LAB REPORT

Lab 4: Control Design Using the Root Locus

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Prelab: 1 marks

Lab Report: 4 marks

Lab Work: 5 marks

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$\boxed{\text{Pre-Lab}}$ Lab 4 Group 3

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$\boxed{1}$ plant has one pole at $-b$, and none at origin. Therefore it is type 0.

Assume $D(s)=0$, $E(s)=R(s)-V(s)$ (1)

From figure 2:

$$V(s)=\left[E(s)C(s)+\cancel{D(s)}^{0}\right]\left(\frac{a}{s+b}\right)=E(s)C(s)\left(\frac{a}{s+b}\right)$$

sub eq. (1)

$$\frac{E(s)}{R(s)}=1-\frac{V(s)}{R(s)}=1-\frac{E(s)C(s)\left(\frac{a}{s+b}\right)}{R(s)}$$

$$\frac{E(s)}{R(s)}\left(1+C(s)\left(\frac{a}{s+b}\right)\right)=1 \quad \text{common factor } \frac{E(s)}{R(s)}$$

sub $C(s)=K\left(1+\frac{1}{T_I s}\right)$

$$\frac{E(s)}{R(s)}=\frac{1}{1+K\left(1+\frac{1}{T_I s}\right)\left(\frac{a}{s+b}\right)}$$

$$=\frac{1}{1+Ka\left(\frac{T_I s+1}{T_I s}\right)\left(\frac{1}{s+b}\right)} \qquad T_I s(s+b)+Ka$$

$$\boxed{=\frac{T_I s(s+b)}{T_I s(s+b)+Ka(T_I s+1)}}$$

$$\left.\frac{E(s)}{D(s)}\right|_{R(s)=0}=?$$

From lecture 12: $E(s)=\dfrac{\cancel{R(s)}^{0}}{1+C(s)G(s)}-\dfrac{D(s)G(s)}{1+C(s)G(s)}$

$$\Rightarrow \frac{E(s)}{D(s)}=\frac{-G(s)}{1+C(s)G(s)}=\frac{-\left(\frac{a}{s+b}\right)}{1+K\left(\frac{T_I s+1}{T_I s}\right)\left(\frac{a}{s+b}\right)}$$

$$= \frac{-\left(\frac{a}{s+b}\right)}{\frac{T_I s(s+b) + Ka(T_I s+1)}{T_I s(s+b)}} = \boxed{\frac{-T_I s a(s+b)}{T_I s(s+b) + Ka(T_I s+1)}}$$

2. $R(s) = \frac{\bar{v}}{s}$ $D(s) = \frac{\bar{d}}{s}$

$$E(s) = \frac{R(s)}{1+C(s)G(s)} - \frac{D(s)G(s)}{1+C(s)G(s)}$$

sub in $\left.\frac{E(s)}{D(s)}\right|_{R(s)=0}$

sub in $\left.\frac{E(s)}{R(s)}\right|_{D(s)=0}$

$$= \frac{\bar{v}\, T_I \cancel{s}(s+b)}{\cancel{s}[T_I s(s+b) + Ka(T_I s+1)]} - \frac{\bar{d}\, T_I \cancel{s} a(s+b)}{\cancel{s}[T_I s(s+b) + Ka(T_I s+1)]}$$

$$= \frac{\bar{v}\, T_I (s+b) - \bar{d}\, T_I\, a(s+b)}{T_I s(s+b) + Ka(T_I s+1)} \qquad \text{(pole-zero cancellation)}$$

3. Final Value Theorem to $E(s)$

$E(s)$ is rational and proper, and has poles in OLHP assuming positive coefficients a, b, K and T_I.

$\therefore$ by FVT $\lim_{t\to\infty} e(t) = e(\infty)$ exists as is finite

$$\text{and } e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s[\bar{v}\, T_I (s+b) - \bar{d}\, T_I\, a(s+b)]}{T_I s(s+b) + Ka(T_I s+1)}$$

$$= \frac{0}{Ka} = 0 \quad \text{zero asymptotic error.}$$

Signatures

system has two poles on the real axis with real part ≤ -20. Prove the truth of the claim by using the root locus plot.

