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THE OUTPUT RESISTANCE OF A POWER SUPPLY

Working with your partner, answer the questions in this exercise and submit them as a single file (including tables, figures, etc.) via Quercus by 11:59 on the 2nd Tuesday following your session (Oct. **4**/11). This is not as formal as Lab 1 and Lab 2 which you will do later in the term but make sure that your submission contains the title, both of your names, and the date.

Objectives: to review basic electrical measurement techniques; analyze uncertainties of direct and indirect measurements; understand error propagation; apply curve fitting to the set of data points; calculate uncertainty in linear fit.

I. Introduction

Any source of electrical energy (generator, battery, thermocouple, etc.) with no load attached to it produces voltage potential across the terminals called an electromotive force (*emf*), or an open-circuit voltage, V_∞.

This number does not completely specify the power supply. In a closed circuit, a current ***I*** will be drawn from the power supply, and the voltage at the terminals, V, called *the terminal voltage*, will typically fall below V_∞. A plot of what the terminal voltage V vs. current I may look like is shown in Fig. 1.

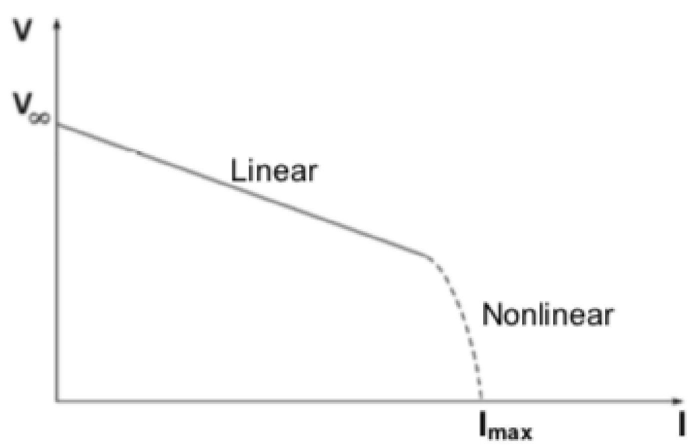

FIG.1: Terminal voltage vs. current

Many power sources will exhibit a linear variation of V for small current values, followed by a nonlinear behavior at higher currents. The linear part of the curve can be described by

$$V = V_{\infty} - RI, \tag{1}$$

where R is the *output resistance of the power source*. In this linear regime, according to Thevenin's theorem, the power source is completely represented by this equivalent circuit as in Fig. 2.

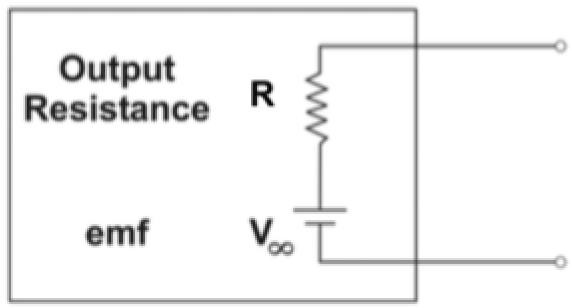

Fig. 2. Equivalent circuit of an electric power source.

The output resistance (R) can be determined by attaching different external resistances of the load (R_l) to the power source, and measuring the current and voltage with a multimeter. Figure 3 shows two possible ways of doing this. Both would be equivalent **if** the multimeter were ideal. However, in this exercise we will measure with real, not ideal, multimeters.

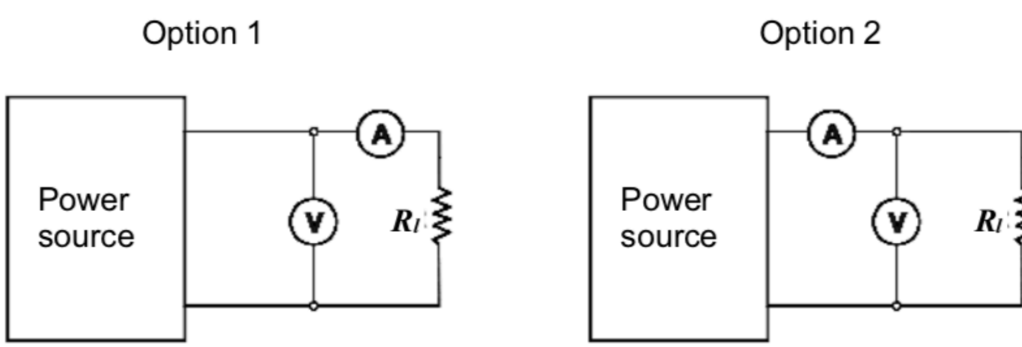

Fig. 3. Possible circuits for determining the output resistance of a power source.

