Pierre Ishak (1007897457), Joaquin Arcilla (1007930820)

Wave Phenomena Lab

Abstract

The objective of this lab is to study and experimentally observe the properties of waves, specifically focusing on reflection, refraction, interference, and diffraction. Through the use of a ripple tank generator, a reflector, and a light source, we will generate ripples and observe the interference patterns formed by various barriers and point sources. The lab will also investigate the Law of Reflection, the relationship between wave speed, gravity, and water depth, the ratio of speed propagation, and the inverse equation of Huygen's Principle. In addition, we will explore the amplitude of the superposition of two point sources and derive the wave equation and the amplitude for the superposition of two point sources that are out of phase. Through these experiments, we will verify various relationships and equations that accurately describe wave behavior. Finally, we will apply these concepts to determine the focal distance and radius of a curved barrier and validate the equations presented in the lab manual. Overall, this lab will provide a comprehensive understanding of wave properties and their applications in real-world situations.

1. Introduction

In this lab, we will delve into the crucial concepts of wave phenomena, which are fundamental to our understanding of the world around us. Waves are at the heart of many scientific and engineering fields, including acoustics, communication systems, and seismology. By conducting experiments focused on wave reflection, refraction, interference, and diffraction, we will gain insight into the behavior of waves as they interact with different barriers and point sources. We will also study the effect of variables such as water depth on wave speed, to better understand the physics governing wave motion. This knowledge will be applicable in a variety of real-world scenarios, from designing soundproof rooms to developing new communication technologies. By investigating these phenomena in a practical setting, we will develop a deeper appreciation for the vital role that wave phenomena play in many aspects of our daily lives.

1.1 Traveling Waves

The waves studied in this experiment are traveling waves generated by disturbances in the water-air boundary. These waves are propagated along the interface between water and air, and are governed by the restoring forces of gravity and surface tension. Any two-dimensional traveling wave equation can be expressed as a periodic function, where v is the wave speed, and x,y, and t are positions and time coordinates.

$$\psi(ax - by - vt) \quad [1]$$

In simple cases where the wave speed is a phase velocity (when the wave is non-dispersive) v can be expressed via equation 2. Where ω is angular frequency, k is wave number ($2\pi/\lambda$), f is frequency, and λ is wavelength.

$$v = \frac{\omega}{k} = f\lambda \quad [2]$$

For shallow water, the phase velocity is independent of wavelength and can be expressed via equation 3. Where g is acceleration due to gravity and d is the depth of the water.

$$v = \sqrt{gd} \quad [3]$$

1.2 Huygens's Principle

Huygens's Principle is a fundamental concept for understanding wave propagation and interaction. It states that every point on a wave front is the source of a new spherical wave that spreads out at the wave speed, and that after some time, the shape of the original wave front is tangent to all its secondary created spherical waves. By visualizing wave behavior using Huygens's Principle, we can accurately approximate the propagation of waves and better understand the wave phenomena studied in these experiments.

1.3 Reflection

Reflection occurs when a wave encounters a boundary between two different media. When a wave collides with a finite plane wall, it generates a new spherical wave that radiates back from its original direction, according to the principle of reflection, which states that the angle of incidence is always equal to the angle of reflection.

This occurs because the wave closest to the barrier is the first to hit it, generating a new wave that bounces back into the original medium. Each subsequent wavefront generates another spherical wave that propagates with the same delay as the previous ones, until all wavefronts add up and propagate in a single direction. By using the principle of reflection, we can accurately predict the behavior and propagation of waves.

$$\theta_i = \theta_r \; [4]$$

1.4 Refraction

Refraction is a phenomenon that occurs when a wave passes through a boundary between two different media with different propagation speeds. As a result of the change in speed, the wave appears to bend. When a plane wave approaches a boundary, new spherical wave sources are created from the portion of the wave that encounters the boundary. The spherical wave generated within the new medium travels with a different speed of propagation compared to the corresponding medium it was generated in. This difference in speed causes the bending effect, and the change in angle between incidence and transmission is proportional to the ratio of the speeds of propagation between the two media.

$$\frac{1}{v_1} sin(\theta_1) = \frac{1}{v_2} sin(\theta_2) \; [5]$$

1.5 Diffraction

Diffraction is a wave phenomenon that occurs when a wave encounters an obstacle or passes through an opening. When a wave passes through a narrow opening, it bends around the edges of the opening and spreads out into the region behind it. This results in the wave appearing to diffract or spread out. In the context of water waves, diffraction can be observed when waves pass through slits in straight barriers. When a wave encounters a barrier with a slit, it will spread out into the region behind the slit, creating a semicircular wave pattern which can be described by equation 6. Where a is the length of the slit opening and θ is the angular spread of the first diffraction maximum as shown in Figure 1.

$$asin(\theta) = \lambda \; [6]$$

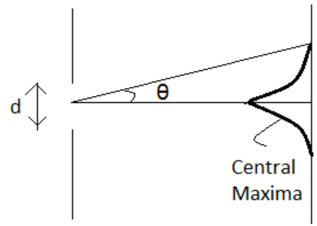

Figure 1: Angular Spread going up to the first diffraction maximum

1.6 Interference

Interference is a phenomenon that occurs when a wavefront passes through a pair of narrow openings, which act as secondary sources for new spherical waves that propagate outward with the same speed as the original wave. As these newly produced waves overlap and combine through a process known as superimposition, interference patterns are formed. In our experiment, interference is studied using two point wave dippers, which act as the narrow openings. A point P(x,y) from the slits is described via the following general equation.

$$\psi(x_p, y_p, t) = A_0 cos[\omega(t - \frac{r_1}{v})] + A_0 cos[\omega(t - \frac{r_2}{v})] = 2A_0 cos(\omega t) cos[\frac{\pi(r_2 - r_1)}{\lambda}] \quad [7]$$

Whenever the wave amplitude is zero the following equation applies.

$$r_1 = r_2 = dsin(\theta_m) = m\lambda \quad [8]$$

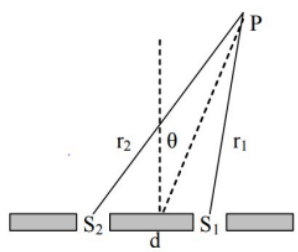

Figure 2: Double slit interference pattern where amplitude is zero

2. Procedure

2.1 Equipment/Apparatus

- Ripple Tank
- Pasco 9896 Ripple Generator
- Light Source
- Reflector
- Projection Screen
- Camera connected to LabVIEW
- Dippers, Barriers and Refractors

2.2 Experimental Method

1. Pinch the draining tube of the ripple tank shut.
2. Pour approximately 800 mL of water into the ripple tank, ensuring uniformity in water depth by using a water level and adjustable legs on the tank.
3. Press the foam edges of the tank into the water until fully saturated to prevent uneven absorption of water.
4. Turn on the light source via the switch on the Pasco 9896 ripple generator and set the settings to strobe.
5. Set the ripple generator to parallel, and make wave dippers arms evenly graze the water.

