Problem 1.3

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Problem 1.3:

5.1 *Linear independence of stacked vectors.* Consider the stacked vectors

$$c_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, \ \ldots, \ c_k = \begin{bmatrix} a_k \\ b_k \end{bmatrix},$$

where $a_1, \ldots, a_k$ are n-vectors and $b_1, \ldots, b_k$ are m-vectors.

(a) Suppose $a_1, \ldots, a_k$ are linearly independent. (We make no assumptions about the vectors $b_1, \ldots, b_k$.) Can we conclude that the stacked vectors $c_1, \ldots, c_k$ are linearly independent?

(b) Now suppose that $a_1, \ldots, a_k$ are linearly dependent. (Again, with no assumptions about $b_1, \ldots, b_k$.) Can we conclude that the stacked vectors $c_1, \ldots, c_k$ are linearly dependent?

a) if $a_1 \ldots a_k$ are linearly independant,

$a_1 \neq a_2 \neq \ldots \neq a_k$ & $a_1 \neq l a_2$

$$\therefore \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \neq \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \neq \ldots \neq \begin{bmatrix} a_k \\ b_k \end{bmatrix}$$

even if $b_1 = b_2 = \ldots b_k$

i.e. $\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\begin{matrix} \} a_1 \\ \} b_1 \end{matrix}$ $\begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\begin{matrix} \} a_2 \\ \} b_2 \end{matrix}$ $\rightarrow$ are linear independant since a_i are linear independant

b) In the same way, we cannot guarantee that $c_1 \ldots c_k$ are independent even if $a_1 \ldots a_k$ are dependant because $b_1 \ldots b_k$ could be independant and then $c_1 \ldots c_k$ would be independant be independant and then $c_1 \ldots c_k$ would be independant