Question 1:

Without connecting circuits and making measurements, how would you expect the readings of the voltmeter and the ammeter to differ between Option 1 and in Option 2? Explain.

A: In Option 2 the ammeter will receive all the current from the power source, whereas in Option 1 according to current division the ammeter will receive some fraction of the total current supplied from the power source.

In Option 1 the voltmeter will have a higher reading than Option 2. There is more resistance over the voltmeter in Option 1 since it is the resistor added with the internal resistance of the ammeter as opposed to just the resistor in Option 2.

Question 2:

To calculate the resistance of the power source, you will need to know the internal resistance of the voltmeter and the ammeter. How can you find these values by making measurements of current and voltage with the Option 1 and Option 2 circuits? Derive the general formulae for the internal resistance of the voltmeter and the ammeter based on the results of these measurements and the known resistance of the load.

A: In option 1:

$$V_{V} = I_{A} R_{A} + I_{A} R_i$$

$$\frac{V_{V}}{I_{A}} = R_{A} + R_i$$

$$R_{A} = \frac{V_{V}}{I_{A}} - R_i$$

Figure [01]: Derivation of Internal Resistance of Ammeter

In Option 2

$$I_{A} = \frac{V}{R_{V}} + \frac{V}{R_i}$$

$$I_{A} = V\left(\frac{1}{R_{V}} + \frac{1}{R_i}\right)$$

$$\frac{I_{A}}{V} = \frac{1}{R_{V}} + \frac{1}{R_i}$$

$$\frac{1}{R_{V}} = \frac{I_{A}}{V} - \frac{1}{R_i}$$

$$R_{V} = \frac{1}{\frac{I_{A}}{V} - \frac{1}{R_i}}$$

$$R_{V} = \frac{V}{I_{A} - \frac{V}{R_i}}$$

Figure [02]: Derivation of Internal Resistance of Voltmeter

II. The Experiment

Measure the resistance of up to 7 resistors with the multimeter set to ohmmeter. Use the data to complete the table below and indicate the (reading) uncertainty for each measurement.

Table 1

R_{l1} $k\Omega$	ΔR_{l1} $k\Omega$	R_{l2} $k\Omega$	ΔR_{l2} $k\Omega$	R_{l3} $k\Omega$	ΔR_{l3} $k\Omega$	R_{l4} $k\Omega$	ΔR_{l4} $k\Omega$	R_{l5} Ω	ΔR_{l5} Ω	R_{l6} Ω	ΔR_{l6} Ω	R_{l7} Ω	ΔR_{l7} Ω
103.	±0.0	26.6	±	2.69	±	0.46	±-0.	215.	±0.0	99.9	±0.0	59.1	±0.0

51	05	12	0.005	2	0.005	5	005	15	05	6	05	7	05

Question 3:

Choose at least 4 of these resistors to construct circuits for this experiment. Briefly justify your choice.

A: We chose $R_3, R_4, R_5,\ R_6$ for a range of Ohmages

Circuit Option 1:

- Assemble circuit Option 1 (Fig. 3). Sketch or photograph it and include this sketch/photograph in your report. List the elements of the circuit in a caption.

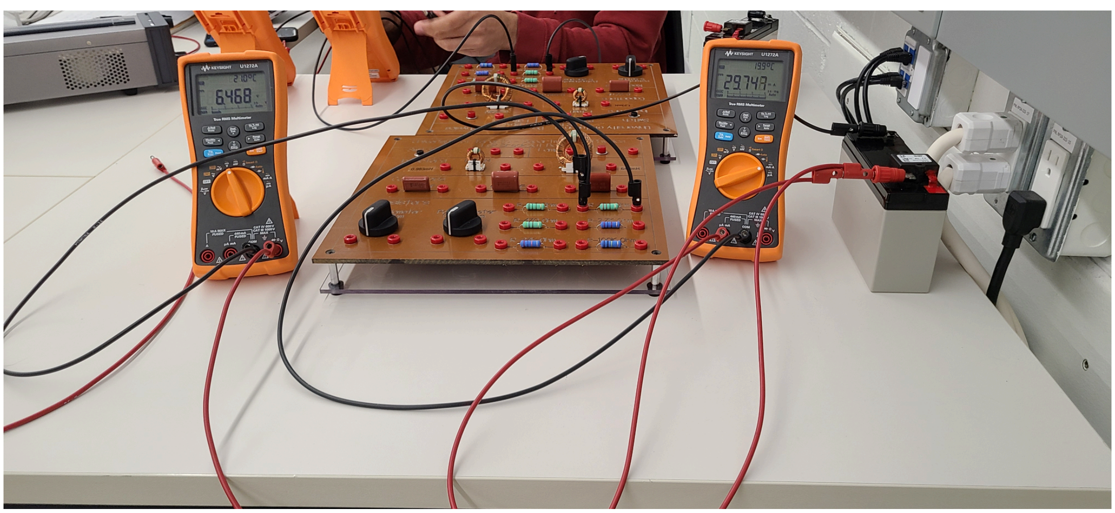

Figure [03]: Circuit for Option 1 using an Ammeter, a Voltmeter, 100Ω resistor, 5V Battery, and wires