Evidently, we need to choose a different value for T_I. Try different (positive) values of T_I. For each choice of T_I, plot the corresponding root locus. By trial and error, find the value of T_I compatible with this requirement: *there exist $K > 0$ such that the closed-loop system has two poles close to $s = -20$ on the negative real axis.* Save the root locus plot.

Once you found $T_I > 0$ satisfying the requirement above, you need to find $K > 0$ for which the closed-loop system has two poles close to $s = -20$. First, plot the root locus corresponding to the value of T_I you just selected. Next, issue the command

```
>> rlocfind(G)
```

Move the mouse cursor over the root locus and click on the desired location of the closed-loop poles on the real axis as $s = -20$. This action will return the value of K you were looking for. Notice that the PI controller with the values of K and T_I you have just found should meet SPEC2-SPEC4.

Have your TA sign here before proceeding to the next step. JM

2. Next, you'll double-check that using the values of K and T_I you just found, SPEC2-SPEC4 are met in simulation. Download the file `lab3.mdl` from Blackboard and open it. Begin by double-checking that your controller indeed meets SPEC4. Enter the values of θ, T_I and K into the Matlab workspace

```
>> a=(your value); b=(your value); theta = 0; TI=(your value); K=(your value);
>> VMAX_UPM=11.75;
```

Run the Simulink block by clicking on `Simulation > Start`. The scopes depict (i) the output $v(t)$ versus the reference signal $r(t)$, (ii) the tracking error $e(t)$, and (iii) the voltage $v_m(t)$. The reference signal is a square wave of frequency 0.5Hz and amplitude 0.2m/s.

Recall that the settling time T_s of $v(t)$ is the time $v(t)$ takes to reach and stay within the range

$$[v(\infty) - 0.02(v(\infty) - v(0)), v(\infty) + 0.02(v(\infty) - v(0))]$$

where $v(\infty)$ is the value $v(t)$ asymptotically settles to. By zooming in on one period of the simulation output, graphically estimate the settling time T_s. Save the plot you used to derive your estimate. Is it true that $T_s \approx 0.2$ sec.? Verify that SPEC5 is approximately met.

Have your TA sign here before proceeding to the next step. JM

3. Now you'll try to design a more "aggressive" PI controller. Similarly to what you did in step 1, use the root locus and trial error to find the value of $T_I > 0$ such that there exists $K > 0$ such that the closed-loop system has two poles close to $s = -30$. Use the command `rlocfind` to find the value of K for which the closed-loop system has two poles close to -30.

Enter the values of K and T_I you just found in the Matlab workspace and run the Simulink diagram `lab3.mdl`. Similarly to what you did in step 2, evaluate the settling time T_s by zooming in on one period of the simulation output. Save the plot. Compare the performance of this "aggressive" controller to that of the controller you evaluated earlier.

for new poles

How do the settling times and overshoots compare? settling time is less

Which controller is best suited to meet the specs?

What is the cause of the differences you observe? → higher gain

Have your TA sign here before proceeding to the next step. JM

4.3 Controller Implementation

To perform the practical experiment for each part, open its Arduino file with ".ino" suffix and the associated Matlab m-file. You will enter your controller in the Arduino code and will record the data using Matlab. Follow your TA's instructions on how to compile the Arduino code and how to perform the experiment. Perform the following steps:

- Open the Arduino code file named as "*Lab3.ino*", The velocity reference is set to a square wave form with the amplitude of 0.2 m/s and frequency of 0.5 Hz.
- In the beginning of the code, modify and adjust the variables "K" and "Ti".
- Enter the values of K and T_I associated with the less "aggressive" controller. Upload the code and launch the Matlab m-file located in the same directory named "*real_time_data_plot_Lab3*".
- Save the obtained plot. Does your controller satisfy the design specifications?

  Have your TA sign here before proceeding to the next step. JM
- Now you'll test the performance of your controller against the unknown disturbance $d(t)$. Using one or more books, raise one end of the track. This corresponds to setting $\theta \neq 0$. While holding the track in a firm position, run the experiment again and compare the results.
- Now enter the values of K and T_I associated with the more "aggressive" controller and repeat all the above steps.

