

Forward Private Searchable Symmetric Encryption with Optimized I/O Efficiency

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Abstract—Recently, several practical attacks raised serious concerns over the security of searchable encryption. The attacks have brought emphasis on forward privacy, which is the key concept behind solutions to the adaptive leakage-exploiting attacks, and will very likely to become a must-have property of all new searchable encryption schemes. For a long time, forward privacy implies inefficiency and thus most existing searchable encryption schemes do not support it. Very recently, Bost (CCS 2016) showed that forward privacy can be obtained without inducing a large communication overhead. However, Bost’s scheme is constructed with a relatively inefficient public key cryptographic primitive, and has poor I/O performance. Both of the deficiencies significantly hinder the practical efficiency of the scheme, and prevent it from scaling to large data settings. To address the problems, we first present FAST, which achieves forward privacy and the same communication efficiency as Bost’s scheme, but uses only symmetric cryptographic primitives. We then present FASTIO, which retains all good properties of FAST, and further improves I/O efficiency. We implemented the two schemes and compared their performance with Bost’s scheme. The experiment results show that both our schemes are highly efficient.

Index Terms—searchable encryption, symmetric primitives, forward privacy, I/O efficiency.

1 INTRODUCTION

SEARCHABLE encryption is perhaps one of the most intensively studied cryptographic primitives. The need for searchable encryption comes from the surge of Storage-as-a-service, a service model in which the clients store their data on remote servers that are managed by external service providers (e.g. Amazon S3, Microsoft Azure Storage, Google Cloud Storage etc.). When data storage is outsourced, data privacy becomes a primary concern because the service providers may not always be trusted. While conventional encryption can be used to protect the outsourced data, it does not allow effective retrieval of data by searching on encrypted data. Searchable encryption was proposed to solve this problem. In general, searchable encryption schemes work by generating an encrypted index, which will be outsourced to the service provider along with the encrypted data. Later the client can generate search tokens that encrypt certain keywords and the server can perform a search algorithm using the tokens and the encrypted index to find matches. In this way, even though data is stored on and searching is handled by an untrusted server, the privacy of the data can still be preserved.

Recently, there has been a major concern over the security of searchable encryption schemes. Since 2012, several attacks [1], [2], [3] have been devised that allow an untrusted server to recover the keywords in the client’s search tokens, and in consequence, to learn a significant amount of information about the outsourced encrypted data. These attacks are normally performed by utilizing

information leaked in the searching and updating phases, and they are pervasive because the information leakage exists inevitably in all searchable encryption schemes. In the most recent work by Zhang et al. [3], the authors showed a simple yet effective adaptive attack that can fully reveal the client’s queries by injecting only a small number (usually less than 100) of files. The result is devastating: not only it enables the server to learn partial information about the encrypted data, the recovered keywords can also help the server in other statistical attacks. The essential idea of the attack is that the server first crafts a set of files (each contains certain keywords), then sends the files to the client and tricks the client into encrypting them. After the client has encrypted and uploaded the injected files, the server can use the tokens previously submitted by the clients to search on the injected files. By knowing which keywords are in each injected file and observing which files match a token, the server can deduce easily which keyword is encrypted in the token. The attack shows that even seemingly harmless small leakage can be exploited, and highlights the importance of forward privacy, which requires that a newly inserted file cannot be linked in anyway with previous search queries. With forward privacy, the attack can be prevented.

While forward privacy is not a new concept, most searchable encryption schemes do not support it. Until recently, forward private searchable encryption implies prohibitively high communication cost. There is a trivial way to achieve forward privacy: when a new file needs to be added, the client downloads all encrypted files from the server, re-indexes them with the new file and re-encrypts everything with new keys. This is obviously impractical. There were also non-trivial solutions using an oblivious data storage, e.g. Oblivious RAM (ORAM) [4], [5] or Distributed Oblivious Data structure (DOD) [6], which hides search patterns and access patterns from the server. In this approach, the server cannot observe which files match the token, therefore cannot learn anything about the query. Nevertheless, the communication cost is still too high unless replying on strong assumptions like multiple non-colluding servers. One notable work by Chang and

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Mitzenmacher [7] achieves forward privacy without using an oblivious data storage. However, in this scheme the search query size grows linearly in the number of updates, which means after the system has run for a period of time, the communication cost for the search operation will become unacceptably high due to the ever-growing query size.

In CCS 2016, Bost [8] proposed an efficient forward secure searchable encryption scheme called Sophos that achieves a low communication cost. In this scheme, the client keeps a state for each keyword and evolves the state when updating a file on the server that contains this keyword. The state is used as an input when generating the encrypted index for the updated file, which means the previous tokens are outdated and cannot be used to match the new index. The state is kept secret from the server until the client performs a search, at which point a constant size token is generated that contains the current state. The token enables the server to recover all previous states from the current state. By knowing all states, the server can search through all updates and find matching files. Sophos uses **trapdoor permutation**, a public key primitive, for evolving the state so that the server, who does not know the private key, cannot predict future states. Thus when new updates come in, the server learns nothing and forward privacy is achieved.

Although Sophos is more efficient than previous forward private schemes, there are still two significant deficiencies in the design. Firstly, the scheme is based on public key cryptography. It has been showed clearly in Bost's paper [8] that if the data is updated frequently, the public key operations become a major performance bottleneck. It would be ideal if only symmetric cryptographic primitives are used. Secondly, the search operation is not I/O efficient. Recently it has been shown that I/O has become a major bottleneck that prevents searchable encryption from scaling up to large data [9], [10]. **In Sophos, the locality, a measure of I/O efficiency, of the search operation increases linearly in the number of updates.** The performance of the search operation is inversely proportional to the locality, which means search performance will degrade inevitably and eventually become unacceptably slow after a long run. The author acknowledged this problem but also argued that the poor locality is a necessary price paid for forward privacy.

1.1 Our Contributions

Our investigation started from two questions: 1) can we construct a communication-efficient forward private searchable encryption scheme using only symmetric key primitives? 2) to what extent can we optimize the I/O efficiency under the security constraint imposed by forward privacy? We present positive answers to these two questions by constructing two new searchable encryption schemes FAST and FASTIO.

- **FAST (Forward privAte searchable Symmetric encrypTion)** is our first attempt to construct a forward private searchable encryption scheme using only symmetric key primitives. The design of FAST follows the state-based approach used by Sophos [8], but with a very different idea. In Sophos, the states are related by a trapdoor permutation. Since the trapdoor permutation is a public key primitive, by giving the server only the public key, the server can generate states backwards, but not forwards. This strategy however is not valid when using symmetric key primitives. The idea in FAST is inspired by singly linked list, a classic data structure. Each update is like

a node in a **singly linked list** and contains a randomly generated **ephemeral key** (encrypted). The key is like the pointer in a list node that points backwards to the previous node. In FAST, a new state is generated by encrypting the previous state using the ephemeral key. Given the current key and the current state, the server can compute the previous state by decryption. The decrypted state will help the server to get the previous update and then the previous key. Iteratively, the server can find all previous states as well as all previous updates. However, since a future key is randomly generated on-the-fly by the client and is independent of the current and past keys, by no means can the server predict the future keys/states given the keys/states it has already known. Thus forward privacy is achieved.

- **FASTIO**, where IO in the name stands for **I/O Optimized**. While FAST is entirely based on symmetric key cryptography which makes it computationally more efficient, it has bad I/O efficiency. The I/O overhead comes from 1) the ephemeral keys that are required to be stored on the server, and 2) the fact that the server is forced to read the index in a very non-local manner in order to achieve forward privacy. Our idea is simple: after each search query, we can let the server store the result. Since the search result has already been revealed to the server, saving it will not leak additional information to the server. The server now needs only the latest state and does not need to store ephemeral keys anymore. In addition, the non-local reads are also minimized because the previous search result can be read continuously.

We implemented the two schemes and conducted experiments to measure their performance. The results confirm that our schemes are more efficient in terms of computation than Sophos, which is the most efficient forward private searchable encryption scheme to date. The results also confirm that FASTIO has a much better I/O efficiency, which makes its much more scalable.

2 RELATED WORK

Searchable encryption The functionality of secure search can be realized by using generic cryptographic primitives such as Fully Homomorphic Encryption (FHE) [11], Oblivious RAM (ORAM)[12], [13] and Secure Multi-Party Computation(SMPC) [14], [15]. With the generic primitives, it is possible to perform secure search with minimum information leakage. However, searchable encryption based on generic primitives is inefficient and is not suitable to be used in practice. Research in practical searchable encryption started from the seminal paper by Song et al. [16], and the goal is to achieve practical efficiency and scalability with the price of accepting a small leakage.

One branch in this research area considers the static setting, in which the client encrypts the data at the setup phase, and does not update the ciphertexts on the server side afterwards. In this setting, Curtmola et al. [17] gave the first reversed-index based scheme with sub-linear efficiency. Chase and Kamara [18] also proposed a similar scheme but with a higher storage complexity. Cash et al. [9] proposed a scheme that supports boolean queries. Static searchable encryption, although can achieve sub-linear efficiency, are not suitable in many application scenarios where updates are required.

Dynamic searchable encryption enables the client to add and delete data items after the dataset is outsourced to the cloud. Actually, the work of Song et al. [16] supports dynamically update, but search and update operations require sequential scan performed

on the full-text. Kamara et al. [19] proposed a dynamic searchable symmetric encryption (DSSE) scheme as an improvement of [17]. Their scheme achieves sub-linear search time but has a high complexity for the *add* and *delete* operations. The scheme also leaks much more information to the server than [17]. Dong et al. [20] proposed a dynamic searchable encryption scheme that supports multiple users. However the scheme is based on public key cryptography. Kamara and Papamanthou [21] proposed a parallel DSSE scheme, which leaks less information but requires multiple interactions between the server and client. Sun et al. [22] proposed a multi-client version of DSSE scheme that supports boolean query. Hahn and Kerschbaum [23] proposed a scheme that only leaks the search pattern and also achieves linear searching complexity. Naveed et al. [24] proposed a new primitive called blind-storage, and based on it, they constructed a DSSE scheme. Instead of processing and computing on the data stored on it, in this scheme, the server just acts as a transmission and storage entity. It achieves less information leakage but requires multiple rounds of interaction between the client and server. Gajek [25] proposed a DSSE scheme based on constrained functional encryption (CFE), which encodes the index as an encrypted binary tree and constructed an inner product functionality to traverse the encrypted tree. Another DSSE scheme with multi-keyword ranked search was given by Xia et al. [26].

Forward Privacy Roughly, forward privacy requires that a newly inserted file cannot be matched with previous search queries. This notion is not applicable to static searchable encryption schemes because they do not support update. Most dynamic searchable encryption schemes do not support forward privacy. To date, most existing forward secure dynamic searchable encryption schemes are constructed based on ORAM-like structure. Among them, the concept of forward privacy was firstly precisely stated by Stefanov et al. [4]. They proposed a forward private DSSE scheme based on hierarchical ORAM. Likewise, Garg et al. [5] proposed the first efficient round-optimal oblivious RAM(ORAM) scheme. As an application of their ORAM scheme, they constructed a DSSE scheme with sub-linear search efficiency that conceals the search pattern. The problem of this approach is the high communication cost that is resulted from using ORAM like structures. In the scheme of Hoang et al. [6], they use a distributed oblivious data structure that is a distributed encrypted incidence matrix created on two non-colluding servers. There are only a few forward private searchable encryption schemes that do not use ORAM. In the scheme proposed by Chang and Mitzenmacher [7], forward privacy was achieved by changing search token after each update. The tokens are unrelated. Thus to perform a search, the client needs to submit all previous tokens in the query. The most efficient forward private searchable encryption scheme was proposed by Bost [8], in which forward privacy is achieved by using a one-way trapdoor permutation to evolve a state stored at the client side.

