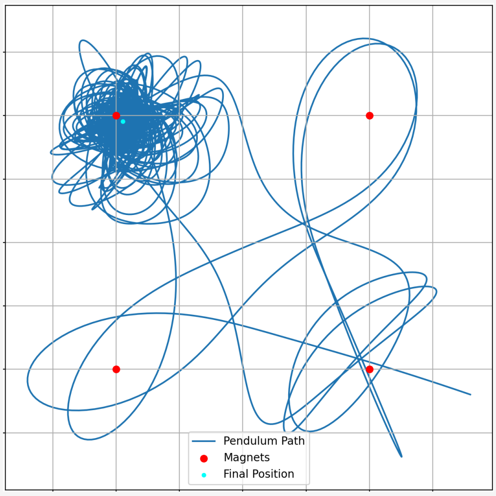
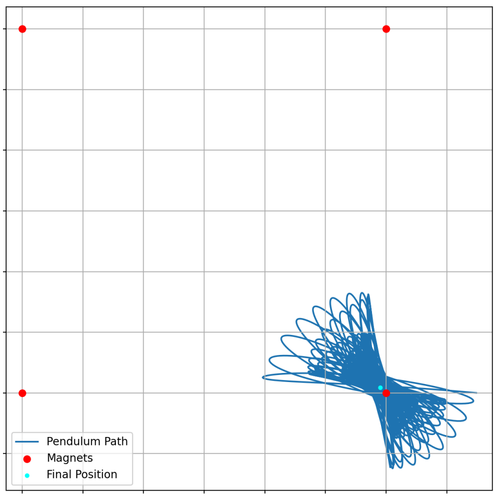
By calculating the current velocity of the pendulum (velocity += acceleration * dt), the new position of the pendulum can be found using Euler’s method (position += velocity * dt).

Implementing this method through python, we can use matplotlib.pyplot, to create plots of the pendulum trajectory, up until the point at which the pendulum reaches an arbitrarily small velocity (< 1e-3 m/s).

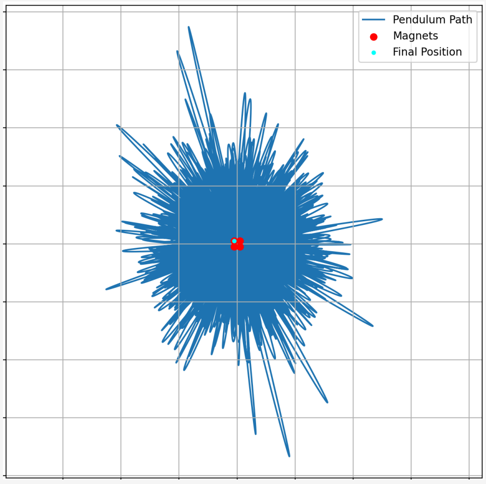
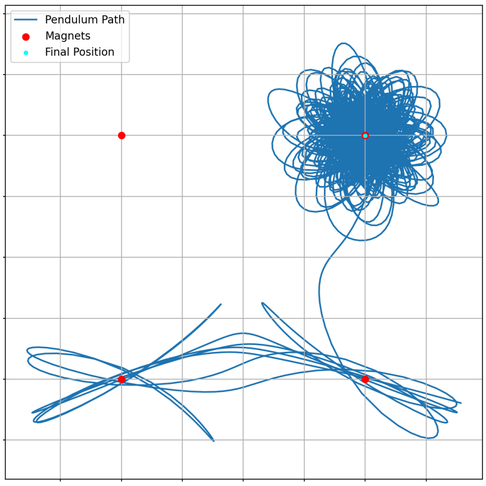
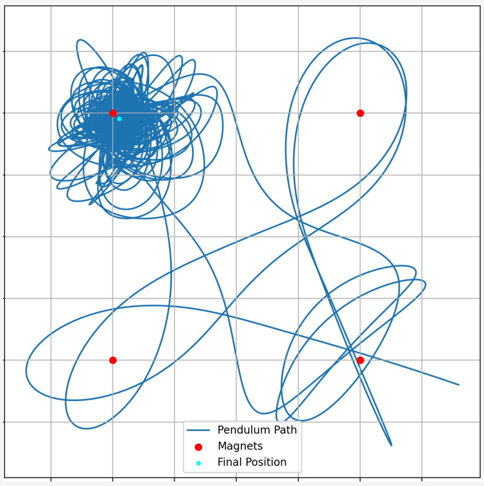
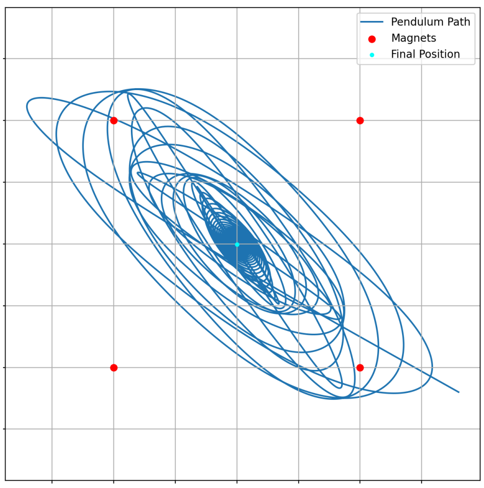
Our four magnets lie at the corner of a square centred at the origin, and thus by substituting in different values for the side length for this square.