  Have your TA sign here before proceeding to the next step. JM

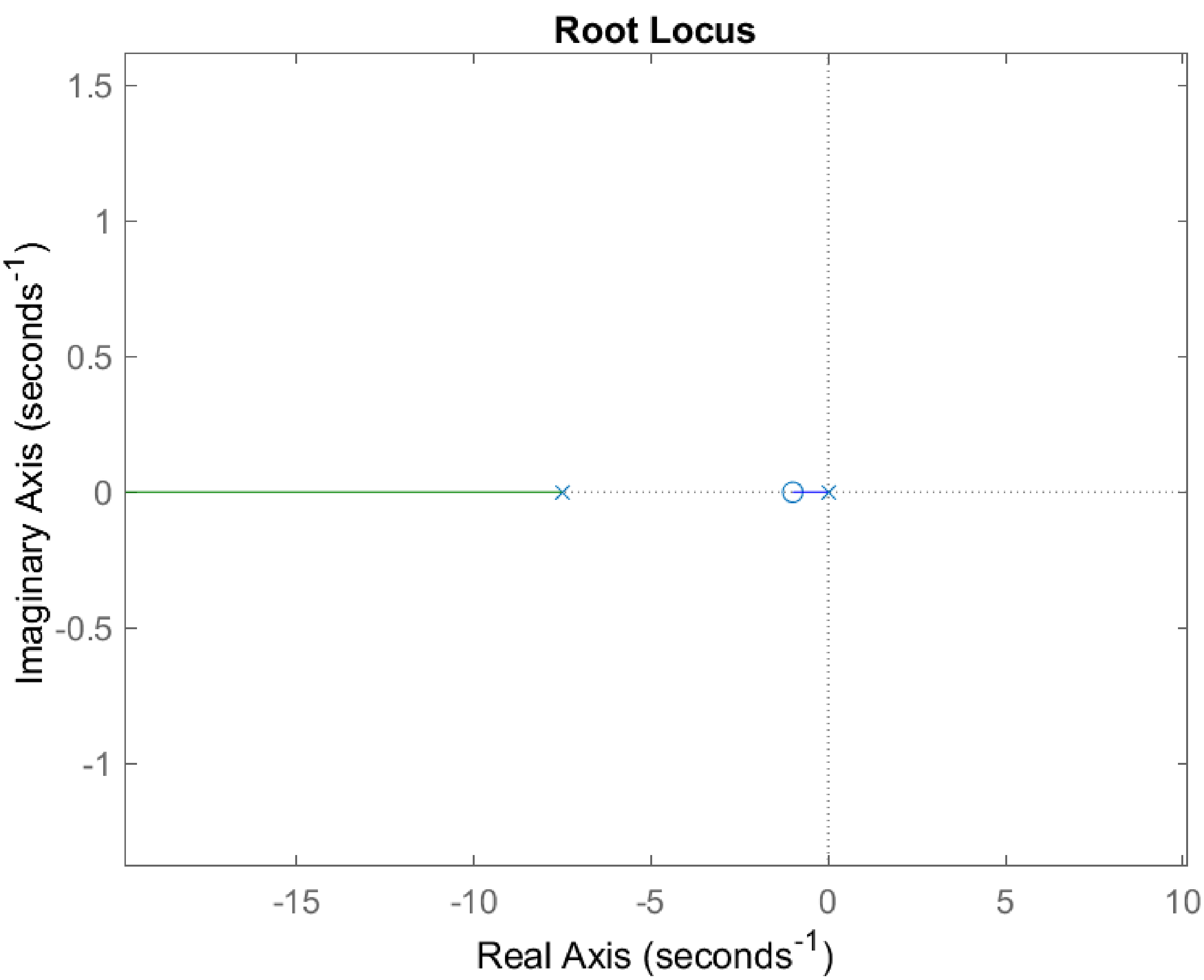
4.1 Identification of model parameters

(0.25 mark) Estimated parameters are: a = 2.5 b = 7.5

4.2.1 Controller design using Matlab, Part1

(0.25 mark) Root locus plot when TI = 1.