- If you are working with a DC power supply, set it to about 6.5V.
- Measure the voltage V and current I for circuits with your four chosen resistors. You can use more than 4 resistors if you like. Use this information to calculate the internal resistance of the ammeter. Organize all of the data in Table 2.

Table 2. For Circuit Option 1

	Resistance Rli,Ω	**Uncertainty Rli,Ω**	**Voltage V,V**	**Uncertainty V,V**	**Current I, mA**	**Uncertainty I, mA**	**Resistance of ammeter RA, Ω**	**Uncertainty RA, Ω**
1	2692	5	6.495	0.0005	2.415	0.0005	-2.559	0.001318
2	465	5	6.48	0.0005	13.863	0.0005	2.43	0.00823
3	215.15	0.005	6.462	0.0005	29.716	0.005	2.308	0.01487
4	99.96	0.005	6.446	0.0005	63.1	0.005	2.2	0.032
						Average:	1.1	0.013

- Plot V vs I and apply a linear fit to determine the slope m_1 and its uncertainty. Check two goodness of fit criteria.

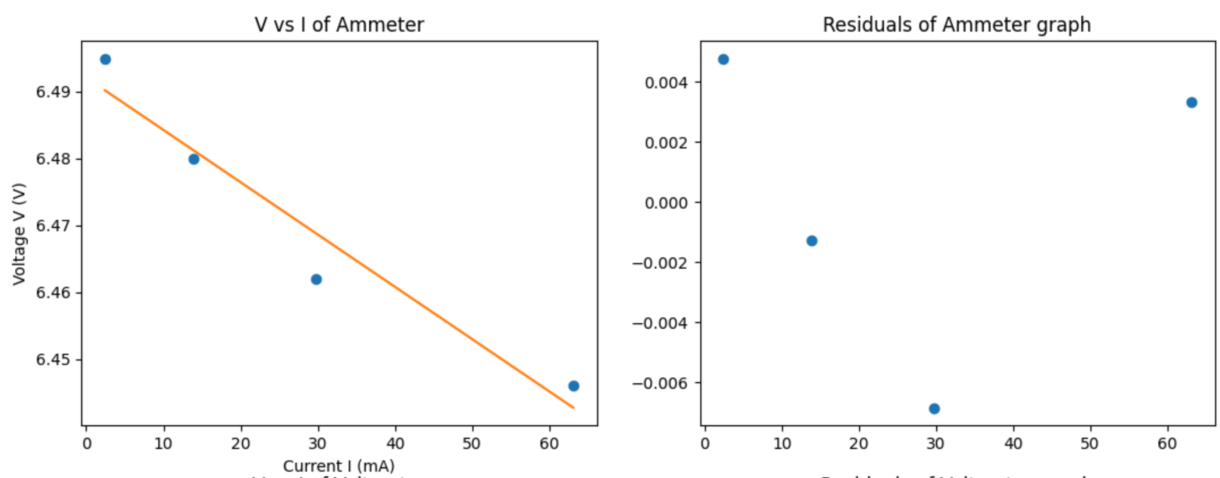

Figure [04]: Plot of Voltage and Current of Ammeter with linear fit, and plot of residuals

Line of best fit: m = -0.0007832850991339173 +/- 0.00014025755905430323, b = 6.492112926151229 +/- 0.0049898317728642285

Goodness to Fit: Chi Squared: 164.24623704824404

Circuit Option 2:

- Assemble circuit Option 2 (Fig. 3).
- Measure the voltage V and current I for circuits with your four (or more) chosen resistors. Use this information to calculate the internal resistance of the voltmeter. Organize all of the data in Table 2.

Table 3. For Circuit Option 2

	Resistance Rli,Ω	**Uncertainty Rli,Ω**	**Voltage V,V**	**Uncertainty V,V**	**Current I, mA**	**Uncertainty I, mA**	**Resistance of voltmeter RV, Ω**	**Uncertainty RV, Ω**
1	2692	5	6.471	0.0005	2.408	0.0005	2.690	5.965 x 10^-4
2	465	5	6.443	0.0005	13.847	0.0005	0.466	4.019 x 10^-5
3	215.15	0.005	6.394	0.0005	29.7	0.05	0.22	3.64 x 10^-4
4	99.96	0.005	6.305	0.0005	63.02	0.005	0.03	1.12 x 10^-5
						Average:	0.868	6.06

- Plot V vs I and apply a linear fit to determine the slope m_2 and its uncertainty. Check two goodness of fit criteria.