I/O Efficiency of Searchable Encryption Focusing on the practical performance of searchable encryption, Cash et al. [9] made a compromise in security and efficiency and proposed a scheme with good I/O efficiency based on T-Set (a data structure of keyword-entry tuple). Their method was extended by Cash et al. [27]. The new scheme has a better performance in both searching and updating phase. Cash and Tessaro [10] investigated the I/O efficiency of searchable encryption and proved that it is impossible to achieve optimal server storage, read efficiency and locality at the same time due to the security requirement of searchable encryption. Following the work of Cash and Tessaro, a tight lower

bound of storage locality was given by Asharov et al. [28]. A tunable SSE scheme was given by Demertzis et al. [29] in which the locality can be tuned. Recently, Demertzis et al. [30] proposed a linear-space searchable encryption scheme with constant locality and sublogarithmic read efficiency. All these schemes are mainly for the static setting and do not support dynamic update. Miers et al. [31] came up with a DSSE scheme by using oblivious update index (OUI) as an intermediate cache and pushing a bucket of document identifiers from OUI to append-only storage when that bucket is full. Their scheme has a higher I/O efficiency compared to using an ORAM scheme and achieves a better locality through the bucket mechanism, but leaks more information and does not support forward privacy.

Recent Work While this manuscript was in submission, four new papers appeared [32], [33], [34], [35]. Some ideas in this paper were independently developed in those papers. Here we provide a brief review of the overlaps and differences between our work and the simultaneous work. All four papers construct a forward private searchable encryption scheme using only symmetric key primitives. As we have discussed in section 1.1, the difficulty of using only symmetric key primitives is how to prevent the server from generating future states. In [32], the strategy is the same as that in [7], thus has a big communication overhead because a search query needs to include one token for each new update since the last search. In [33], they observe that updates are naturally ordered in a sequence. By using a range-constrained PRF, a search token can be restricted to only applicable to updates within a specified range. Thus, future updates are secure as they are out of the range of past search tokens. However, there is an upper bound on the number of updates that can be performed, and this scheme has a higher computational and communication overhead compared to our schemes. In [34], [35], they both generate a new key for future updates after each search query (the current key is given to the server as part of the search token). A major disadvantage of this rekeying strategy is that the search operation is not read-only and the search result has to be re-encrypted and written back, which increases computational and I/O costs. Re-encryption is done by the server in [34], and by the client in [35] (which requires an additional round and more data to be transferred). While in FAST, our search is read-only because we use an ephemeral key for each update, which means rekeying and re-encryption are not needed. Communication and/or computational wise, our schemes are better than the schemes in the four papers. The idea of caching previous search results to improve efficiency, which we use in FASTIO, is also used/mentioned in [32], [33].

3 PRELIMINARIES

3.1 Notations

We use $r \xleftarrow{\$} X$ to denote that an element r is sampled uniformly at random from a set X , $\{0, 1\}^l$ to denote the set of all l -bit strings, $\{0, 1\}^*$ to denote the set of arbitrary length strings, and $a||b$ to denote the concatenation of two strings a and b . Let λ be a security parameter, we say a function $\nu : \mathbb{N} \rightarrow \mathbb{R}$ is negligible in λ if for every positive polynomial p , $\nu(\lambda) < 1/p(\lambda)$ for sufficiently large λ . We use $\text{poly}(\lambda)$ and $\text{negl}(\lambda)$ to represent the unspecified polynomial and negligible functions in λ , respectively. For a set S , we use $|S|$ to denote S 's cardinality. For a string a , we use $|a|$ to denote a 's bit length.

3.2 Searchable Encryption

A *dynamic searchable symmetric encryption* (DSSE) scheme enables a client to outsource its data to an untrusted server in an encrypted form. Later the server can execute search or update queries on the encrypted data. We only consider the dynamic setting in this paper because forward privacy is not applicable to the static setting.

Let a database $DB = \{(\text{ind}_i, W_i)\}_{i=1}^D$ be a D -vector of identifier/keyword-set pairs, where $\text{ind}_i \in \{0, 1\}^l$ is a document identifier and $W_i \subseteq \mathcal{P}(\{0, 1\}^*)$. The universe of keywords of the database DB is $W = \cup_{i=1}^D W_i$. We use $N = \sum_{i=1}^D |W_i|$ to denote the number of document/keyword pairs. We use $DB(w) = \{\text{ind}_i | w \in W_i\}$ to denote the set of documents that contain the keyword w . A DSSE scheme $\Pi = \{\text{Setup}, \text{Search}, \text{Update}\}$ consists of three protocols ran by the client and the server:

- $((K, \sigma); \text{EDB}) \leftarrow \text{Setup}(\lambda, DB; \perp)$: It takes a security parameter λ and a database DB as inputs and outputs (K, σ) to the client and EDB to the server, where K is a secret key, σ is the client's state, and EDB is the encrypted database.
- $((\sigma', \text{DB}(w)); \text{EDB}') \leftarrow \text{Search}(K, \sigma, w; \text{EDB})$: The client's input consists of its secret key K , the state σ and a keyword w , the server's input is the encrypted database EDB . The client's output includes a possibly updated state σ' and $\text{DB}(w)$, i.e. the set of the identifiers of the documents that contain the keyword w . The server's output is the possibly updated encrypted database EDB' .
- $(\sigma'; \text{EDB}') \leftarrow \text{Update}(K, \sigma, \text{ind}, w, \text{op}; \text{EDB})$: The client's input is the secret key K , the state σ , a document identifier ind , a keyword w and an operation type op . The server's input is EDB . The operation op is taken from the set $\{\text{add}, \text{del}\}$, which means the client wants to add or delete a document/keyword pair. The client updates its state and the server updates EDB as requested by the client.

3.3 Security Definition

All existing searchable encryption schemes leak more or less some information to the server, as a tradeoff to gain efficiency. Thus, the security of searchable encryption is defined in the sense that no more information is leaked than allowed. This is captured by using the simulation paradigm and providing the simulator a set of predefined leakage functions $\mathcal{L} = \{\mathcal{L}_{\text{Setup}}, \mathcal{L}_{\text{Search}}, \mathcal{L}_{\text{Update}}\}$.

Definition 1 (Adaptively Secure Searchable Encryption). Let $\Pi = (\text{Setup}, \text{Search}, \text{Update})$ be a searchable encryption scheme, \mathcal{A} be an adversary, \mathcal{S} be a simulator parameterized with leakage function $\mathcal{L} = \{\mathcal{L}_{\text{Setup}}, \mathcal{L}_{\text{Search}}, \mathcal{L}_{\text{Update}}\}$. We define the following two probabilistic experiments:

- $\text{Real}_{\mathcal{A}}^{\Pi}(\lambda)$: \mathcal{A} chooses a database DB , the experiment runs $\text{Setup}(\lambda, DB; \perp)$ and returns EDB to \mathcal{A} . Then, the adversary adaptively chooses queries q_i . If q_i is a search query then the experiment answers the query by running $((\sigma_{i+1}, \text{DB}_i(w_i)), \text{EDB}_{i+1}) \leftarrow \text{Search}(K, \sigma_i, q_i; \text{EDB}_i)$. If q_i is an update query, then the experiment answers the query by running $(\sigma_{i+1}, \text{EDB}_{i+1}) \leftarrow \text{Update}(K, \sigma_i, q_i; \text{EDB}_i)$. Finally, the adversary \mathcal{A} outputs a bit $b \in \{0, 1\}$.
- $\text{Ideal}_{\mathcal{A}, \mathcal{S}}^{\Pi}(\lambda)$: \mathcal{A} chooses a database DB . Given the leakage function $\mathcal{L}_{\text{Setup}}(DB)$, the simulator \mathcal{S} generates an encrypted database $\text{EDB} \leftarrow \mathcal{S}(\mathcal{L}_{\text{Setup}}(DB))$ and returns it to \mathcal{A} . Then, the adversary adaptively chooses queries q_i . If q_i is

a search query, the simulator answers the query by running $\mathcal{S}(\mathcal{L}_{\text{Search}}(q_i))$. If q_i is an update query, the simulator answers the query by running $\mathcal{S}(\mathcal{L}_{\text{Update}}(q_i))$. Finally, the adversary \mathcal{A} outputs a bit $b \in \{0, 1\}$.

We say Π is an \mathcal{L} -adaptively-secure searchable encryption scheme if for any probabilistic, polynomial-time (PPT) adversary \mathcal{A} , there exists a PPT simulator \mathcal{S} such that:

$$|\Pr(\text{Real}_{\mathcal{A}}^{\Pi}(\lambda) = 1) - \Pr(\text{Ideal}_{\mathcal{A}, \mathcal{S}}^{\Pi}(\lambda) = 1)| \leq \text{negl}(\lambda)$$

3.4 Leakage Functions and Forward Privacy

A good searchable encryption scheme should reveal as little information as possible. The leakage is captured by the leakage functions. Usually we require $\mathcal{L}_{\text{Setup}} = (D, N)$, i.e. the Setup protocol only leaks the size of DB and the number of document/keyword pairs. For the Search protocol, we usually require $\mathcal{L}_{\text{Search}} = (\text{ap}, \text{qp})$, where ap is the *access pattern* and qp is the *query pattern*. More formally, $\mathcal{L}_{\text{Search}}$ keeps a history $\text{Hist} = \{(DB_i, q_i)\}_{i=0}^Q$, which contains all queries q_i so far and the snapshot of the database DB_i corresponding to q_i . The access pattern is defined as $\text{ap}(\text{Hist}) = (t_1, \dots, t_Q)$ where $t_i = (i, \text{DB}_i(w_i))$ if $q_i = w_i$ is a search query, or $t_i = (i, \text{op}_i, \text{ind}_i)$ if $q_i = (\text{ind}_i, w_i, \text{op}_i)$ is an update query. The query pattern is defined as $\text{qp}(w) = \{j | q_j \text{ contains } w \text{ for each } q_j \text{ in Hist}\}$.

Note that in Section 3.2, 3.3 and this section, we only consider the encrypted index, but not the encrypted files. The operations for manipulating the actual stored files are not included in the protocols and information leaked through file operations are not modeled explicitly. This simplified treatment of SSE is one of the influential formalizations and is often seen in the literature, e.g. see [27], [9], [8], [33]. The rationale behind this simplification is that the actual files are always encrypted using a semantically secure encryption scheme thus the ciphertext of a file leak no information about its content except size. If operations on files are limited to only add and delete, then the information leaked from file operation is limited to the size of the file. The size information is assumed to be implicitly included in the identifier (ind) and can be computed from it if needed.

