

https://github.com/LizzieSparling/magnetic_pendulum

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Euler's Method to plot pendulum trajectory:

Set height of the pendulum bob above the x-y plane to be 0.5cm. In order for our derived equations to hold, we need to assume that small angle approximations can be used, and thus we are making the assumption that the length of the pendulum is always very large in comparison to the radius of the magnets. This allows us to plot the trajectory by adopting the plan view of the system, projecting the bob's position onto the x-y plane.

Given initial point (0.9, -0.6). This point was chosen using the *numpy.random.uniform* method, selecting x and y from -1 to 1, increments of 0.1).

Initial velocity set to 0.0m/s, releasing the pendulum from rest.

Magnetic force (from each of the four magnets) is proportional to $1 / ((x_n - x)^2 + (y_n - y)^2 + h^2)$. Magnetic strength is kept at a constant such that the magnetic force constant of proportionality is 5.

Total magnetic force is the sum of the 4 magnetic forces provided by each magnet.

Gravitational restoring force acting on the pendulum is proportional to $-(x_0, y_0)$.

Damping force is proportional to chosen constant ($b = 0.05$) multiplied by the velocity of the pendulum at that point in time.

The total force acting on the pendulum is, therefore, given by the sum of these three force components.

Using Newton's Second Law, assuming the pendulum has unit mass, we equate the acceleration of the pendulum to the total force acting upon it.

Euler's Method gives us the following algorithm to compute the velocity of our pendulum with steps in time $dt = 0.01$: $y_1 = y_0 + f(t_0, y_0) * (t - t_0) = y_0 + f(t_0, y_0) * (0.01)$

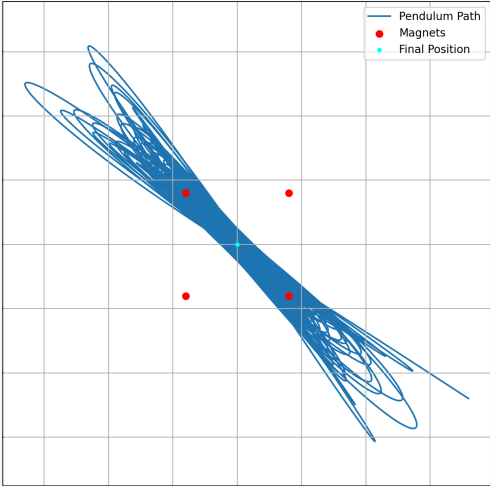
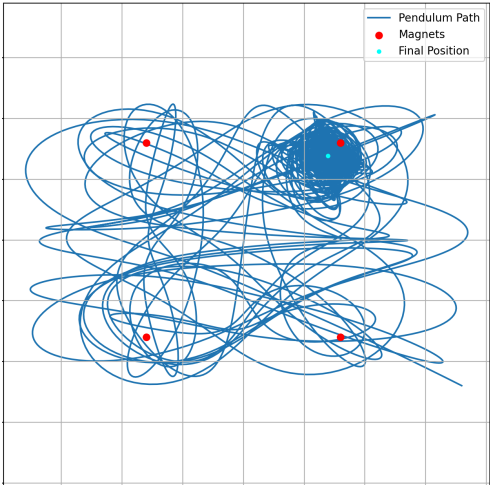
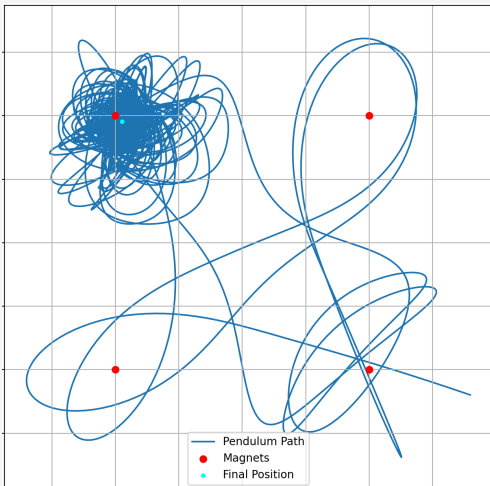
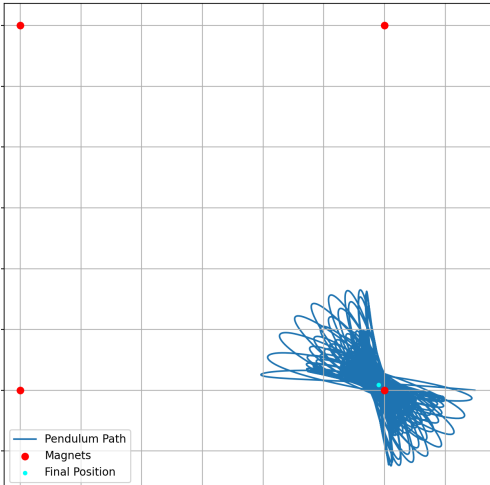
By calculating the current velocity of the pendulum ($velocity += acceleration * dt$), the new position of the pendulum can be found using Euler's method ($position += velocity * dt$).