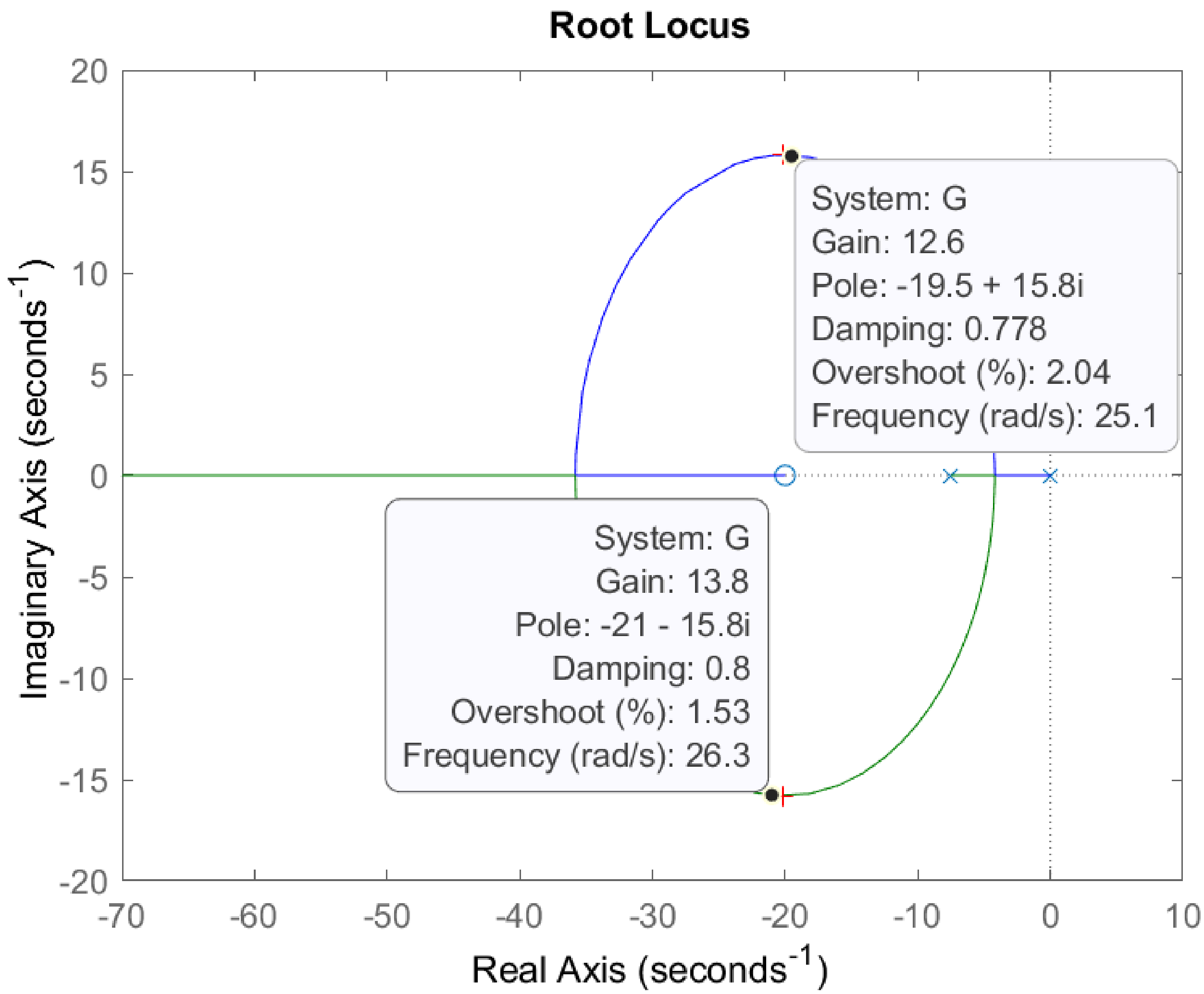
Using the plot, prove that there doesn't exist K>0 such that the closed-loop system has two poles on the real axis with real part<-20.



(0.25 mark) Value of TI and K for which the closed-loop system has two poles at S = -20,

TI = 0.05   K = 15

(0.25 mark) Root locus plot for the value of TI you just found.



