

Frank-Hertz Lab Report

Experimentally confirming Energy Quantization in Atoms

1. Introduction

The Frank Hertz experiment is a monumental scientific investigation that helped confirm the Bohr model of the atom and the idea of quantized energy. The Bohr Model was proposed by Ernest Rutherford and Niels Bohr in 1913 [1]. This model of atomic structure suggests that electrons orbit the nucleus of an atom in distinct shells with energy that increases in discrete quantized increments, implying that electrons have a minimum possible energy and distance from the nucleus. James Franck and Gustav Hertz verified this model empirically in 1914 by shooting vaporized mercury with electrons. They hypothesized that by analyzing the momentum transfer of the electrons with the mercury, it could be determined whether the atom's electrons gained energy in discrete/quantized amounts.

In the experiment, electrons are passed through a tube containing vaporized mercury, which is heated to the point of vaporization. The collisions between electrons and mercury are predicted to change based on the constantly increasing voltage referred to as the accelerating voltage. If the electrons' kinetic energy is less than the mercury's first excitation energy, the collision will be perfectly elastic, and no energy or momentum will be transferred. On the other hand, if an electron's kinetic energy is at or above the first excitation energy, then the electrons will collide inelastically, and the mercury atom will absorb the energy. This behavior of the collisions can be directly observed by measuring the flow of electrons out of the tube using an electrometer.

Furthermore, as the voltage is increased, the electrons will have enough kinetic energy to inelastically collide with several mercury atoms, transferring a discrete excitation energy with each collision. As a result, the current will steadily increase but sharply drop off at discrete, constant intervals equal to this excitation energy. Based on the original experiment, the first excitation energy of mercury is $4.92 \pm 0.04 \text{ eV}$ and the emitted wavelength is 227 nm [1]. This experiment provided strong support for Bohr's quantum model of the atom and is regarded as a significant contribution to the development of modern atomic physics.

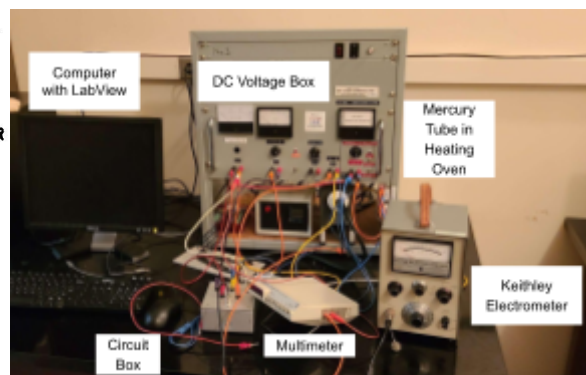
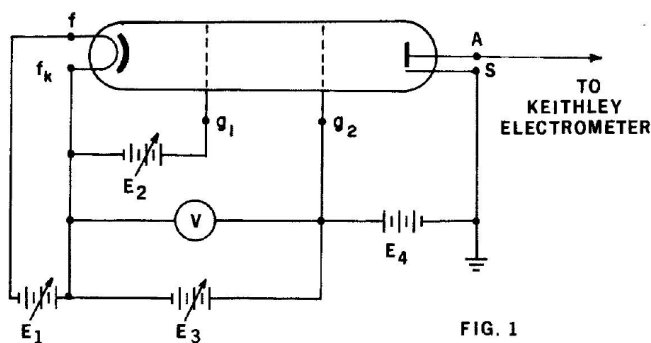
2. Procedure

2.1 Equipment

- Circuit Box
- Frank-Hertz Mercury Tube
- Heating Oven
- Keithley Electrometer
- Multimeter
- DC Power Supply with fixed (E_1 , E_4), and adjustable (E_2 , E_3) which included E_3 having sweeping voltage
- Computer with LabView Analysis Software

2.2 Apparatus

Figures 1 and 2: Experimental Setup from lab handout and in the actual lab location)



2.3 Experimental Method

The experimental Method was followed using the lab manual [2].