Implementing this method through python, we can use matplotlib.pyplot, to create plots of the pendulum trajectory, up until the point at which the pendulum reaches an arbitrarily small velocity ($< 1e-3$ m/s).

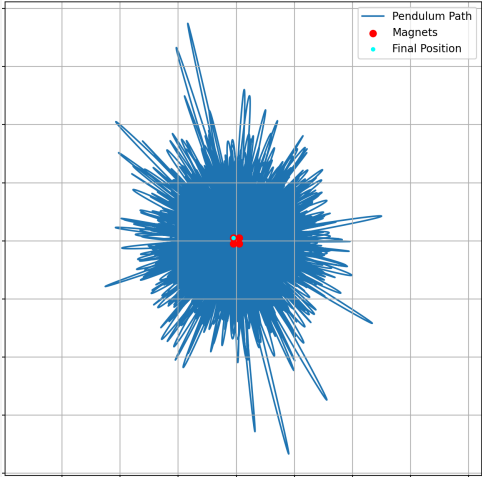
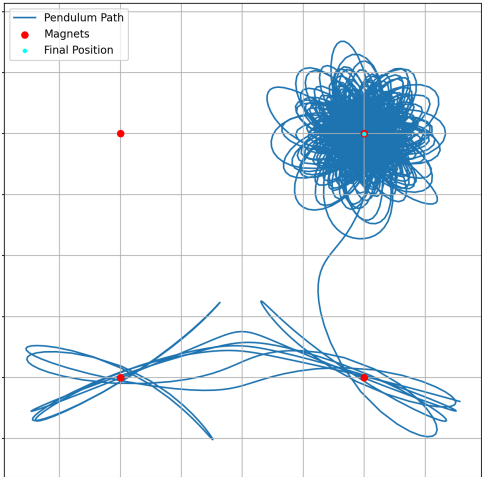
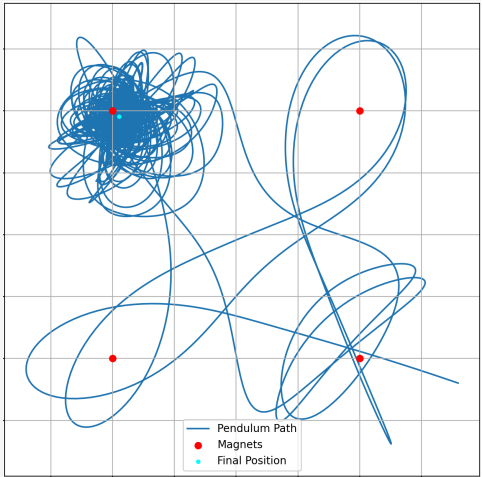
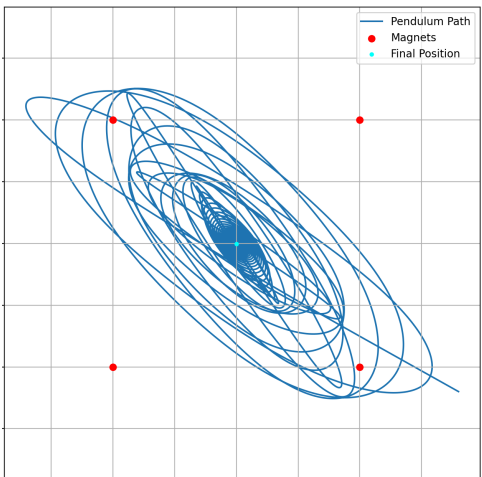
Our four magnets lie at the corner of a square centred at the origin, and thus by substituting in different values for the side length for this square.