Forward privacy is a strong property regarding the leakage of the update operation in DSSE. Intuitively, it requires that given previous search tokens, an update query leaks no information about the keywords being searched in the past. If a searchable encryption scheme is not forward private, a search token can be used to retrieve documents added after the token being issued. Thus, after a token has been given to a curious server, the server can save it and test it on every new update to see whether the new update matches the old token. This is exactly what has been exploited in the file-injection attack: the server tricks the client into encrypting files containing certain chosen keywords, then test previously received tokens over the injected files to find for each token which files it matches. Then the server can deduce which keyword is encoded in this token. With forward privacy, this attack can be prevented because the previous tokens cannot be used on new updates. The formal definition is as the following:

Definition 2 (Forward Privacy). An \mathcal{L} -adaptively-secure SSE scheme is forward private if for an update query $q_i = (\text{ind}_i, w_i, \text{op}_i)$, the update leakage function $\mathcal{L}_{\text{Update}}(q_i) = (i, \text{op}_i, \text{ind}_i)$.

For the convenience of proof, in our definition the update query contains only one document/keyword pair. This definition

is actually equivalent to the one in [8], where the update query contains multiple keywords. Our definition also supports adding or removing full document, which can be regarded as iteratively adding or removing the corresponding document/keyword pairs. The number of updated pairs to a document is also leaked and can be computed easily from our defined leakage.

3.5 Searchable Encryption and I/O Efficiency

Scalability is a key consideration when designing a searchable encryption scheme because the target application is data storage outsourcing, which usually implies that there is a large amount of data. Recent searchable encryption schemes are often very efficient in terms of computation so one consequence is that I/O efficiency becomes the bottleneck for scaling [9], [10], [31], [29].

The I/O efficiency of searchable encryption is characterized by three factors: server side index size, the locality and the read efficiency. The first is self-explanatory. The locality is the number of non-contiguous reads the server must perform when answering a search query. High locality means more random access I/O requests and a longer latency due to the inter-requests latency. The read efficiency is quantified by the amount of unnecessary or irrelevant data read. More formally, we have:

Definition 3 (Locality). Let Π be a searchable encryption scheme, K be a secret key, σ be the client's state and EDB be a database encrypted by Π under K with regard to σ . For every $w \in \mathcal{W}$, we say the locality is r for the search query $\text{Search}(K, \sigma, w; \text{EDB})$ if the server can answer the query by reading at most r intervals from EDB.

Definition 4 (Read Efficiency). Let Π be a searchable encryption scheme, K be a secret key, σ be the client's state and EDB be a database encrypted by Π under K with regard to σ . For every $w \in \mathcal{W}$, we say the read efficiency is c for the search query $\text{Search}(K, \sigma, w; \text{EDB})$ if the server can answer the query by reading at most $c \cdot (\sum_{s_i \in \text{DB}(w)} |s_i|)$ bits.

Cash et al. [10] noticed that the security property has a negative effect on I/O efficiency and pointed out that, for the sake of security, a searchable encryption scheme must either extend the encrypted index to an overwhelming large size, or perform searching in a non-local way (e.g. in a random manner), or read much more bits than actually needed. Afterwards Bost [8] additionally observed that locality and forward privacy are two irreconcilable notions. On the one hand, forward privacy requires that in an update operation for a keyword w , the locations in the EDB that are modified should be unrelated to the locations that are already known to match w . On the other hand, to reduce locality, it is necessary to organize entries in EDB relating to w together so that they can be read continuously. Bost concluded, which is not entirely true as we will show later, that under the constraint of forward privacy, no locality optimization is possible for keyword w unless large modification is done to the encrypted database, either during searches or updates.

4 FAST: FORWARD PRIVATE SEARCHABLE SYMMETRIC ENCRYPTION

In this section, we present FAST, our first forward private searchable encryption scheme. The main design goal of this scheme is to eliminate public key operations while retaining optimal communication complexity.

Recall that a forward private searchable encryption scheme can be based on ORAM. While ORAM can be purely symmetric key based [12], [13], [36], [37], it has a large communication overhead. Therefore, we turned to the state-based approach that was used in a few existing schemes [7], [8]. The main challenge of this approach is how to allow the server to re-generate all previous states (so that communication cost can be reduced), but not later states (so that forward privacy can be achieved). Sophos [8] solved this problem by using a public key primitive (trapdoor permutation), so that the client can evolve the state forward using a private key and the server can re-generate the whole history of the states using the public key. However, there exists a huge gap in adopting this strategy with symmetric key cryptography. Symmetric key means there is only one key and once it is released to the server, the server can compute future states and there is no forward privacy any more. This is why in [7], the client has to keep the key secret and generate the history of states locally rather than giving the key to the server and letting the server to generate the history. However since the query contains the full history of the state, this results in a huge expansion in the token size and a large communication overhead.

A naive symmetric key based solution with low communication overhead is to use a hash chain for the states. More specifically, for each keyword, the client chooses the last state $st_L \xleftarrow{\$} \{0, 1\}^\lambda$, then generates all other states $st_i \leftarrow H(st_{i+1}), 0 \leq i < L$. When performing update queries, the client starts from st_0 then st_1 and so on so forth. When performing a search query, a state st_c is revealed to the server. By the one-wayness of the hash function, the server can recover st_{c-1}, \dots, st_0 but not states after st_c , thus forward privacy is guaranteed. However, there are two **intrinsic** problems. The first problem is that the hash chain parameter L must be fixed at the beginning, which is inflexible because it limits the number of update operations the client can perform. The second problem is that, if the client stores the hash chains, then the storage overhead is too high; on the other hand, if the client only stores st_L and computes the state st_c on-the-fly, then it will need $L - c$ hash invocations per search or update query, which is computationally too expensive. Therefore we need a better way to generate and evolve the states.

The idea of FAST is that rather than pseudorandomly generating all states using a fixed key as in Sophos or use a hash chain as in the naive solution, the client chooses a random ephemeral key every time to evolve the state. The current state is essentially the encryption of the previous state under the ephemeral key. The ephemeral key is then stored on the server side, encrypted using the current state. The server can only get the ephemeral key if it knows the current state. Then by decrypting the current state with the ephemeral key, the server can derive the previous state. Iteratively, the server obtains all states, which enable it to search. Forward privacy is guaranteed since the server cannot infer unknown states from currently known states and keys, and the search token size can be made constant because the client only needs to give the latest state to the server.

Fig. 1 shows the protocols of the scheme. We also depict in Fig. 2 how the states evolve forward and backward in the update and search protocols. In the **Setup** protocol, the client generates k_s and Σ , where k_s is a λ -bit long-term key that will be used to encrypt keywords and Σ is an empty map that will be used to store the states on the client side. The long-term key k_s is necessary to prevent the server from generating tokens by itself.

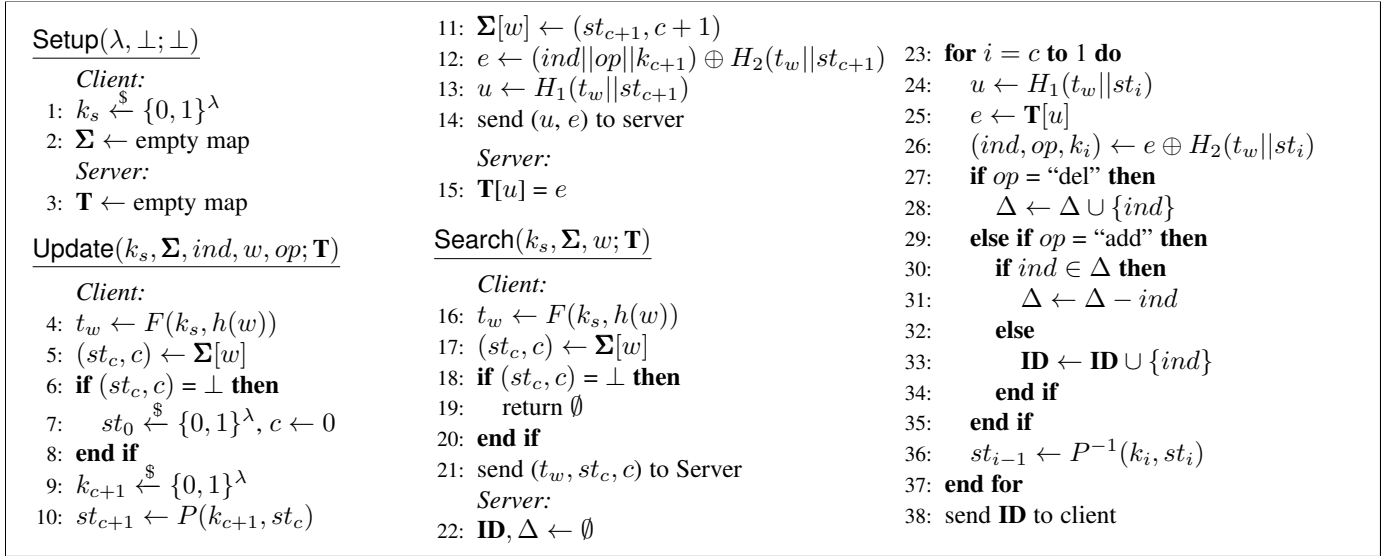


Fig. 1: Pseudocode of Protocols in FAST

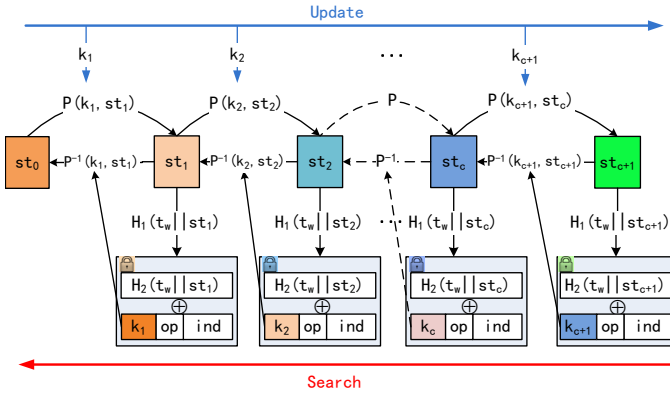


Fig. 2: Update and Search in FAST

The server generates \mathbf{T} , which is an empty map to be used to store the encrypted index. In the **Update** and the **Search** protocols, F is a pseudorandom function, P is a pseudorandom permutation (and P^{-1} is the inverse permutation), and h, H_1, H_2 are secure hash functions with appropriate output lengths. When updating a file that contains a keyword w and whose identifier is ind , the client needs to retrieve the previous state st_c from the local state store Σ (line 5-8), then generates a random ephemeral key k_{c+1} and evolves the state forward to the current state st_{c+1} using the pseudorandom permutation (line 9-11). The ephemeral key k_{c+1} is not stored on the client side, but is embedded in the encrypted index entry e that will be stored on the server side (line 12). The client also generates a reference u from the current state and the keyword (line 13). The pair (u, e) is sent to the server and the server updates its map \mathbf{T} accordingly (line 15). To search a keyword w , the client retrieves the current state from Σ and sends a search token that contains the encrypted keyword and the current state to the server. Given the current state, the server needs to generate all previous states st_i and find the corresponding update sequence. In the for loop of the algorithm, the server can recover the ephemeral key k_i (line 26), which can later be used to recover the previous state (line 36). Because both “add” and “del” are allowed update operations, the server needs to make sure

not including deleted files in the result set. To do so, the server maintains a set Δ that contains deleted file identifiers during the search process. When the server sees a delete update, the server puts the file identifier ind into Δ . When the server sees an add update and the file identifier ind is in Δ , the server removes ind from Δ . The idea is that the server searches backwards with regard to the update sequence, therefore $\text{ind} \in \Delta$ means the added file later was deleted. So the two operations cancelled, then ind should be removed from Δ and should not be added to the result set. If the server sees an add update and the file identifier is not in Δ then the file was not deleted and the identifier is added to the result set.