The experiment is conducted in a setup process that involves setting up the oven to 170°C and placing the Franck-Hertz tube inside. The Keithley Electrometer is set to a scale of 10^{-9} Amperes, and the power supply is enabled, and the LabView Franck-Hertz application is opened.

After setting up, the experiment starts with a manual check, which involves placing the E3 voltage plugs in the Manual ports. The E3 voltage is manually swept and adjusted while the E2 voltage is tuned so that smooth, clear peaks appear on the electrometer. E2 voltage was set to 2.5 ± 0.05 V was selected to produce clear peaks.

Then, the sweep voltage stage begins, where the E3 voltage plugs are placed in the Sweep ports, and the electrometer wires are connected to the X computer data ports. The data collection process is then carried out. The sweep voltage is turned to Reset, and the data is recorded through LabView as the sweep voltage is turned to On. These steps are repeated multiple times each time with a different accelerating voltage speed.

Finally, the data is saved, and the experiment is concluded.

3. Results and Analysis

3.1 Uncertainties, Error Propagation and Sources of Error

The uncertainties in this lab were generated from the devices used to measure the voltages of both the accelerating potential, as well as the collection voltage. Since the digital devices gave values with ten significant figures, we would've usually used the convention of \pm half the last significant digit. But in this case that felt really unreasonable because it wasn't realistic for the device to be that accurate. So we based the uncertainty on the electrometer which was ± 0.05 V. For the uncertainties of the peaks, the standard deviation of the different experiments was used. This equation is:

$$\sigma = \sqrt{\frac{1}{N-1} \sum (x_i - \Delta x_i)^2} \quad (1)$$

For error propagation we used two equations depending on whether or not the formula being calculated used an addition/subtraction, or if the formula used multiplication or division. These formulae are shown below:

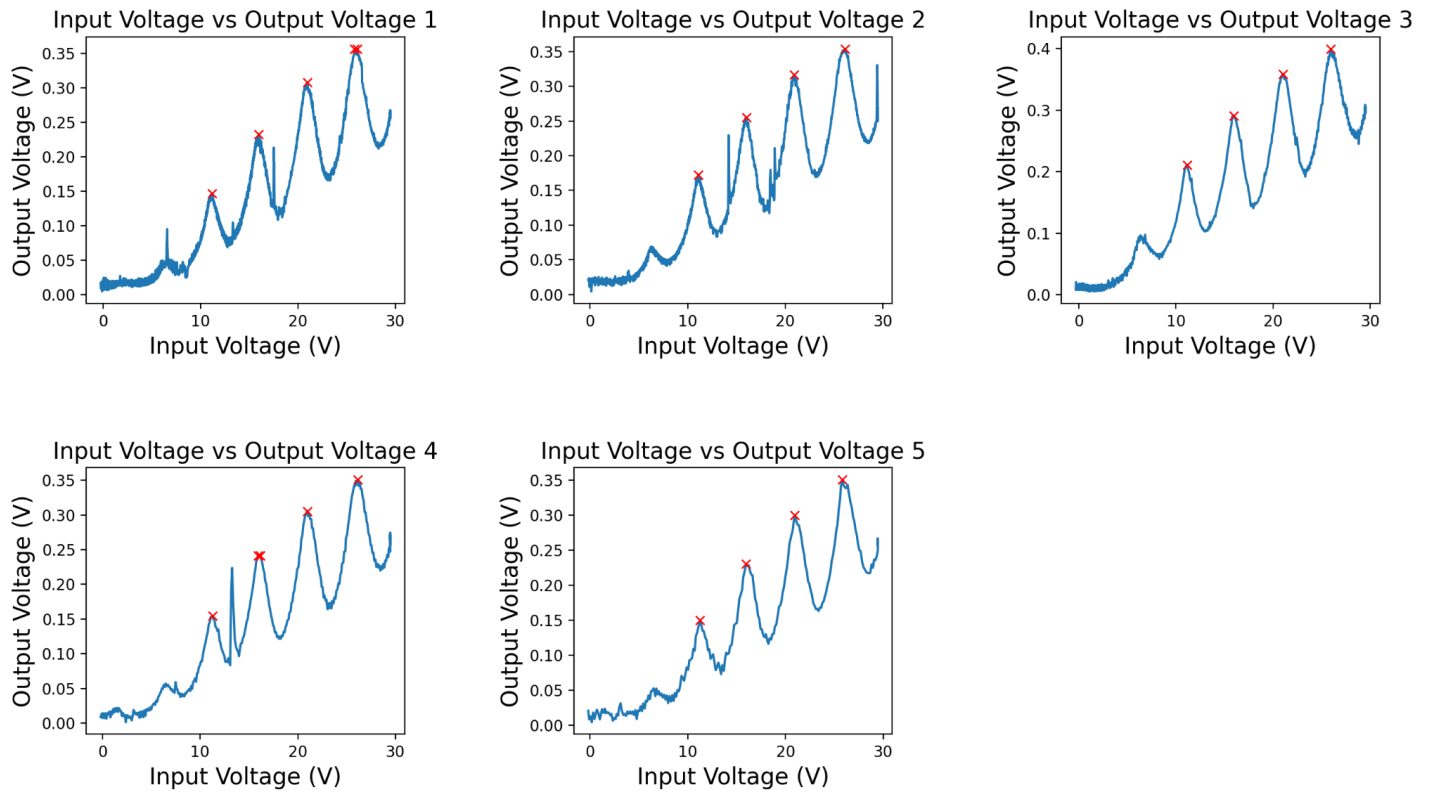
$$\Delta z = z \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \quad (2)$$