2.2.1 Reflection

1. Place the long straight barrier at an angle in the middle of the tank.
2. Use the plane wave dipper and set the frequency to 20 Hz.
3. Measure the incidence and reflection angles to verify Equation 4.
4. Replace the long straight barrier with the curved barrier.
5. Record the reflected wave pattern for the barrier curved towards and away from the plane wave generator.
6. Estimate the focal distance and radius of the curved barrier.

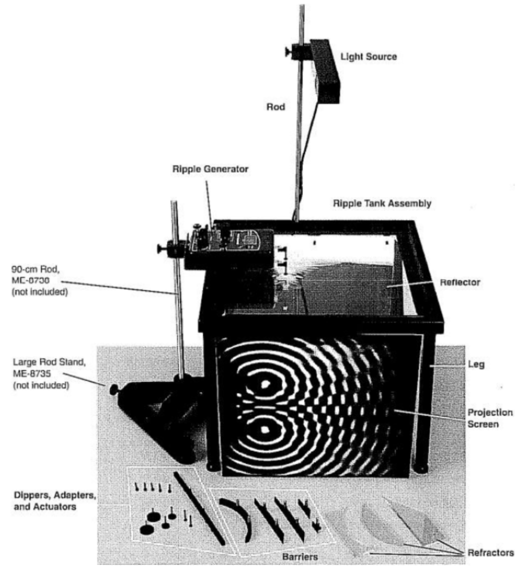

Figure 3: Diagram of the Experiment Apparatus

2.2.2 Wave Speed

1. Measure the wavelength five times at 5 Hz and calculate the average and standard deviation.
2. Repeat step 1 for four other frequencies and graph the average wavelength vs frequency.
3. Use Equation 2 to calculate the velocity of the wave for each frequency.
4. Repeat the wavelength measurements at water depths ranging from 2 - 10 mm and calculate the wave speed for each depth.
5. Graph the wave velocity vs wavelength and verify the Equation 3.

2.2.3 Refraction

1. Positioned the trapezoidal refractor in the center of the tank, orienting the triangular end toward the plane wave dipper.
2. Checked to make sure that the water level in the tank was high enough to cover the refractor by 2 millimeters.
3. Observed the pattern of refraction generated by the refractor in the water.
4. Measured the angles of the refracted waves to verify the accuracy of Equation 5.

2.2.4 Diffraction

1. Created a 3-cm "slit" in the ripple tank using barriers.
2. Captured the diffraction pattern that resulted after the "slit" using a frequency of 20 Hz.
3. Calculated the angular spread of the circular waves present in the pattern.
4. Repeated the process for four different "slit" sizes.
5. Verified the accuracy of Equation 6 through these experiments.

2.2.5 Interference

1. Turn off the ripple generator and carefully position two standard dippers, just barely touching the surface of the water.
2. Measure the angles between the first and second points of constructive and destructive interference at a frequency of 20 Hz.
3. Measure the wavelength five times and calculate the average wavelength.

4. Confirm the separation between the dippers using Equation 8.
5. Repeat the above steps with different separation distances to observe the effect on the interference pattern.
6. Adjust the dippers to be out of phase, observe the new pattern, and reformulate Equation 7 and 8 to explain the observed pattern.

3. Results and Analysis

3.1 Uncertainties, Error Propagation and Sources of Error

Most of the uncertainty from this lab comes from LabVIEW and how accurate those measurement tools are. Since it is a digital system, we set our uncertainties for any length and/or angle measurement to be +/- 0.05 units, depending on the unit. For the uncertainties of averages, we used the standard error formula presented below:

$$\sigma = \frac{1}{\sqrt{N}}\sqrt{\frac{1}{N-1}\Sigma(x_i - \Delta x_i)^2} \tag{9}$$

For error propagation we used two equations depending on whether or not the formula being calculated used an addition/subtraction, or if the formula used multiplication or division. These formulae are shown below:

$$\Delta z = z\sqrt{(\frac{\Delta x}{x})^2 + \frac{\Delta y}{y})^2} \text{ (10)} \qquad \Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2} \text{ (11)}$$

Possible sources of errors are fully described per experiment in the sections below. Section 3.2.1 describes the LabVIEW errors we encountered, which affected each of our experiments.

3.2 Reflection of Waves

3.2.1 Straight Barrier

Theoretically, the incident angle and reflected angle should be the same. By conducting this experiment and taking an image of the incident and reflected waves, we were able to verify whether or not this is true. Below is an example image of the straight barrier images:

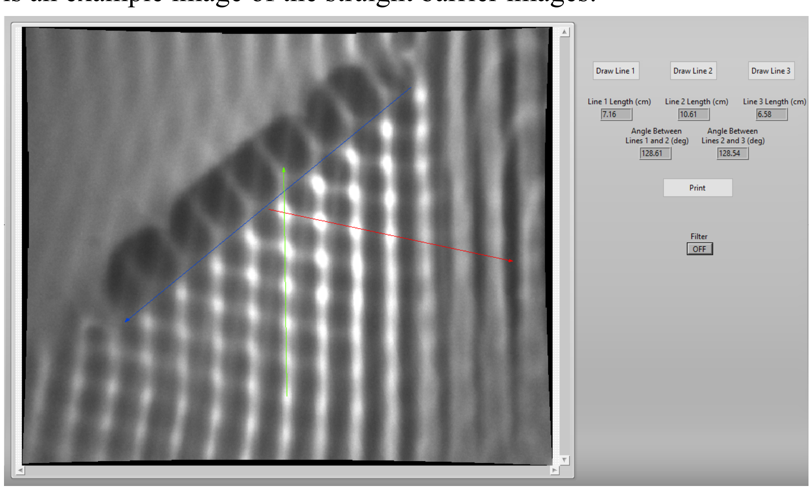

Figure 4: Incident and Reflected wave from a straight barrier

By taking three of these images at various incident angles, and measuring the corresponding reflection angle, we were able to graph these points on an x-y graph and attempt to draw a line of best fit for them. Theoretically, this line would be one to one, which is essentially x = y or in standard line notation, y = 1x + 0. Below is our graphed data and the line of best fit generated:

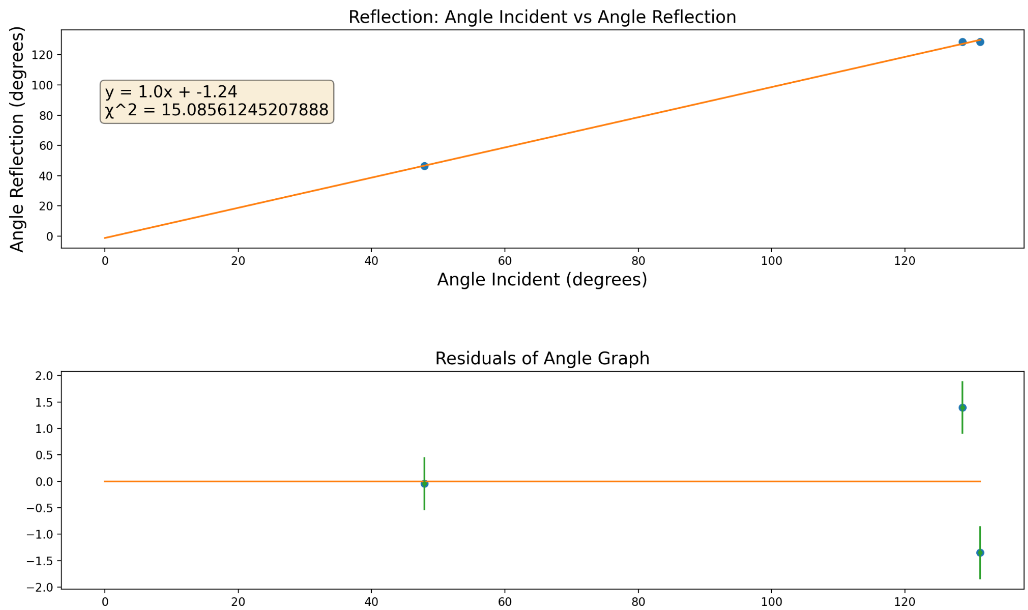

Figure 5: Incident angle vs Reflected angle

Based on our line of best fit, one can see that the two parameters (a and b) are close to the theoretical parameters. Based on the Chi-squared and residual values, we can see that the line of best fit fits the data quite well as the Chi-squared value is close to 1 and the residuals are only 1-3 degrees off from each other. Thus our experiment agrees with the theoretical values and equation [4] is experimentally correct.

In the process of collecting data, certain errors may have occurred. One such error might have been due to the method used to measure angles in LabVIEW, which could have been more user-friendly. Another potential source of error could have been the unclear displays on the LabVIEW software, which may have led to inaccuracies in the collected measurements. These LabVIEW issues exist for every experiment conducted during this lab. Additionally, the warping effect from the camera may make it challenging to gather data accurately.

3.2.2 Curved Barrier

Using our setup outlined above, we captured images of reflected waves from a curved barrier. From these images, we were able to estimate the location of the barrier's focal point, when we drew lines in the directions of the reflected wave propagation. For the concave barrier, the image and focal point are shown below:

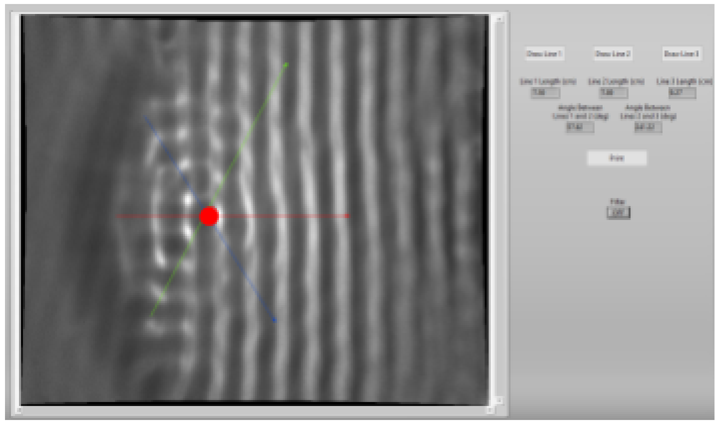

Figure 6: Focal point of a concave barrier

For the convex barrier, we drew the lines backwards through the barrier in order to find the focal point. This focal point is not where all the waves meet, but where all the waves seem to be coming from if you were to imagine removing the barrier. This case is shown below:

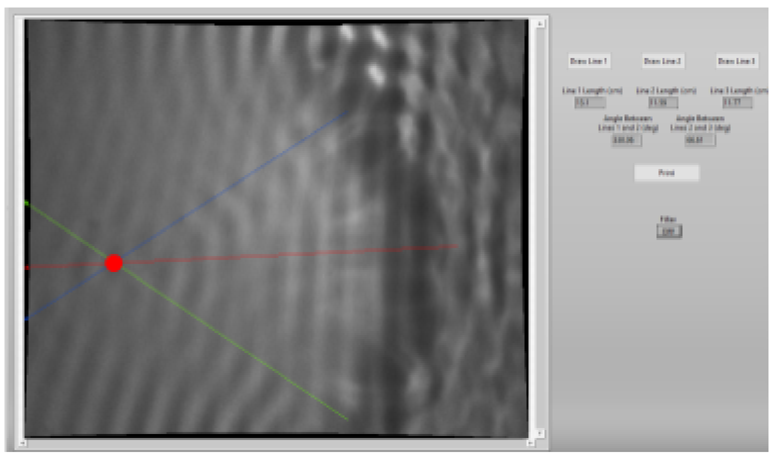

Figure 7: Focal point of a concave barrier

Based on these two cases, we can once again see that the angle of incident and the angle of reflections are all the same, since all the waves converge or diverge (this one is not real) from the same location. This means that for all the reflected waves, the angle of travel is the same, therefore proving equation [4].