Complexity Analysis On the client side, the computational complexity for update and search is $O(1)$. The client stores a λ -bit key and a map Σ whose size is $O(|W|)$, where $|W|$ is the total number of keywords. On the server side, the computational complexity for update is $O(1)$, and for search is $O(c_w)$ where c_w is the total number of updates (add + del) that contain the keyword being searched since the initialization of the system. The server stores a map \mathbf{T} whose size is $O(N_u)$, where $N_u = \sum_{w \in W} c_w$ is the total number of updated document/keyword pairs since setup. The communication complexity is $O(1)$ for update. For search, the communication complexity is $O(1)$ for the query and $O(|\text{DB}(w)|)$ for the result set, which contains $|\text{DB}(w)|$ matching files. Complexity-wise, FAST is optimal (given the security constraints).

Security Analysis FAST satisfies the adaptive security definition (Definition 1). The search query leaks only the access pattern and query pattern, which is standard in searchable encryption. The update query leaks no information about the keywords, thus satisfies forward privacy. We have the following theorem regarding the security of FAST:

Theorem 1. Let F be a pseudorandom function, P be a pseudorandom permutation. Let H_1 and H_2 be two hash functions modeled as random oracles. Define leakage $\mathcal{L} = (\mathcal{L}_{\text{Setup}}, \mathcal{L}_{\text{Search}}, \mathcal{L}_{\text{Update}})$ as

$$\mathcal{L}_{\text{Setup}} = \perp$$

$$\mathcal{L}_{\text{Search}}(w) = (\text{ap}(w), \text{qp}(w))$$

$$\mathcal{L}_{\text{Update}}(i, \text{op}_i, w, \text{ind}_i) = (i, \text{op}_i, \text{ind}_i)$$

Then FAST is a \mathcal{L} -adaptively-secure dynamic SSE with forward privacy.

The proof can be found in Appendix A.

5 FASTIO: I/O OPTIMIZED SCHEME

In this section, we present FASTIO, our second forward private searchable encryption scheme. FASTIO retains all good properties of FAST, and has been optimized for I/O efficiency, which enables it to scale up to very large datasets.

5.1 I/O Deficiencies in FAST

Let us compare Sophos and FAST in term of I/O efficiency. The criteria are the server side index size, read efficiency and locality. In Sophos, the server side index size is $N \cdot l$ bits where N is the number of document/keyword pairs in the index and l is the identifier's size. This is optimal. To answer a search query with keyword w , Sophos needs to read $c_w \cdot l$ bits, where c_w is the number of updates that contain w since the initialization of the system. The read efficiency is 1, which means no unnecessary data is read and the read efficiency is optimal. The locality of Sophos is c_w because the c_w entries are located randomly in the encrypted index. In FAST, the server side index size is $N \cdot (l + 1 + \lambda)$ bits where λ is the size of the random ephemeral key and the additional 1 bit is used to indicate the operation ("add" or "del"). To answer a search query with keyword w , FAST needs to read $c_w \cdot (l + 1 + \lambda)$ bits, which means the read efficiency is $(l + 1 + \lambda)/l$. The locality of searching a keyword in FAST is c_w , which is the same as Sophos.

As we can see, in FAST the server side index size and read efficiency are worse than those in Sophos. And the locality is as poor as Sophos. Take a closer look, we can find that the I/O deficiencies in FAST are related to its symmetric key construction. To use symmetric key primitives, FAST has to store the ephemeral key in the index. The key size λ is often similar to l , which means a concrete overhead of 100% or so. However, if we do not store the key in the index, the server cannot re-generate previous states and cannot search. The question is: do we have to sacrifice I/O efficiency for computational efficient (i.e. using symmetric key primitives)?

5.2 The Construction

The idea of FASTIO is to store the search query result, then when the server answers a new search query, it can use the saved result and only needs to search through new updates since the last search query. The protocols of FASTIO can be found in Fig. 3. In the Setup protocol, the client generates a long-term key for blinding keywords, and an empty map Σ for storing states. The server generates two empty maps, \mathbf{T}_e is to be used to store the encrypted index and \mathbf{T}_c is to store the last search query results. For each keyword w , the client stores in Σ two things: a state st that is generated randomly at the first time w is encountered or after each search query of w , and a counter c to produce a sub-state for each update after a search query (until the next search query). Note that unlike in FAST that the state st must evolve each time an update is made, in FASTIO st stays unchanged in between two search queries even though there might be many updates. Although two updates may be based on the same st , in the Update protocol, the client can still make them unlinkable by hashing a sub-state that is the concatenation of st and a counter (line 9 - 12). In the

Search protocol, if there are some documents on the server side that contain the keyword w , the client generates t_w (line 18). The purpose of t_w is to enable the server to find the last search query result of w in \mathbf{T}_c . The client also retrieves the stored state as well as the counter, and checks whether they should be sent to the server as part of the search token (line 19 - 25). In this process, if the counter c is non-zero, which means there have been updates since the last search query, then st will be sent to the server and the client must generate a new random state st and reset the counter at this point so that future updates can be made unlinkable. If the counter c is zero, then there is no update since the last search query and the query result should be the same as the previous search. Therefore in this case t_w alone is enough to retrieve the result and the client keeps st secret. Search on the server side consists of two steps. The first step is to retrieve the last search query result using t_w (line 27). Then the server may proceed to the second step (line 31 - 40). If there have been updates since the last search query, the server finds all updates using k_w (the state) and the counter. Then the server finds the document identifier from each update and either removes it from or adds it to the result set depending on whether the update operation is del or add. The server also removes the update from \mathbf{T}_e since its content has been revealed and is no long secret to the server. After all updates have been processed, the server returns the result set to the client and also stores it in \mathbf{T}_c (line 41 - 42).

Complexity Analysis Complexity wise, FASTIO is almost the same as FAST, except that on the server side, the computational complexity for search now is $O(\bar{c}_w)$ where \bar{c}_w is the total number of updates that contain the keyword w since the last time w was searched.

Security Analysis FASTIO achieves the same security level as FAST. We have the following theorem regarding the security of FASTIO:

Theorem 2. Let F be a pseudorandom function, H_1 and H_2 be two hash functions modeled as random oracles. Define leakage functions $\mathcal{L} = (\mathcal{L}_{\text{Setup}}, \mathcal{L}_{\text{Search}}, \mathcal{L}_{\text{Update}})$ as

$$\begin{aligned} \mathcal{L}_{\text{Setup}} &= \perp \\ \mathcal{L}_{\text{Search}}(w) &= (\text{ap}(w), \text{qp}(w)) \\ \mathcal{L}_{\text{Update}}(i, \text{op}_i, w, \text{ind}_i) &= (i, \text{op}_i, \text{ind}_i) \end{aligned}$$

Then FASTIO is an \mathcal{L} -adaptively-secure dynamic SSE with forward privacy

The proof can be found in Appendix B.

5.3 I/O Efficiency Analysis

FASTIO has a near optimal server side index size and read efficiency. In FASTIO, the server stores the index in two maps \mathbf{T}_e and \mathbf{T}_c . In \mathbf{T}_e , each entry is $l + 1$ bits where l is the size of the document identifier. In \mathbf{T}_c , each entry is a set of document identifiers, thus the size is a multiple of l . In total, when there are N_u document/keyword pairs, the size of the whole index is between $N_u \cdot l$ and $N_u \cdot (l + 1)$ bits. To answer each search query, the server needs to read relevant entries in the index. The server needs to read in total $|\text{DB}(w)|$ document identifier ($|\text{DB}(w)|$ is the number of documents matching the keyword w). In a plaintext search, the server must read in at least all identifiers of the documents matching the keyword, which is in total $|\text{DB}(w)| \cdot l$ bits. In FASTIO, some identifiers are read from \mathbf{T}_e and some are read from \mathbf{T}_c . Entries in \mathbf{T}_e adds 1 bit overhead per identifier for the operation type, and entries in \mathbf{T}_c have no overhead. Therefore, the read efficiency of the search query is between 1 and $1 + \frac{1}{l}$.

<p>Setup($\lambda, \perp; \perp$)</p> <p><i>Client:</i></p> <ol style="list-style-type: none"> $k_s \xleftarrow{\\$} \{0, 1\}^\lambda$ $\Sigma \leftarrow$ empty map <p><i>Server:</i></p> <ol style="list-style-type: none"> $T_e, T_c \leftarrow$ empty map <p>Update($k_s, \Sigma, ind, w, op; T_e$)</p> <p><i>Client:</i></p> <ol style="list-style-type: none"> $(st, c) \leftarrow \Sigma[w]$ if $(st, c) = \perp$ then $st \xleftarrow{\\$} \{0, 1\}^\lambda$ $c \leftarrow 0$ end if $u \leftarrow H_1(st (c + 1))$ $e \leftarrow (ind op) \oplus H_2(st (c + 1))$ $\Sigma[w] \leftarrow (st, c + 1)$ 	<ol style="list-style-type: none"> send (u, e) to server <p><i>Server:</i></p> <ol style="list-style-type: none"> $T_e[u] = e$ <p>Search($k_s, \Sigma, w; T_e, T_c$)</p> <p><i>Client:</i></p> <ol style="list-style-type: none"> $(st, c) \leftarrow \Sigma[w]$ if $(st, c) = \perp$ then return \emptyset end if $t_w \leftarrow F(k_s, h(w))$ if $c \neq 0$ then $k_w \leftarrow st, st \xleftarrow{\\$} \{0, 1\}^\lambda$ $\Sigma[w] \leftarrow (st, 0)$ else $k_w \leftarrow \perp$ end if send (t_w, k_w, c) to Server 	<p><i>Server:</i></p> <ol style="list-style-type: none"> $ID \leftarrow \emptyset$ $ID.add(T_c[t_w])$ if $k_w = \perp$ then return ID end if for $i = 1$ to c do $u_i \leftarrow H_1(k_w i)$ $(ind, op) \leftarrow T_e[u_i] \oplus H_2(k_w i)$ if $op = \text{"del"}$ then $ID \leftarrow ID - \{ind\}$ else if $op = \text{"add"}$ then $ID \leftarrow ID \cup \{ind\}$ end if delete $T_e[u_i]$ end for $T_c[t_w] \leftarrow ID$ send ID to client
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Fig. 3: Pseudocode of Protocols in FASTIO

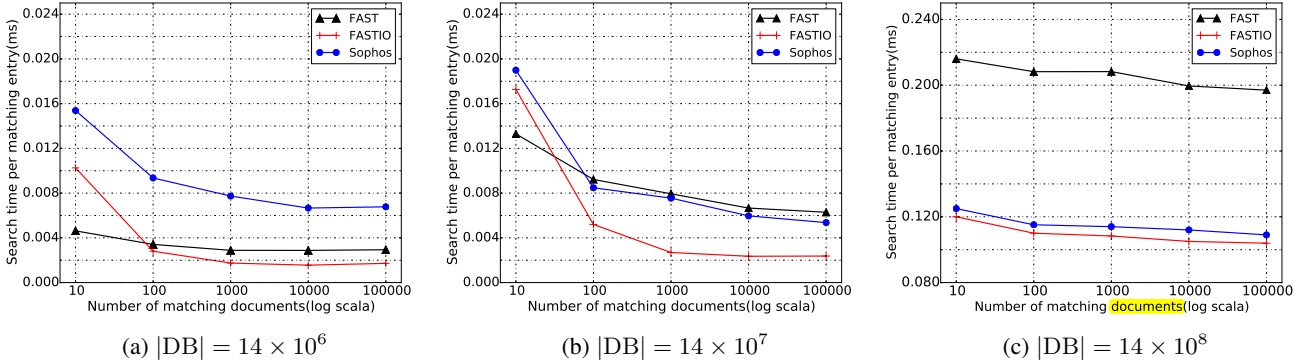


Fig. 4: Search time per matched document for FAST, FASTIO and Sophos.