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2} \quad (3)$$

There are several sources of error in this lab, including the equipment provided and the Labview Data acquisition. One issue we encountered was the presence of random spikes in our labview data at a certain lower accelerating voltage, which introduced uncertainty and made it difficult to determine whether a peak was present or not. Furthermore, the labview data we obtained from E3 had a very low uncertainty, which may have provided us with up to 10 digits of accuracy, but this is not reflective of the actual measured values. This led us to align our uncertainties with those obtained from the measured electrometer as opposed to those obtained from Labview. Another issue we encountered was that the heated oven often fluctuated from its set point of 170°C, fluctuating around 3 degrees due to continuous overshooting and cooling below the set point. This fluctuation could have caused the mercury to exist in vapor or liquid form, further complicating our data analysis.

3.2 Calculating Peaks

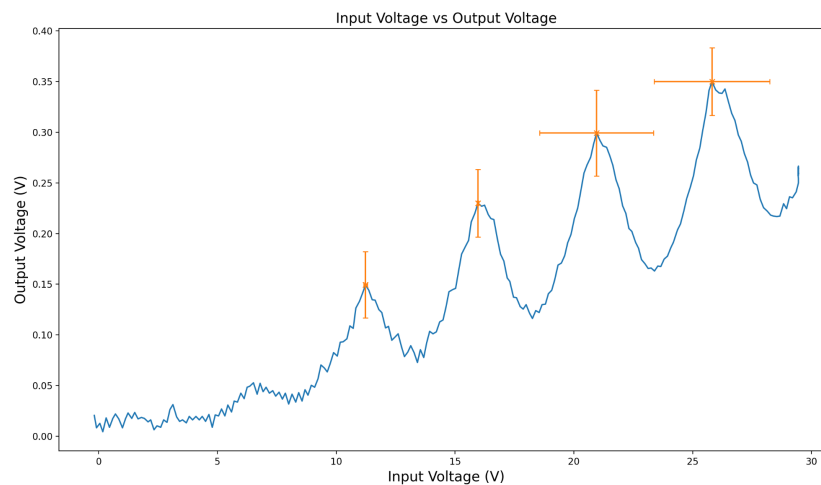
During our experiment, we collected five sets of data, each with an E2 voltage of 2.5V, at different speeds of voltage increase. Taking the raw data from the labview files, we wrote Python code to take the data and display it in a human-readable form. A key component for this was calculating the peaks in the data, which was done with the scipy python library. The result from this part of the code is shown below:



(Figure 3: Peaks of Voltage of each test)

3.3 Graph of Peaks

Taking the average of each test and calculating the standard deviations of each peak, we got a graph that represents the quantized energy levels of the mercury. The error bars of the graph below are derived from the standard deviations of the peaks. The full graph is shown below:



(Figure 4: Average data with peaks and uncertainties labeled)

The peaks of the data are represented in table form below:

Peak Index Number	Accelerating Voltage (V)	Collection Voltage (V)
1	11.23 ± 0.08	0.15 ± 0.03
2	15.96 ± 0.03	0.23 ± 0.03
3	20.9 ± 2.4	0.30 ± 0.04
4	25.8 ± 2.4	0.35 ± 0.03

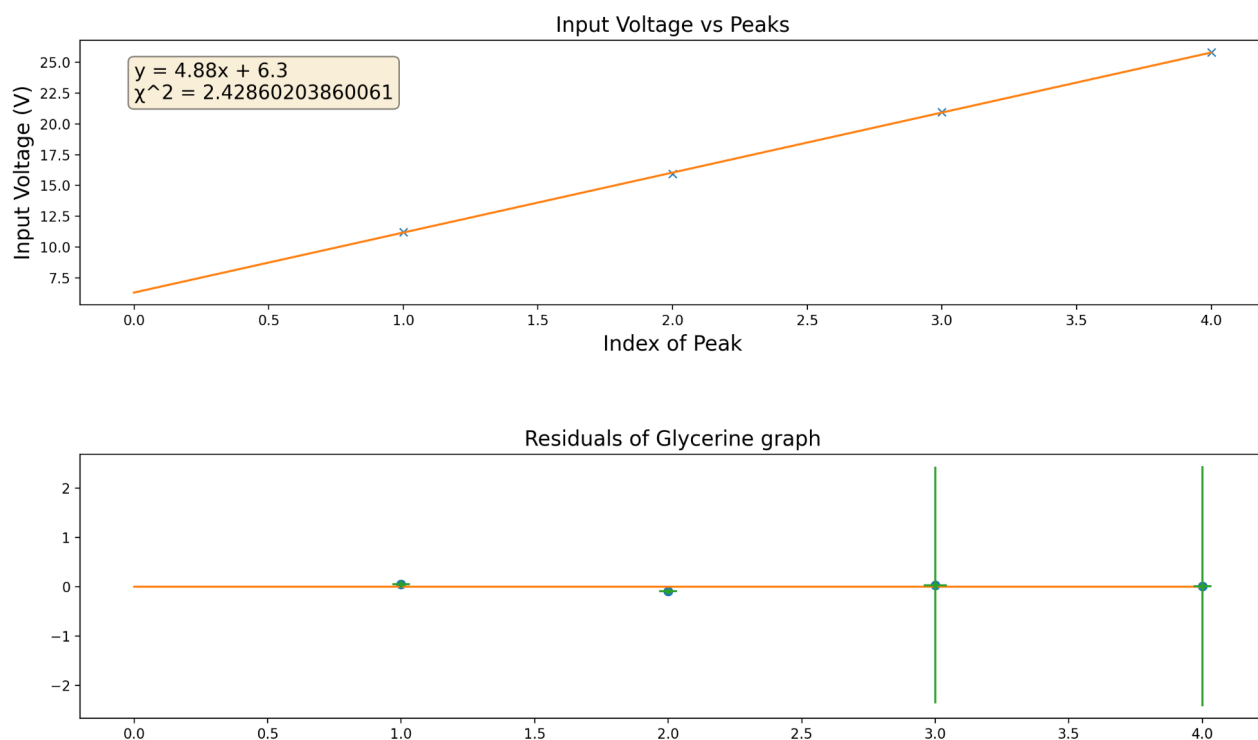
(Table 1: Average Peaks of Collector Voltage)