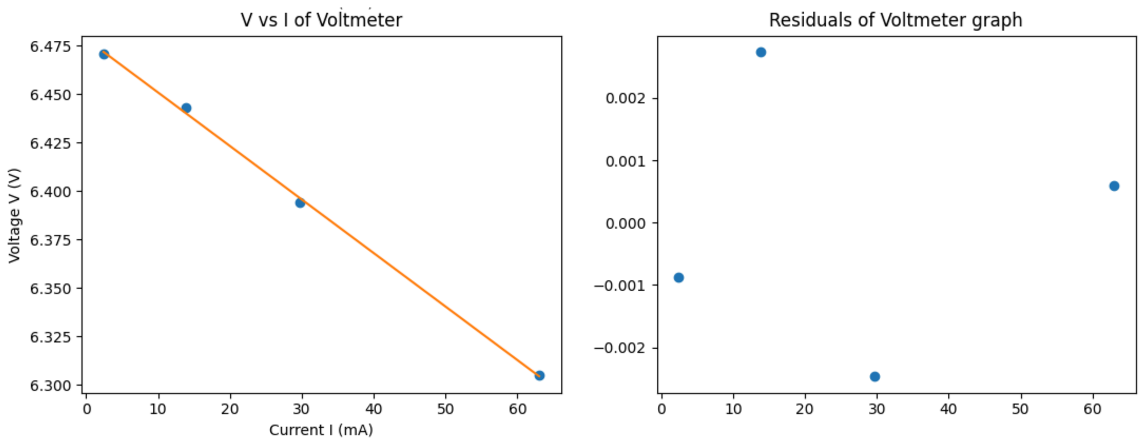

Figure [05]: Plot of Voltage and Current of Voltmeter with linear fit, and plot of residuals

Line of best fit: m = -0.00276280485916265 +/- 5.934326164443371e-05, b = 6.478519164881813 +/- 0.0021088052560866164

Goodness to Fit: Chi Squared: 29.331745042704373

III. Analysis

- Derive a relationship among m_1, R_V and R_1 to find the output resistance with its uncertainty. Show the steps of your error propagation calculation for the uncertainty of R_1.

$$m_1 = \frac{I}{V} \qquad R_1 = \frac{V}{I_1}$$

$$I_1 = I + \frac{V}{R_v}$$

$$R_1 = \frac{V}{I + \frac{V}{R_v}} \cdot \frac{\frac{1}{V}}{\frac{1}{V}}$$

$$R_1 = \frac{1}{\frac{I}{V} + \frac{1}{R_v}}$$

$$R_1 = \frac{1}{m_1 + \frac{1}{R_v}}$$

Figure [06]: Derivation based on m_1, R_v and R_1

- Derive a relationship among m_2, R_A and R_2 to find the output resistance with its uncertainty. Show the steps of your error propagation calculation for the uncertainty of R_2.

$$V_A + V = IR_2$$

$$IR_a + V = IR_2$$

$$R_A + \frac{V}{I} = R_2$$

$$R_2 = R_A + m_2$$

Figure [07]: Derivation based on m_2, R_A and R_2

- Show the output resistance of the battery (or DC power supply) as: R_1 = (___ ± ___) Ω and R_2 = (___ ± ___) Ω.

  $R_1 = 1.094 \pm 0.01339\,\Omega$

  $R_2 = 0.865 \pm 0.000260\,\Omega$

- Write a brief conclusion on the difference between the two results.

These resistances are basically a measure of how hard it is to draw a certain voltage from the power source based on current. This is shown in Ohm's law V = IR, where R is the ratio between the voltage and the current. In our circuit, this value is dependent on the rest of the circuit, as the impedances in the circuit affect how much voltage is delivered from the power source. The different placements of the voltmeter and the ammeter change the impedance of the circuit, since parallel and series circuits are different from each other, thus we have two different values for the resistance of the power supply.

Another factor is the experimental setup. Because the voltmeter and ammeter have such low resistances (since they don't want to mess with the circuit they are measuring), the measured values are unpredictable and some of the values gathered theoretically don't make sense until you consider uncertainties and margins of errors. This in turn can throw off the calculations. An Ideal lab set up to measure the resistance of the power source, only requires one version of the circuit, and has a voltmeter and ammeter with an internal resistance of zero. However, that is impossible so we must make do with this lab setup.