In practice, the document identifiers need to be long enough to be unique and l is often large, e.g. 128. Then the server side index size and read efficiency in FASTIO are only less than 1% worse than optimal.

The locality can be improved because reading the previous search query result can be done in one go. If there have been \bar{c}_w new updates after the last search query, we need \bar{c}_w non-contiguous reads (this part cannot be improved by caching previous result). Therefore the overall locality is $\bar{c}_w + 1$ (in contrast to c_w). As we have discussed in Section 3.5, locality and forward privacy are two irreconcilable notions. The implication is that, as Bost observed, the *worst-case* locality cannot be improved unless large modifications is done to the encrypted database. If the keyword is rarely searched, the optimization has little effect.

6 PERFORMANCE EVALUATION

In this section, we evaluate the performance of FAST and FASTIO, and compare the results with Sophos, which is the most efficient forward private searchable encryption to date.

6.1 Implementation and Experiment Settings

We implemented FAST and FASTIO using C++¹. We use Crypto++ library² for the cryptographic operations: SHA256 for H_1 and H_2 , and AES-128 for F and P . To make the comparison fair, in our implementation, we use the same underlying libraries as in the Sophos implementation: Rocksdb³ for storing key-value pairs and gRPC⁴ for communication. We used the C++ implementation of Sophos⁵ by the author in our comparison. The identifier length is set to 64-bit in all schemes. The server is deployed on an Alibaba Cloud ECS.i1.xlarge instance located in US West, which has 4 cores (Intel Xeon E5-2682v4, 2.5 GHz), 16GB RAM and 2×104 GB SSD disks. The client is deployed on a desktop PC located in China, which has 4 cores (Intel Core i5-3470, 3.7GHz), 4 GB RAM and 500 GB hard disk.

- <https://github.com/BintaSong/DistSSE/tree/2.0>
- <https://cryptopp.com>
- <http://rocksdb.org>
- <http://www.grpc.io>
- <https://gitlab.com/sse/sophos>

6.2 Experiment Results

6.2.1 Update Efficiency

We first show the performance of the update operation. As we explained earlier, the update operation in Sophos is based on a public key primitive, while update operations in FAST and FASTIO are all symmetric key based. We first measured the update efficiency of FAST, FASTIO and Sophos in a local setting in which we ran the client and the server on the same cloud instance. In this setting, the time we measured does not include latency caused by network, therefore it gives a better picture of the difference in computational efficiency. We then measured the update efficiency in a WAN setting, in which the server (in US) and the client (in China) are distributed. The results are shown in Table 1.

As we can see, in the local setting, the throughput of FAST is about $11\times$, and FASTIO is about $15\times$, that of Sophos. In the WAN setting, the throughput of FAST is about $7\times$, and FASTIO about $10\times$, that of Sophos. The results confirm that FAST and FASTIO are superior in terms of update efficiency, compared to public key based Sophos.


			FAST	FASTIO	Sophos
Local	Throughput (ops/s)		54060	76100	4890
	Single update time (ms)		0.018	0.013	0.20
WAN	Throughput (ops/s)		21650	31080	2990
	Single update time (ms)		0.046	0.032	0.334

TABLE 1: Update efficiency for FAST, FASTIO and Sophos

6.2.2 Search Efficiency – Index Processing

For FAST, FASTIO and Sophos, the search operation on the server side requires processing a list of encrypted index entries to find matching documents. The search efficiency thus depends crucially on the efficiency of processing the index. In Fig. 4, we show the performance of index processing in the three schemes.

We conducted three experiments with different database sizes 14×10^6 , 14×10^7 and 14×10^8 . In each experiment, we measured the total time on the server side (i.e. without counting network latency and token generation time on the client side) for searching keywords that have $10 - 10^5$ matching documents. We repeated 30 times and took the average, then divided it by the number of matching documents to get the time for processing a single entry. As we can see in the figure, the time for processing a single entry decreases when the number of entries increases. This is because there is a fixed cost for initializing the search, which is amortized into the per entry processing time. As the number of entries increases, the amortized initialization cost becomes less significant. When the database size is 14×10^6 and 14×10^7 , FAST performs better than FASTIO and Sophos in the cases where the number of entries is small (10). This is because FAST has a smaller initialization cost, which is the result of an implementation level optimization.

We can observe the impact of I/O from Fig. 4. As we can see, for the same scheme and the same index size, the time we measured with the largest database (14×10^8) is much higher than the time we measured with the smaller databases (14×10^6 and 14×10^7). The performance degradation is about 2 orders of magnitude. The observation is in line with the results from previous studies (see Section 2). We can also see that FAST was impacted the most as it has the worst I/O efficiency.

In smaller databases where I/O does not dominates the processing time, FASTIO performs much better than Sophos. The

performance difference is about 1 - 2 times. This is mainly because FASTIO uses only symmetric key operations. In large databases, the performance difference is not that large, but FASTIO still performs marginally better than Sophos.

Even though FAST is based on symmetric primitives, we can observe that compared to Sophos, the efficiency improvement for search (around $2 \times$ faster) is not as significant as that with update ($10 \times$ faster). One reason is that in the Sophos implementation, the trapdoor permutation evaluation (for generating previous states) on the server side is done by a dedicated thread, and there are multiple threads taking the states produced and searching in parallel. While in FAST, due to its inherent structure (conceptually a linked list), the search cannot be parallelized because the previous state cannot be found until the search has gone through the current node. FASTIO does not have this problem.

6.2.3 Search Efficiency – Trace Simulation

The experiments in Section 6.2.2 do not fully reflect the search performance of FASTIO. In fact, it shows the *worst-case* performance of FASTIO. Recall that in FASTIO, the previous search results are stored to make search more efficient. In order to see how significant this improvement is, we also simulated the dynamic setting using traces. We generated 3 traces. Each trace is a list of update and search queries for a certain keyword. We fixed the length of the trace to 100,000. Each trace has a parameter α that is the probability of search queries, i.e. each query in the trace has a probability of α to be a search query and a probability of $1 - \alpha$ to be an update query. In the experiment, we let the client to replay the traces to simulate a real-world setting where updates and search queries are interleaved. We recorded the total time on the server side for each search query (from receiving search token to obtaining the results) in the trace.

In Fig. 5, we show the comparison of search efficiency with regard to the random traces. We used three different database sizes and three different query probabilities. In each sub-figure, the x -axis shows the sequence number of the query in the trace and the y -axis shows the search time. For FAST and Sophos, the search time increases almost monotonically in accordance to the number of update queries performed so far. Recall that in these two schemes, their indexes contain all entries from all previous updates. The index entries have to be stored in random locations in order to ensure forward privacy, this results in the decrease of I/O efficiency as the locality will increase monotonically. For FASTIO, as we can see, the search performance is much better, especially when search is more frequent. The difference is more significant for large databases. This is because in FASTIO, the index contains only entries since the last search query. The locality is much better than in the other two schemes.

In Fig. 6, we show the search time of FASTIO with regard to the traces. As we can see, despite the large number of updates, FASTIO's search time is always kept low. Obviously the more frequent the search queries are performed, the more performance gain we will see. This suggest that FASTIO has a better scalability, as frequent updates and large database size has a much less impact on its search performance than in the other two schemes.

7 CONCLUSION

Designing searchable symmetric encryption is not trivial if we want to combine efficiency and forward privacy, which are two irreconcilable properties. In this paper, we designed two forward

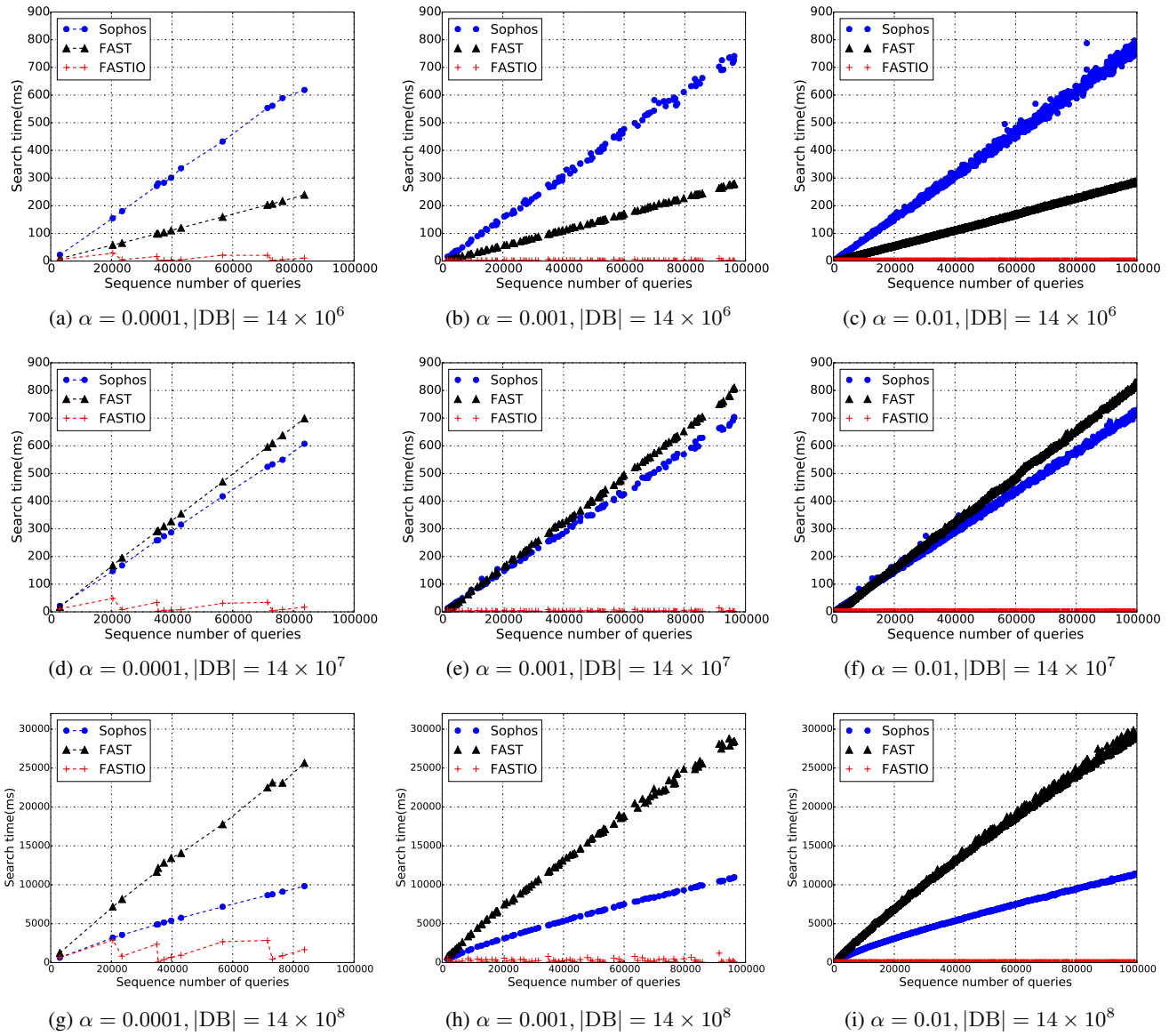


Fig. 5: Search efficiency by trace simulation.