3.3 Wave speed

3.3.1 Waves speed vs Frequency

Using the setup described above, we measured corresponding wavelengths for different frequencies. Using equation [2] we calculated wave speed from these wavelengths. These results are summarized in the table below:

Frequency (Hz):	Average Wave Speed (m/s)
5 +/- 0.05	26.290 +/- 1.156
8 +/- 0.05	22.240 +/- 0.695
11 +/- 0.05	20.878 +/- 0.227

14 +/- 0.05	21.028 +/- 0.260
17 +/- 0.05	21.284 +/- 0.173
20 +/- 0.05	21.400 +/- 0.335

Table 1: Frequency vs Wave Speed

This data is graphed below:

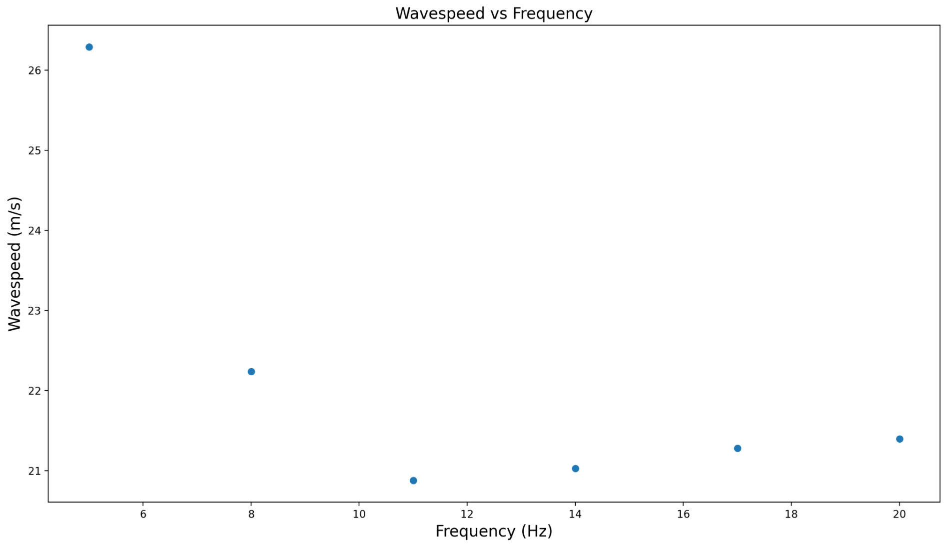

Figure 8: Frequency vs Wave Speed

Possible errors that may be influencing this data are the fact our wavelength measurements may not be accurate, since the amplitudes used to measure the wavelength were quite wide in the images and the fact that the device changing the frequency wasn't in the best condition and it was difficult to manipulate the frequency exactly.

3.3.2 Wave speed vs Depth

From the setup described above, we measured different wavelengths for different water depths. Since we had a constant frequency, we could use these wavelength measurements to calculate the wave speed similar to section 3.3.1 using equation [2]. Now that we have wave speed and the depth, we could graph it and see if the relationship described by equation [3] was true. We first graphed depth vs wave speed, which is shown below:

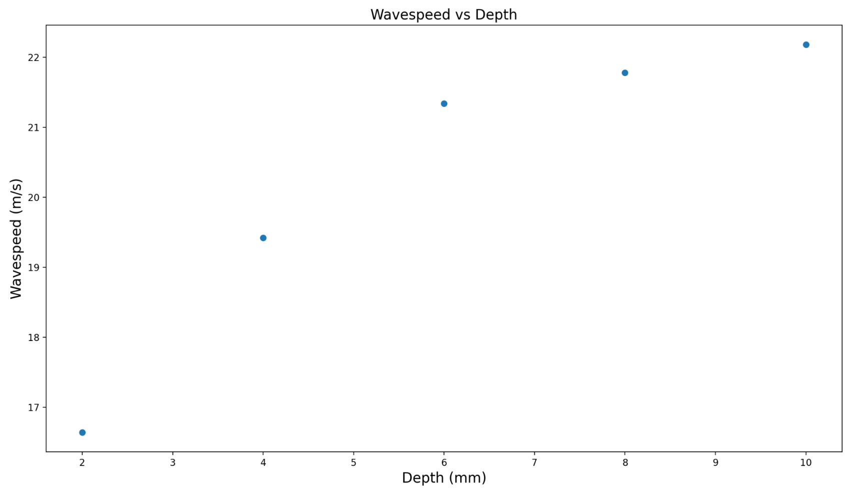

Figure 9: Depth vs Wave Speed

In order to see if the relationship shown in equation [3] is true, we first square rooted the depth values of our data. This would allow us to use a line of best fit to see if there was a linear relationship between wave speed and the square root of depth, since the equation became a linear equation with this change in variable. The result of this is shown below:

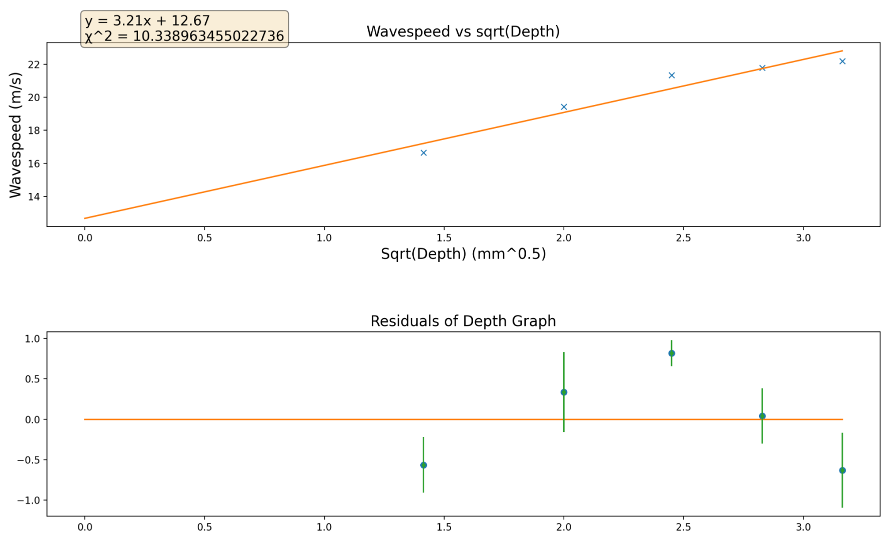

Figure 10: √Depth vs Wave Speed

Based on this line of best fit, we can see that the relationship between the square root of depth and wave speed is indeed linear, which means that relationship between depth and wave speed is reciprocal, as is shown in equation [3]. With a chi-squared value of 10.3 and residuals only between 1 and -1 off, this line of best fit is quite good and a fair representation of our data. All of this indicates that equation [3] is true and can be used in further experiments.