3.4 Relationship between Peaks and Accelerating Voltage

In order to determine whether or not the excitation voltage has a linear relationship, we fitted the peak data to a linear function (Equation 3). If y is the accelerated voltage at the peak, then there are two parameters to this function. The parameter m is the excitation voltage required to excite the mercury at each energy level. If the linear fit is accurate, the excitation voltage should be the same at each excitation level. The parameter b is the minimum voltage the electrometer receives.

$$y = mx + b \quad (4)$$

The peaks and fit function are graphed below with the chi-squared value. Another measure of “goodness of fit” is the residuals and is graphed below the fit graph:



(Figure 5: Fit Function with Residuals)

With a chi-squared value of 2.42 (pretty close to 1) and a residual graph looking really close, we can say that there is a linear relationship between the different excitation peaks and that the m value of 4.88V is the voltage required to excite the mercury electron to the next energy level.

3.5 Energy Transfer in an inelastic Collision

In order to calculate the energy transferred in an inelastic collision between the electron and the Mercury Atom, we used the conservation of Momentum equation (Equation 5) and the conservation of Energy equation (Equation 6). In Equation 5 we took the initial velocity of mercury to be zero.

$$\Delta p = 0 = (m_e v_{ei} + m_{Hg} v_{Hgi}) - (m_e + m_{Hg})v_f \rightarrow v_f = \frac{m_e}{m_e + m_{Hg}}v_{ei} \quad (5)$$

$$\Delta KE = \frac{1}{2}(m_e v_{ei}^2 + m_{Hg} v_{Hgi}^2 - (m_e + m_{Hg})v_f^2) \quad (6)$$

Through electrostatics, we know that the kinetic energy of the electron is also based on the voltage difference causing the acceleration. This voltage was calculated in the line of fit as the m value which is $4.88 \pm 0.001V$. This is represented in Equation 7.

$$\Delta KE = qV = \frac{1}{2}m_e v_{ei}^2 \quad (7)$$

Substituting equation 5 and 7 into equation 6 creates a new equation where we know all the unknowns:

$$\Delta KE = qV(1 - \frac{m_e}{m_e + m_{Hg}}) \quad (7)$$

Using the known values of the mass of an electron [4], mass of mercury [5], and the charge of an electron [4], we can use Equation 7 to calculate the kinetic energy. This yields $\Delta KE = 4.87 \pm 0.05$ eV (using the uncertainty propagation in section 3.1). This is the amount of energy being transferred into the Mercury atom through a collision. This value is pretty close to the theoretical value of $4.92V \pm 0.04$ eV and is within the uncertainty.

3.5 Emission Wavelength

To calculate the Emission wavelength of the light emitted by mercury from the first excited state to the ground state, we use Planck's equation:

$$E = h\nu = \frac{hc}{\lambda} \quad (7)$$

Where h is Planck's constant [6]. Using this equation, we calculated that the wavelength of the emitted light to be: $\lambda = 254 \pm 0.03$ nm. This value is nearly exact to the theoretical value of 227 nm.

3.6 Use of Vaporized Mercury

Franck and Hertz chose to use vaporized mercury in their experiment instead of hydrogen gas. This is because hydrogen gas consists of two bonded atoms, which makes it difficult to excite individual atoms with electrons. This means that it requires more energy to break the bond between hydrogen atoms than to excite a single

hydrogen atom. As a result, the observed photoelectric emissions do not match the excitation energies of hydrogen.