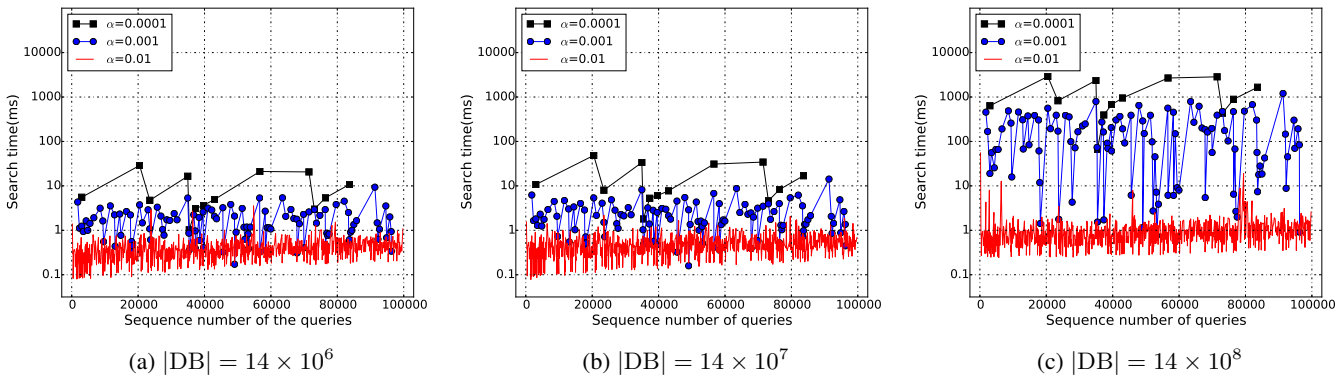


Fig. 6: Trace simulation for FASTIO. Search time is presented in log scale.

private searchable symmetric encryption schemes, both achieve optimal computational and communicational complexity. As the

first attempt, FAST utilized a state based approach and is computation friendly for its symmetric construction. Based on the idea

from FAST and utilizing intrinsic leakage, our second construction FASTIO improves I/O efficiency significantly. Our experiment results demonstrated that our schemes are efficient and scalable.

Future work and Open questions For static searchable encryption, Cash et al. [10] gave theoretical bounds of locality, storage, and read efficiency and showed the relationship between them and the security notion. For dynamic searchable encryption, the relationship has not been well-understood and there has not been any theoretical study to establish the bounds. We would like to investigate in this direction so that we can understand better the limit and margin of further performance improvement.

In addition, modern data management system leverage distributed and parallel architecture to handle large amount of data. For SSE, search efficiency will sharply drop when the database is out of the capability of the single machine. In fact, even for plaintext database, it's not practical to be deployed on a single server. We would like to investigate searchable encryption in distributed and parallel settings.

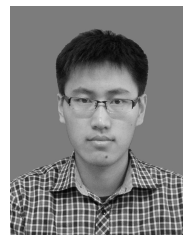
ACKNOWLEDGMENT

The authors would like to thank the editor and the anonymous reviewers for their help in improving the paper. This work is supported by the National Natural Science Foundation of China (Grant No. 61572294, Grant No. 61632020), the Special Fund for General Scientific Research in Shandong University (Grant No. 2017JC019), and by the Engineering and Physical Sciences Research Council of UK (Grant No. EP/M013561/2). Qiuliang Xu and Changyu Dong are corresponding authors.

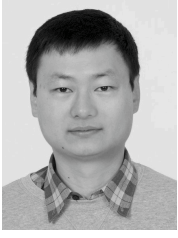
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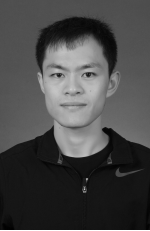
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APPENDIX A PROOF OF THEOREM 1

Proof: We model the two hash functions H_1 and H_2 as random oracles. The oracles for H_1 and H_2 are identical except for different output lengths. Each oracle maintains a mapping \mathbf{H}_i that stores input/output pairs (in, out) where $in \in \{0, 1\}^*$ and $out \in \{0, 1\}^\ell$ where ℓ is the length of the hash function's output. Given an input string x , the oracle first checks mapping \mathbf{H}_i , if there is an entry for x then it returns the value associated with x and terminates here. If there is not an entry for x in \mathbf{H}_i , the oracle randomly picks a string y from $\{0, 1\}^\ell$, then stores (x, y) in \mathbf{H}_i and returns y .

We prove through a sequence of games. We start from $\mathbf{Real}_A^\Pi(\lambda)$ and construct a sequence of games that differs slightly from the previous game and show they are indistinguishable. Eventually we reach the last game that is $\mathbf{Ideal}_{A,S}^\Pi(\lambda)$. By the transitive property of the indistinguishability, we conclude that $\mathbf{Real}_A^\Pi(\lambda)$ is indistinguishable from $\mathbf{Ideal}_{A,S}^\Pi(\lambda)$ and complete our proof.

Hybrid G_1 : G_1 is the same as $\mathbf{Real}_A^\Pi(\lambda)$ except that instead of generating t_w using F , the experiment maintain a mapping **Token** to store $(h(w), t_w)$ pairs. In the search protocol, when t_w is needed, the experiment first checks whether there is an entry in **Token** for $h(w)$, if so returns the entry; otherwise randomly picks a t_w in $\{0, 1\}^l$ and stores the $(h(w), t_w)$ pair in **Token**. It's trivial to see that G_1 and $\mathbf{Real}_A^\Pi(\lambda)$ are indistinguishable, otherwise we can distinguish a pseudo-random function F and a truly random function.

Hybrid G_2 : G_2 differs from G_1 in two places. Firstly, the experiment maintains a mapping **Key**. In the update protocol (line 9 in Fig. 1), after k_{c+1} is generated, the experiment stores $(w || (c+1), k_{c+1})$ in **Key**. Secondly, instead of querying H_1 in the update protocol, a random string is used. The random string is used to program the random oracle when a search query is issued. More specially, Instead of doing $u \leftarrow H_1(t_w || st_{c+1})$, which is line 13 in the update protocol in Fig. 1, the experiment does the following:

$$u \xleftarrow{\$} \{0, 1\}^\ell \\ \mathbf{L}[t_w || st_{c+1}] \leftarrow u$$

where \mathbf{L} is a mapping maintained by the experiment. The second change is in the search protocol. The following is added after line 20 and line 21 of the search protocol:

```
for  $i = 1$  to  $c$  do
   $\mathbf{H}_1[t_w || st_{c+1}] \leftarrow \mathbf{L}[t_w || st_{c+1}]$ 
   $k_i \leftarrow \mathbf{Key}[w || i]$ 
   $st_{i-1} \leftarrow P^{-1}(k_i, st_i)$ 
end for
```

where \mathbf{H}_1 is the table for random oracle H_1 .

Now G_1 and G_2 behaves exactly the same except that in G_2 , with some probability inconsistency in random oracle query results can be observed. In G_2 , \mathbf{H}_1 is not updated immediately. For $t_w || st_{c+1}$, the corresponding value u is generated in the update protocol. However, u is only pushed into \mathbf{H}_1 when a search query is issued. After the update with state $t_w || st_{c+1}$, if the adversary queries \mathbf{H}_1 with $t_w || st_{c+1}$ before the next search query, it will get a value u' such that with a overwhelming probability $u' \neq u$ because $\mathbf{H}_1[t_w || st_{c+1}]$ has not been updated and a random string u' is chosen by the oracle in this case. If the adversary queries \mathbf{H}_1 with $t_w || st_{c+1}$ again after the next search query, u will be updated to the \mathbf{H}_1 and the query result will be u . If the inconsistency is observed (we denote this event as **Bad**), the adversary knows it is in G_2 . We have:

$$\Pr[G_1 = 1] - \Pr[G_2 = 1] \leq \Pr[\mathbf{Bad}]$$

Note that the event **Bad** can only happen if the adversary can query the oracle with $t_w || st_{c+1}$. Since st_{c+1} is pseudorandom, the probability of the adversary choosing st_{c+1} by chance is $2^{-\lambda} + \mathbf{negl}(\lambda)$. A PPT adversary can make at most $q_1 = \mathbf{poly}(\lambda)$ guesses, then $\Pr[\mathbf{Bad}] \leq \frac{q_1}{2^\lambda} + q_1 \cdot \mathbf{negl}(\lambda)$. The probability is negligible and G_1 and G_2 are indistinguishable.

Hybrid G_3 : G_3 is obtained from G_2 in a similar way. In the update protocol, we replace the line $e \leftarrow (ind || op || k_{c+1}) \oplus H_2(t_w || st_{c+1})$ (line 12 in Fig. 1) with the following:

$u \xleftarrow{\$} \{0, 1\}^{l+\lambda+1}$
 $\mathbf{E}[t_w || st_{c+1}] \leftarrow u$
 $e \leftarrow (ind || op || k_{c+1}) \oplus u$

where \mathbf{E} is a mapping maintained by the experiment. Similarly, the following is added after line 20 and line 21 of the search protocol:

for $i = 1$ **to** c **do**
 $\mathbf{H}_2[t_w || st_{c+1}] \leftarrow \mathbf{E}[t_w || st_{c+1}]$
 $k_i \leftarrow \mathbf{Key}[w || i]$
 $st_{i-1} \leftarrow P^{-1}(k_i, st_i)$
end for

Using the same argument, we can conclude that G_2 and G_3 are indistinguishable.

