Problem 1.1

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a) show the functions $l_1 : R^n \to R$ is a norm

Definition of a norm:

$\|\cdot\|$ is a function that : $U \to R$ that satisfies:

i) $\|V\| \geq 0$, $\forall v \in V$, $\|V\| = 0$ iff $V = 0$

ii) $\|\alpha V\| = \|\alpha\| \|V\|$ $\forall v \in V$, $\alpha \in \mathbb{R}$

iii) $\|u + V\| \leq \|u\| + \|V\|$

l_1 definition:

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}}, \quad 1 \leq p < \infty$$

Where $p = 1$

$$l_1 = \|x\|_1 = \left(\sum_{i=1}^{n} |x_i|^1 \right)^{\frac{1}{1}} = \sum_{i=1}^{n} |x_i|$$

i) if $\vec{V} = 0$

then, $\|\vec{V}\|$

$$= \sum^{n} |V_i| = (V_1 + V_2 + \dots V_n)$$

$$= \sum_{i=1}^{n} |V_i| = (V_1 + V_2 + \dots V_n)$$

$$= (0 + 0 + 0 \dots 0)$$

$$= 0$$

$\therefore \|\vec{V}\| = 0$

Other way:

if $\|\vec{V}\|$ then $\sum_{i=0}^{n} |V_i| = 0$

$$(V_1 + V_2 \dots V_n) = 0$$

$$V_i = 0 \quad \forall i$$

$$\therefore \vec{V} = 0$$

$\therefore$ Property satified

ii) $\|\alpha V\|$

$$= \sum_{i=1}^{n} |\alpha V_i|$$

Property of absolute functions

$$= \sum_{i=1}^{n} |\alpha| \, |V_i|$$

$$= \sum_{i=1}^{n} |\alpha| \sum_{i=1}^{n} |V_i|$$

$$= \|\alpha\| \, \|V\|$$

iii) $\|u + V\|$ Triangle inequality: $|x + y| \le |x| + |y|$

$$\sum^{n} |u_i \; V_i|$$

$$= \sum_{i=1}^{n} |u_i + v_i|$$

$$\sum_{i=1}^{n} |u_i + v_i| \leq \sum_{i=1}^{n} |u_i| + |v_i| = \sum_{i=1}^{n} |u_i| + \sum_{i=1}^{n} |v_i|$$

$$||u+v|| \leq ||u|| + ||v||$$

$\therefore$ Property holds

b) $l_\infty : R^n \to R$

Defn: $l_\infty = \max\limits_{i=1 \dots n} |x_i|$

1) if $\vec{v} = 0$ then,

$$||\vec{v}|| = \max_{i=1,\dots n} |v_i|$$

$$= \max_{i=1 \dots n} |0, 0, 0, \dots 0|$$

$$= 0$$

other way:

if $||\vec{v}|| = 0$ then,

$$\max_{i=1 \dots n} |v_i| = 0$$

$$v_i = 0 \quad \forall i$$

$$\therefore \vec{v} = 0$$

$\therefore \vec{V} = 0$

$\therefore$ Property holds

2) $\|\alpha V\|$

$= \max_{i \dots n} |\alpha V_i|$

$= \max_{i \dots n} |\alpha| |V_i|$

due to Scaling

$= |\alpha| \max_{i \dots n} |V_i|$

$= \|\alpha\| \|V\|$

Property holds

3) $\|u + V\| = \max_{i \dots n} |w_i + V_i|$

$\leq \max_{i \dots n} |u_i| + \max_{i \dots n} |V_i|$

the max of the sum will be less than or equal to the sum of ax of w, and the max of V

$\leq \|u\| + \|V\|$