In contrast, mercury is a monatomic gas, which means that its atoms are not bonded together in pairs. This makes it easier to excite individual mercury atoms with electrons, as there are no bonds between the atoms that need to be broken. Furthermore, mercury has a unique property of being a liquid at room temperature, unlike other elements with similar masses that are typically solid. This characteristic of mercury means that it requires less energy to partially vaporize, which reduces the energy required to heat the Franck-Hertz tube. This lower energy requirement makes the use of mercury more practical than other options.

3.7 Shape of Current vs. Accelerating Voltage Graph

Looking at our data in Figure 4, one can see that the electrometer voltage rises and falls with noticeable maxima and minima and does not follow a sawtooth pattern for several reasons. A sawtooth pattern would mean that electrometer current scales linearly as voltage accelerating the electrons towards the mercury vapor increases. This is not true since the electrons being shot can travel through the tube without interacting with any mercury atoms due to size difference between atoms and electrons, and just pure luck.

Moreover, not all excitations increase the energy of mercury's outermost electron into its first excited state, although this electron requires the least energy to be excited. Electrons may also excite other electrons in other orbitals with higher excitation energies, although this is less likely. However, despite these factors, the determined peaks are minimally affected due to their prominence.

4. Conclusion

In this experiment, we aimed to replicate the experimental results of James Franck and Gustav Hertz, who confirmed Bohr's model of the atom, which hypothesized the existence of discrete energy levels for electron shells. To do this, we utilized a setup where electrons were directed towards vaporized mercury in a heated tube, while the voltage was gradually increased. As the electrons collided with the mercury atoms, an electrometer measured the voltage of the exiting electrons. We observed dips in voltage at equal sweep voltages, which indicated inelastic collision and where the corresponding excitation energy levels were.

Our observations provided further support for Bohr's model of the atom and its postulation of discrete energy levels within electron shells. Specifically, we found that the first excitation energy and corresponding wavelength emission, measured at $E = 4.81 \pm 0.05 \text{ eV}$ and $\lambda = 254 \pm 0.03 \text{ nm}$, respectively, were in close agreement with theoretical values (those detailed in the introduction). We also explained why mercury was a more suitable choice for our experiment than hydrogen, and why the plot of electrometer voltage vs accelerating voltage did not display sawtooth shapes. Goodness of fit metrics indicated that the residuals plot fit the line well, and the chi-squared value was reasonably low at 2.43.

References

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Appendices

A Equations

Chi-Squared Formula:

$$\chi^2 = \sum_{i=1}^N \frac{[y_i - f(x_i)]^2}{\sigma_{y_i}^2}$$

B Empirical values

Mass of an electron: $9.11 \times 10^{-31} \text{ kg}$ [4]

Mass of a Mercury Atom: $3.33 \times 10^{-25} \text{ kg}$ [5]

Charge of an electron: $1.602 \times 10^{-19} \text{ C} = 1 \text{ eV}$ [4]