Setup(λ, \perp, \perp)

Client:

- 1: $\mathbf{L}, \mathbf{E} \leftarrow$ empty map
- 2: $v \leftarrow 0$

Update($k_s, \Sigma, ind, w, op; \mathbf{T}$)

Client:

- 3: $v \leftarrow v + 1$
- 4: Append (v, op, ind) to $\mathbf{Updates}[w]$
- 5: $\mathbf{L}[v] \xleftarrow{\$} \{0, 1\}^\ell$
- 6: $\mathbf{E}[v] \xleftarrow{\$} \{0, 1\}^{l+\lambda+1}$
- 7: send ($\mathbf{L}[v], \mathbf{E}[v]$) to server

Search($k_s, \Sigma, w; \mathbf{T}$)

Client:

- 8: **if** $\mathbf{Token}[h(w)] = \perp$ **then**
- 9: $\mathbf{Token}[h(w)] \xleftarrow{\$} \{0, 1\}^\lambda$
- 10: **end if**
- 11: $t_w \leftarrow \mathbf{Token}[h(w)]$
- 12: **if** $\Sigma[w] = \perp$ **then**
- 13: $\Sigma[w] \xleftarrow{\$} \{0, 1\}^\lambda$
- 14: **end if**
- 15: $st_0 \leftarrow \Sigma[w]$
- 16: $c \leftarrow |\mathbf{Updates}[w]|$
- 17: $[(u_0, op_0, ind_0), \dots, (u_c, op_c, ind_c)] \leftarrow \mathbf{Updates}[w]$
- 18: **if** $c = 0$ **then**
- 19: return \emptyset
- 20: **end if**
- 21: **for** $i = 1$ **to** c **do**
- 22: **if** $\mathbf{Key}[w || i] \neq \perp$ **then**
- 23: $k_i \leftarrow \mathbf{Key}[w || i]$
- 24: **else**
- 25: $k_i \xleftarrow{\$} \{0, 1\}^\lambda$
- 26: $\mathbf{Key}[w || i] \leftarrow k_i$
- 27: **end if**
- 28: $st_i \leftarrow P(k_i, st_{i-1})$
- 29: $\mathbf{H}_1(t_w || st_i) \leftarrow \mathbf{L}[u_i]$
- 30: $\mathbf{H}_2(t_w || st_i) \leftarrow \mathbf{E}[u_i] \oplus (ind_i || op_i || k_i)$
- 31: **end for**
- 32: send (t_w, st_c, c) to Server

Fig. 7: Description of hybrid G_4

Hybrid G_4 : In G_4 , the client side algorithms are changed as shown in Fig. 7. The main difference between G_4 and G_3 is that in G_4 , st and k is generated on the fly when search is performed. In G_4 a mapping $\mathbf{Updates}$ is maintained by the experiment to record all the update requests since the last search query. Unlike in G_3 where the random oracle query results are chosen according to $t_w || st_{c+1}$, in G_4 the values are chosen randomly without the knowledge of st (line 5 and 6, Fig. 7). Then when performing a search, st_0 is generated if this the first search query of w (line 12 - 15, Fig. 7). Then k_i, st_i are generated retrospectively and then the oracle is updated accordingly

$\mathcal{S}.\text{Setup}()$

Client:

- 1: $\mathbf{L}, \mathbf{E} \leftarrow$ empty map
- 2: $v \leftarrow 0$

$\mathcal{S}.\text{Update}()$

Client:

- 3: $v \leftarrow v + 1$
- 4: $\mathbf{L}[v] \xleftarrow{\$} \{0, 1\}^\ell$
- 5: $\mathbf{E}[v] \xleftarrow{\$} \{0, 1\}^{l+\lambda+1}$
- 6: send ($\mathbf{L}[v], \mathbf{E}[v]$) to Server

$\mathcal{S}.\text{Search}(\mathbf{sp}(w), \mathbf{uh}(w))$

Client:

- 7: $w \leftarrow \min \mathbf{sp}(w)$
- 8: **if** $\mathbf{Token}[w] = \perp$ **then**
- 9: $\mathbf{Token}[w] \xleftarrow{\$} \{0, 1\}^\lambda$
- 10: **end if**
- 11: $t_w \leftarrow \mathbf{Token}[w]$
- 12: **if** $\Sigma[w] = \perp$ **then**
- 13: $\Sigma[w] \xleftarrow{\$} \{0, 1\}^\lambda$
- 14: **end if**
- 15: $st_0 \leftarrow \Sigma[w]$
- 16: $c \leftarrow |\mathbf{uh}(w)|$
- 17: $[(u_0, op_0, ind_0), \dots, (u_c, op_c, ind_c)] \leftarrow \mathbf{uh}(w)$
- 18: **if** $c = 0$ **then**
- 19: return \emptyset
- 20: **end if**
- 21: **for** $i = 1$ **to** c **do**
- 22: **if** $\mathbf{Key}[w || i] \neq \perp$ **then**
- 23: $k_i \leftarrow \mathbf{Key}[w || i]$
- 24: **else**
- 25: $k_i \xleftarrow{\$} \{0, 1\}^\lambda$
- 26: $\mathbf{Key}[w || i] \leftarrow k_i$
- 27: **end if**
- 28: $st_i \leftarrow P(k_i, st_{i-1})$
- 29: $\mathbf{H}_1(t_w || st_i) \leftarrow \mathbf{L}[u_i]$
- 30: $\mathbf{H}_2(t_w || st_i) \leftarrow \mathbf{E}[u_i] \oplus (ind_i || op_i || k_i)$
- 31: **end for**
- 32: send (t_w, st_c, c) to Server

Fig. 8: Description of simulator \mathcal{S}

(line 21 - 31, Fig 7). The change however are not observable by the adversary. From the adversary's perspective, G_3 and G_4 behaves exactly the same: in both games the update protocol outputs two uniformly random strings, and the search protocol outputs (t_w, k_w, c) that has the same distribution in the two games. Therefore they are perfectly indistinguishable.

$$\Pr[G_3 = 1] = \Pr[G_4 = 1]$$

Ideal $_{\mathcal{A}, \mathcal{S}}^\Pi(\lambda)$: in this game, a simulator must generate a view given only the leakage functions, but not the actual data and queries. For convenience, in the proof we will use *search pattern* and *update history* instead of using the search leakage function $\mathcal{L}_{\text{Search}}(w)$ directly. The two can be constructed from the leakage function. The search pattern $\mathbf{sp}(w) = \{j | q_j = w \text{ in Hist}\}$ reveals which queries in the history are search queries with regard to w . The update history $\mathbf{up}(w) = \{(j, op_j, ind_j) | q_j = (ind_j, w, op_j) \text{ for each } q_j \text{ in Hist}\}$ reveals which queries in the history are update queries with regard to w , as well as the update types and the document identifiers. We use $\mathbf{up}^{>k}(w)$ to denote the partial update history after the k -th query. It is clear that $\mathbf{sp}(w)$ can be obtained from $\mathbf{qp}(w)$ by throwing away all update requests, and $\mathbf{up}(w)$ can be obtained from $(\mathbf{ap}(w), \mathbf{qp}(w))$ by combining the information about update queries. They do not require additional information beyond what the leakage function allowed.

The simulator \mathcal{S} is shown in Fig. 8. The simulator maintains two mappings for simulating random oracle queries, and a counter to record the number of updates since the initialization of the system. For each update query, two random strings are chosen. In the search protocol, we use $\underline{w} = \min \mathbf{sp}(w)$ to denote the very first index that w appeared in search pattern. Each \underline{w} uniquely identify an unknown keyword w and the simulator can just use \underline{w} without knowing w . The token t_w is associated with \underline{w} . The simulator can use the update history to decide the update queries with regard to w and their sequence number. Then states and keys can be generated given the update history. Then the random oracles are updated. The view produced by the simulator is perfectly indistinguishable from the one produced in G_4 .

$$\Pr[\mathbf{Ideal}_{A,S}^{\Pi}(\lambda) = 1] = \Pr[G_4 = 1]$$

Summing up, we have

$$\Pr[\mathbf{Real}_{A,S}^{\Pi}(\lambda) = 1] - \Pr[\mathbf{Ideal}_{A,S}^{\Pi}(\lambda) = 1] \leq \mathbf{negl}(\lambda)$$

Q.E.D. \square

APPENDIX B

PROOF OF THEOREM 2

Proof: We model the two hash functions H_1 and H_2 as random oracles and prove through a sequence of games.

Hybrid G_1 : G_1 is the same as $\mathbf{Real}_{A,S}^{\Pi}(\lambda)$ except that instead of generating t_w using F , the experiment maintain a mapping **Token** to store $(h(w), t_w)$ pairs. In the search protocol, when t_w is needed, the experiment first checks whether there is an entry in **Token** for $h(w)$, if so returns the entry; otherwise randomly picks a t_w in $\{0, 1\}^l$ and stores the $(h(w), t_w)$ pair in **Token**. It's trivial to see that G_1 and $\mathbf{Real}_{A,S}^{\Pi}(\lambda)$ are indistinguishable, otherwise we can distinguish a pseudo-random function F and a truly random function.

Hybrid G_2 : G_2 differs from G_1 in that instead of querying H_1 in the update protocol, a random string is used. The random string is used to program the random oracle when a search query is issued. More specially, Instead of doing $u \leftarrow H_1(st||c+1)$, which is line 9 in the update protocol in Fig. 3, the experiment does the following:

$$u \xleftarrow{\$} \{0, 1\}^{\ell}$$

$$\mathbf{UT}[st||c+1] \leftarrow u$$

where **UT** is a mapping maintained by the experiment. The second change is in the search protocol. The following is added in between line 19 and line 20 of the search protocol:

$$\text{for } i = 1 \text{ to } c \text{ do}$$

$$\mathbf{H}_1[st||i] \leftarrow \mathbf{UT}[st||i]$$

$$\text{end for}$$

where \mathbf{H}_1 is the table for random oracle H_1 .

Now G_1 and G_2 behaves exactly the same except that in G_2 , with some probability inconsistency in random oracle query results can be observed. In G_2 , \mathbf{H}_1 is not updated immediately. For $st||c+1$, the corresponding value u is generated in the update protocol. However, u is only pushed into \mathbf{H}_1 when a search query is issued. After the update with state st and counter $c+1$, if the adversary queries \mathbf{H}_1 with $st||c+1$ before the next search query, it will get a value u' such that with a overwhelming probability $u' \neq u$ because $\mathbf{H}_1[st||c+1]$ has not been updated and a random string u' is chosen by the oracle in this case. If the adversary queries \mathbf{H}_1 with $st||c+1$ again after the next search query, u will be updated to the \mathbf{H}_1 and the query result will be u . If the inconsistency is observed (we denote this event as **Bad**), the adversary knows it is in G_2 . We have:

$$\Pr[G_1 = 1] - \Pr[G_2 = 1] \leq \Pr[\mathbf{Bad}]$$

Note that the event **Bad** can only happen if the adversary can query the oracle with $st||c+1$. Since $st \xleftarrow{\$} \{0, 1\}^{\lambda}$, the probability of the adversary choosing st by chance is $2^{-\lambda}$. A PPT adversary can make at most $q_1 = \mathbf{poly}(\lambda)$ guesses, then $\Pr[\mathbf{Bad}] \leq \frac{q_1}{2^{\lambda}}$. The probability is negligible and G_1 and G_2 are indistinguishable.