Randomly-chosen initial position : [0.9, -0.6]
Damping constant (b) = 0.05

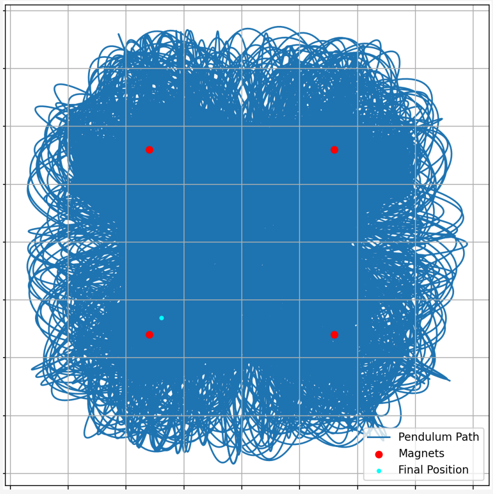
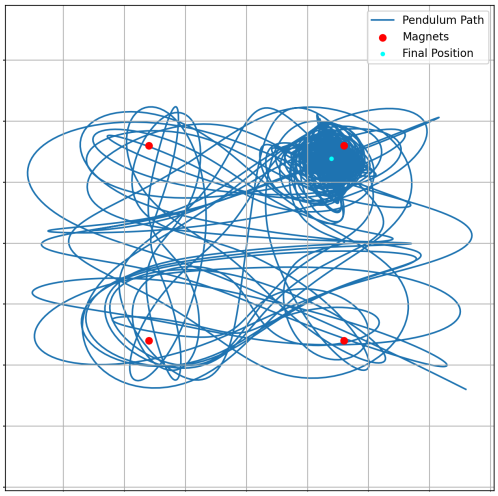
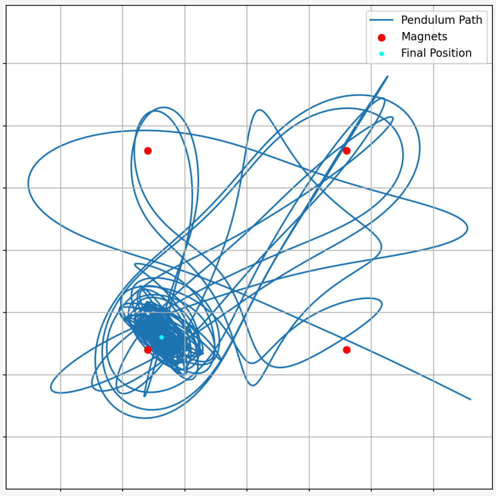
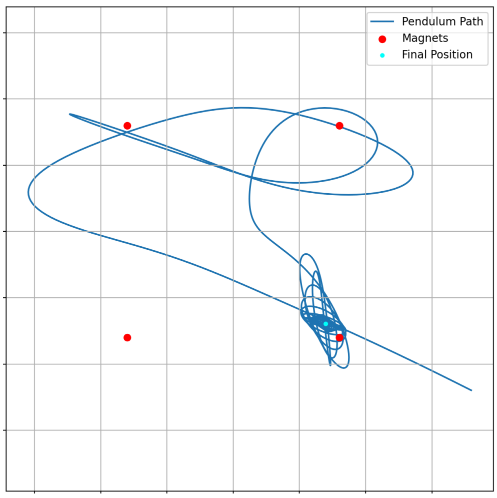
Changing magnet radius, height above x-y plane = 0.5cm