**Randomly-chosen initial position : [0.9, -0.6]
Damping constant (b) = 0.05**

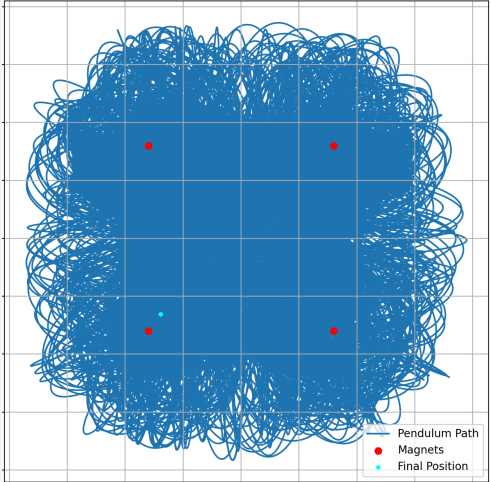
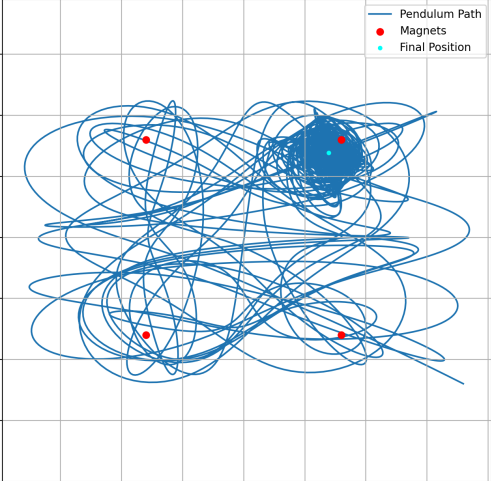
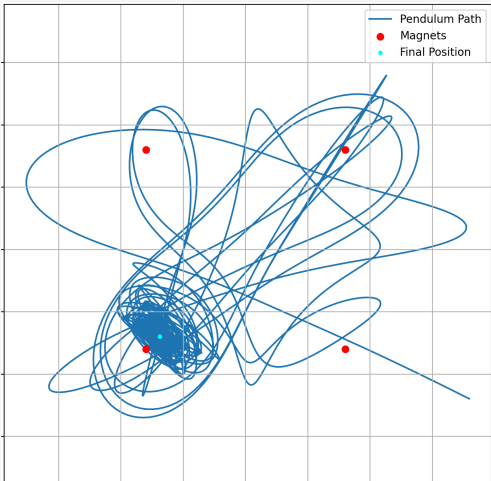
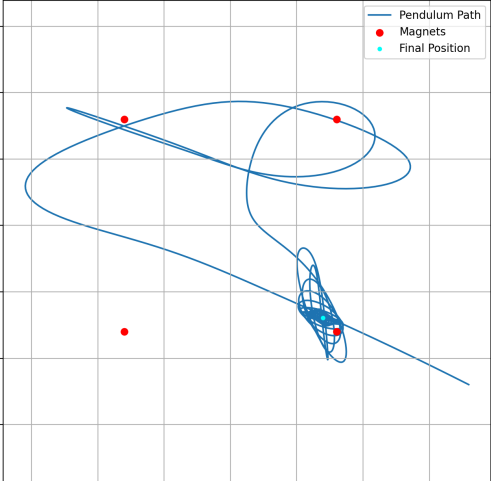
Changing magnet radius, height above x-y plane = 0.5cm