In this experiment, our possible error could be from the fact that we measured the water depth using the ruler provided in our lab kit which isn't the most accurate method of measuring distances. We also started with the maximum water depth we wanted and drained the water level for each iteration of the experiment. This could've created an uneven water level so our measurements still could be wrong. Similar to section 3.3.1, our wavelength measurements may have also been incorrect due to the wideness of the amplitudes on the images.

3.4 Refraction

For our refraction test, we used the images we had collected to verify equation [5]. A sample of this collected image is shown below:

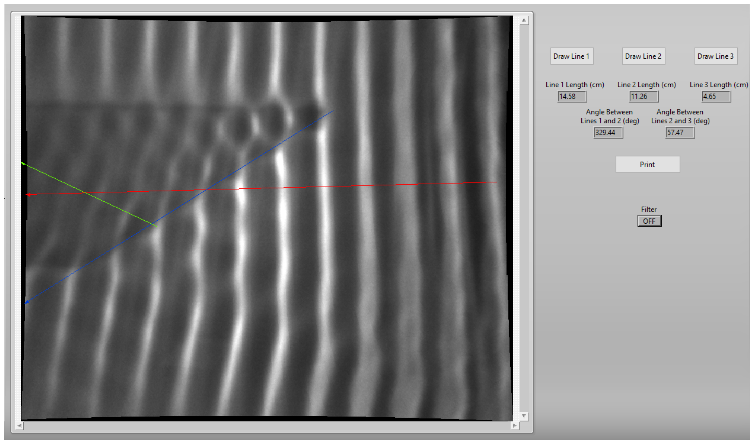

Figure 11: Refraction measurement

In order to verify equation [5] we first had to use equation [3] to get wave speed. As this equation was verified in section 3.3, there were no issues in simply calculating the wave speed from the water depth. Since we had both the angle and wave speed of both mediums, we were able to calculate the constant on either side of equation [5] and compare them. The data is shown below:

Trial 1			
	Wave Speed (m/s)	Angle (degrees)	$\frac{1}{v_1} sin(\theta_1)$
Incident Wave	9.4 +/- 0.4	59.44 +/- 0.05	0.092 +/- 0.009
Refracted Wave	5.4 +/- 0.2	32.56 +/- 0.05	0.099 +/- 0.009
Trial 2			
	Wave Speed (m/s)	Angle (degrees)	$\frac{1}{v_1} sin(\theta_1)$
Incident Wave	9.4 +/- 0.3	59.56 +/- 0.05	0.0917 +/- 0.009
Refracted Wave	5.4 +/- 0.1	38.06 +/- 0.05	0.11 +/- 0.009

Table 2: Verifying equation [5]

In both trails, the constants are pretty close to each other, within uncertainties. This seems to indicate that equation [5] is true and the refraction relationship presented in the equation is a real phenomena.

Possible errors arise from the fact that we once again measured depth using a ruler which isn't the most accurate, and from using LabVIEW to measure the angles, which didn't make sense to us while we were doing the lab.

3.5 Diffraction

Using the setup described above, we took the images we had collected and measured the angular spread from them. This was done by observing the boundary layer around the precipice of the slit opening and the waves formed from the slit. While doing this, we also measured the experimental wavelength of the waves. An example of this is shown below:

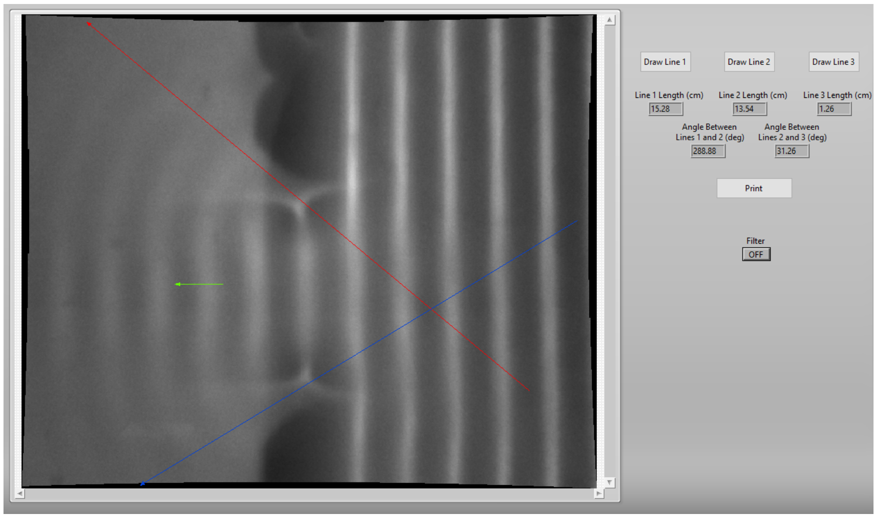

Figure 12: Diffraction measurement

Collecting this data, we used equation [6] to calculate a theoretical wavelength, and then graphed this wavelength against the slit size. We also put the experimental wavelength on this graph in order to compare the accuracy of the theory to the measured values. This graph is shown below:

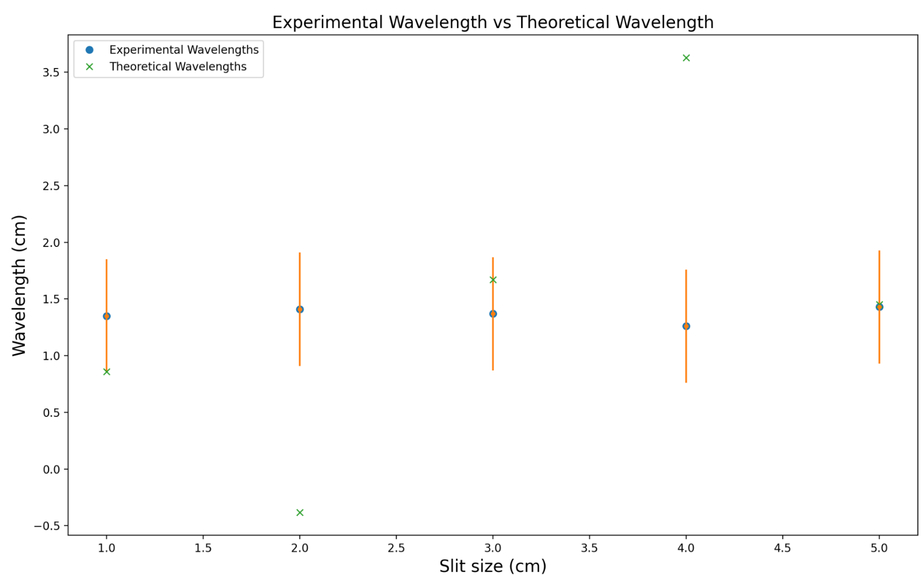

Figure 13: Experimental Wavelength vs Theoretical Wavelength

From this graph, we can see that the theoretical wavelength is not accurate to the experimental one. This is most likely caused by lab errors, especially when trying to measure the angular spread. When we measured angular spread, we not only had to deal with actually finding the angular spread, but also the LabVIEW user interface being difficult. This doesn't disprove equation [6]

3.6 Interference

Using the methodology outlined above, we measured the constructive and deconstructive angles from the center node. We graphed this data against the space between the point sources, shown below:

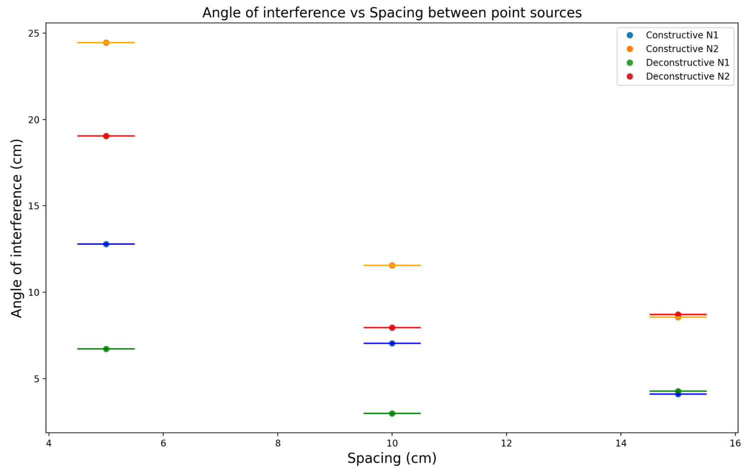

Figure 14: Angle of Interference vs Spacing

Using equation [8] we used the data to calculate the theoretical spacing of the two point sources which is summarized in the table below:

Experimental Spacing	Theoretical Spacing
5.0 +/- 0.5	5.14 +/- 0.2
10.0 +/- 0.5	9.98 +/- 0.2
15.0 +/- 0.5	15.08 +/- 0.2

Table 3: Experimental Spacing vs Theoretical Spacing

Based on the data, equation [8] yielded accurate results when compared to the experimental spacings. They are within uncertainties of each other. Thus it would seem that equation [8] is also true and accurate to reality.

Possible errors for this experiment is from how we measured the angles and viewed the constructive and deconstructive interference. During the lab it was very confusing and we may not have done the measurements completely accurately. However the data seems to indicate we weren't too far off.

4. Conclusion

In this set of experiments, we investigated the fundamental principles of reflection, wave speed and dispersion, refraction, diffraction, and interference. Using a ripple tank, ripple generator and different dippers, we generated traveling water waves and explored how they interact with various barriers, refractive media, and slits. Theoretical equations that describe these wave phenomena were validated by the results of this experiment, except for diffraction (equation 6), where the results were ambiguous compared to theoretical values calculated.

References

[1] J. J. Sinclair, “Wave phenomena: Ripple tank experiments,” 2016.
[2] Vedantu.com, 2023.
https://www.vedantu.com/question-sets/db735b51-52d9-472e-8bc3-c8abea9d635f3709179071670777581.png (accessed Apr. 11, 2023).

Appendices

A Equations

Chi-Squared Formula:

$$\chi^2 = \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma_{y_i}^{2}}$$

B Python Code

```
#Imports
import numpy as np
import matplotlib.pyplot as plt
from pylab import loadtxt
import scipy.optimize as optimize
from scipy.stats import chisquare
import scipy.signal as signal

# Constants
fontsize = 15

### Uncertainties
def uncertainty_sum(dx, dy):
    return np.sqrt(dx**2 + dy**2)

def uncertainty_prod(x, dx, y, dy, z):
    return np.sqrt((dx/x)**2 + (dy/y)**2) * z

def import_data(filename,skip_rows=2, cols=(0,1)):
    data=loadtxt(filename, usecols=cols, skiprows=skip_rows, unpack=True)

    return data[0], data[1]

def mean_same_uncern(list):
    return sum(list)/len(list)

def standard_deviation(value, list):
    return np.sqrt((1/(len(list)-1))*sum((value - list)**2))

def standard_error(value, list):
    return standard_deviation(value, list) / np.sqrt(len(list))

def mean_diff_uncern(list, uncern_list):
    return sum(list/uncern_list**2) / sum(1/uncern_list**2)

# Modify depending on fit function
def chi_squared(func, a, b, x_data, y_data, dy_data): # Goodness of Fit
    fx = func(x_data, a, b)
    N = len(x_data)
    sum = 0

    for i in range(N):
        sum += (((y_data[i] - fx[i])**2) / (dy_data[i]**2))

    v = N-2

    return sum / v

def r_squared(y_data, y_graph):
    y_mean = np.mean(y_data)
    ss_total = np.sum((y_data - y_mean)**2)
    ss_residual = np.sum((y_data - y_graph)**2)
    r_squared = 1 - (ss_residual / ss_total)
    return r_squared

def residuals(func, a, b, x, y): # Residuals
    fx = func(x, a, b)
    resid = y - fx

    return x, resid

def calc_curve_of_fit(func, xdata, ydata, init_guess):
    param, uncern = optimize.curve_fit(func, xdata, ydata, p0=init_guess)
    a = param[0]
    b = param[1]
    a_uncern = uncern[0]
    b_uncern = uncern[1]
    x_graph = np.linspace(0, np.max(xdata))
    y_graph = func(x_graph, a, b)

    return x_graph, y_graph, a, a_uncern, b, b_uncern

def fit_func(x, a, b):
    x = np.array(x)
    return a*x + b

### lab functions
def wavespeed(wavelength, frequency):
    return wavelength*frequency

def diffraction_function(a, theta):
    return a*np.sin(theta)

def refraction_function(depth, theta):
    wavespeed = np.sqrt(9.81*depth) # equation 3
    return wavespeed, (1/wavespeed)*np.sin(theta*(np.pi/180))

### Exercise 1:
print("Exercise 1: Reflection of Straight Barrier")
print("Graphed")
angle_inc, angle_refl = import_data("exercise1_data.txt", skip_rows=1) # Import data
y_uncern = [0.5, 0.5, 0.5]

# Line of best fit
graph_x, graph_y, graph_a, graph_a_uncern, graph_b, graph_b_uncern = calc_curve_of_fit(fit_func, angle_inc, angle_refl, [0.5, 0.5])
# Residuals
```

```
graph_resid_x, graph_resid_y = residuals(fit_func, graph_a, graph_b, angle_inc, angle_refl)
# Chi squared and R squared:
chi = chi_squared(fit_func, graph_a, graph_b, np.array(angle_inc), np.array(angle_refl), np.array(y_uncern))

graph_reflection = False
# Graph
# Plot 1 Normal
plt.subplot(2,1,1)
plt.title(f"Reflection: Angle Incident vs Angle Reflection",fontsize=fontsize)
plt.plot(angle_inc, angle_refl, 'o')
plt.plot(graph_x, graph_y)
textstr = f"y = {round(graph_a,2)}x + {round(graph_b,2)} \nχ^2 = {chi}"
props = dict(boxstyle='round', facecolor='wheat', alpha=0.5)
plt.text(0, 100, textstr, fontsize=14, verticalalignment='top', bbox=props)
plt.xlabel("Angle Incident (degrees)",fontsize=fontsize) # Labels
plt.ylabel("Angle Reflection (degrees)",fontsize=fontsize)
plt.tight_layout()

# Plot 2 Residuals
plt.subplot(2,1,2)
y_zeroline = [0,0]
x_zeroline = [0, max(graph_resid_x)]
plt.plot(graph_resid_x, graph_resid_y, 'o')
plt.plot(x_zeroline, y_zeroline)
plt.errorbar(graph_resid_x, graph_resid_y, xerr=None, yerr=y_uncern, fmt=".") #error bars
plt.title("Residuals of Angle Graph", fontsize=fontsize)
plt.tight_layout()
if graph_reflection == True:
    plt.show()

### Exercise 2:
print("\nExercise 2: Wave Speed")
print("Wave Speed vs Frequency")
# Import data
freq = [5, 8, 11, 14, 17, 20]
wavelength_freq = []
for i in range(len(freq)):
    dummy, wavelength_freq_curr = import_data("exercise2_freq_data.txt", skip_rows=1, cols=(0, i))
    wavelength_freq.append(wavelength_freq_curr)

# Calculations
wavespeed_freq = []
for i in range(len(freq)):
    wavespeed_freq.append(wavespeed(wavelength=wavelength_freq[i], frequency=freq[i]))

# Mean and std dev
wavespeed_freq_mean = []
wavespeed_freq_std_error = []
for i in range(len(freq)): # calculate means and std dev for each freq
    wavespeed_freq_mean.append(mean_same_uncern(wavespeed_freq[i]))
    wavespeed_freq_std_error.append(standard_error(wavespeed_freq_mean[i] ,wavespeed_freq[i]))
for i in range(len(freq)):
    print(f"Frequency: {freq[i]}Hz, Wavespeed: {wavespeed_freq_mean[i]:.3f} +/- {wavespeed_freq_std_error[i]:.3f}")

wavespeed_graph = False
# Graph
plt.clf()
plt.title(f"Wavespeed vs Frequency",fontsize=fontsize)
plt.plot(freq, wavespeed_freq_mean, 'o')
plt.xlabel("Frequency (Hz)",fontsize=fontsize) # Labels
plt.ylabel("Wavespeed (m/s)",fontsize=fontsize)
plt.tight_layout()
if wavespeed_graph == True:
    plt.show()

print("Wave Speed vs Depth")
# Import data
freq = 10
depth = [10, 8, 6, 4, 2]
wavelength_depth = []
for i in range(len(depth)):
    dummy, wavelength_depth_curr = import_data("exercise2_depth_data.txt", skip_rows=1, cols=(0, i))
    wavelength_depth.append(wavelength_depth_curr)

# Calculations
wavespeed_depth = []
for i in range(len(depth)):
    wavespeed_depth.append(wavespeed(wavelength_depth[i], freq))

# Mean and std dev
wavespeed_depth_mean = []
wavespeed_depth_std_error = []