Planck’s Constant: $4.11 \times 10^{-15} \text{ eV} \cdot \text{s}$

C Code

```

#Imports
import numpy as np
import matplotlib.pyplot as plt
from pylab import loadtxt
import scipy.optimize as optimize
from scipy.stats import chisquare
import scipy.signal as signal

#Uncertainties
def uncertainty_sum(dx, dy):
    return np.sqrt(dx**2 + dy**2)

def uncertainty_prod(x, dx, y, dy, z):
    return np.sqrt((dx/x)**2 + (dy/y)**2) * z

def import_data(filename, skip_rows=2, cols=(0,1)):
    data=loadtxt(filename, usecols=cols, skiprows=skip_rows, unpack=True)

    return data[0], data[1]

def mean_same_uncern(list):
    return sum(list)/len(list)

def standard_deviation(value, list):
    return np.sqrt((1/(len(list)-1))*sum((value - list)**2))

def mean_diff_uncern(list, uncern_list):
    return sum(list/uncern_list**2) / sum(1/uncern_list**2)

def chi_squared(func, a, b, x_data, y_data, dy_data): # Goodness of Fit
    fx = func(x_data, a, b)
    N = len(x_data)
    sum = 0

    for i in range(N):
        sum += ((y_data[i] - fx[i])**2) / (dy_data[i]**2)

    v = N-2

    return sum / v

def r_squared(y_data, y_graph):
    y_mean = np.mean(y_data)
    ss_total = np.sum((y_data - y_mean)**2)
    ss_residual = np.sum((y_data - y_graph)**2)
    r_squared = 1 - (ss_residual / ss_total)
    return r_squared

def residuals(func, a, b, x, y): # Residuals
    fx = func(x, a, b)
    resid = y - fx

    return x, resid

def calc_curve_of_fit(func, xdata, ydata, init_guess):
    param, uncern = optimize.curve_fit(func, xdata, ydata, p0=init_guess)
    a = param[0]
    b = param[1]
    a_uncern = uncern[0]
    b_uncern = uncern[1]
    x_graph = np.linspace(0, xdata[len(xdata)-1])
    y_graph = func(x_graph, a, b)

    return x_graph, y_graph, a, a_uncern, b, b_uncern

# Functions for the lab
def find_peaks(data):
    peaks, _ = signal.find_peaks(data, width=5, prominence=0.05) # Returns the index value of the peaks
    return peaks

def peaks_each_graph(data):
    # data is a list of lists
    peaks = []
    for i in range(len(data)):
        peaks_lite = find_peaks(data[i][1])
        peaks.append(peaks_lite)
    #print(f'Index Peaks: {peaks}')
    return peaks

def convert_peaks_to_values(data, peaks, axis=1):
    # peaks is the index values
    # actual_peaks is the real values
    actual_peaks = []
    for i in range(len(peaks)):
        peaks_lite = data[i][axis][peaks[i]]
        actual_peaks.append(peaks_lite)
    #print(f'Actual Peaks: {actual_peaks}')
    return actual_peaks

```



```

def peaks_for_std_dev(peaks):
    peaks_lis = []
    for i in range(4):
        list_lite = []
        for j in range(len(peaks)):
            list_lite.append(peaks[j][1])
        peaks_lis.append(list_lite)
    return peaks_lis

def fit_func(x, a, b):
    x = np.array(x)
    return a*x + b

def KE(q, V, me, mhq):
    return q*V*(1-(me/(me+mhq)))

def planck(KE):
    h = 4.14e-15
    c = 3e8
    return (h*c)/KE

### Import data
files = ["FH_Trial_V2.5_SW20.txt",
        "FH_Trial_V2.5_SW30.txt",
        "FH_Trial_V2.5_SW50.txt",
        "FH_Trial_V2.5_SW70.txt",
        "FH_Trial_V2.5_SW100.txt"]
data = []
for file in files:
    data_lite = import_data("data/" + file)
    data.append(data_lite)

### Error bars
# Peaks
index_peaks = peaks_each_graph(data)
peaks_y = convert_peaks_to_values(data, index_peaks)
peaks_x = convert_peaks_to_values(data, index_peaks, 0)

fontsize = 15

# Graph peaks visually to compare
test_peaks = True
if test_peaks == True:
    for i in range(5):
        print(i)
        #Test to see if peaks are good
        plt.subplot(2,3,i+1)
        plt.title(f"Input Voltage vs Output Voltage {i+1}",fontsize=fontsize)
        plt.plot(data[i][0], data[i][1])
        plt.plot(data[i][0][index_peaks[i]], data[i][1][index_peaks[i]], "xr")
        plt.xlabel("Input Voltage (V)",fontsize=fontsize) # Labels
        plt.ylabel("Output Voltage (V)",fontsize=fontsize)
        plt.tight_layout()
        i += 1
    plt.show()
# When graphed, there are 4 peaks in each graph except for the first one which has a fifth 'ghost one'
# I can just ignore it

# Form Error bars
# Error in y
peaks_in_order = peaks_for_std_dev(peaks_y) # rearranges
main_peaks = peaks_y[4]
errorBars_y = []
for i in range(len(peaks_in_order)):
    errorBars_y.append(standard_deviation(main_peaks[i], peaks_in_order[i])) # uses std deviation to make error bars

# Error in x
peaks_in_order = peaks_for_std_dev(peaks_x) # rearranges
main_peaks = peaks_x[4]
errorBars_x = []
for i in range(len(peaks_in_order)):
    errorBars_x.append(standard_deviation(main_peaks[i], peaks_in_order[i])) # uses std deviation to make error bars

### Final Graph
final_graph = True
if final_graph == True:
    plt.title("Input Voltage vs Output Voltage",fontsize=fontsize)
    plt.plot(data[4][0], data[4][1])
    plt.errorbar(data[4][0][index_peaks[4]], data[4][1][index_peaks[4]], yerr=errorBars_y, xerr=errorBars_x, fmt="x", capsize=3) #error bars
    plt.xlabel("Input Voltage (V)",fontsize=fontsize) # Labels
    plt.ylabel("Output Voltage (V)",fontsize=fontsize)
    plt.tight_layout()
    plt.show()

### Chart for graph
for i in range(len(peaks_x[4])):
    print(f"Index: {i+1}, Input: {peaks_x[4][i]} +/- {errorBars_x[i]}, Output: {peaks_y[4][i]} +/- {errorBars_y[i]}")

### Fit