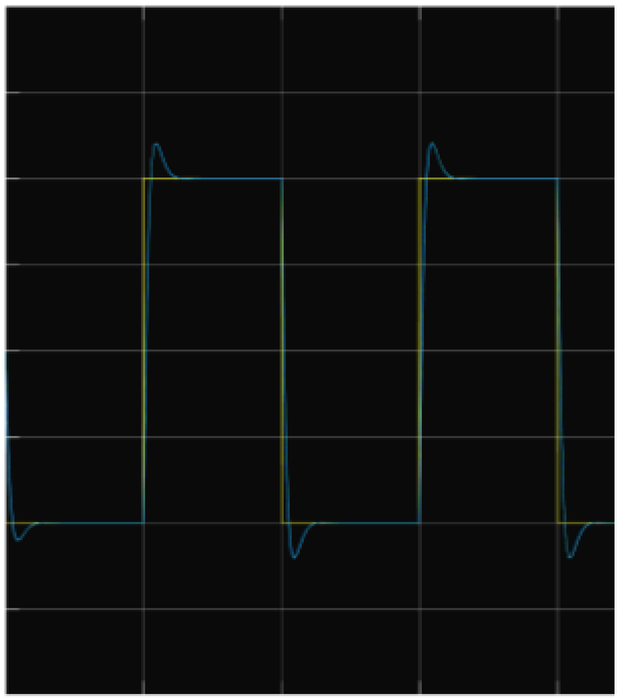
4.2.2 Controller design using Matlab, Part2

(0.25 mark) Plot showing one period of the simulation output (with proper labels).

What is the estimated value of the settling time: Ts = 0.2 sec

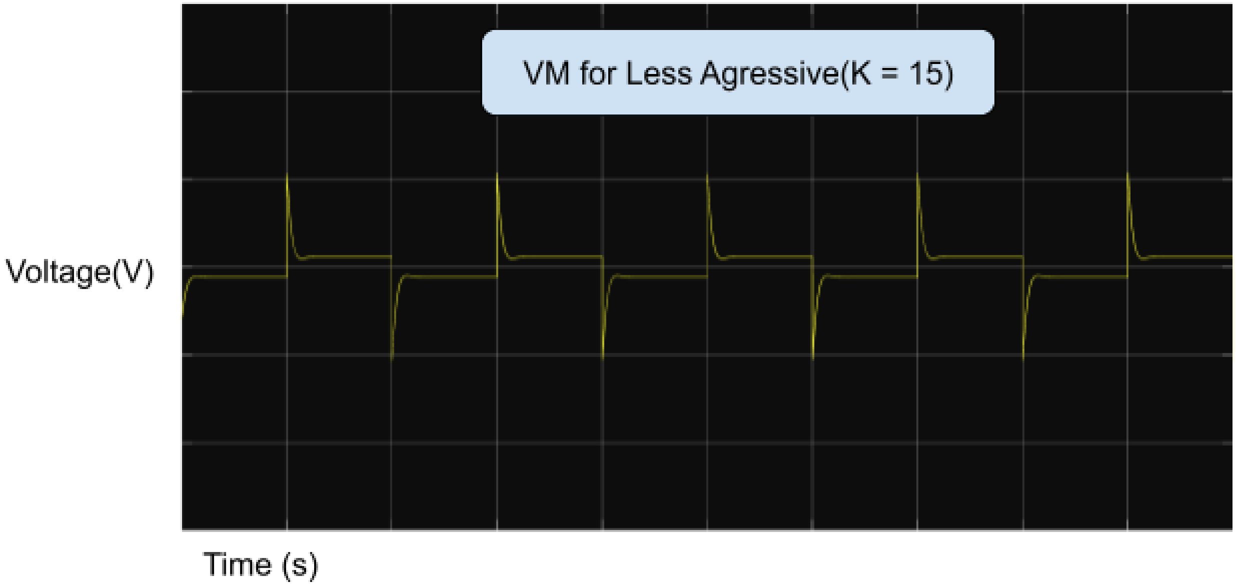
Speed vs. reference, less aggressive controller

Velocity (m/s)



Time (sec)

(0.25 mark) Plot showing the control input voltage Vm(t) (with proper labels).