Side length of square / Magnet radius (cm)	Trajectory plot
0.2	
0.4	
0.5	
0.6	

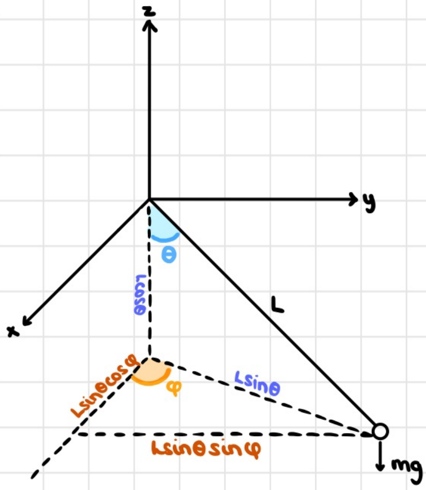
Changing height above plane, magnet radius = 0.5cm

Height above x-y plane	Trajectory plot
0.125	
0.25	
0.5	
1.0	

Magnet radius = 0.4cm , height above x-y plane = 0.5cm, changing damping

Damping constant (b)	Trajectory plot
0	
0.05	
0.1	
0.5	

Modelling system with spherical coordinates:



- pendulum attached to fixed point $(0,0,0)$
- pendulum length $= L$
- magnet positions $\begin{pmatrix}0\\R\\-(L+h)\end{pmatrix}, \begin{pmatrix}0\\R\\-(L+h)\end{pmatrix}, \begin{pmatrix}R\\0\\-(L+h)\end{pmatrix}, \begin{pmatrix}R\\0\\-(L+h)\end{pmatrix}$

SPERICAL COORDINATES:

- more accurate representation of pendulum motion
- fully incorporate 3D dynamics (dependence on both azimuthal and polar angles)

POSITION VECTOR: $x = \begin{bmatrix} L\sin\theta\cos\varphi \\ L\sin\theta\sin\varphi \\ -L\cos\theta \end{bmatrix}$

VELOCITY: $\dot{x} = L \begin{bmatrix} \dot\theta\cos\theta\cos\varphi - L\dot\varphi\sin\theta\sin\varphi \\ \dot\theta\cos\theta\sin\varphi + L\dot\varphi\sin\theta\cos\varphi \\ \dot\theta\sin\theta \end{bmatrix}$

KINETIC ENERGY: $E = {}^{m}/_{2} \, |\dot{x}| \cdot |\dot{x}| = \frac{mL^2}{2}\left(\dot\theta^2 + \dot\varphi^2 \sin^2\theta\right)$

POTENTIAL ENEGY $= U_{grav} + V_{mag}$

$$U_{grav} = mg\left(L(1-\cos\theta)+h\right)$$

$$V_{mag} = \sum_{i=1}^{4} \frac{-p}{|x-x_i|^4}$$ magnitude of magnetic force predicted to vary as $^1/_{dist^4}$

LAGRANGIAN: $L = KE - U_{grav} - V_{mag}$

$$= \tfrac{1}{2} mL^2\left(\dot\theta^2 + \dot\varphi^2\sin^2\theta\right) - mg\left(L(1-\cos\theta)+h\right) + \sum^{4} \frac{p}{|x-x_i|^4}$$

Linear damping proportional to velocity, use Rayleigh dissipation function $F = \tfrac{1}{2} b\dot{x}^2$ where $b =$ constant of proportionality $\Rightarrow$ $F = \tfrac{1}{2} bL^2\left(\dot\theta^2 + \dot\varphi^2\sin^2\theta\right)$

EULER LAGRANGE:

① $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = -\frac{\partial F}{\partial\dot\theta}$

② $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\varphi}\right) - \frac{\partial L}{\partial\varphi} = -\frac{\partial F}{\partial\dot\varphi}$