for i in range(len(depth)): # calculate means and std dev for each freq
    wavespeed_depth_mean.append(mean_same_uncern(wavespeed_depth[i]))
    wavespeed_depth_std_error.append(standard_error(wavespeed_depth_mean[i] ,wavespeed_depth[i]))
for i in range(len(depth)):
    print(f"Depth: {depth[i]}mm, Wavespeed: {wavespeed_depth_mean[i]:.3f} +/- {wavespeed_depth_std_error[i]:.3f}")

# Line of best fit
#wavespeed_depth_squared = np.array(wavespeed_depth_mean)**2
depth_sqrt = np.sqrt(depth)
graph_x, graph_y, graph_a, graph_a_uncern, graph_b, graph_b_uncern = calc_curve_of_fit(fit_func, depth_sqrt, wavespeed_depth_mean, [0.5, 0.5])
# Residuals
graph_resid_x, graph_resid_y = residuals(fit_func, graph_a, graph_b, depth_sqrt, wavespeed_depth_mean)
# Chi squared and R squared:
chi = chi_squared(fit_func, graph_a, graph_b, np.array(depth_sqrt), np.array(wavespeed_depth_mean), np.array(wavespeed_depth_std_error))
```

```
graph_wavespeed = True
plt.clf()
# Graph
# Plot 1 Normal
plt.title(f"Wavespeed vs Depth",fontsize=fontsize)
plt.plot(depth, wavespeed_depth_mean, 'o')
plt.xlabel("Depth (mm)",fontsize=fontsize) # Labels
plt.ylabel("Wavespeed (m/s)",fontsize=fontsize)
plt.tight_layout()
plt.show()

plt.subplot(2,1,1)
plt.title("Wavespeed vs sqrt(Depth)", fontsize=fontsize)
plt.xlabel("Sqrt(Depth) (mm^0.5)",fontsize=fontsize) # Labels
plt.ylabel("Wavespeed (m/s)",fontsize=fontsize)
plt.plot(depth_sqrt, wavespeed_depth_mean, 'x')
plt.plot(graph_x, graph_y)
textstr = f"y = {round(graph_a,2)}x + {round(graph_b,2)} \nχ^2 = {chi}"
props = dict(boxstyle='round', facecolor='wheat', alpha=0.5)
plt.text(0, 25, textstr, fontsize=14, verticalalignment='top', bbox=props)
plt.tight_layout()

# Plot 3 Residuals
plt.subplot(2,1,2)
y_zeroline = [0,0]
x_zeroline = [0, max(graph_resid_x)]
plt.plot(graph_resid_x, graph_resid_y, 'o')
plt.plot(x_zeroline, y_zeroline)
plt.errorbar(graph_resid_x, graph_resid_y, xerr=None, yerr=wavespeed_depth_std_error, fmt=".") #error bars
plt.title("Residuals of Depth Graph", fontsize=fontsize)
plt.tight_layout()
if graph_wavespeed == True:
    plt.show()

### Exercise 3:
print("\nExercise 3: Refraction")
# Calculate wave speed from depth (what is depth???)
# Use angles from images to check whether or not the two values are the same
# we have two points of data for it so we can graph it to see if there is a constant value?
# Import data
refraction = []
for i in range(3):
    dummy, refraction_curr = import_data("exercise3_data.txt", skip_rows=1, cols=(0, i))
    refraction.append(refraction_curr)
theta_inc, theta_ref, depth = refraction

# Calculate the constant
# Incident wave
wavespeed_inc, inc_constant = refraction_function(depth[0], theta_inc)
wavespeed_ref, ref_constant = refraction_function(depth[1], theta_ref)

# Print results
for i in range(2):
    print(f"Wave speed incident: {wavespeed_inc}, Theta_Inc: {theta_inc[i]}, Constant: {inc_constant[i]} \n Wave speed Refraction: {wavespeed_ref}, Theta_Inc

### Exercise 4:
y_uncern = [0.5, 0.5, 0.5, 0.5, 0.5]
print("\nExercise 4: Diffraction")
print("Graphed")
diffraction = []
for i in range(3):
    dummy, diffraction_curr = import_data("exercise4_data.txt", skip_rows=1, cols=(0, i))
    diffraction.append(diffraction_curr)
slit, exp_angular_spread, exp_wavelength = diffraction
theo_wavelength = diffraction_function(slit, exp_angular_spread)

plot_exercise_4 = True
# Plot experimental wavelength vs actual wavelength
plt.clf()
plt.title("Experimental Wavelength vs Theoretical Wavelength",fontsize=fontsize)
plt.xlabel("Slit size (cm)",fontsize=fontsize) # Labels
plt.ylabel("Wavelength (cm)",fontsize=fontsize)
plt.plot(slit, exp_wavelength, 'o', label='Experimental Wavelengths')
plt.errorbar(slit, exp_wavelength, xerr=None, yerr=y_uncern, fmt=".") #error bars
plt.plot(slit, theo_wavelength, 'x', label='Theoretical Wavelengths')
plt.legend(loc="upper left")
if plot_exercise_4 == True:
    plt.show()

### Exercise 5:
y_uncern = [0.05, 0.05, 0.05]
x_uncern = [0.5, 0.5, 0.5]
print("\nExercise 5: Interference")
# Import data
interference = []
for i in range(5):
    dummy, interference_curr = import_data("exercise5_data.txt", skip_rows=1, cols=(0, i))
    interference.append(interference_curr)
spacing, const_n1, const_n2, dec_n1, dec_n2 = interference

plot_exercise_5 = True
plt.clf()
plt.title("Angle of interference vs Spacing between point sources",fontsize=fontsize)
plt.xlabel("Spacing (cm)",fontsize=fontsize) # Labels
plt.ylabel("Angle of interference (cm)",fontsize=fontsize)
```

```python
plt.plot(spacing, const_n1, 'o', label='Constructive N1')
plt.plot(spacing, const_n2, 'o', label='Constructive N2')
plt.plot(spacing, dec_n1, 'o', label='Deconstructive N1')
plt.plot(spacing, dec_n2, 'o', label='Deconstructive N2')
plt.errorbar(spacing, const_n1, xerr=x_uncern, yerr=y_uncern, fmt=".", color='blue')
plt.errorbar(spacing, const_n2, xerr=x_uncern, yerr=y_uncern, fmt=".", color='orange')
plt.errorbar(spacing, dec_n1, xerr=x_uncern, yerr=y_uncern, fmt=".", color='green')
plt.errorbar(spacing, dec_n2, xerr=x_uncern, yerr=y_uncern, fmt=".", color='red')
plt.legend(loc="upper right")
if plot_exercise_5 == True:
    plt.show()

# Calculate theoretical spacing
wavelength = 1.1 # To be removed
distance_const = (((1*wavelength)/np.sin((const_n1*np.pi)/180)) + ((2*wavelength)/np.sin((const_n2*np.pi)/180)))/2
distance_dec = (((1*wavelength)/np.sin((dec_n1*np.pi)/180)) + ((2*wavelength)/np.sin((dec_n2*np.pi)/180)))/2
for i in range(3):
    print(f"Experimental Spacing: {spacing[i]}, Theoretical Spacing: {distance_const[i]}")
```