Side length of square / Magnet radius (cm)	Trajectory plot
0.2	 <p>This plot shows a pendulum path (blue line) that is highly elongated and narrow, indicating a strong magnetic influence. The path is concentrated around a central point, with several red dots representing magnet positions and a cyan dot representing the final position. The legend indicates: Pendulum Path, Magnets, and Final Position.</p>
0.4	 <p>This plot shows a pendulum path (blue line) that is more complex and spread out than the 0.2 cm case, indicating a moderate magnetic influence. The path is concentrated around a central point, with several red dots representing magnet positions and a cyan dot representing the final position. The legend indicates: Pendulum Path, Magnets, and Final Position.</p>
0.5	 <p>This plot shows a pendulum path (blue line) that is even more complex and spread out than the 0.4 cm case, indicating a weak magnetic influence. The path is concentrated around a central point, with several red dots representing magnet positions and a cyan dot representing the final position. The legend indicates: Pendulum Path, Magnets, and Final Position.</p>
0.6	 <p>This plot shows a pendulum path (blue line) that is highly elongated and narrow, indicating a strong magnetic influence. The path is concentrated around a central point, with several red dots representing magnet positions and a cyan dot representing the final position. The legend indicates: Pendulum Path, Magnets, and Final Position.</p>

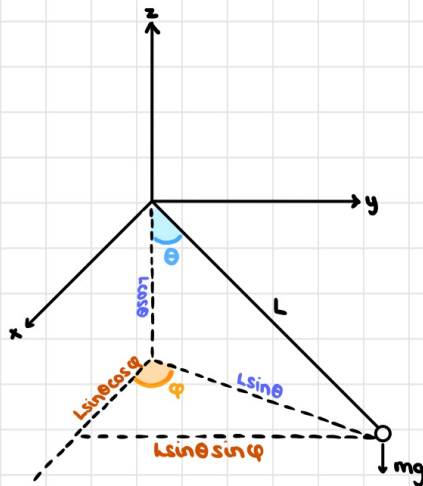
Changing height above plane, magnet radius = 0.5cm

Height above x-y plane	Trajectory plot
0.125	 <p>A plot showing the pendulum path (blue lines) and magnet positions (red dots) at a height of 0.125. The path is highly oscillatory and dense, centered around a magnet located at the center of the plot. A legend in the top right corner identifies the 'Pendulum Path' (blue line), 'Magnets' (red dots), and 'Final Position' (cyan dot).</p>
0.25	 <p>A plot showing the pendulum path (blue lines) and magnet positions (red dots) at a height of 0.25. The path is more spread out and less dense than at 0.125, with a magnet located at the center. A legend in the top left corner identifies the 'Pendulum Path' (blue line), 'Magnets' (red dots), and 'Final Position' (cyan dot).</p>
0.5	 <p>A plot showing the pendulum path (blue lines) and magnet positions (red dots) at a height of 0.5. The path is more spread out and less dense than at 0.25, with a magnet located at the center. A legend in the bottom right corner identifies the 'Pendulum Path' (blue line), 'Magnets' (red dots), and 'Final Position' (cyan dot).</p>
1.0	 <p>A plot showing the pendulum path (blue lines) and magnet positions (red dots) at a height of 1.0. The path is more spread out and less dense than at 0.5, with a magnet located at the center. A legend in the top right corner identifies the 'Pendulum Path' (blue line), 'Magnets' (red dots), and 'Final Position' (cyan dot).</p>