What is the peak value of Vm(t) ? 5V

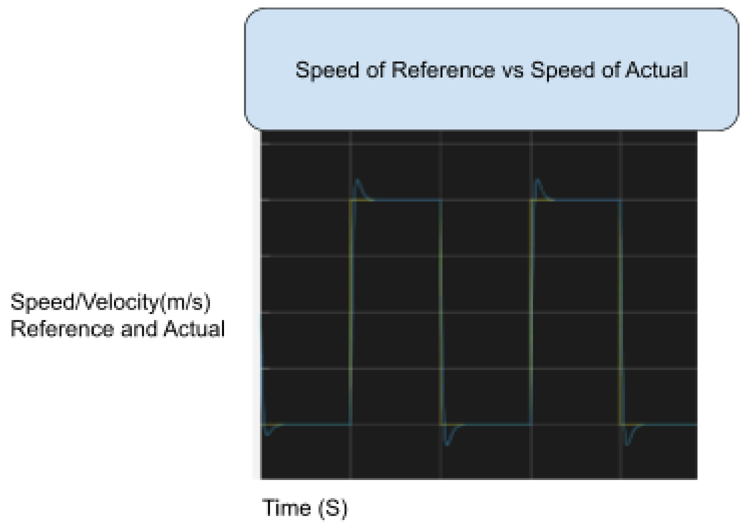
4.2.3 Controller design using Matlab, Part3

(0.25 mark) Value of TI and K for the more aggressive controller (S = -30)

TI = 0.05 K = 20

(0.25 mark) Plot showing one period of the simulation output (with proper labels).

What is the estimated value of the settling time: Ts = 0.15 sec



(0.25 mark) Plot showing the control input voltage Vm(t) (with proper labels).

What is the peak value of Vm(t) ? *Peak value of Vm(t) is 7.5 volts.*

Vm(t) for Aggressive Controller

Voltage
(V)

Time (sec)

(0.75 mark) Compare the performance of the two controllers you designed earlier.

How do settling time and overshoots compare? How about the maximum value of Vm(t)?

> The settling time is less for the more aggressive controller, as seen in the error graph. The overshoot however is very similar. The max value of Vm(t) is larger for the more aggressive controller (7.5 V to 5V).

Which controller is best suited to meet the specifications?

> The more aggressive controller is better suited to meet the specifications because it has less overshoot and less settling time, meaning it follows the reference more closely. The aggressive controller also has max Vm(t) < 11.75 V, meeting SPEC5.

What is the cause of the differences between the two controllers?

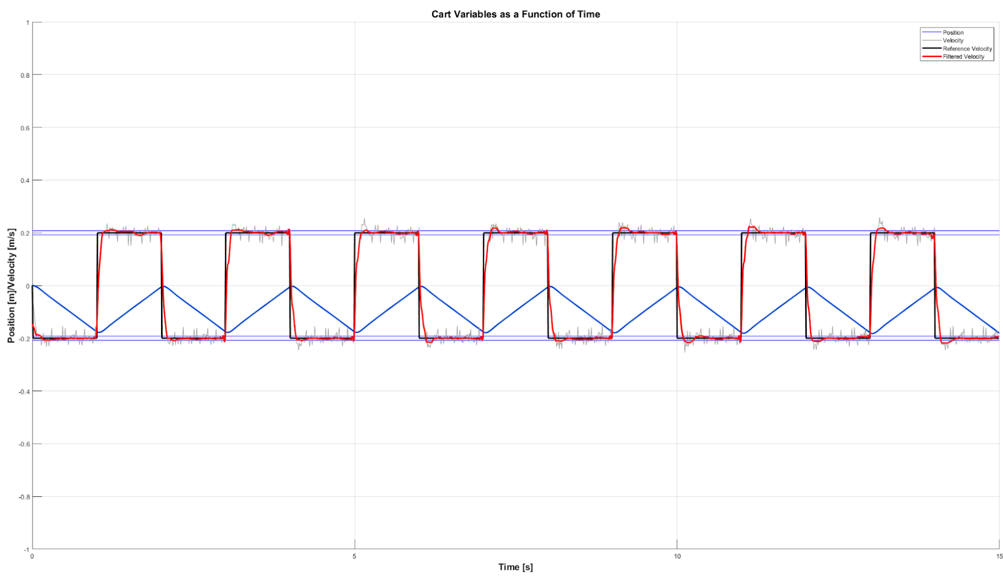
> Since the gain is higher, the value of the poles are more negative (in the real axis) which means that the system converges to the reference a lot faster than in the less aggressive one. This makes it better able to handle disturbances such as the ramp.

4.3 Controller Implementation

(0.5 mark) Normal controller, with no disturbance:

Plot showing actual cart speed V(t) and reference r(t) (with proper labels).

What is the estimated value of the settling time: Ts = 0.2 sec



(0.5 mark) Normal controller, when cart is tilted:

Plot showing actual cart speed V(t) and reference r(t) (with proper labels).