```

```

# Data
index_of_peaks = [1, 2, 3, 4]
accel_voltage_peaks = data[4][0][index_peaks[4]]
y_uncern = [0.05, 0.05, 0.05, 0.05]

# Curve fitting
graph_x, graph_y, graph_a, graph_a_uncern, graph_b, graph_b_uncern = calc_curve_of_fit(fit_func, index_of_peaks, accel_voltage_peaks, [0.5, 0.5])

# Residuals
graph_resid_x, graph_resid_y = residuals(fit_func, graph_a, graph_b, index_of_peaks, accel_voltage_peaks)

# Chi squared and R squared:
chi = chi_squared(fit_func, graph_a, graph_b, np.array(index_of_peaks), np.array(accel_voltage_peaks), np.array(y_uncern))

# Plot 1 Normal
plt.subplot(2,1,1)
plt.title(f"Input Voltage vs Peaks", fontsize=fontsize)
plt.plot(index_of_peaks, accel_voltage_peaks, 'x')
plt.plot(graph_x, graph_y)
plt.xlabel("Index of Peak", fontsize=fontsize) # Labels
plt.ylabel("Input Voltage (V)", fontsize=fontsize)
textstr = f"y = {round(graph_a,2)}x + {round(graph_b,2)} \n x^2 = {chi}"
props = dict(boxstyle='round', facecolor='wheat', alpha=0.5)
plt.text(0, 25, textstr, fontsize=14, verticalalignment='top', bbox=props)
plt.tight_layout()

# Plot 2 Residuals
plt.subplot(2,1,2)
y_zeroline = [0,0]
x_zeroline = [0, graph_resid_x[len(graph_resid_x)-1]]
plt.plot(graph_resid_x, graph_resid_y, 'o')
plt.plot(x_zeroline, y_zeroline)
plt.errorbar(graph_resid_x, graph_resid_y, xerr=errorBars_x, yerr=errorBars_y, fmt=".") #error bars
plt.title("Residuals of Glycerine graph", fontsize=fontsize)
plt.tight_layout()
plt.show()

print(f"m = {graph_a} +/- {graph_a_uncern}")
print(f"b = {graph_b} +/- {graph_b_uncern}")

### Calculate KE and Wavelength
q = 1.602e-19
V = graph_a
me = 9.11e-31
mhg = 3.33e-25
KE = KE(1, V, me, mhg)
lambda = planck(KE)
print(f"KE of Inelastic Collision: {KE}")
print(f"Wavelength emitted is: {lambda*1e9}")

```