Hybrid G_3 : G_3 is obtained from G_2 in a similar way. In the update protocol, we replace the line $e \leftarrow (ind||op) \oplus H_2(st||c+1)$ with the following:

$$u \xleftarrow{\$} \{0, 1\}^{l+1}$$

$$\mathbf{E}[st||c+1] \leftarrow u$$

$$e \leftarrow (ind||op) \oplus u$$

where \mathbf{E} is a mapping maintained by the experiment. Similarly, the following is added in after line 19 of the search protocol:

$$\text{for } i = 1 \text{ to } c \text{ do}$$

$$\mathbf{H}_2[st||i] \leftarrow \mathbf{E}[st||i]$$

$$\text{end for}$$

Using the same argument, we can conclude that G_2 and G_3 are indistinguishable.

Setup(λ, \perp, \perp)

Client:

- 1: **UT**, **E** \leftarrow empty map
- 2: $v \leftarrow 0$

Update($k_s, \Sigma, ind, w, op; \mathbf{T}_e$)

Client:

- 3: $v \leftarrow v + 1$
- 4: Append (v, op, ind) to **Updates**[w]
- 5: $\mathbf{UT}[v] \xleftarrow{\$} \{0, 1\}^{\ell}$
- 6: $\mathbf{E}[v] \xleftarrow{\$} \{0, 1\}^{l+1}$
- 7: send $(\mathbf{UT}[v], \mathbf{E}[v])$ to server

Search($k_s, \Sigma, w; \mathbf{T}_e, \mathbf{T}_c$)

Client:

- 8: **if** **Token**[$h(w)$] = \perp **then**
- 9: **Token**[$h(w)$] $\xleftarrow{\$} \{0, 1\}^{\lambda}$
- 10: **end if**
- 11: $t_w \leftarrow \mathbf{Token}[h(w)]$
- 12: **if** **Updates**[w] = \perp **then**
- 13: $k_w \leftarrow \perp$
- 14: **else**
- 15: $k_w \xleftarrow{\$} \{0, 1\}^{\lambda}$
- 16: $c \leftarrow |\mathbf{Updates}[w]|$
- 17: $\{(v_1, op_1, ind_1), \dots, (v_c, op_c, ind_c)\} \leftarrow \mathbf{Updates}[w]$
- 18: **for** $1 \leq i \leq c$ **do**
- 19: $\mathbf{H}_1(k_w||i) \leftarrow \mathbf{UT}[v_i]$
- 20: $\mathbf{H}_2(k_w||i) \leftarrow \mathbf{E}[v_i] \oplus (op_i||ind_i)$
- 21: **end for**
- 22: **end if**
- 23: send (t_w, k_w, c) to Server
- 24: **Updates**[w] $\leftarrow \perp$

Fig. 9: Description of hybrid game G_4

Hybrid G_4 : In G_4 , the client side protocols are changed as shown in Fig. 9. The main difference between G_4 and G_3 is that in G_4 , st is generated on the fly when search is performed. In G_4 a mapping **Updates** is maintained by the experiment to record all the update requests since the last search query. Unlike in G_3 where the random oracle query results are chosen according to $st||c+1$, in G_4 the values are chosen randomly without the knowledge of st (line 5 and 6, Fig 9). Then when performing a search, k_w (i.e. st) is generated retrospectively and then the oracle is updated accordingly (line 15 - 21, Fig 9). The change however are not observable by the adversary. From the adversary's perspective, G_3 and G_4 behaves exactly the same: in both games the update protocol outputs two uniformly random strings, and the search protocol outputs (t_w, k_w, c) that has the same distribution in the two games. Therefore they are perfectly indistinguishable

$$\Pr[G_3 = 1] = \Pr[G_4 = 1]$$

Ideal $_{A,S}^{\Pi}(\lambda)$: in this game, a simulator must generate a view given only the leakage functions, but not the actual data and queries. For convenience, in the proof we will use *search pattern* and *update history* instead of using the search leakage function $\mathcal{L}_{\text{Search}}(w)$


```

S.Setup()
  Client:
  1: UT, E  $\leftarrow$  empty map
  2:  $v \leftarrow 0$ 

S.Update()
  Client:
  3:  $v \leftarrow v + 1$ 
  4: UT[ $v$ ]  $\xleftarrow{\$}$   $\{0, 1\}^\ell$ 
  5: E[ $v$ ]  $\xleftarrow{\$}$   $\{0, 1\}^{l+1}$ 
  6: send (UT[ $v$ ], E[ $v$ ]) to Server

S.Search(sp(w), uh(w))
  Client:
  7:  $\underline{w} \leftarrow \min \mathbf{sp}(w)$ 
  8:  $\overline{w} \leftarrow \max \mathbf{sp}(w)$ 
  9: if Token[ $\underline{w}$ ] =  $\perp$  then
  10:   Token[ $\underline{w}$ ]  $\xleftarrow{\$}$   $\{0, 1\}^\lambda$ 
  11: end if
  12:  $t_w \leftarrow \mathbf{Token}[\underline{w}]$ 
  13: if  $\mathbf{uh}^{>\overline{w}}(w) = \perp$  then
  14:    $k_w \leftarrow \perp$ 
  15: else
  16:    $k_w \xleftarrow{\$} \{0, 1\}^\lambda$ 
  17:    $c \leftarrow |\mathbf{uh}^{>\overline{w}}(w)|$ 
  18:    $\{(v_1, \text{op}_1, \text{ind}_1), \dots, (v_c, \text{op}_c, \text{ind}_c)\} \leftarrow \mathbf{uh}^{>\overline{w}}(w)$ 
  19:   for  $1 \leq i \leq c$  do
  20:      $\mathbf{H}_1(k_w || i) \leftarrow \mathbf{UT}[v_i]$ 
  21:      $\mathbf{H}_2(k_w || i) \leftarrow \mathbf{E}[v_i] \oplus (\text{op}_i || \text{ind}_i)$ 
  22:   end for
  23: end if
  24: send ( $k_w, t_w, c$ ) to Server

```

Fig. 10: Description of simulator \mathcal{S}

directly. The two can be constructed from the leakage function. The search pattern $\mathbf{sp}(w) = \{j | q_j = w \text{ in Hist}\}$ reveals which queries in the history are search queries with regard to w . The update history $\mathbf{up}(w) = \{(j, \text{op}_j, \text{ind}_j) | q_j = (\text{ind}_j, w, \text{op}_j) \text{ for each } q_j \text{ in Hist}\}$ reveals which queries in the history are update queries with regard to w , as well as the update types and the document identifiers. We use $\mathbf{up}^{>k}(w)$ to denote the partial update history after the k -th query. It is clear that $\mathbf{sp}(w)$ can be obtained from $\mathbf{qp}(w)$ by throwing away all update requests, and $\mathbf{up}(w)$ can be obtained from $(\mathbf{ap}(w), \mathbf{qp}(w))$ by combining the information about update queries. They do not require additional information beyond what the leakage function allowed.

The simulator \mathcal{S} is shown in Fig. 10. The simulator maintains two mappings for simulating random oracle queries, and a counter to record the number of updates since the initialization of the system. For each update query, two random strings are chosen. In the search protocol, we use $\underline{w} = \min \mathbf{sp}(w)$ to denote the very first index that w appeared in search pattern and $\overline{w} = \max \mathbf{sp}(w)$ to denote the last index that w been searched. Each \underline{w} uniquely identify an unknown keyword w and the simulator can just use \underline{w} without knowing w . The token t_w is associated with \underline{w} . The simulator can use the update history to decide whether there have been new updates since the last search query. If so, a random k_w is generated and the number of updates after the last search query can be decided given the update history. Then the random oracles are updated. The view produced by the simulator is perfectly indistinguishable from the one produced in G_4 .

$$\Pr[\mathbf{Ideal}_{\mathcal{A}, \mathcal{S}}^\Pi(\lambda) = 1] = \Pr[G_4 = 1]$$

Summing up, we have

$$\Pr[\mathbf{Real}_{\mathcal{A}}^\Pi(\lambda) = 1] - \Pr[\mathbf{Ideal}_{\mathcal{A}, \mathcal{S}}^\Pi(\lambda) = 1] \leq \mathbf{negl}(\lambda)$$

Q.E.D. \square

APPENDIX C A BRIEF SUMMARY OF SOPHOS

```

Setup()
  Client:
  1:  $K_S \xleftarrow{\$} \{0, 1\}^\lambda$ 
  2:  $(\mathbf{SK}, \mathbf{PK}) \leftarrow \text{KeyGen}(1^\lambda)$ 
  3: W, T  $\leftarrow$  empty map
  4: return  $((\mathbf{T}, \mathbf{PK}), (K_S, \mathbf{SK}), \mathbf{W})$ 

Update(add, w, ind,  $\sigma$ ; EDB)
  Client:
  1:  $K_w \leftarrow F_{K_S}(w)$ 
  2:  $(ST_c, c) \leftarrow \mathbf{W}[w]$ 
  3: if  $(ST_c, c) = \perp$  then
  4:    $ST_0 \xleftarrow{\$} c \leftarrow -1$ 
  5: else
  6:    $ST_{c+1} \leftarrow \pi_{\mathbf{SK}}^{-1}(ST_c)$ 
  7: end if
  8:  $\mathbf{W}[w] \leftarrow (ST_{c+1}, c + 1)$ 
  9:  $UT_{c+1} \leftarrow H_1(K_w, ST_{c+1})$ 
  10:  $e \leftarrow \text{ind} \oplus H_2(K_w, ST_{c+1})$ 
  11: Send  $(UT_{c+1}, e)$  to the server
  Server:
  12:  $\mathbf{T}[UT_{c+1}] \leftarrow e$ 

Search(w,  $\sigma$ ; EDB)
  Client:
  1:  $K_w \leftarrow F_{K_S}(w)$ 
  2:  $(ST_c, c) \leftarrow \mathbf{W}[w]$ 
  3: if  $(ST_c, c) = \perp$  then
  4:   return  $\emptyset$ 
  5: end if
  6: Send  $(K_w, ST_c, c)$  to the server.
  Server:
  7: for  $i = c$  to 0 do
  8:    $UT_i \leftarrow H_1(K_w, ST_i)$ 
  9:    $e \leftarrow \mathbf{T}[UT_i]$ 
  10:    $\text{ind} \leftarrow e \oplus H_2(K_w, ST_i)$ 
  11:   Output each ind
  12:    $ST_{i-1} \leftarrow \pi_{\mathbf{PK}}(ST_i)$ 
  13: end for

```

Fig. 11: Pseudocode of Protocols in Sophos

Fig. 11 shows the pseudocode of Sophos. The state in Sophos is evolved using a trapdoor permutation π . The client evaluate $\pi_{\mathbf{SK}}^{-1}$ to generate a new state ST_{c+1} to perform an update. The client only reveals ST_{c+1} (then constant communication obtained) during a search. At that point, the server can reconstruct all previous states by evaluating $\pi_{\mathbf{PK}}$, which enables it to perform search. Because only the client has the secret key SK, the server has no idea about ST_{c+1} before it revealed, thus forward privacy is obtained.