What is the estimated value of the settling time: Ts = 0.25 sec

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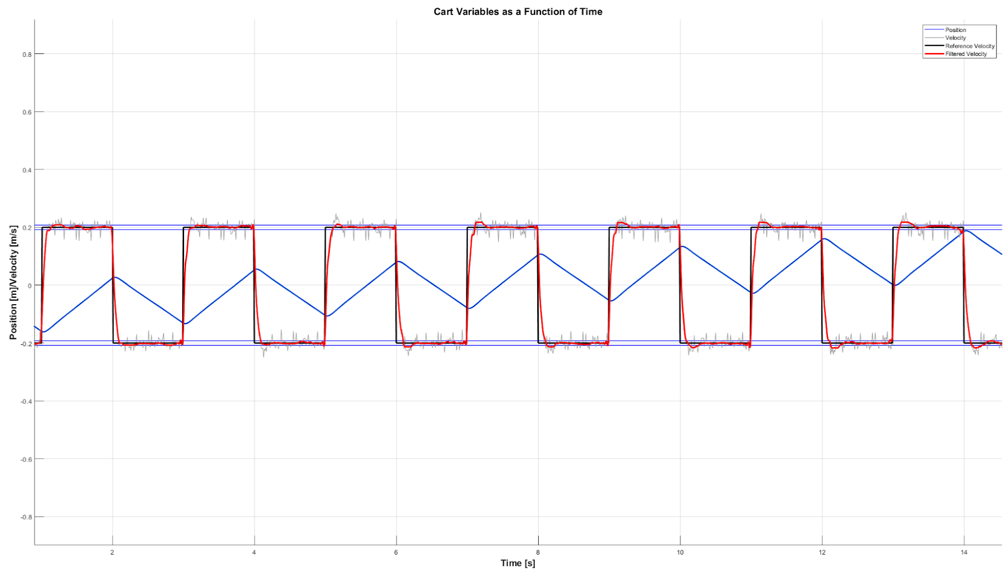
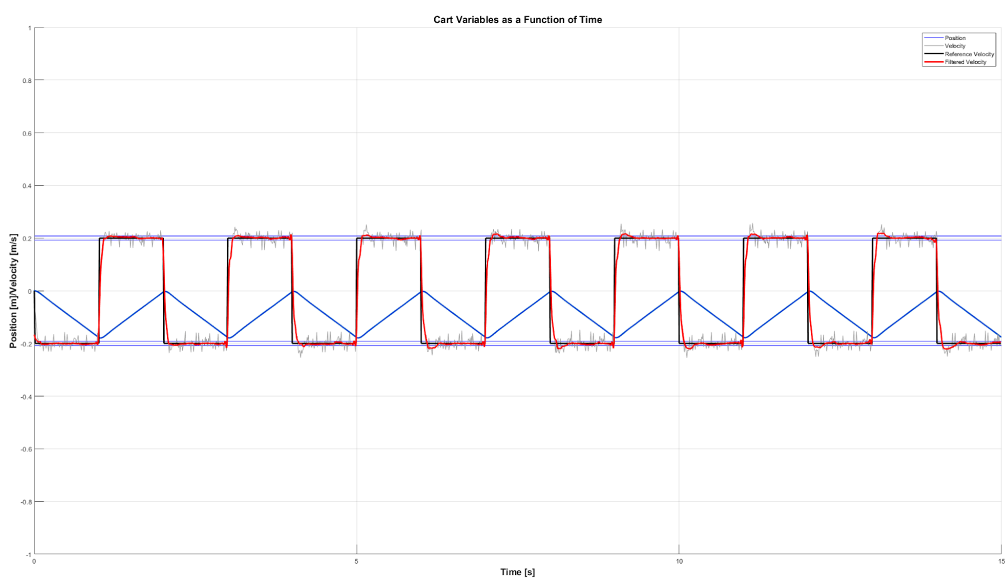


(0.5 mark) Aggressive controller, with no disturbance:

Plot showing actual cart speed V(t) and reference r(t) (with proper labels).

What is the estimated value of the settling time: Ts = 0 to 0.15 sec



(0.5 mark) Aggressive controller, when cart is tilted:

Plot showing actual cart speed V(t) and reference r(t) (with proper labels).

What is the estimated value of the settling time: Ts = 0 to 0.2 sec