Problem 1.5

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Exercise 2.4 (Inner product) Let $x, y \in \mathbb{R}^n$. Under which condition on $\alpha \in \mathbb{R}^n$ does the function

$$f(x,y) = \sum_{k=1}^{n} \alpha_k x_k y_k$$

define an inner product on $\mathbb{R}^n$?

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$f(x,y) = \sum_{k=1}^{n} \alpha_k x_k y_k$$

Definition of inner product:

$$\langle x, y \rangle = x^T y = [x_1 \dots x_n] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$= x_1 y_1 + x_2 y_2 + \dots x_n y_n$$

$$= \sum_{i=1}^{n} x_i y_i$$

$$f(x,y) = \sum_{k=1}^{n} a_k x_k y_k$$

$$f(x,y) = \sum_{k=1} a_k x_k y_k$$

If $a_k = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

$$f(x,y) = \sum_{k=1}^{n} x_k y_k = \langle x, y \rangle$$

$\therefore$ the condition for α is if $\alpha = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$,

then $f(x,y)$ defines the inner product operation