Magnet radius = 0.4cm , height above x-y plane = 0.5cm, changing damping

Damping constant (b)	Trajectory plot
0	 <p>The plot for b=0 shows a highly dense, circular blue trajectory. Four red dots representing magnets are located at approximately (2, 2), (8, 2), (2, 8), and (8, 8) on a 10x10 grid. A cyan dot representing the final position is at the center (5, 5). The legend in the bottom right corner identifies the blue line as 'Pendulum Path', the red dots as 'Magnets', and the cyan dot as 'Final Position'.</p>
0.05	 <p>The plot for b=0.05 shows a blue trajectory that is less dense than the b=0 case, with more distinct loops. The magnets and final position are in the same locations. The legend is in the top right corner.</p>
0.1	 <p>The plot for b=0.1 shows a blue trajectory with even fewer loops, appearing more open. The magnets and final position are in the same locations. The legend is in the top right corner.</p>
0.5	 <p>The plot for b=0.5 shows a blue trajectory that is very sparse, with only a few loops visible. The magnets and final position are in the same locations. The legend is in the top right corner.</p>

Modelling system with spherical coordinates:



• Pendulum attached to fixed point $(0,0,0)$

• Pendulum length = L

• magnet positions $\begin{pmatrix} 0 \\ R \\ -(L+h) \end{pmatrix}, \begin{pmatrix} 0 \\ R \\ -(L+h) \end{pmatrix}, \begin{pmatrix} R \\ 0 \\ -(L+h) \end{pmatrix}, \begin{pmatrix} R \\ 0 \\ -(L+h) \end{pmatrix}$

SPHERICAL COORDINATES:

- more accurate representation of pendulum motion
- fully incorporate 3D dynamics (dependence on both azimuthal and polar angles)

POSITION VECTOR: $\mathbf{x} = \begin{bmatrix} L \sin \theta \cos \varphi \\ L \sin \theta \sin \varphi \\ -L \cos \theta \end{bmatrix}$

VELOCITY: $\dot{\mathbf{x}} = L \begin{bmatrix} \dot{\theta} \cos \theta \cos \varphi - L \dot{\varphi} \sin \theta \sin \varphi \\ \dot{\theta} \cos \theta \sin \varphi + L \dot{\varphi} \sin \theta \cos \varphi \\ \dot{\theta} \sin \theta \end{bmatrix}$

KINETIC ENERGY: $E = \frac{m}{2} |\dot{\mathbf{x}}| \cdot |\dot{\mathbf{x}}| = \frac{mL^2}{2} (\dot{\theta}^2 + \dot{\varphi}^2 \sin^2 \theta)$

POTENTIAL ENERGY = $U_{\text{grav}} + V_{\text{mag}}$

$U_{\text{grav}} = mg(L(1 - \cos \theta) + h)$

$V_{\text{mag}} = \sum_{i=1}^4 \frac{-P}{|\mathbf{x} - \mathbf{x}_i|^4}$ magnitude of magnetic force predicted to vary as $1/\text{dist}^4$

LAGRANGIAN: $L = KE - U_{\text{grav}} - V_{\text{mag}}$

$= \frac{1}{2} mL^2 (\dot{\theta}^2 + \dot{\varphi}^2 \sin^2 \theta) - mg(L(1 - \cos \theta) + h) + \sum_{i=1}^4 \frac{P}{|\mathbf{x} - \mathbf{x}_i|^4}$

Linear damping proportional to velocity, use Rayleigh dissipation function $F = \frac{1}{2} b \dot{\mathbf{x}}^2$

where b = constant of proportionality $\Rightarrow F = \frac{1}{2} b L^2 (\dot{\theta}^2 + \dot{\varphi}^2 \sin^2 \theta)$

EULER LAGRANGE:

① $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = - \frac{\partial F}{\partial \dot{\theta}}$

② $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varphi}} \right) - \frac{\partial L}{\partial \varphi} = - \frac{\partial F}{\partial \dot{\varphi}}$